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BOOK REVIEW

Semigroups, Automata and Languages

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This book contains 24 papers presented at the conference "Semigroups, Automata and Languages" held at the Faculty of Sciences of the University of Porto (Portugal) on June 20–24, 1994. This represents about one half of the talks given at the conference, and about one third of the number of participants.

In the past fifty years the interactions between semigroups, automata and languages have been increasing in such a way that for many papers it is now difficult to tell if the main theme belongs to one or the other of these three areas of mathematics and computer science. Among the many results that have promoted these interactions, I shall only mention three that have been crucial. The reader will certainly perceive from my analyses of most of the papers contained in the book the influence – direct or indirect – of these results:

i) Krohn-Rhodes' Decomposition Theorem (1965). Any finite semigroup S divides a wreath-product $A_0 \circ G_1 \circ A_1 \circ \cdots \circ A_{n-1} \circ G_n \circ A_n$ of finite groups G_i and finite aperiodic semigroups A_i. In fact each A_i can be taken to be U_3, the 2-element right zero semigroup with an adjoined identity, or a semigroup in the pseudovariety generated by U_3.

ii) Schützenberger's Theorem (1965). A rational language L is star-free if and only if the syntactic monoid of L is aperiodic (i. e. with trivial subgroups).

iii) Eilenberg's Theorem (1974) establishing a one-one correspondence, via syntactic semigroups, between "varieties" of rational languages and pseudo-varieties (i. e. classes closed under taking subsemigroups, quotients, and *finite* direct products).

Due to the fact that the papers are presented in the alphabetical order of their authors' names some readers may have some difficulty in getting a global picture of developments in specific areas. Thus I shall distribute my reviews in the three areas of the title of the book, although, as I mentioned above, many papers may belong to several of these areas.

For undefined terminology we refer the reader to the standard textbooks listed in the references.

Semigroups

One class of semigroups that has been the object of intense investigations since the very beginnings of the history of semigroups is the class **RS** of regular semigroups. Some subclasses of **RS**, like inverse semigroups or unions of groups, are amenable to a variety approach à la Birkhoff by introducing a unary operation $x \to x^{-1}$ in addition to their binary operation. Such an approach for studying the full class **RS** has seen somewhat limited results. In 1989–90, T. E. Hall and independently J. Kadourek–M. Szendrei introduced the concept of existence variety for regular semigroups. This is a class of regular semigroups closed under direct products, quotients, and the formation of *regular* subsemigroups. The first paper to read on this subject in the book is by P. G. Trotter. It contains a very complete survey of results on e-varieties up to 1994, with an emphasis on results on semidirect products of e-varieties obtained jointly with P. R. Jones. One sign that e-varieties are indeed an efficient approach to a study of **RS** is the possibility of creating the analog of the identities and the free objects of Birkhoff's theory for important e-varieties.

The topic of decomposition of individual semigroups in **RS** into semi-direct products is surveyed in a paper by Mária B. Szendrei. A regular extension of a semigroup $K \in \mathbf{RS}$ by a group G is defined as a pair (S, θ) where θ is a congruence on $S \in \mathbf{RS}$ such that S/θ is isomorphic to G and $\ker \theta$ is isomorphic to K. A general question, voluntarily left imprecise here, is the following: Is a regular extension (S, θ) of K by G embeddable into a semi-direct product of a semigroup related to K (from an e-varietal point of view) and a group? A review of results on this type of problem in Szendrei's paper is followed by the presentation of more advanced results (obtained jointly with G. M. S. Gomes) on the embedding of regular extensions of semilattices by inverse semigroups.

The collection of all e-varieties forms a complete lattice $\mathcal{L}_e$ under inclusion. A new method based on the notion of divisor systems for **RS** allows K. Auinger to construct complete congruences on $\mathcal{L}_e$ giving, among other properties, a unified approach to former congruences on $\mathcal{L}_e$ found by N. Reilly and S. Zhang.

Closing on regular semigroups, I mention the paper of F. Otto with a very different flavor than the preceding papers, discussing the following problems: Given a finite presentation $\langle A; R \rangle$ of a monoid M, is M a regular monoid? Does $w \in A^*$ represent a regular element of M? Does M contain a non-trivial regular element? All these problems are undecidable in general, and Otto's paper presents decidability results for restricted classes of finite presentations.

There are three papers on (pseudo)varieties of (finite) semigroups. M. Zeitoun surveys recent results on the membership problem and the finite basis problem for the joint of two pseudovarieties of finite semigroups. In his paper M. V. Volkov reviews the main results and problems about the cover property in the lattice of semigroup varieties, and proves that the pseudovariety EA of all finite semigroups whose idempotent generated subsemigroups are aperiodic has no cover in the lattice of all pseudovarieties of finite semigroups. V. A. Molchanov introduces new methods to study pseudovarieties of finite algebras based on nonstandard analysis. In his paper he presents the basic notions of nonstandard analysis needed for his approach, and explores the correspondence between pseudovarieties of finite Ω-algebras and congruences on certain nonstandard Ω-algebra extensions.

Other papers are in more disparate areas of semigroups than the papers reviewed above.

D. A. Bredikhin characterizes by a very simple identity those semilattice-ordered involuted monoids that are isomorphic to similar monoids of binary relations.

V. Fleischer and U. Knauer study under what conditions the monoid of all endomorphisms of a free S-act has projective principal ideals.

When trying to write every element of a semigroup Q as $a^{-1}b$ where a, b are in a subsemigroup S of Q, and a^{-1} stands for the inverse of a in a subgroup of Q one is naturally led to the concept of Q being a semigroup of left quotients of S, and S being a left order in Q. In his paper J. B. Fountain introduces equivalence of orders, and maximal orders as suggested by similar concepts in a ring theory.

By introducing left-hereditary systems of identities (roughly speaking, these are identity systems containing certain left sections of other identities in the systems) G. Mashevitzky gives a solution of the finite basis problem for identities of certain ideal extensions, and of the semigroups of all transformations of rank ≤ 2 on sets with $n \geq 5$ elements.

J. S. Ponizovskii generalizes a theorem of Y. Zalcstein on the finiteness of periodic linear semigroups of bounded period over a field of characteristic zero.

Finally G. I. Zhitomirsky shows how to construct and study sheaves from restrictive semigroups (these are bands satisfying the identity $abc = bac$) or inverse semigroups, following ideas and results of V. V. Vagner from the late 1970's.

Automata

The paper by J. F. P. Hudson studies regular rewrite systems. A rewrite system is the analog of a presentation $\langle A; R\rangle$ when the relation R is considered non-symmetric. Regular rewrite systems are rewrite systems where related pairs of words are recognized by various types of automata.

Among the most interesting papers the one by D. Giammarresi, S. Mantaci, F. Mignosi, and A. Restivo shows a deep connection between the equivalence problem of finite automata and the classical Fine-Wilf theorem [If a word w has two periods p_1, p_2 and if w is long enough then it has period $gcd(p_1, p_2)$]. The connection is achieved by an algorithmic graph construction of $\mathcal{R}_1 \vee \mathcal{R}_2$ where $\mathcal{R}_1$ and $\mathcal{R}_2$ are right congruences of finite index on A^* (A finite).

A language $L \subseteq A^*$ is called a zigzag [resp. stack] language if $uv, v, vw \in L$ imply $uvw \in L$ [resp. if L is a submonoid of A^* and $uv, v \in L$ imply $u \in L$]. These languages correspond to an extension of concatenation of words allowing to come back (or zigzaging) when concatenating, or allowing (limited) popping or pushing in a stack when concatenating. In their paper B. LeSaëc, I. Litovsky and B. Patrou study recognizability, mostly via monoids, of zigzag and stack languages.

M. Delorme and J. Mezoyer present the basic definitions and properties of cellular automata, as well as various classes of languages they recognize. Some of these classes go beyond languages in the classical Chomsky hierarchy (rational or regular, algebraic or context-free, context-sensitive), others are not comparable to them.

Languages

In their paper A. Cherubini and P. San Pietro present an introduction to k-depth grammars. These are grammars similar to context-free grammars but at every step of a sequence of derivations we are allowed k rewriting points for a single non-terminal words.

Another generalization of context-free grammars (coupled-context-free grammars) is studied by G. Hotz and G. Pitsch in a paper in which they generalize the well known representation theorem of Chomsky–Schützenberger.

When the Krohn-Rhodes decomposition theorem is applied to an aperiodic semigroup S then S divides a wreath-product $A_1 \circ A_2 \circ \cdots \circ A_n$ where each A_i is a direct product of copies of U_3 (or belongs to the pseudovariety generated by U_3. In his paper C. L. Nehaniv uses as definition of the aperiodic complexity of S the least n over all decompositions of S as above. He shows then (using as a crucial step a result of J. Almeida on the free objects in the semi-direct product of two varieties of semigroups) that aperiodic complexity is decidable, and hence (via Schützenberger's theorem) that aperiodic complexity (with the ad-hoc definition) of a star-free language is effectively computable. This paper also contains an interesting set of axioms of a complexity function for any pseudovariety.

Recall however that the first proposal to classify star-free languages (J. Brzozowski and R. Cohen, 1971) was according to their dot-depth (this is the smallest number of dots - i.e. products - needed to write a star-free language). The decidability of dot-depth is still open in general for dot-depth ≥ 2. By Eilenberg's theorem this problem is equivalent to the determination of the dot-depth (defined in terms of pseudovarieties) of a finite aperiodic monoid. In his paper D. Cowan obtains a tight approximation of the pseudovariety of inverse monoids of dot-depth $\leq n$ allowing him to recover an earlier result of his that dot-depth two is decidable for aperiodic inverse monoids.

In her paper C. de Felice reports on the difficult problem of characterizing finite codes that can be embedded into a *finite* maximal code. These are said to have finite completion. A remarkable result is that for codes of the type $a^n \cup a^P b \cup ba^Q \cup b$ where P and Q are sets of integers, the finite completion problem is related to the sets P, Q being embeddable into a factorization of $\mathbb{Z}_n$ ($P \oplus Q$ is a factorization of $\mathbb{Z}_n$ if every $z \in \mathbb{Z}_n$ can be written uniquely as $p+q$, $p \in P$, $q \in Q$). The problem of finding these factorizations due to G. Hajós, 1950, is still open.

The next two papers are on infinite words:

An ω-rational language on an alphabet A is obtained from the letters by applying a finite number of times the operations $\cup, \cdot, *$, and ω, where L^ω stands for the set of all infinite words obtained from concatenating sequences of words in L. Such languages can be recognized by finite automata (e.g. requiring for a successful infinite path to go through a terminal state infinitely many times). Some chains of states in automata recognizing ω-rational languages serve as syntactic invariants for these languages. This is the topic of the paper of O. Carton and D. Perrin.

For an infinite word $w \in A^\omega$ let $F(w)$ be the set of all its finite factors. The subword complexity of w is the function $g_w(n) = card(F(w) \cap A^n)$. The word w is called an infinite Sturmian word if $g_w(n) = n + 1$ for all $n \in \mathbb{N}$. Infinite Sturmian words obtained by an "approximating sequence" are said to be standard. Finite factors of infinite standard Sturmian word are related to the set PER of all words having two periods p, q (which are coprime) and are of length $p + q - 2$. The deep paper by A. de Luca contains an inductive construction of PER allowing, among other applications, the construction of all finite and infinite standard Sturmian words.

As these analyses show, this book contains papers in many diverse areas. Due to the fact that almost all of the papers are written in a survey mode - even when they contain complete proofs of new results - I think that the book will be useful to both advanced researchers wanting to get a quick glance at an unfamiliar area of development, and graduate students about to select a research topic for their Ph.D.s. Both will appreciate the very complete reference lists presented in many of the articles as well as the contribution this book makes to the history of the interactions of the three fields in the title.

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