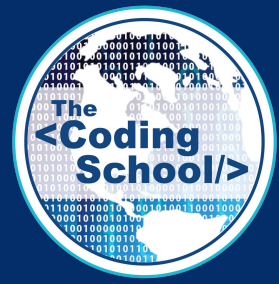
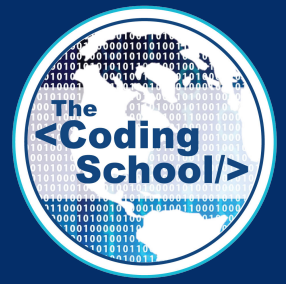




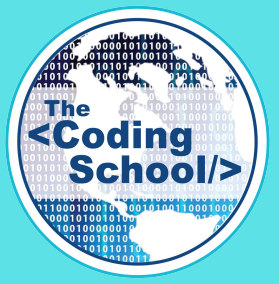
INTRO TO QUANTUM COMPUTING

LECTURE #18

QUANTUM COMPUTATION PT. 4 :
SUPERDENSE CODING & QUANTUM TELEPORTATION

FRANCISCA VASCONCELOS

3/7/2021



ANNOUNCEMENTS

QUANTUM COMPUTATION LECTURE SERIES

Lecture 1 – The Quantum Circuit Model *How can we perform computation with quantum systems?*	CONCEPTS
Lecture 2 – Qiskit Tutorial *How can we program quantum circuits?*	PROGRAMMING
Lecture 3 – Quantum Circuit Mathematics *How can we represent quantum circuits mathematically?*	MATH
Lectures 4-6 – Introductory Quantum Protocols and Algorithms *How can we leverage quantum for cryptography, teleportation, and algorithms?*	APPLICATION

TODAY'S LECTURE

1. Superdense Coding
    a) What is Superdense Coding?
    b) Superdense Coding Protocol
        a) Preparation
        b) Travel
        c) Encoding
        d) Transmission
        e) Decoding

2. Quantum Teleportation
    a) What is Quantum Telportation?
    b) Backstory
    c) Quantum Teleportation Circuit
    d) Quantum Teleportation Protocol
        a) Input
        b) Entangling Circuit
        c) Measurement
        d) Transmission
        e) Receiver Correction

KEY IDEAS

Today, we are going to leverage our knowledge of quantum circuits to implement two really cool ideas:

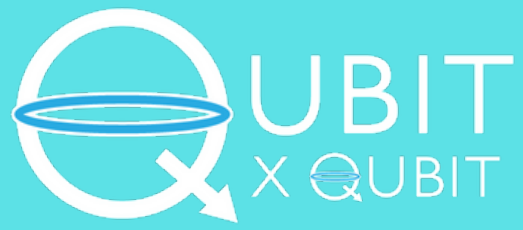
(1) **Superdense Coding**:

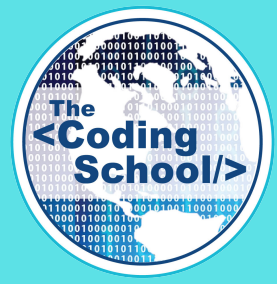
Transmit **two** bits of classical information with **one** qubit! *

(2) **Quantum teleportation**:

Transmit quantum information without physically sending a qubit! *

* Assuming an entangled bell pair has already been split between the sender and receiver...





SUPERDENSE CODING

2 CLASSICAL BITS WITH 1 QUBIT

“SUPERDENSE”

What do you think of when you hear the word “superdense”?

A gym weight or dumb-bell.

Super = very

Dense = closely compacted

Has a lot of content, despite its small size.

“CODING”

Coding refers to the fact that we will be...

Encoding classical states in quantum states for transmission:

Decoding quantum states to obtain the original classical states:

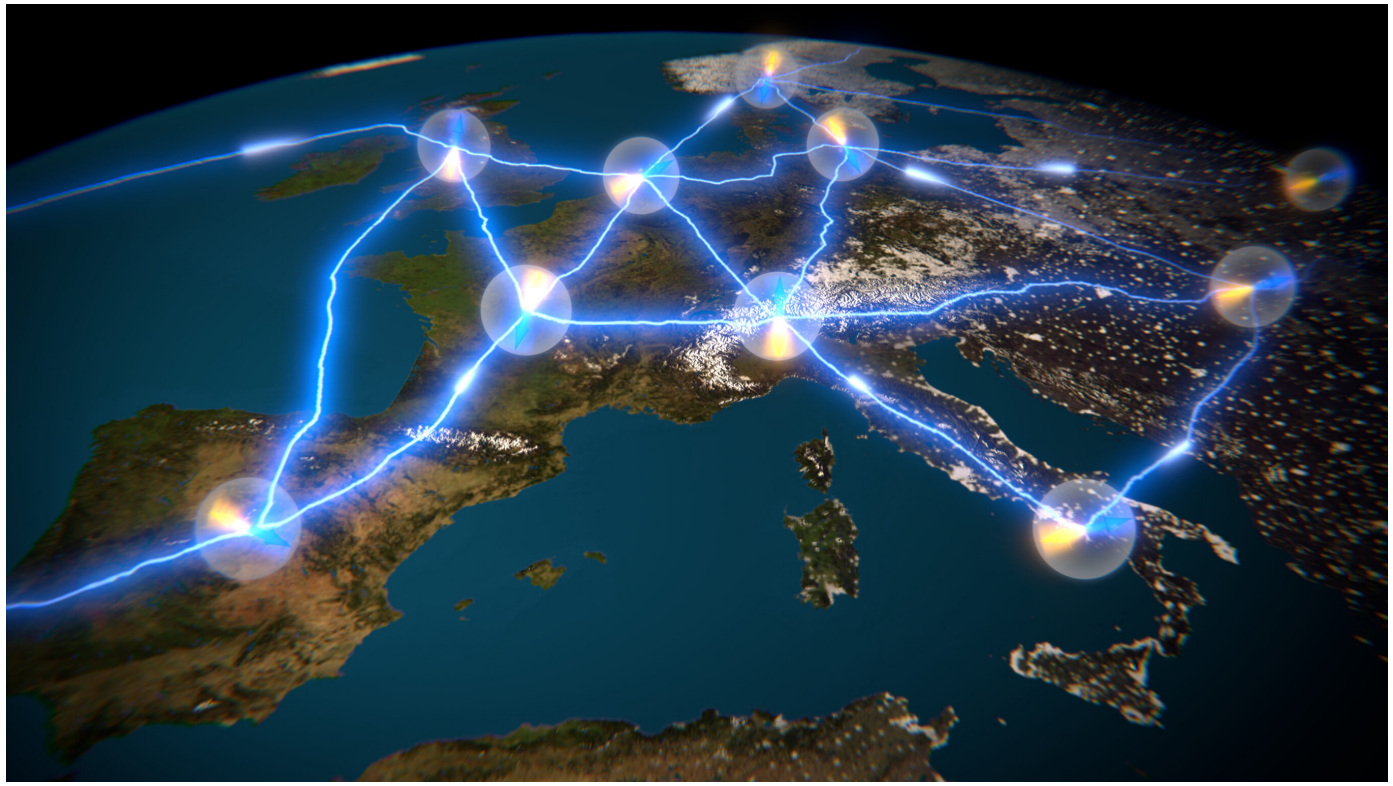
WHY SUPERDENSE CODING?

Superdense coding is cool because it leverages quantum mechanics to more **efficiently**

transmit classical information!!

A QUANTUM INTERNET...

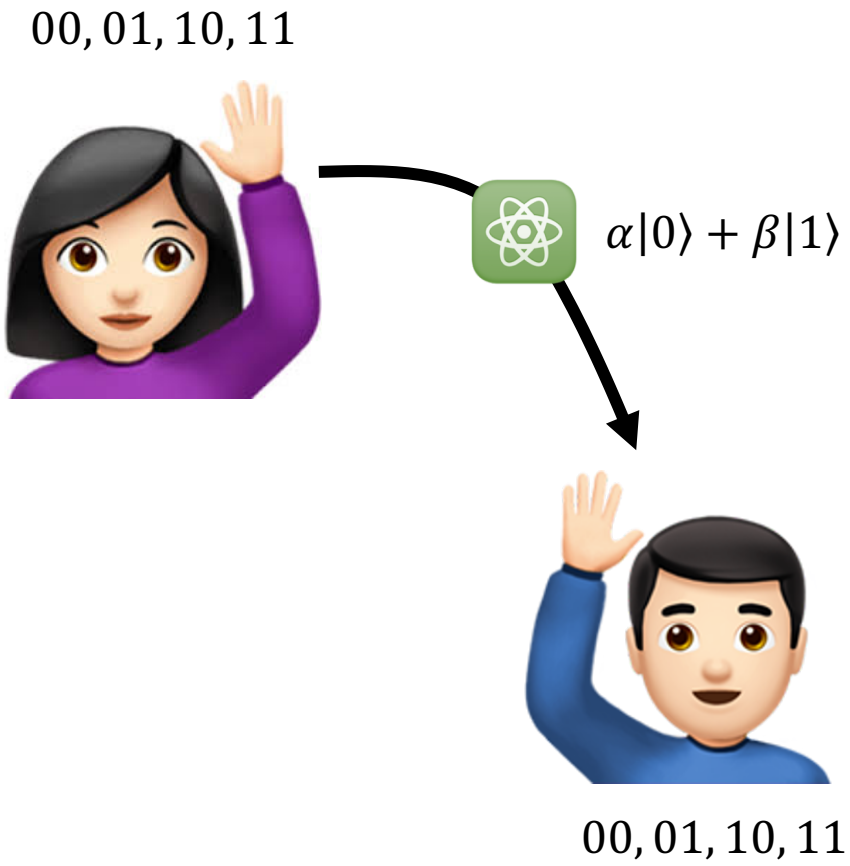
Two goals in building a quantum internet would be (1) more secure and (2) more efficient communications.

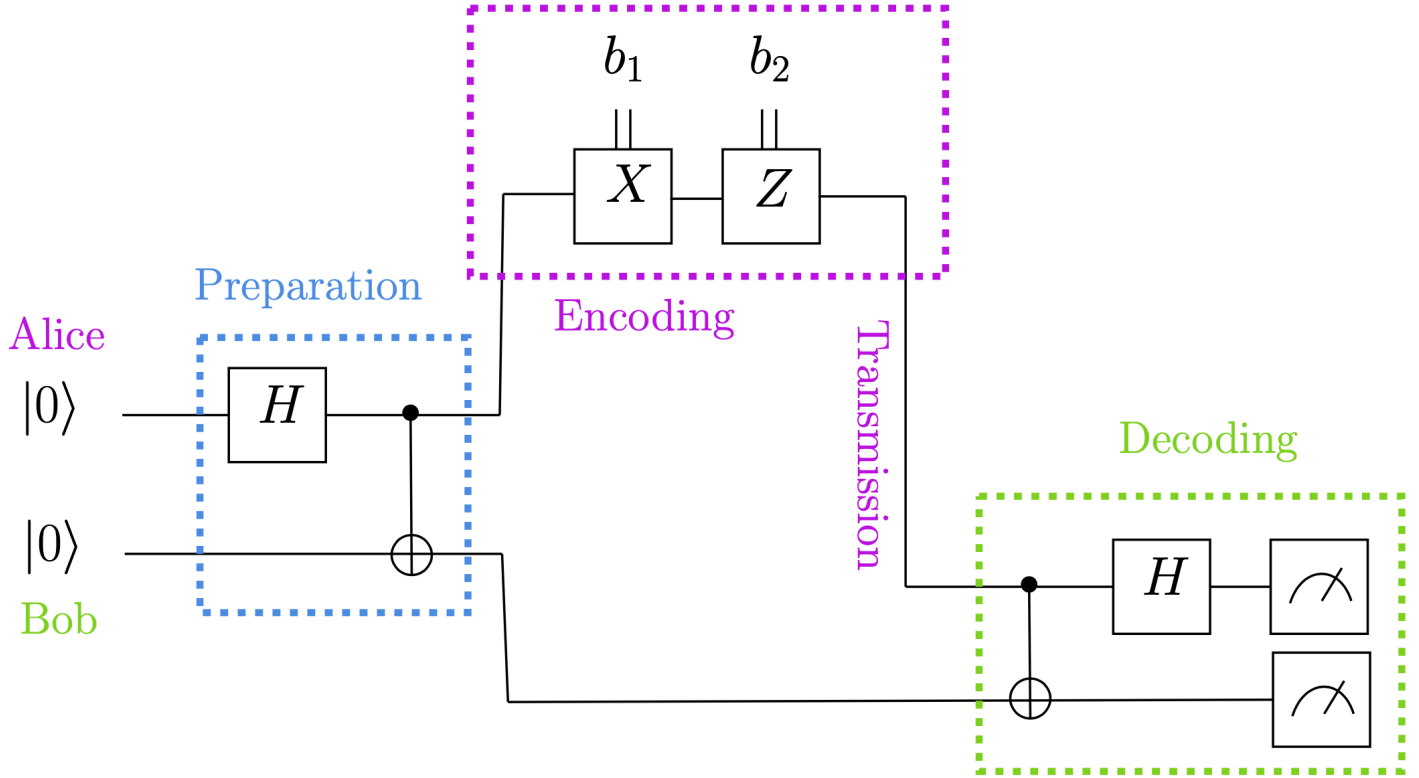


Although superdense coding transfers 2 classical bits with only 1 qubit, it assumes the sender and receiver already share an entangled bell state, meaning entangled bell states would need to be widely distributed (which is very challenging...)

SUPERDENSE CODING

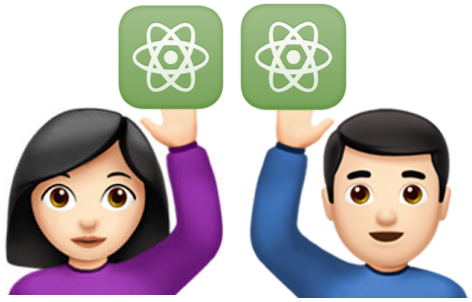
Superdense coding is a quantum communication protocol which transmits **2** classical bits of information with only **1** qubit!



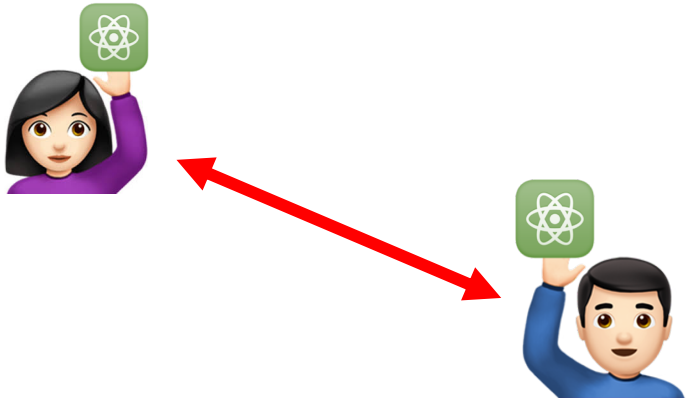


SUPERDENSE CODING PROTOCOL

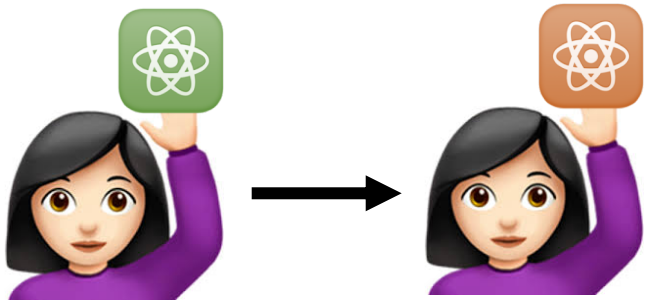
1. **Preparation:** Alice and Bob meet up and split an entangled bell state.



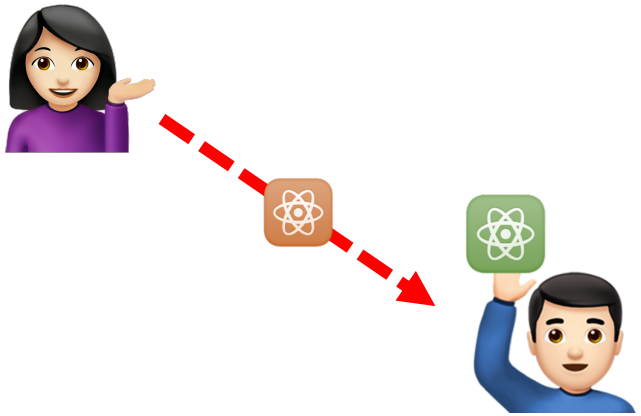
2. Alice and Bob travel far away.



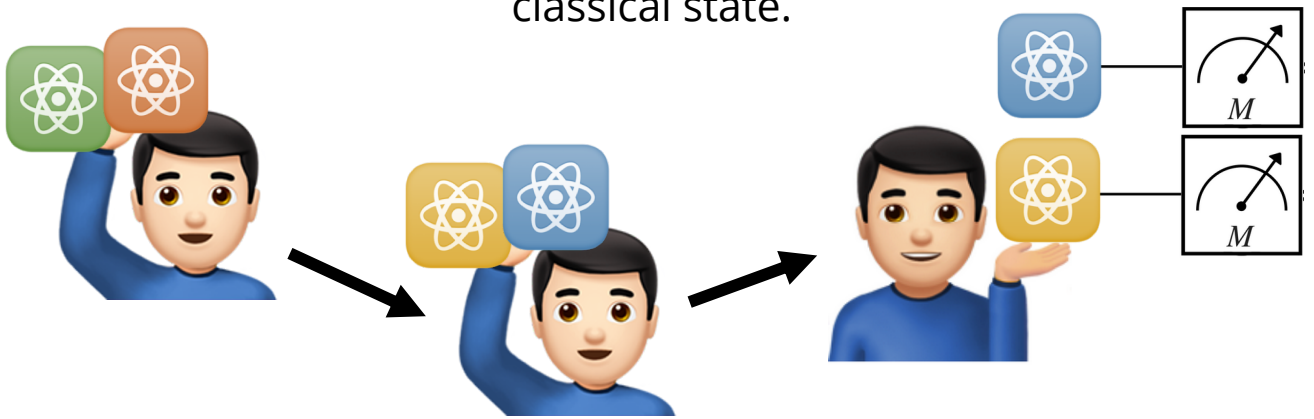
3. **Encoding:** Alice encodes her classical state in her qubit.



4. **Transmission:** Alice sends her qubit to Bob.

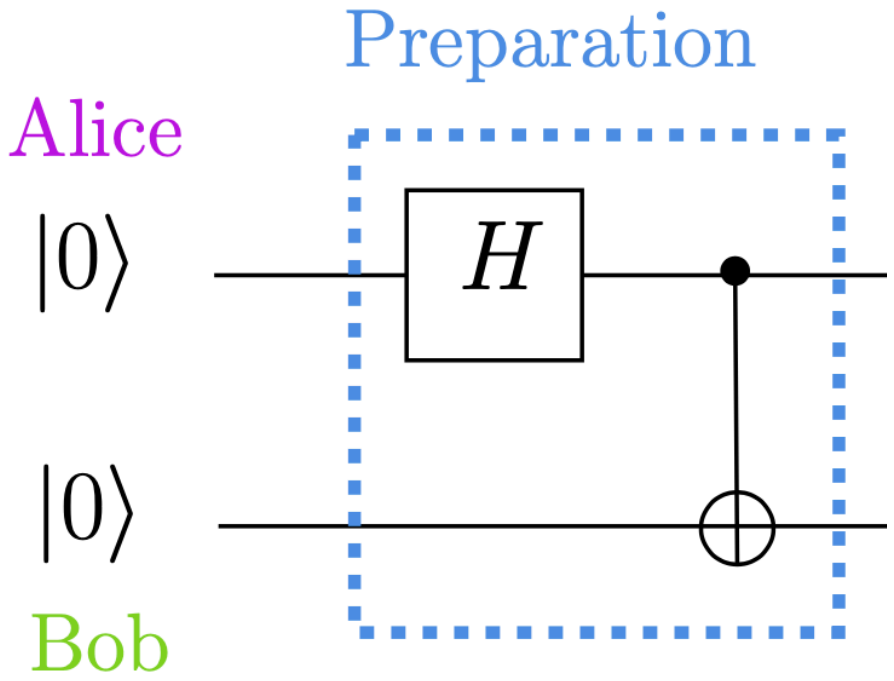


5. **Decoding:** Bob decodes & measures his qubits to retrieve Alice's classical state.



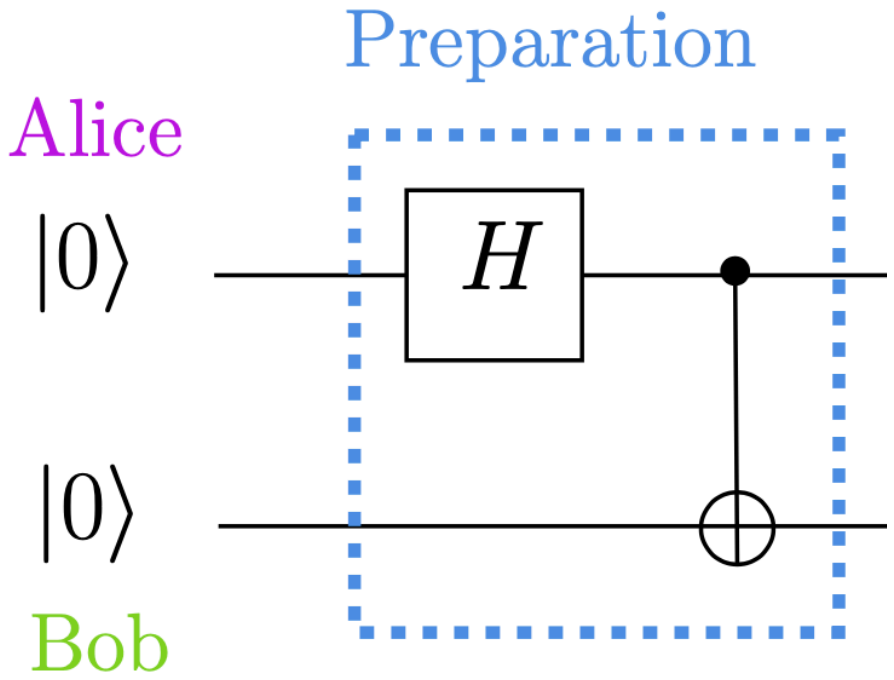
QUANTUM PRACTICE TIME!

What is the output of the following quantum circuit?



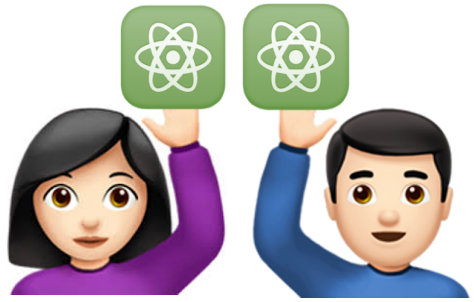
QUANTUM PRACTICE SOLUTION!

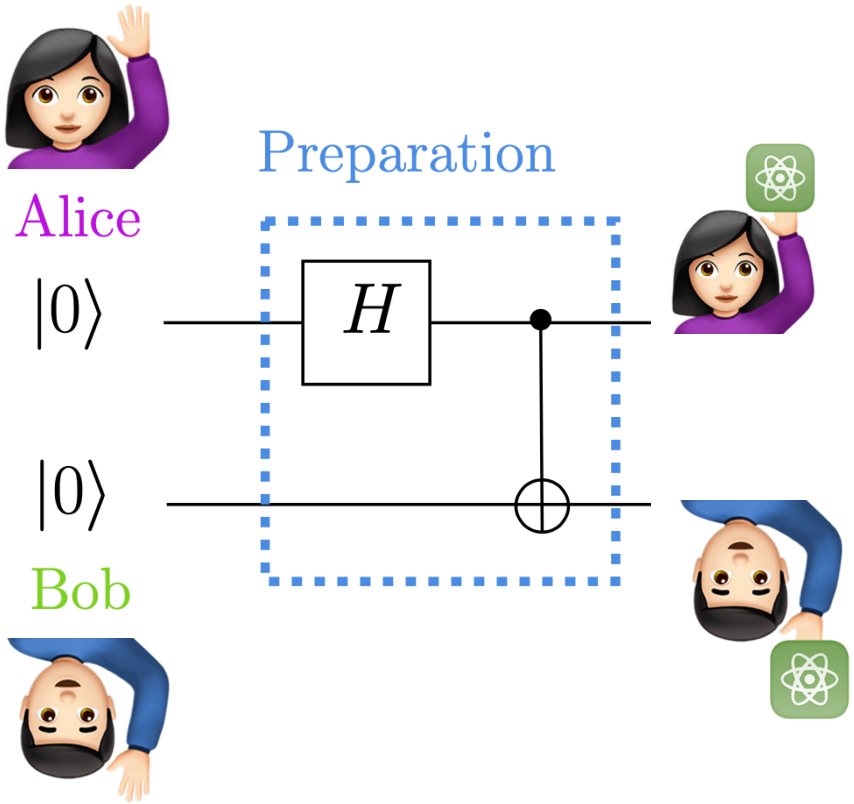
What is the output of the following quantum circuit?



SUPERDENSE CODING: PREPARATION

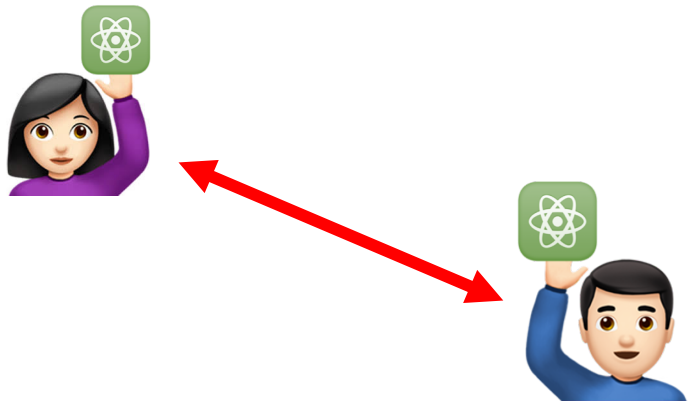
1. **Preparation:** Alice and Bob meet up and split an entangled bell state.

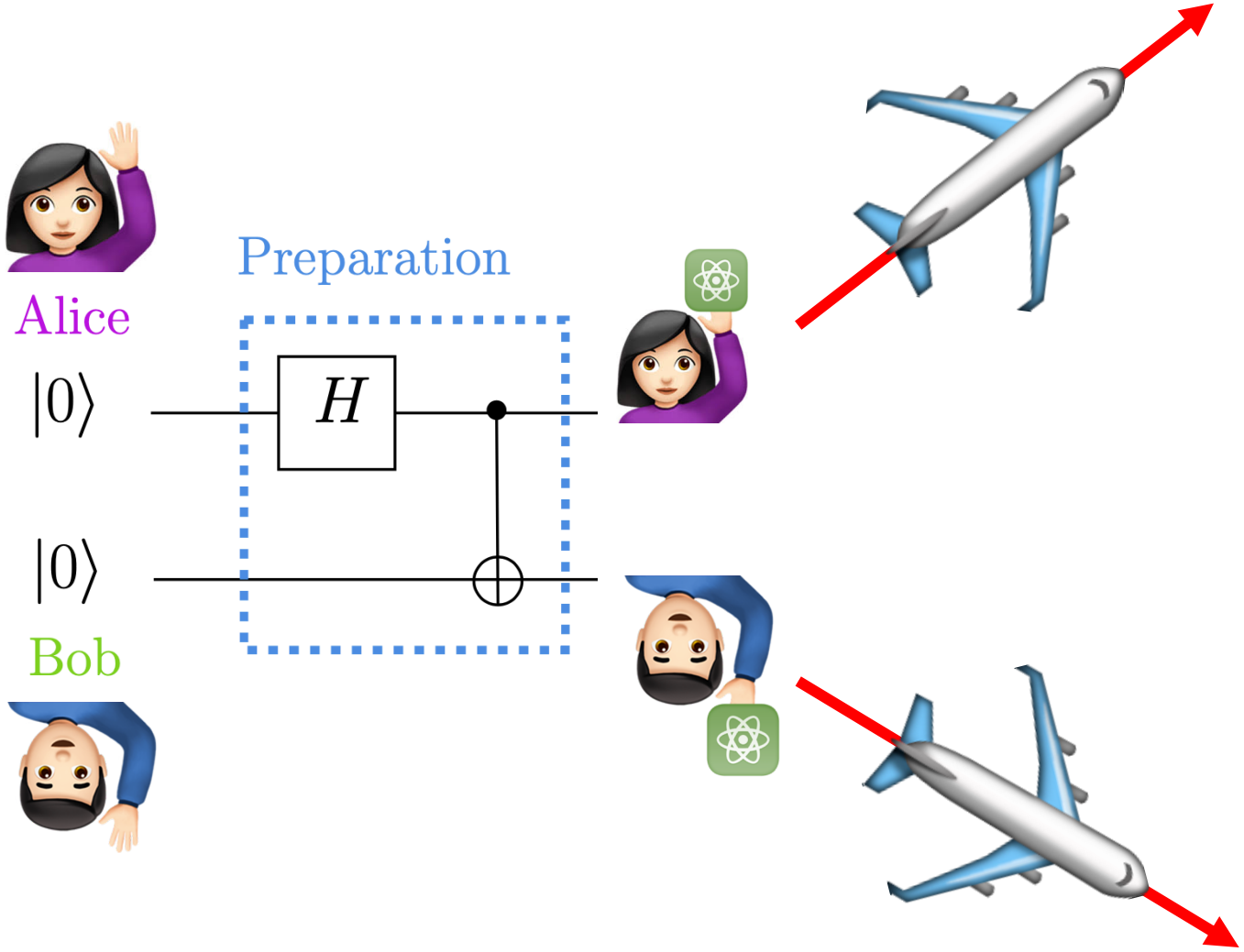




SUPERDENSE CODING: TRAVEL

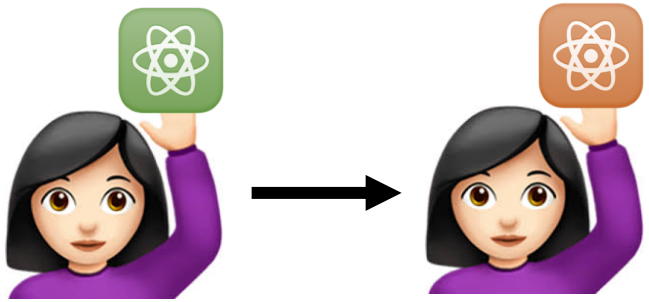
2. Alice and Bob travel far away.

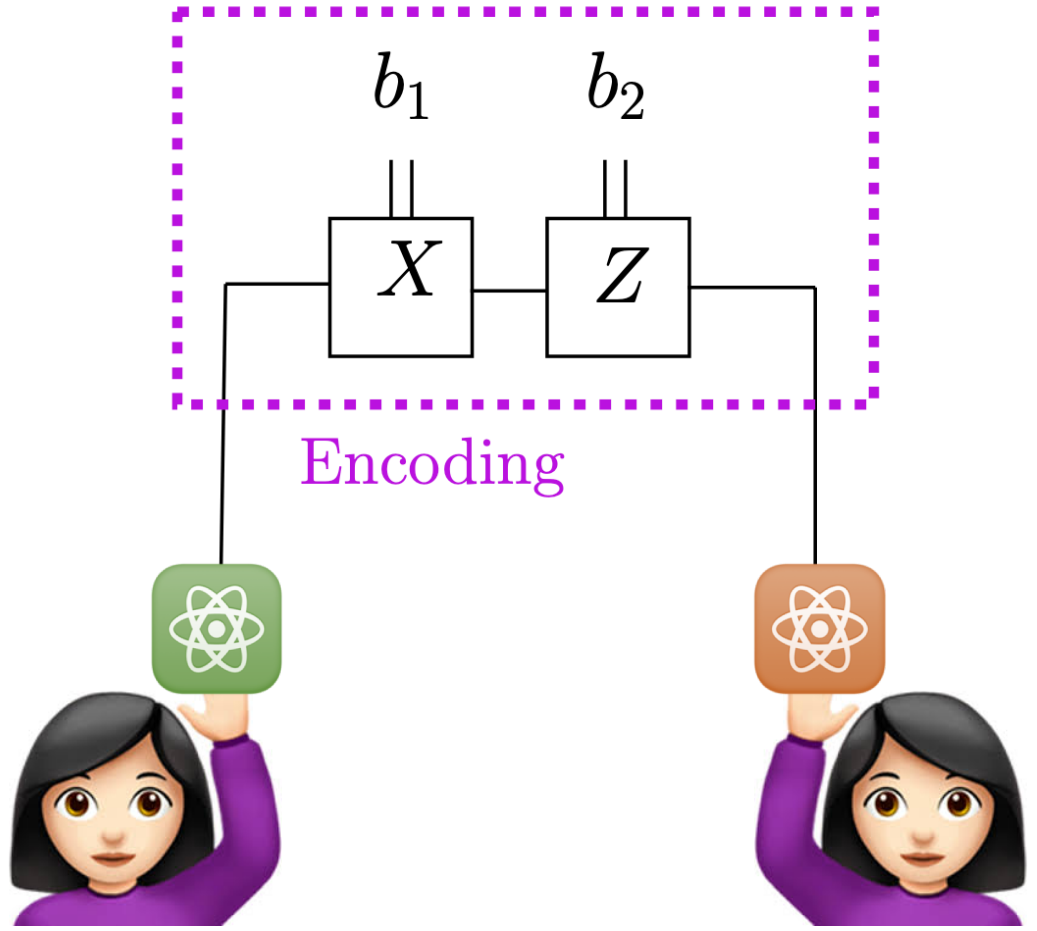




SUPERDENSE CODING: ENCODING

3. **Encoding:** Alice encodes her classical state in her qubit.





SUPERDENSE CODING: ENCODING

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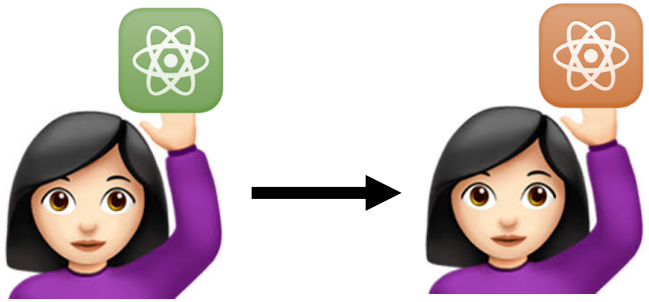
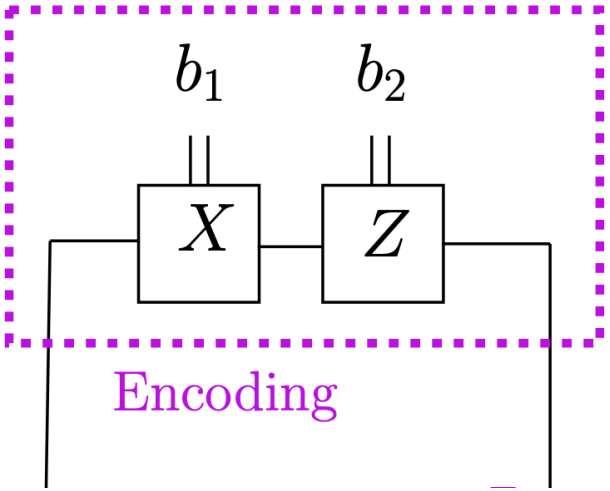


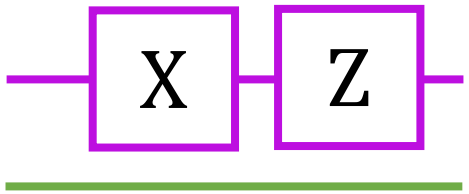


0 (00)	$b_1 = 0$, $b_2 = 0$	$\frac{\|00\rangle + \|11\rangle}{\sqrt{2}}$		$\frac{\|00\rangle + \|11\rangle}{\sqrt{2}}$
1 (01)	$b_1 = 1$, $b_2 = 0$	$\frac{\|00\rangle + \|11\rangle}{\sqrt{2}}$	X	$\frac{\|10\rangle + \|01\rangle}{\sqrt{2}}$
2 (10)	$b_1 = 0$, $b_2 = 1$	$\frac{\|00\rangle + \|11\rangle}{\sqrt{2}}$	Z	$\frac{\|00\rangle - \|11\rangle}{\sqrt{2}}$
3 (11)	$b_1 = 1$, $b_2 = 1$	$\frac{\|00\rangle + \|11\rangle}{\sqrt{2}}$	X Z	???

QUANTUM PRACTICE TIME!

What is the output of the following quantum circuit?

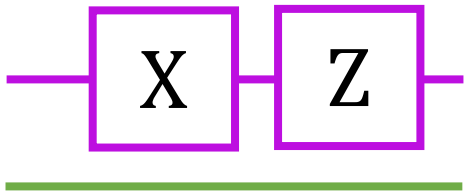
$$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$$



QUANTUM PRACTICE SOLUTION!

What is the output of the following quantum circuit?

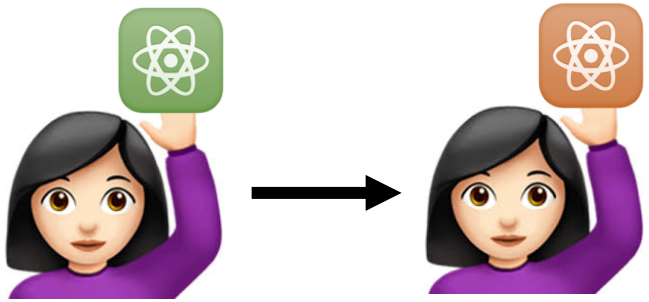
$$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

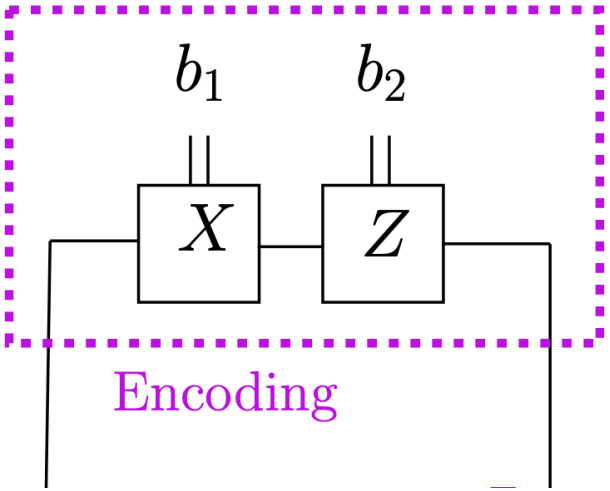


$$\frac{|10\rangle - |01\rangle}{\sqrt{2}}$$

SUPERDENSE CODING: ENCODING

3. **Encoding:** Alice encodes her classical state in her qubit.





0 (00)	$b_1 = 0$, $b_2 = 0$	$\frac{\|00\rangle + \|11\rangle}{\sqrt{2}}$		$\frac{\|00\rangle + \|11\rangle}{\sqrt{2}}$
1 (01)	$b_1 = 1$, $b_2 = 0$	$\frac{\|00\rangle + \|11\rangle}{\sqrt{2}}$	X	$\frac{\|10\rangle + \|01\rangle}{\sqrt{2}}$
2 (10)	$b_1 = 0$, $b_2 = 1$	$\frac{\|00\rangle + \|11\rangle}{\sqrt{2}}$	Z	$\frac{\|00\rangle - \|11\rangle}{\sqrt{2}}$
3 (11)	$b_1 = 1$, $b_2 = 1$	$\frac{\|00\rangle + \|11\rangle}{\sqrt{2}}$	X Z	$\frac{\|10\rangle - \|01\rangle}{\sqrt{2}}$

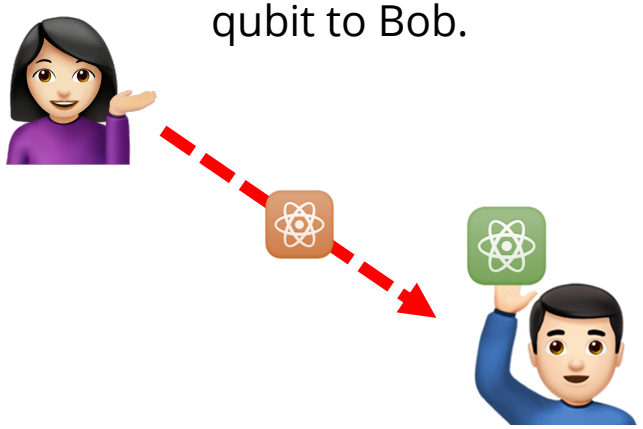
BELL STATES

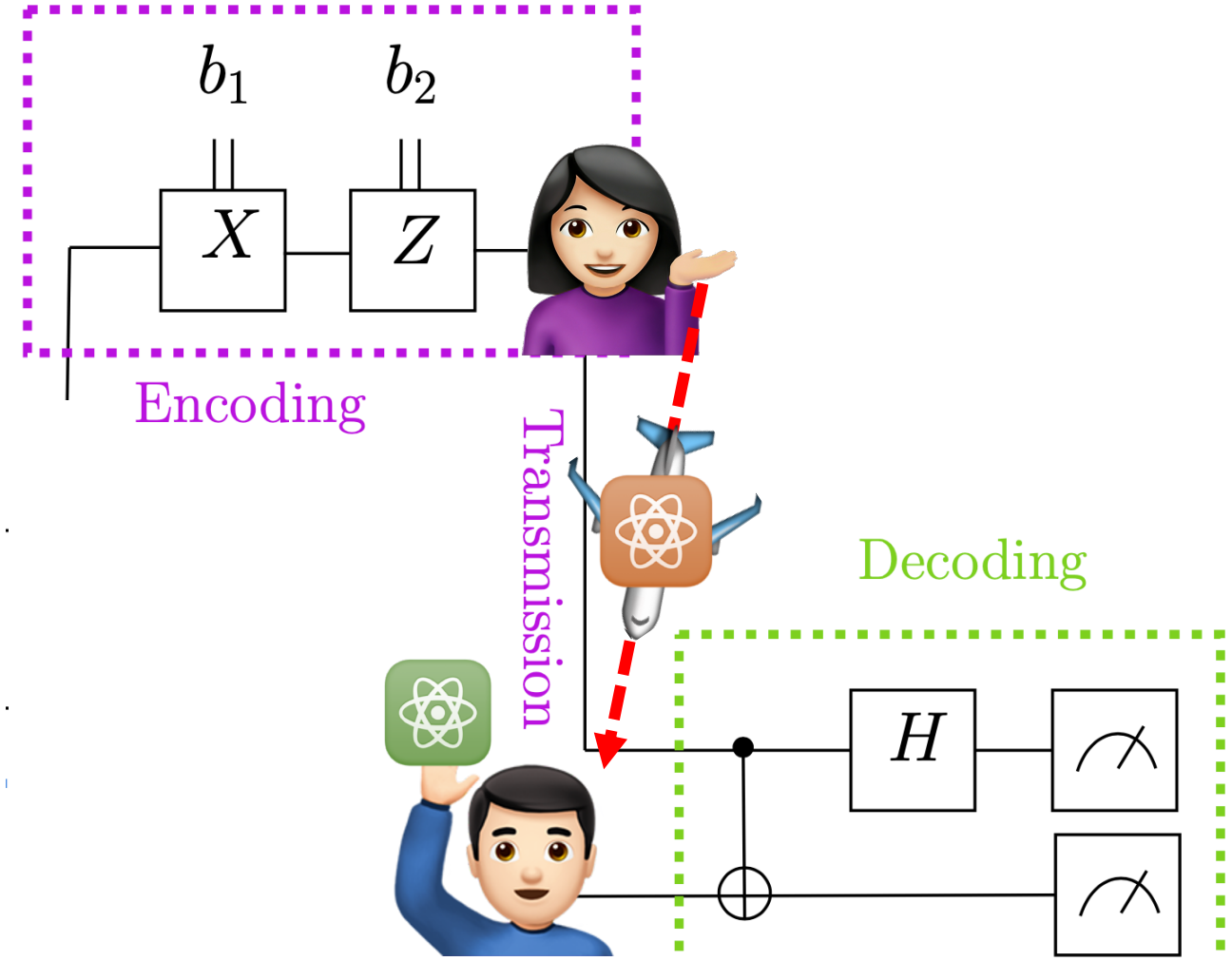
The ***Bell states*** are the 4 possible maximally entangled 2-qubit states.

message	transformation	result
00	$\mathbb{I}_A \otimes \mathbb{I}_B$	$\frac{\|00\rangle+\|11\rangle}{\sqrt{2}} = \|\beta_{00}\rangle$
01	$X_A \otimes \mathbb{I}_B$	$\frac{\|10\rangle+\|01\rangle}{\sqrt{2}} = \|\beta_{01}\rangle$
10	$Z_A \otimes \mathbb{I}_B$	$\frac{\|00\rangle-\|11\rangle}{\sqrt{2}} = \|\beta_{10}\rangle$
11	$X_A, Z_A \otimes \mathbb{I}_B$	$\frac{\|10\rangle-\|01\rangle}{\sqrt{2}} = \|\beta_{11}\rangle$

SUPERDENSE CODING: TRANSMISSION

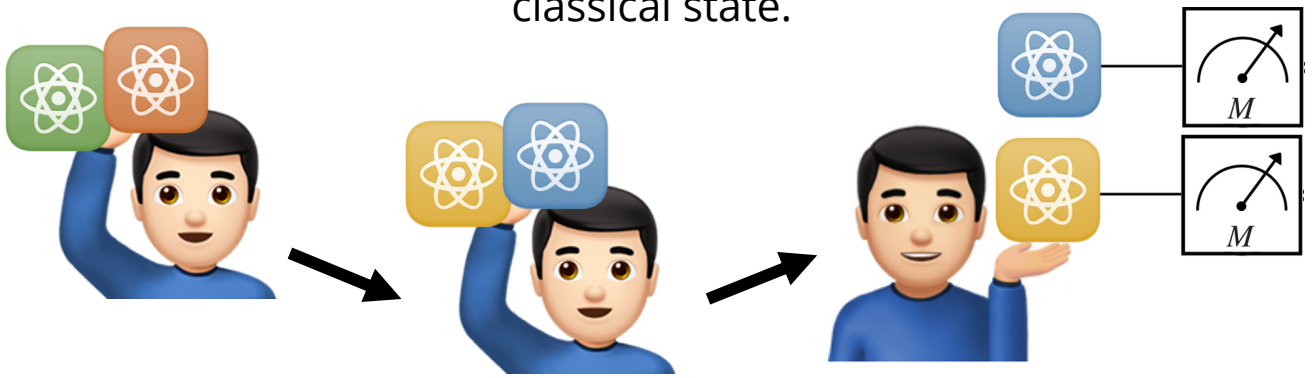
4. **Transmission:** Alice sends her qubit to Bob.

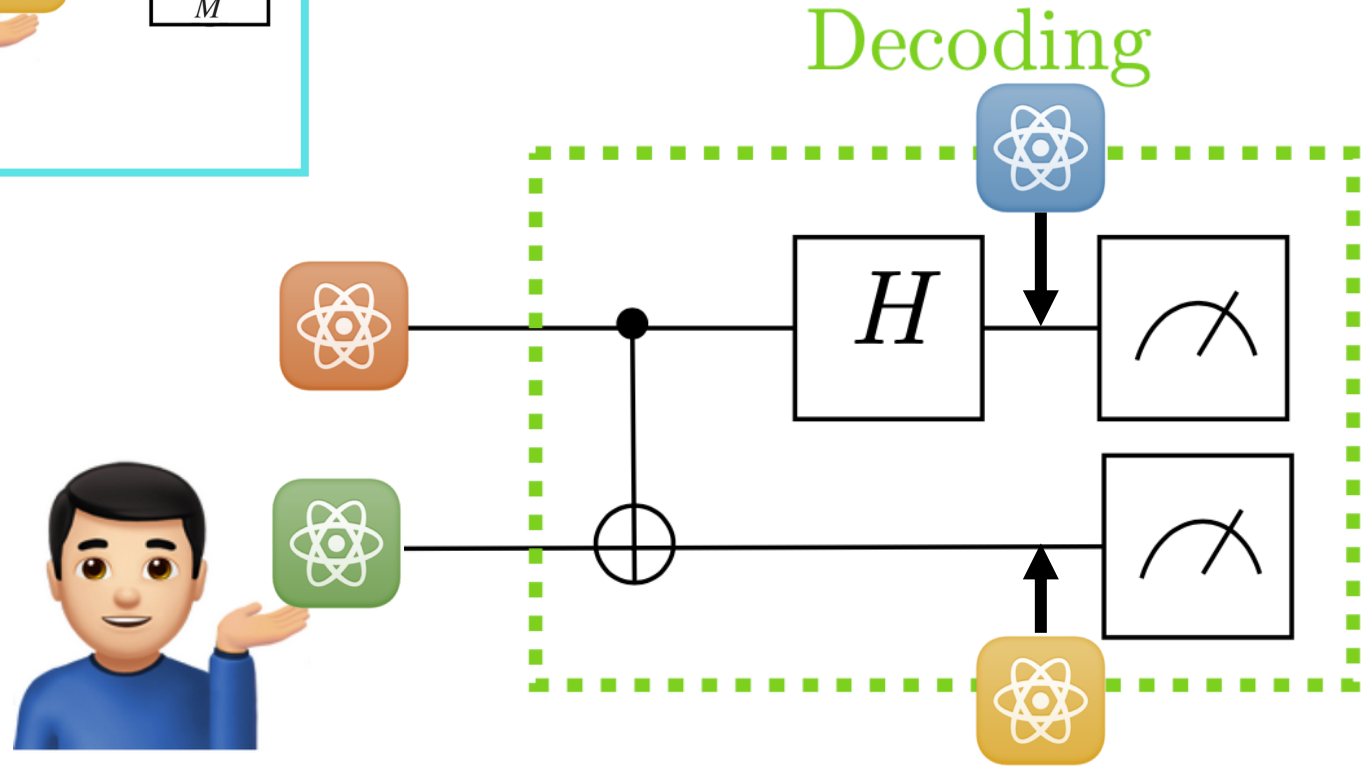




SUPERDENSE CODING: DECODING

5. **Decoding:** Bob decodes & measures his qubits to retrieve Alice's classical state.



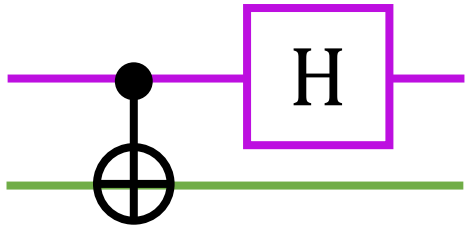


QUANTUM PRACTICE TIME!

What is the output of each of the following quantum circuits?

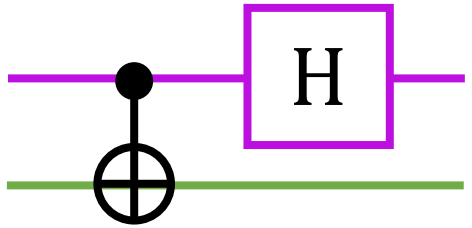
(1)

$$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$$



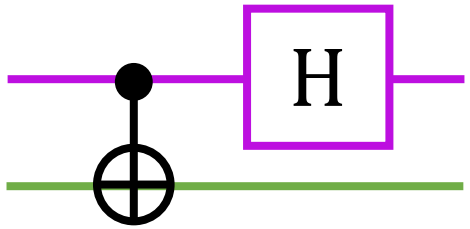
(2)

$$\frac{|10\rangle + |01\rangle}{\sqrt{2}}$$



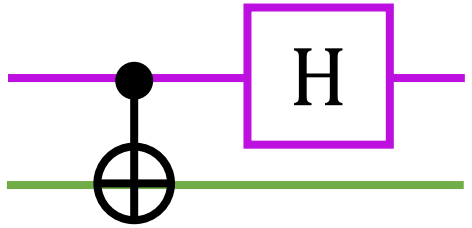
(3)

$$\frac{|00\rangle - |11\rangle}{\sqrt{2}}$$



(4)

$$\frac{|10\rangle - |01\rangle}{\sqrt{2}}$$

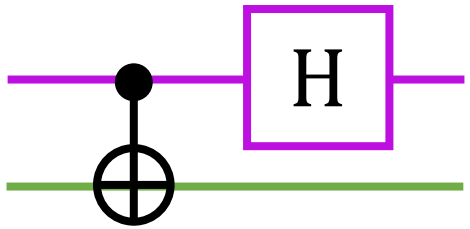


QUANTUM SOLUTION TIME!

What is the output of each of the following quantum circuits?

(1)

$$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

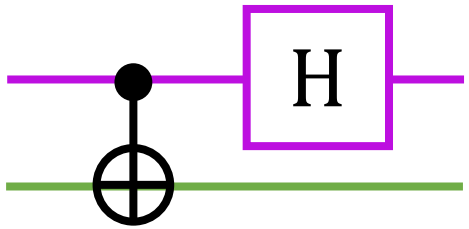


QUANTUM SOLUTION TIME!

What is the output of each of the following quantum circuits?

(2)

$$\frac{|10\rangle + |01\rangle}{\sqrt{2}}$$

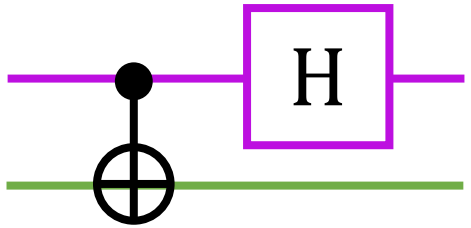


QUANTUM SOLUTION TIME!

What is the output of each of the following quantum circuits?

(3)

$$\frac{|00\rangle - |11\rangle}{\sqrt{2}}$$

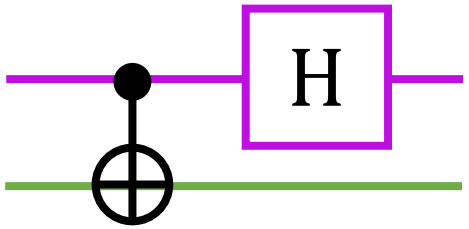


$|10\rangle$

Verify these two yourselves!

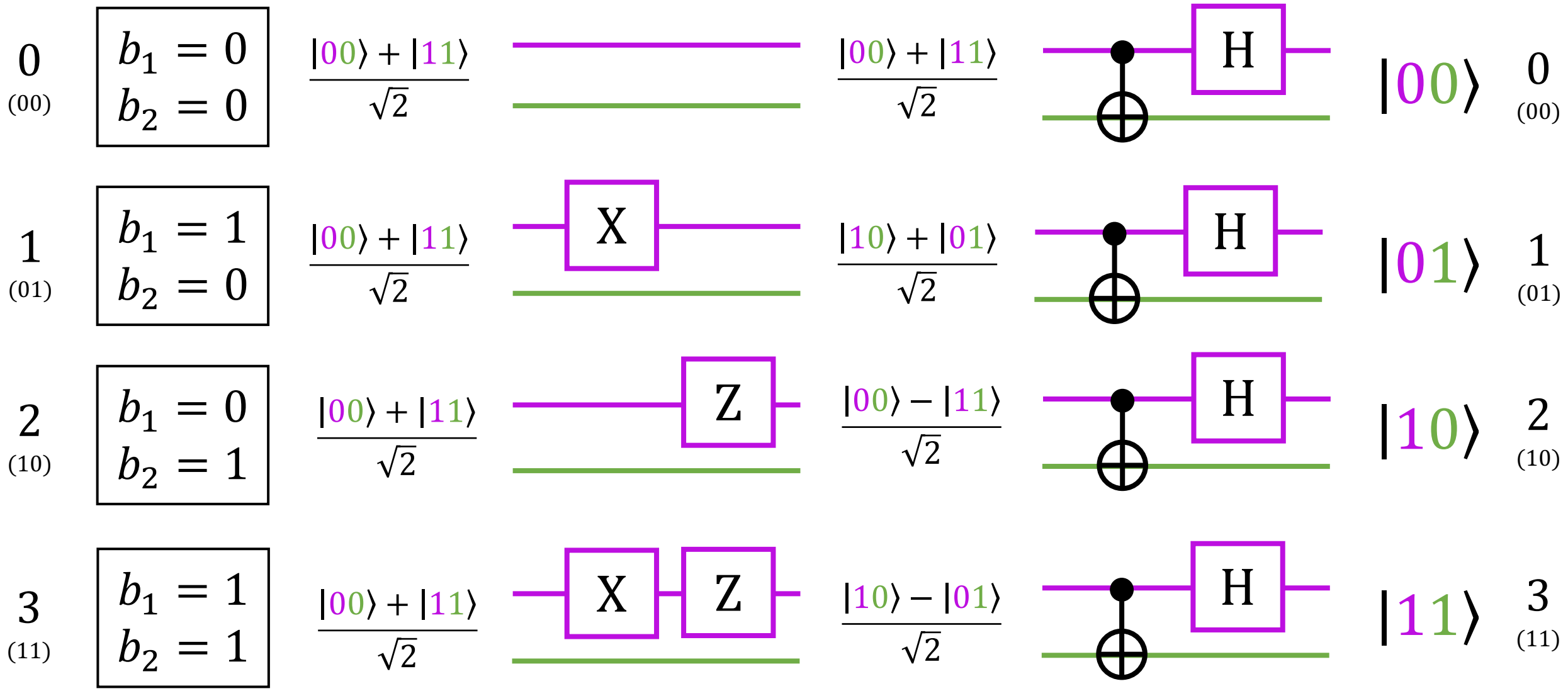
(4)

$$\frac{|10\rangle - |01\rangle}{\sqrt{2}}$$



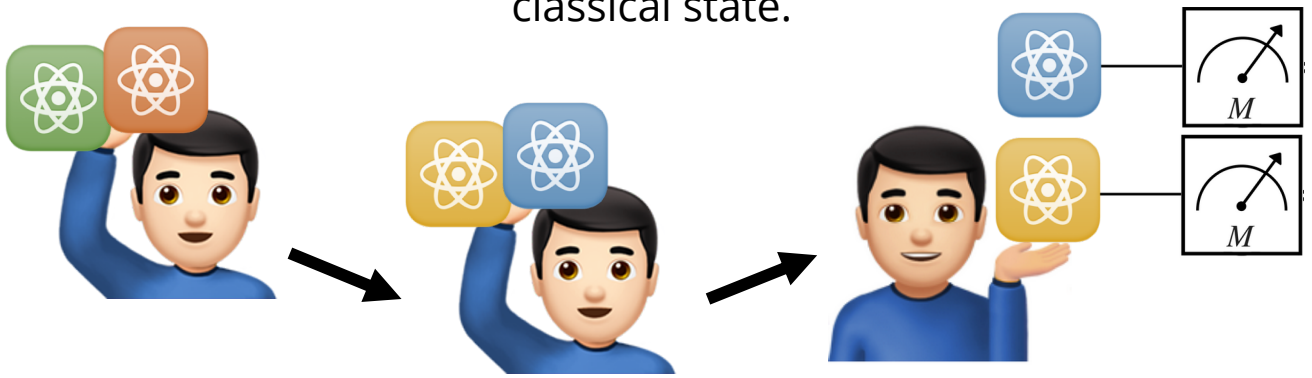
$|11\rangle$

SUPERDENSE CODING: DECODING



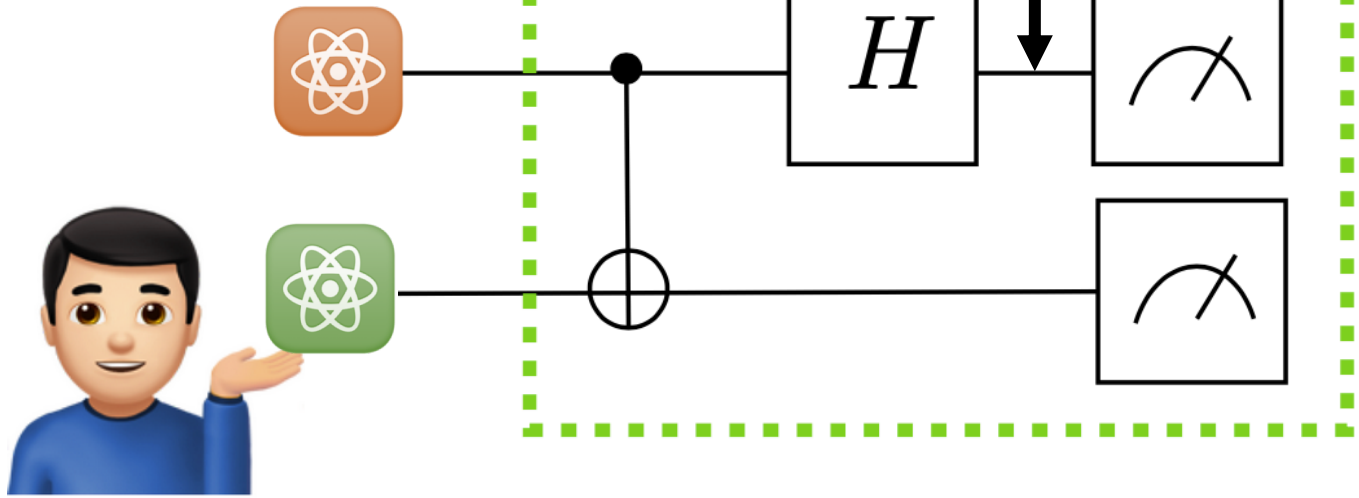
SUPERDENSE CODING: DECODING

5. **Decoding:** Bob decodes & measures his qubits to retrieve Alice's classical state.



$|00\rangle, |01\rangle, |10\rangle, |11\rangle$

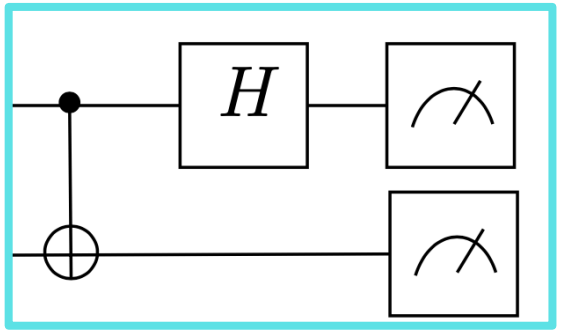
Decoding



BELL MEASUREMENT

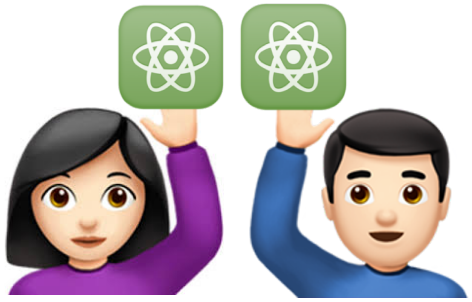
A ***Bell measurement*** is a joint quantum mechanical measurement of 2 qubits, that determines which of the 4 Bell states the qubits are in.

message	transformation	result		BELL MEASUREMENT	message
00	$\mathbb{I}_A \otimes \mathbb{I}_B$	$\frac{\|00\rangle+\|11\rangle}{\sqrt{2}}$	$= \|\beta_{00}\rangle$	→	00
01	$X_A \otimes \mathbb{I}_B$	$\frac{\|10\rangle+\|01\rangle}{\sqrt{2}}$	$= \|\beta_{01}\rangle$	→	01
10	$Z_A \otimes \mathbb{I}_B$	$\frac{\|00\rangle-\|11\rangle}{\sqrt{2}}$	$= \|\beta_{10}\rangle$	→	10
11	$X_A, Z_A \otimes \mathbb{I}_B$	$\frac{\|10\rangle-\|01\rangle}{\sqrt{2}}$	$= \|\beta_{11}\rangle$	→	11

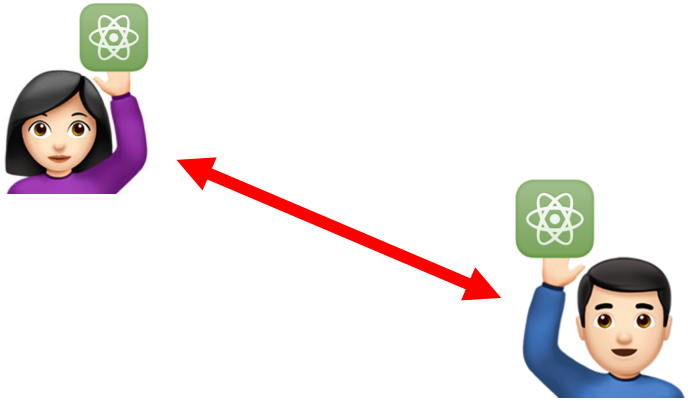


SUPERDENSE CODING PROTOCOL

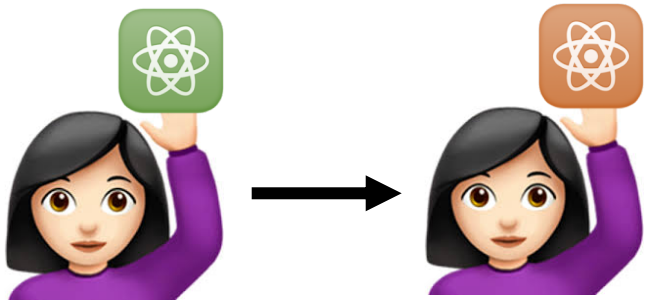
1. **Preparation:** Alice and Bob meet up and split an entangled bell state.



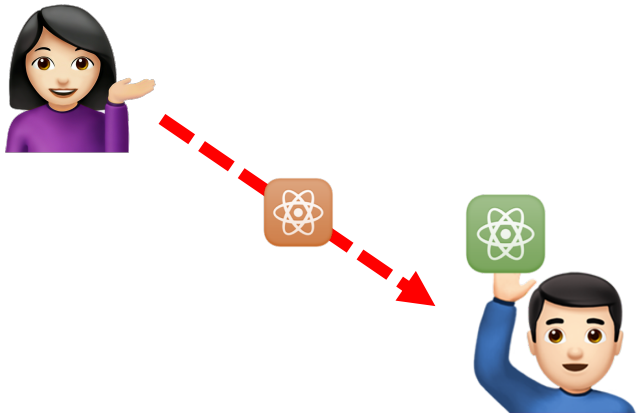
2. Alice and Bob travel far away.



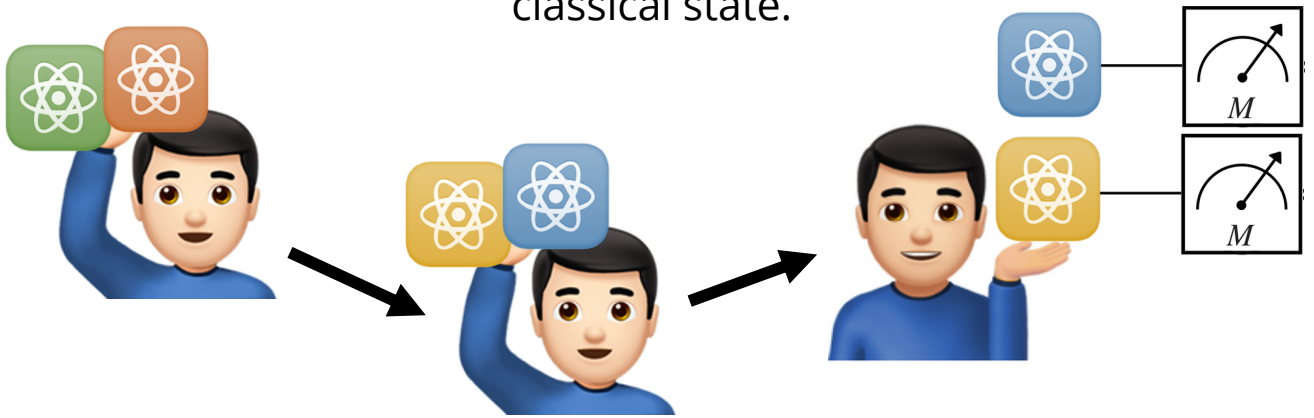
3. **Encoding:** Alice encodes her classical state in her qubit.



4. **Transmission:** Alice sends her qubit to Bob.

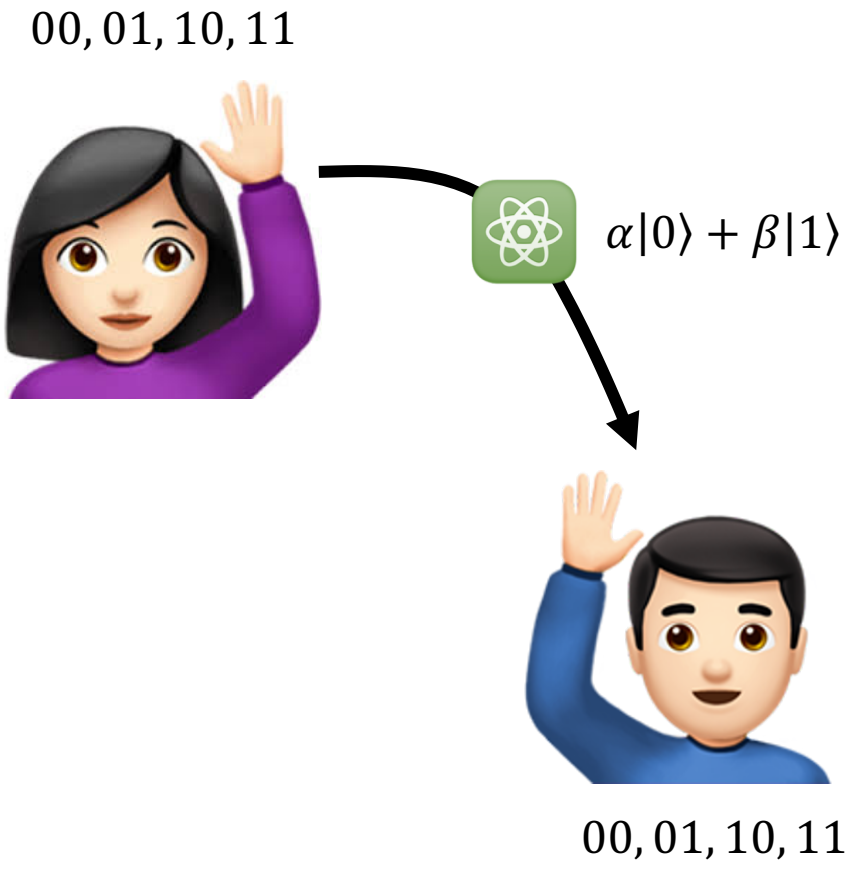


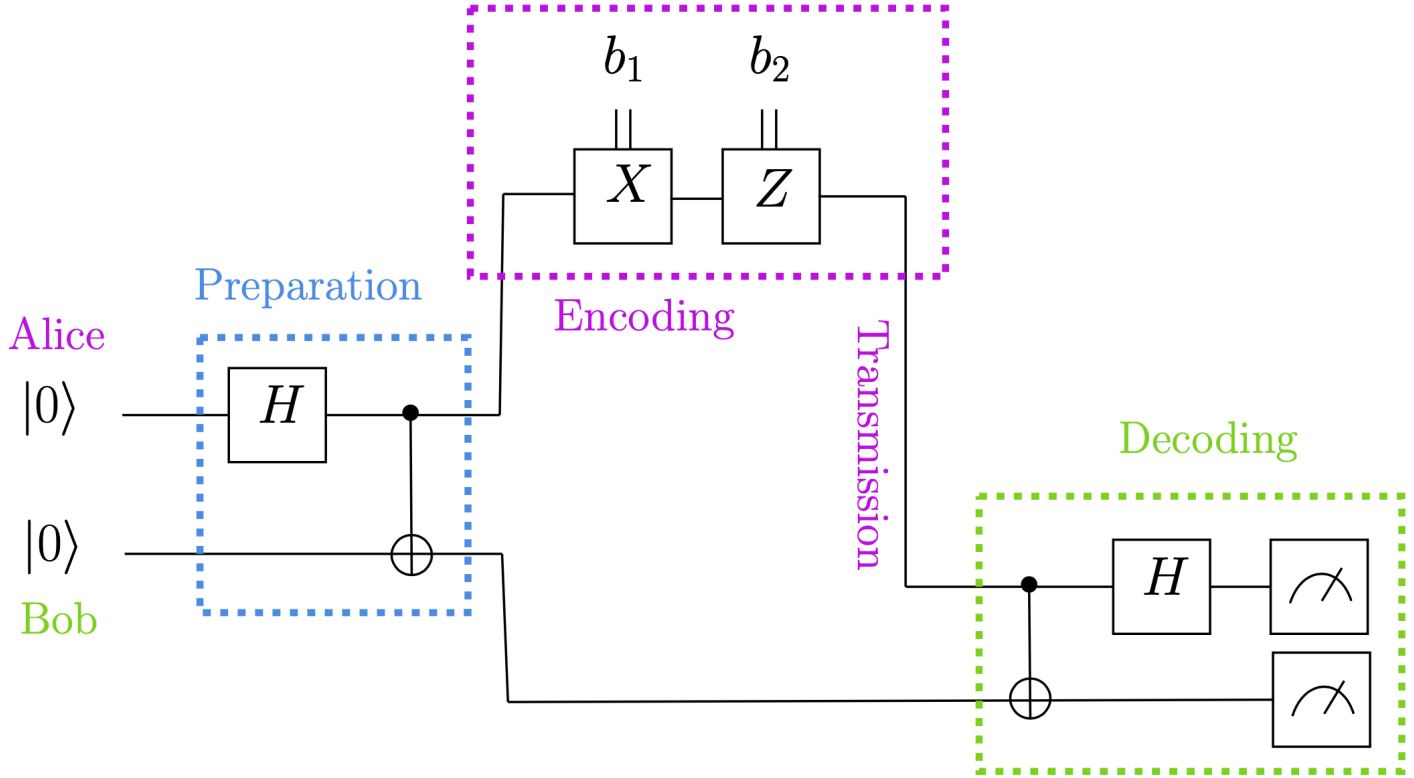
5. **Decoding:** Bob decodes & measures his qubits to retrieve Alice's classical state.

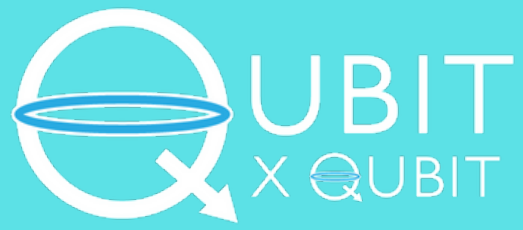


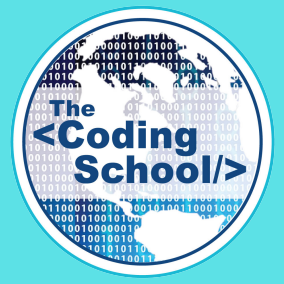
SUPERDENSE CODING

Superdense coding is a quantum communication protocol which transmits **2** classical bits of information with only **1** qubit!







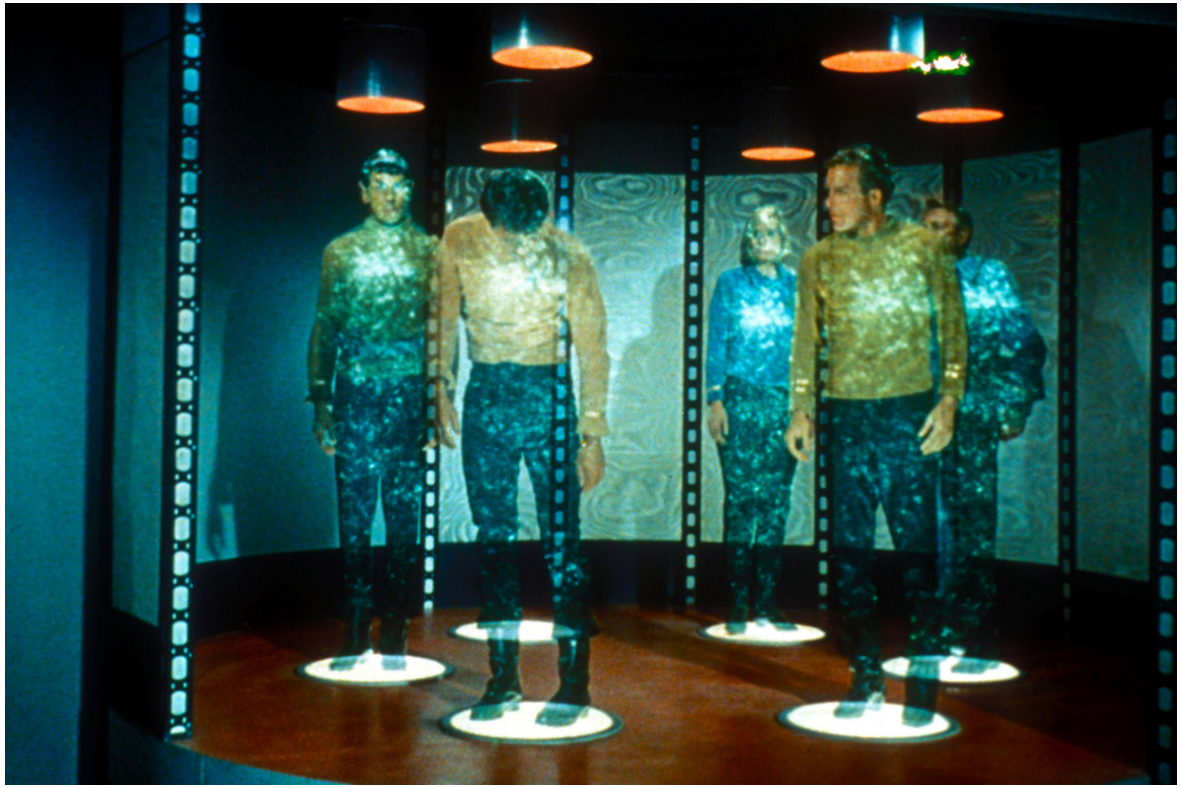


QUANTUM TELEPORTATION

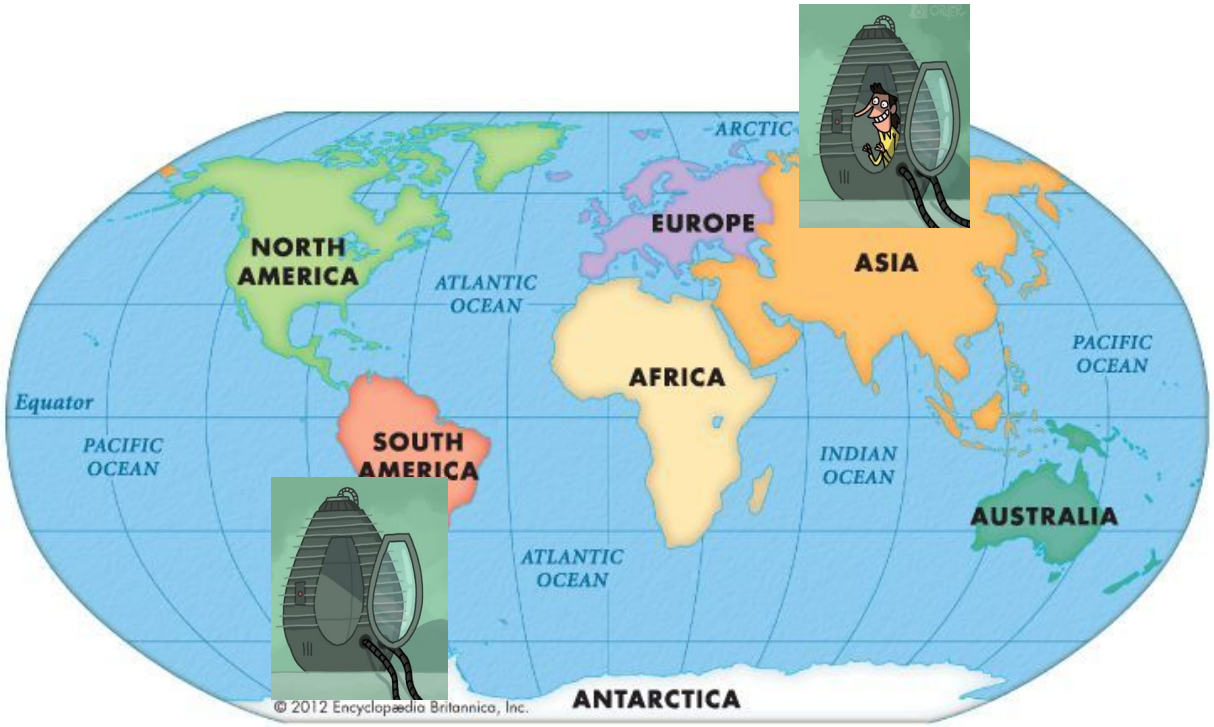
SENDING QUANTUM INFO WITHOUT PHYSICALLY SENDING QUBITS

“TELEPORTATION”

Teleportation is the instantaneous transfer of matter and energy from one location to another, without physically traversing the distance in-between.



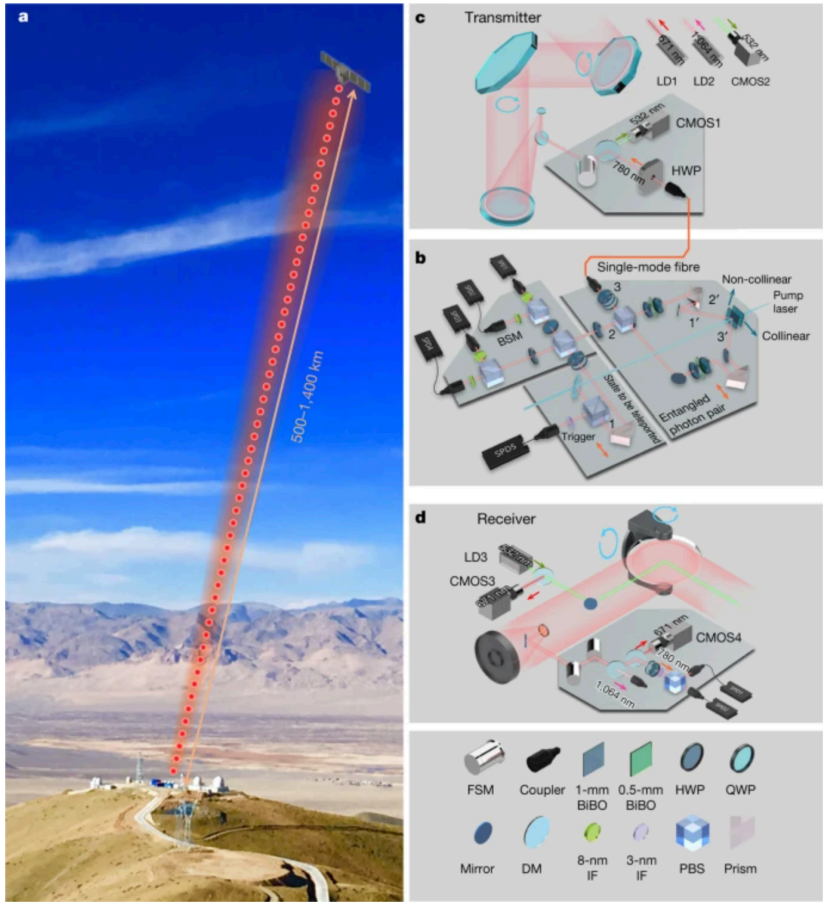
Trekkie Transport



Instantaneously transport from Asia to Antarctica?!

QUANTUM TELEPORTATION

Today we will be learning about ***quantum teleportation***, a technique for transferring quantum information from a sender at one location to a receiver some distance away.

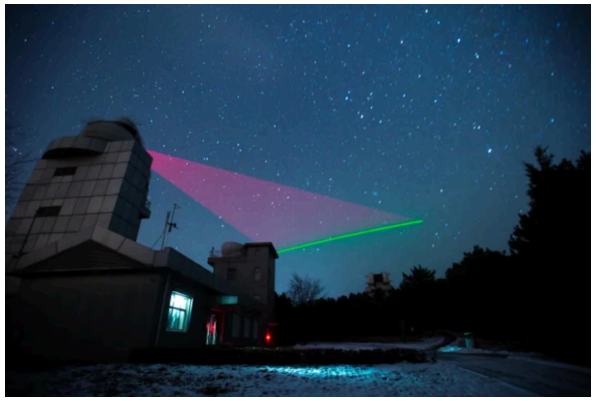


Space July 10, 2017

First Object Teleported from Earth to Orbit

Researchers in China have teleported a photon from the ground to a satellite orbiting more than 500 kilometers above.

MIT Technology Review



SCIENTIFIC AMERICAN®

PHYSICS

China Reaches New Milestone in Space-Based Quantum Communications

The nation's Micius satellite successfully established an ultrasecure link between two ground stations separated by more than 1,000 kilometers

By Karen Kwon on June 25, 2020 أعرض هذا باللغة العربية

QUANTUM TELEPORTATION LIMITATIONS

Although quantum teleportation may bring to mind popular culture ideas of teleportation, let's clarify some ways in which it differs:

(1) Quantum teleportation requires that an **entangled bell state be split** between the sending and receiving destinations.

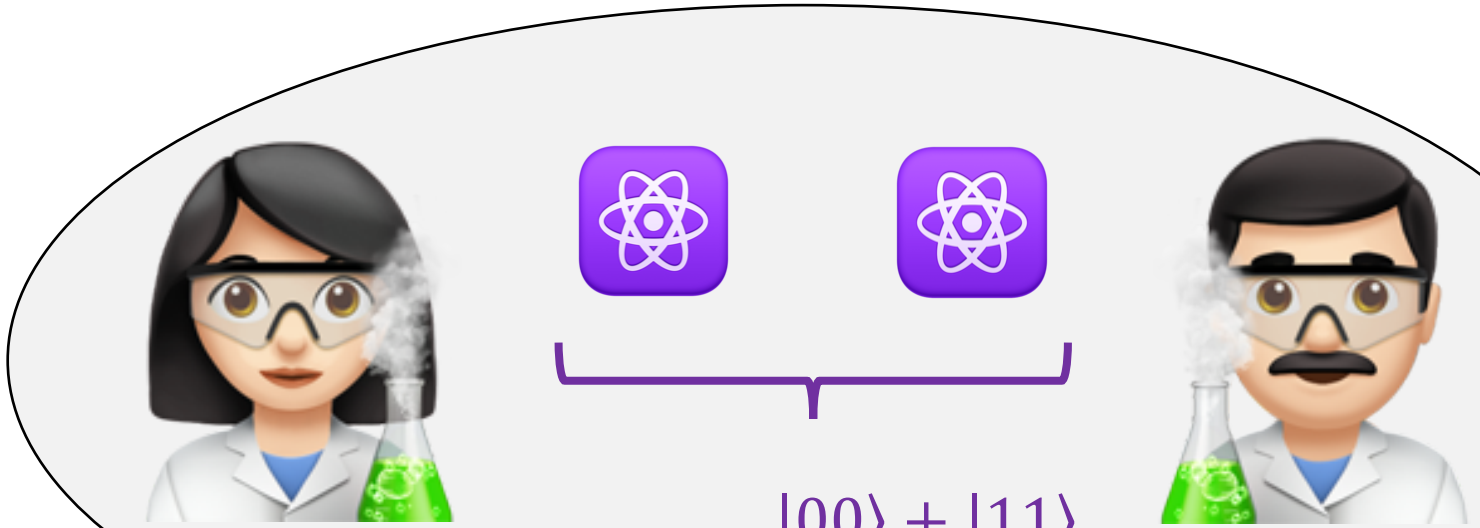
(2) Since quantum teleportation requires measuring the bell state, each time you want to perform teleportation you have to **share a new bell state**.

(3) Quantum teleportation is **not instantaneous**. It requires classical communication (i.e. a phone call or email) between the sender and receiver.

(4) Quantum teleportation is meant for sending one qubit at a time, not a full person!

BACKSTORY

Two scientists, Alice and Bob, met in the United States to create a Bell state (with 2 photons).

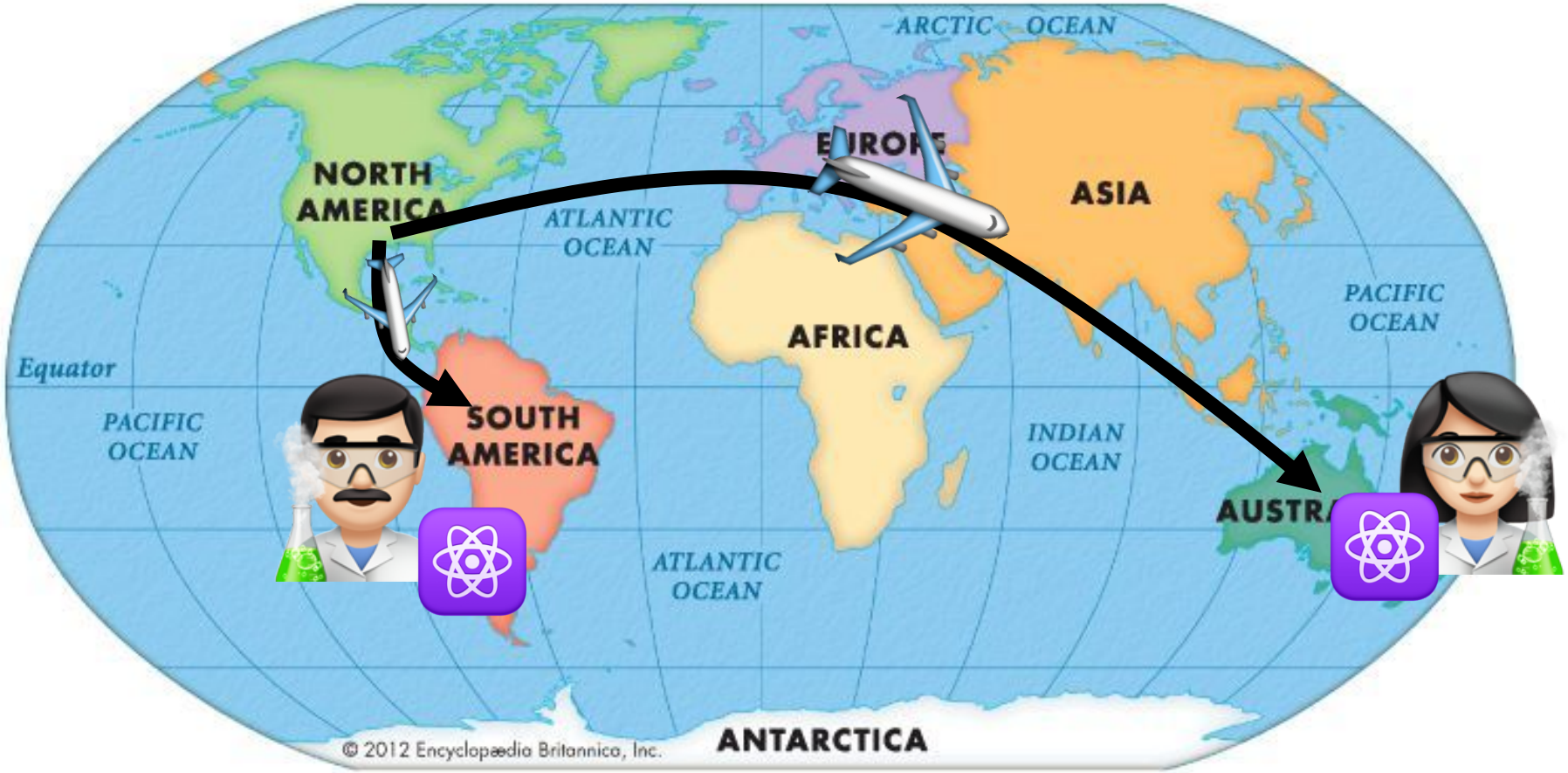


$$|\beta_{00}\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

BACKSTORY

However, Alice had to go back to her lab in Australia and Bob to his lab in Brazil.

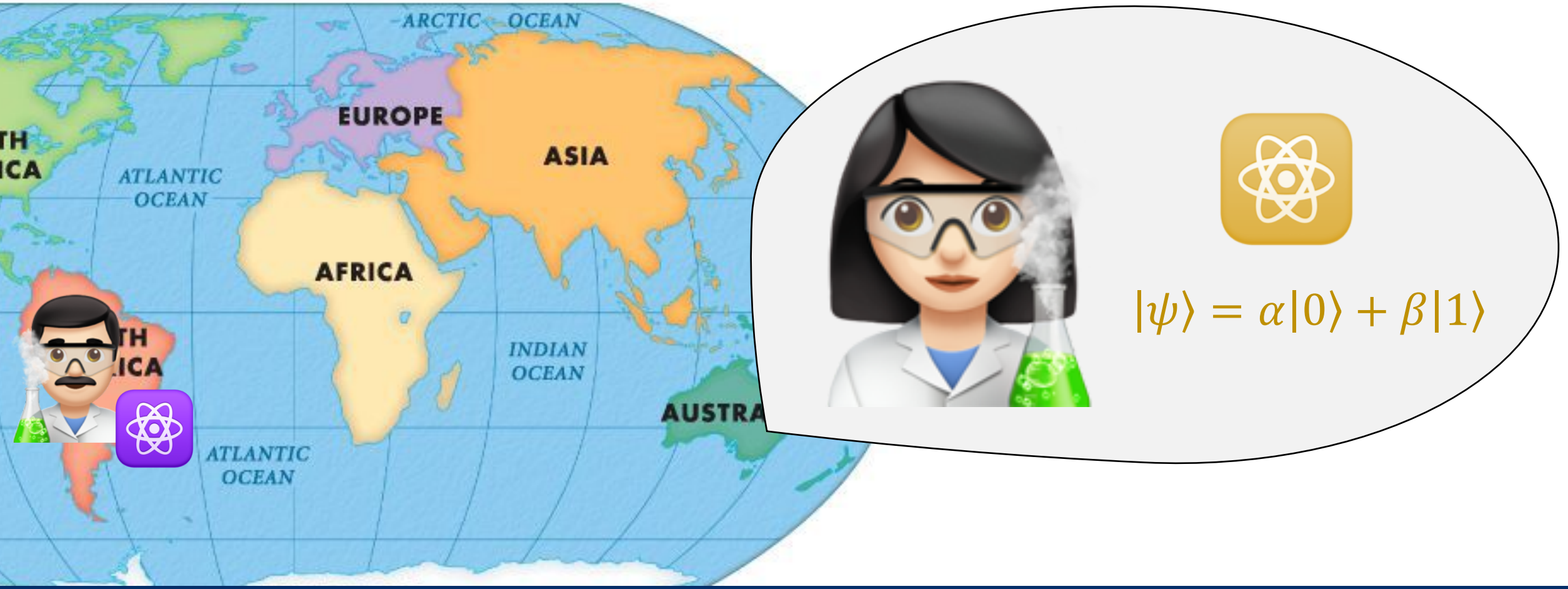
Each took one of the entangled photons back with them...



BACKSTORY

Back to work in her Australian lab, Alice generated all kinds of exotic quantum states.

One day, she managed to create the extremely rare single-qubit state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$.

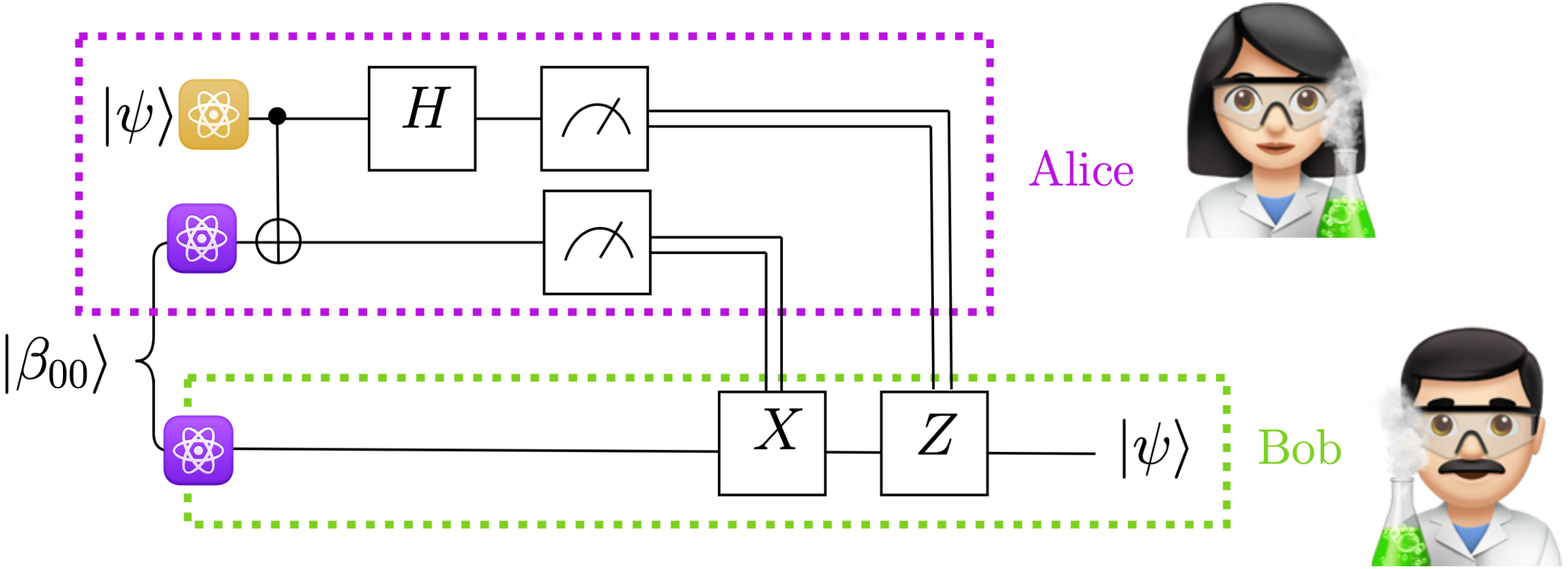


BACKSTORY

Meanwhile, in his Brazilian lab, Bob discovered he needed rare qubit state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ for an important experiment. So, he asked Alice to ***teleport*** $|\psi\rangle$ using their shared Bell states....

QUANTUM TELEPORTATION CIRCUIT

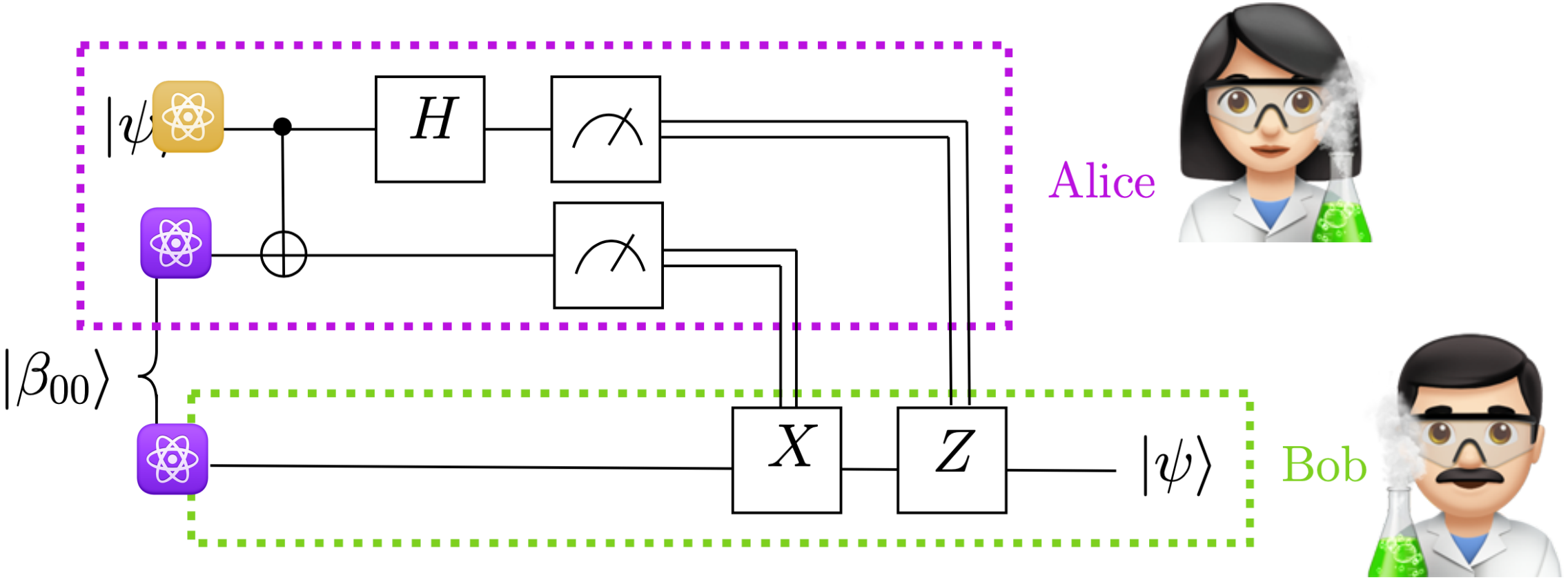
This is the quantum circuit they will use to quantum teleport state $|\psi\rangle$ ...



Let's see why this works...

QUANTUM TELEPORTATION PROTOCOL

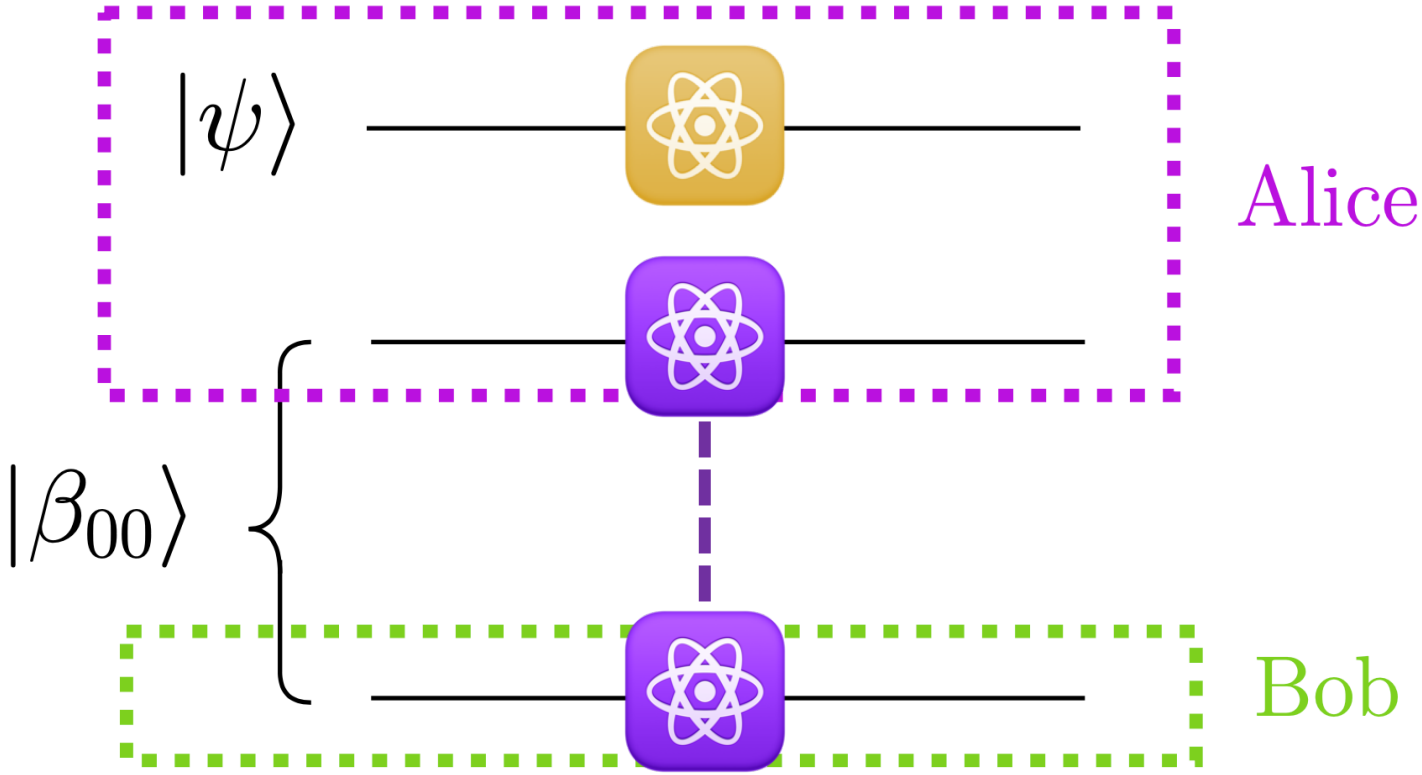
This is the quantum circuit they will use to quantum teleport state $|\psi\rangle$ ...



Let's see why this works...

STEP 0: INPUT

In order to teleport a quantum state, Alice and Bob only need access to an ***entangled bell pair*** and a classical communication channel by which they can transmit ***2 classical bits*** of information!



STEP 0: INPUT

The full mathematical description of the input state $|\phi_0\rangle$ is,

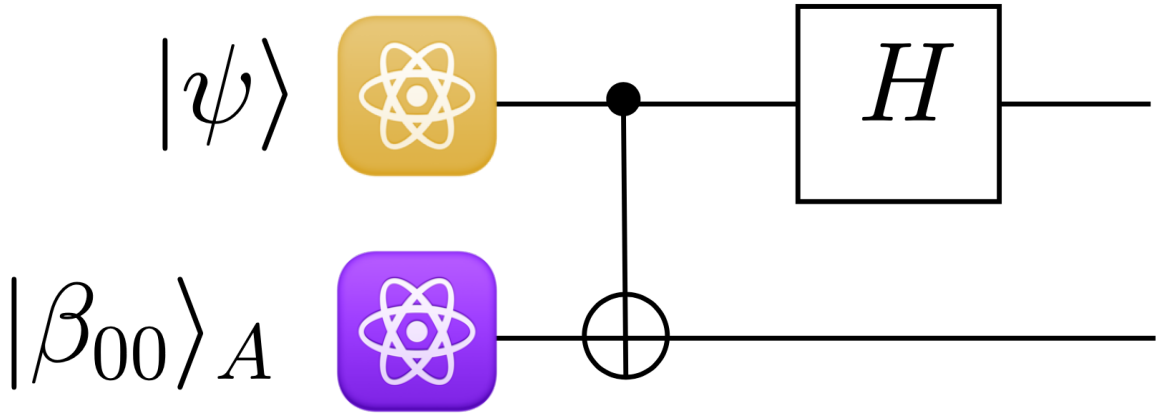
$$|\phi_0\rangle = |\psi\rangle_A |\beta_{00}\rangle_{AB} = \Big(\alpha|0\rangle_A + \beta|1\rangle_A\Big)\frac{1}{\sqrt{2}}\Big(|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B\Big)$$

or, less explicitly,

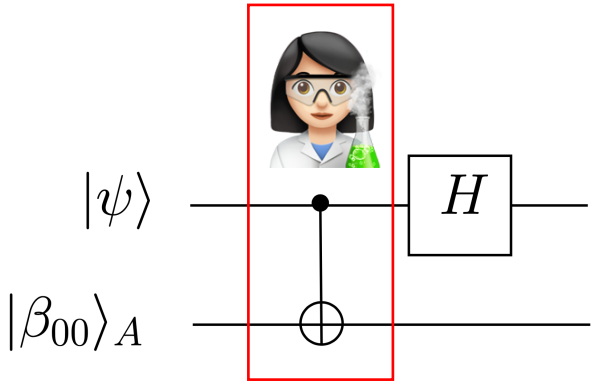
$$|\phi_0\rangle = \frac{1}{\sqrt{2}}\Big[\alpha|0\rangle\Big(|00\rangle + |11\rangle\Big) + \beta|1\rangle\Big(|00\rangle + |11\rangle\Big)\Big]$$

STEP 1: ENTANGLING CIRCUIT

In the first step of the quantum teleportation procedure, Alice performs a CNOT and Hadamard, to entangle her exotic quantum state $|\psi\rangle$, with her portion of the shared Bell state, $|\beta_{00}\rangle_A$.



STEP 1: ENTANGLING CIRCUIT

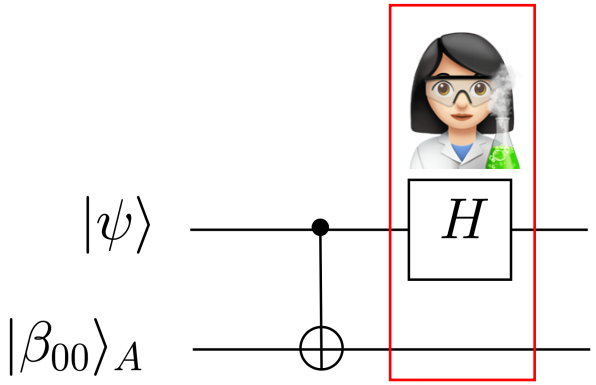


What does Alice's entangling circuit do to the input state?

$$|\phi_0\rangle = \frac{1}{\sqrt{2}}\left[\alpha|0\rangle\Big(|00\rangle + |11\rangle\Big) + \beta|1\rangle\Big(|00\rangle + |11\rangle\Big)\right]$$

First, Alice performs the **CNOT** operation:

STEP 1: ENTANGLING CIRCUIT



After the CNOT, we have state:

$$|\phi_1\rangle = \frac{1}{\sqrt{2}}\left[\alpha|0\rangle\Big(|00\rangle + |11\rangle\Big) + \beta|1\rangle\Big(|10\rangle + |01\rangle\Big)\right]$$

Now, Alice applies a Hadamard to the 1st qubit:

STEP 1: ENTANGLING CIRCUIT

After the Hadamard, Alice is in the state:

$$|\phi_2\rangle = \frac{1}{\sqrt{2}}\left[\alpha\frac{1}{\sqrt{2}}\Big(|0\rangle + |1\rangle\Big)\Big(|00\rangle + |11\rangle\Big) + \beta\frac{1}{\sqrt{2}}\Big(|0\rangle - |1\rangle\Big)\Big(|10\rangle + |01\rangle\Big)\right]$$

And regrouping terms, this is equivalent to:

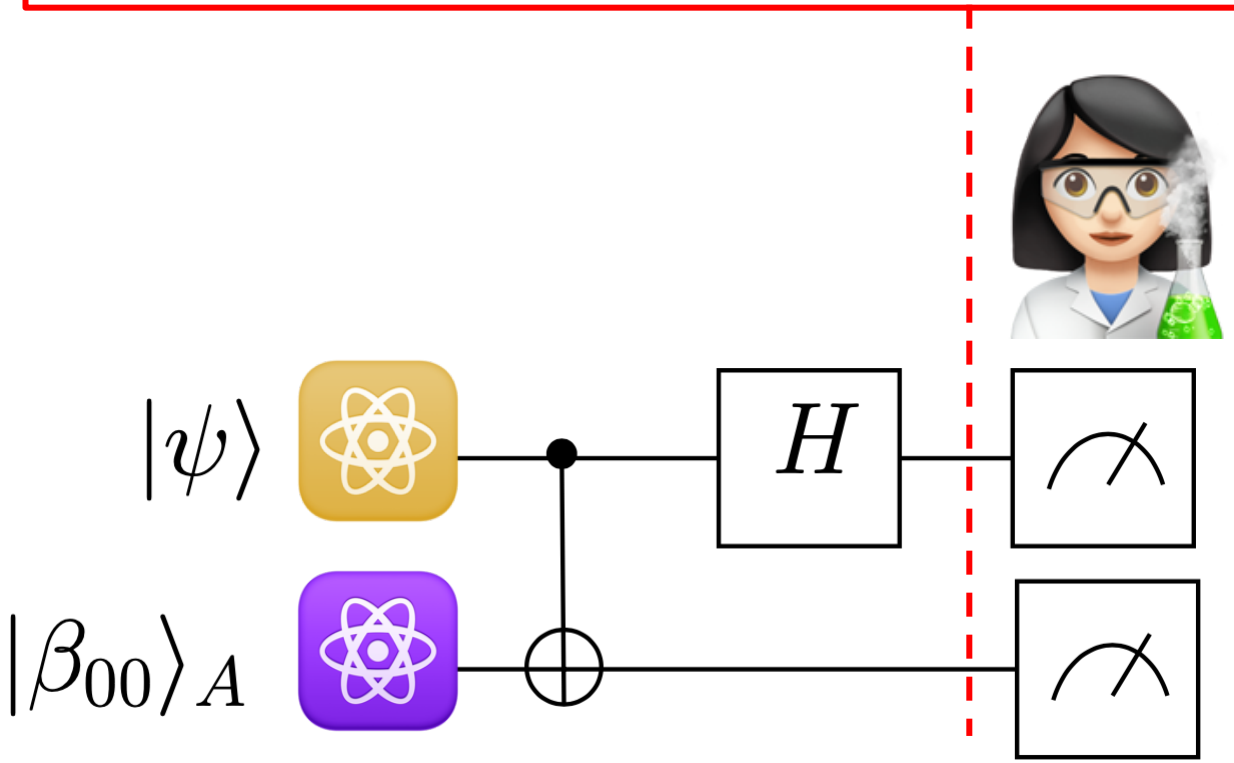
$$|\phi_2\rangle = \frac{1}{2}\Big[|00\rangle\Big(\alpha|0\rangle+\beta|1\rangle\Big)+|01\rangle\Big(\alpha|1\rangle+\beta|0\rangle\Big)+|10\rangle\Big(\alpha|0\rangle-\beta|1\rangle\Big)+|11\rangle\Big(\alpha|1\rangle-\beta|0\rangle\Big)\Big]$$

The Hadamard had an entangling effect on Bob's qubit!

STEP 2: MEASUREMENT

Now, Alice performs a measurement on her qubits, causing them to collapse to classical bit states.

$$|\phi_2\rangle = \frac{1}{2}\Big[|00\rangle\Big(\alpha|0\rangle+\beta|1\rangle\Big)+|01\rangle\Big(\alpha|1\rangle+\beta|0\rangle\Big)+|10\rangle\Big(\alpha|0\rangle-\beta|1\rangle\Big)+|11\rangle\Big(\alpha|1\rangle-\beta|0\rangle\Big)\Big]$$



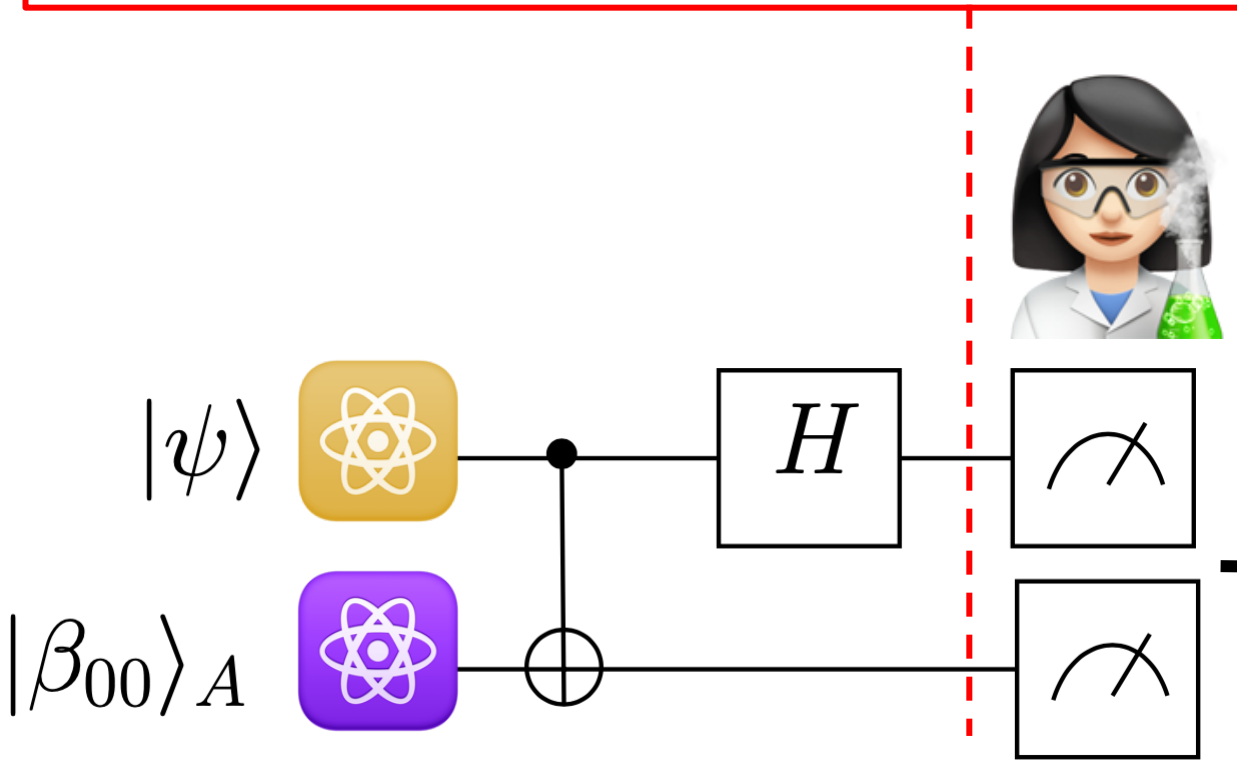
What are Alice's possible classical bit states?

Depending on what Alice measures, can we tell what state Bob's qubit will be in?

STEP 2: MEASUREMENT

Now, Alice performs a measurement on her qubits, causing them to collapse to classical bit states.

$$|\phi_2\rangle = \frac{1}{2}\Big[|00\rangle\Big(\alpha|0\rangle+\beta|1\rangle\Big)+|01\rangle\Big(\alpha|1\rangle+\beta|0\rangle\Big)+|10\rangle\Big(\alpha|0\rangle-\beta|1\rangle\Big)+|11\rangle\Big(\alpha|1\rangle-\beta|0\rangle\Big)\Big]$$



Alice	Bob
$\|00\rangle$	$\alpha\|0\rangle + \beta\|1\rangle$
$\|01\rangle$	$\alpha\|1\rangle + \beta\|0\rangle$
$\|10\rangle$	$\alpha\|0\rangle - \beta\|1\rangle$
$\|11\rangle$	$\alpha\|1\rangle - \beta\|0\rangle$

STEP 2: MEASUREMENT

Alice	Bob
$\|00\rangle$	$\alpha\|0\rangle + \beta\|1\rangle$
$\|01\rangle$	$\alpha\|1\rangle + \beta\|0\rangle$
$\|10\rangle$	$\alpha\|0\rangle - \beta\|1\rangle$
$\|11\rangle$	$\alpha\|1\rangle - \beta\|0\rangle$

If Alice measures the state $|00\rangle$, Bob's qubit is guaranteed to be in the desired state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$!

If Alice measures the state $|00\rangle$, Bob will need to perform operations on his qubit to achieve the desired state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ ...

QUANTUM PRACTICE TIME!

What quantum operations does Bob need to perform on the following states to transform each to the desired $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ state?

(1) $\alpha|1\rangle + \beta|0\rangle$

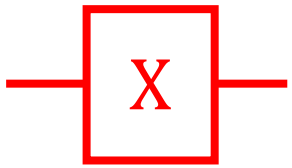
(2) $\alpha|0\rangle - \beta|1\rangle$

(3) $\alpha|1\rangle - \beta|0\rangle$

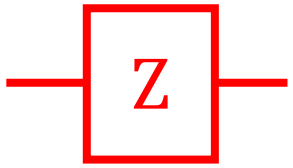
QUANTUM PRACTICE SOLUTIONS!

What quantum operations does Bob need to perform on the following states to transform each to the desired $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ state?

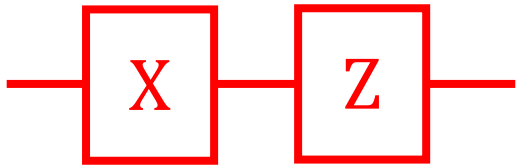
(1) $\alpha|1\rangle + \beta|0\rangle$



(2) $\alpha|0\rangle - \beta|1\rangle$

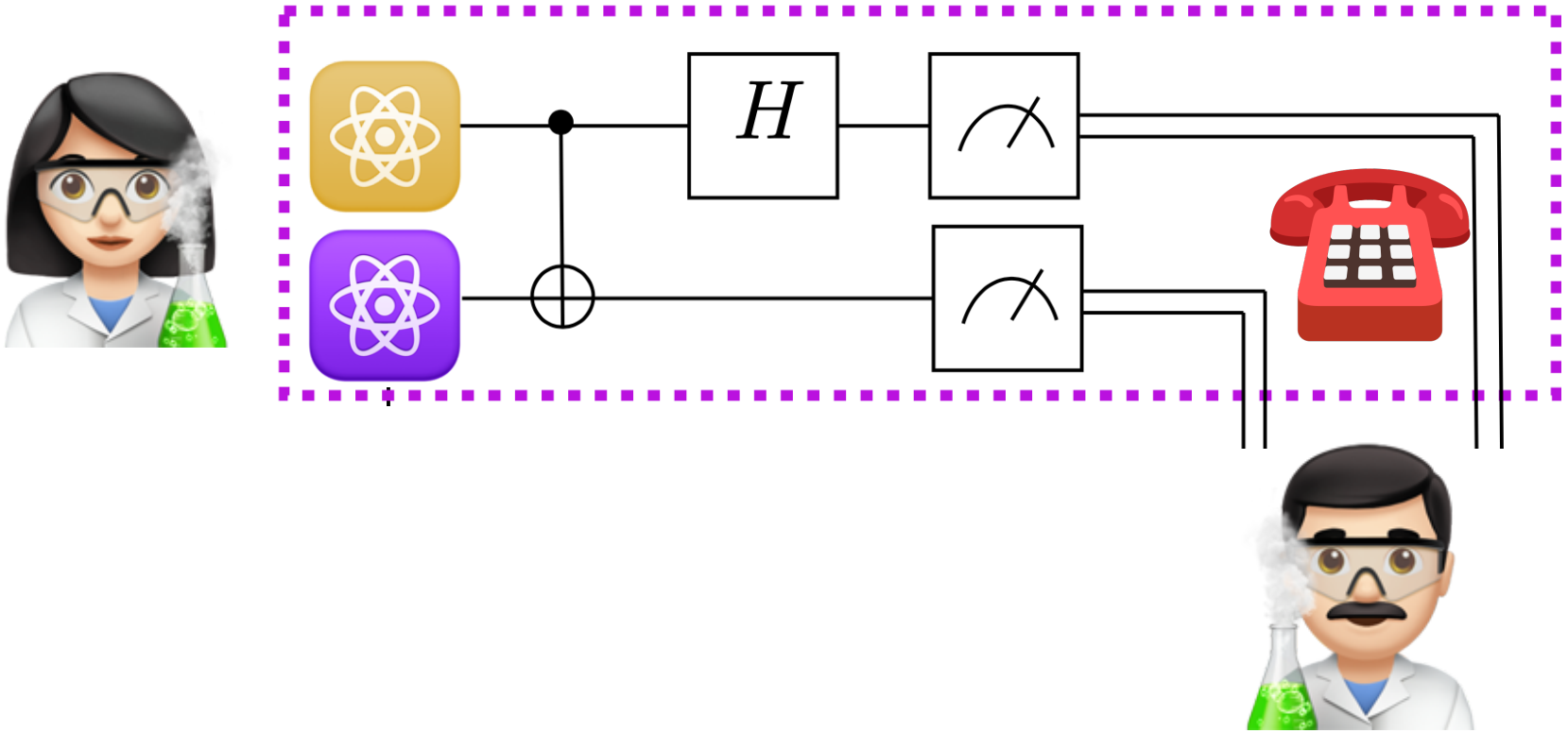


(3) $\alpha|1\rangle - \beta|0\rangle$



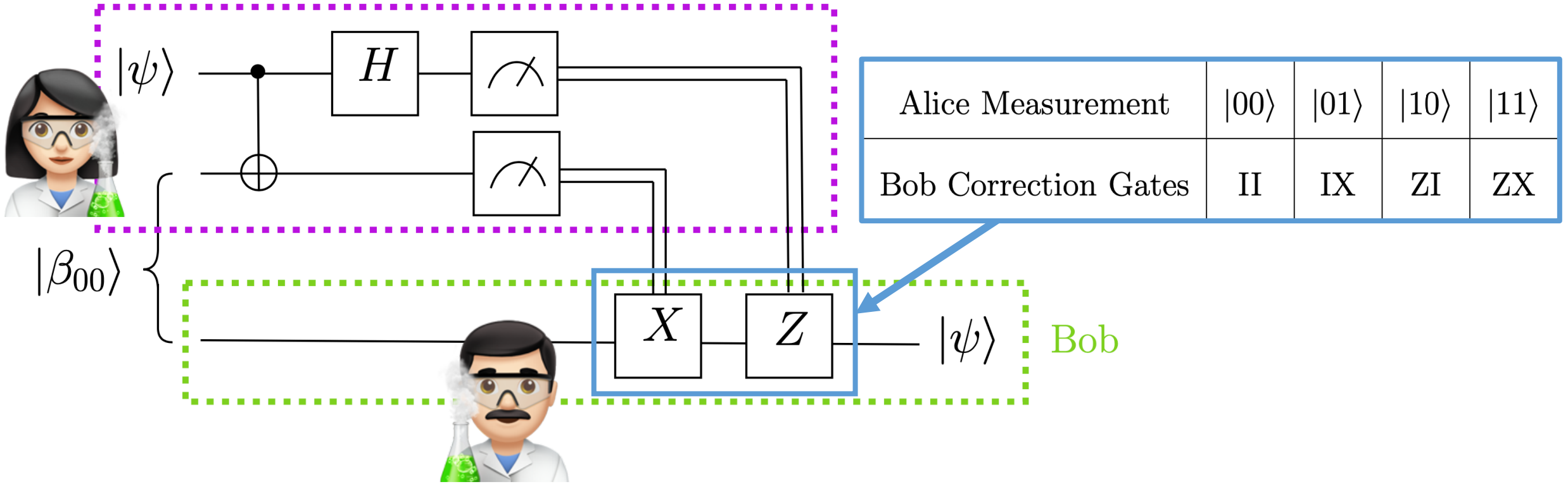
STEP 3: CLASSICAL TRANSMISSION

For Bob to correct his qubit state to $|\psi\rangle$, he needs to know the measurement results of Alice's qubits. Thus, Alice must transmit her measurement results via a classical communication channel.



STEP 4: RECEIVER CORRECTION

Dependent on the classical bit states communicated by Alice, Bob will need to perform two quantum operations from the set $\{I, X, Z\}$, to correct his qubit to state $|\psi\rangle$



Alice Measurement	$\|00\rangle$	$\|01\rangle$	$\|10\rangle$	$\|11\rangle$
Bob Correction Gates	II	IX	ZI	ZX

QUANTUM TELEPORTATION

(0) Input, (1) Entangling, (2) Measurement, (3) Classical Transmission, (4) Receiver Corrections