

Lecture #15: Causal Inference

aka STAT109A, AC209A, CSCIE-109A

CS109A Introduction to Data Science

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Lecture Outline

- Introduction
- Simpson's Paradox
- Causal Structure
- Correlation and causation
- Causal Effects
- Randomized Control
- Adjusting for Confounders

Association vs. Causation

In many of our methods (regression, for example) we often want to measure the association between two variables: the response, Y , and the predictor, X . For example, this association is modeled by a β coefficient in regression, or amount of increase in R^2 in a regression tree associated with a predictor, etc...

If β is *significantly different* from zero (or amount of R^2 is greater than by chance alone), then there is evidence that the response is associated with the predictor.

How can we determine if β is *significantly different* from zero in a model?

Association vs. Causation (cont.)

But what can we say about a **causal association**? That is, can we manipulate X in order to influence Y ?

Not necessarily. Why not?

There is potential for **confounding factors** to be the driving force for the observed association.

Controlling for confounding

How can we fix this issue of confounding variables?

There are 2 main approaches:

1. Model all possible confounders by including them into the model (multiple regression, for example). Or use sophisticated **'causal methods'** to account for the confounders.
2. A **randomized experiment** can be performed where the scientist manipulates the levels of the predictor (now called the **treatment**) to see how this leads to changes in the response.

What are the advantages and disadvantages of each approach?

Controlling for confounding: advantages/disadvantages

1. Modeling the confounders:
 - **Advantages:** cheap
 - **Disadvantages:** not all confounders may be measured.
2. Performing an experiment:
 - **Advantages:** confounders will be **balanced**, on average, across treatment groups
 - **Disadvantages:** expensive, can be an artificial environment

What is Causal Inference?

Options

- A. A special type of boosting method?
- B. The casual way of doing modeling?
- C. Finding the relationships between predictors and response variables
- D. Inferring the effects of any treatment/policy/intervention/etc

What is Causal Inference?

Options

- A. A special type of boosting method?
- B. The casual way of doing modeling?
- C. Finding the relationships between predictors and response variables
- D. Inferring the effects of any treatment/policy/intervention/etc

Which of these are examples of causal inference?

Options

- A. Effect of treatment on a disease
- B. Effect of social media on mental health
- C. Effect of climate change policy on emissions
- D. Effect of going to labs and lectures on final grade
- E. All of the above

Which of these are examples of causal inference?

Options

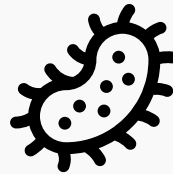
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Simpson's paradox

Example: PyCA-109a



(Python Coding Anxiety)

Treatment T: A (Raderzole) and B (Gumboxin)

Condition Severity C: mild (0) or severe (1)

Outcome Y: alive (0) or dead (1)



Simpson's paradox

Overall Mortality Rate Table:

Treatment	Total
A (Raderzole)	19% 190/1000
B (Gumboxin)	20% 100/500

$$\mathbb{E}[Y|T]$$

Which treatment is best?

Options

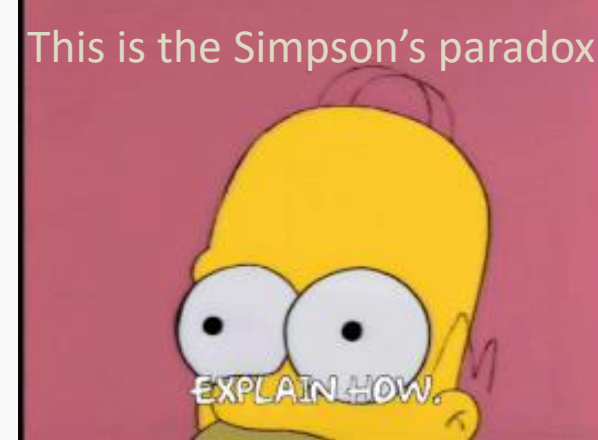
- A. Raderzole because a smaller percentage of the people died
- B. Gumboxin because fewer people died
- C. This is an imbalanced dataset, I need more info
- D. What about the severity conditions of the people who took Raderzole or Gumboxin?

Which treatment is best?

Options

- A. Raderzole because a smaller percentage of the people died
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Simpson's paradox



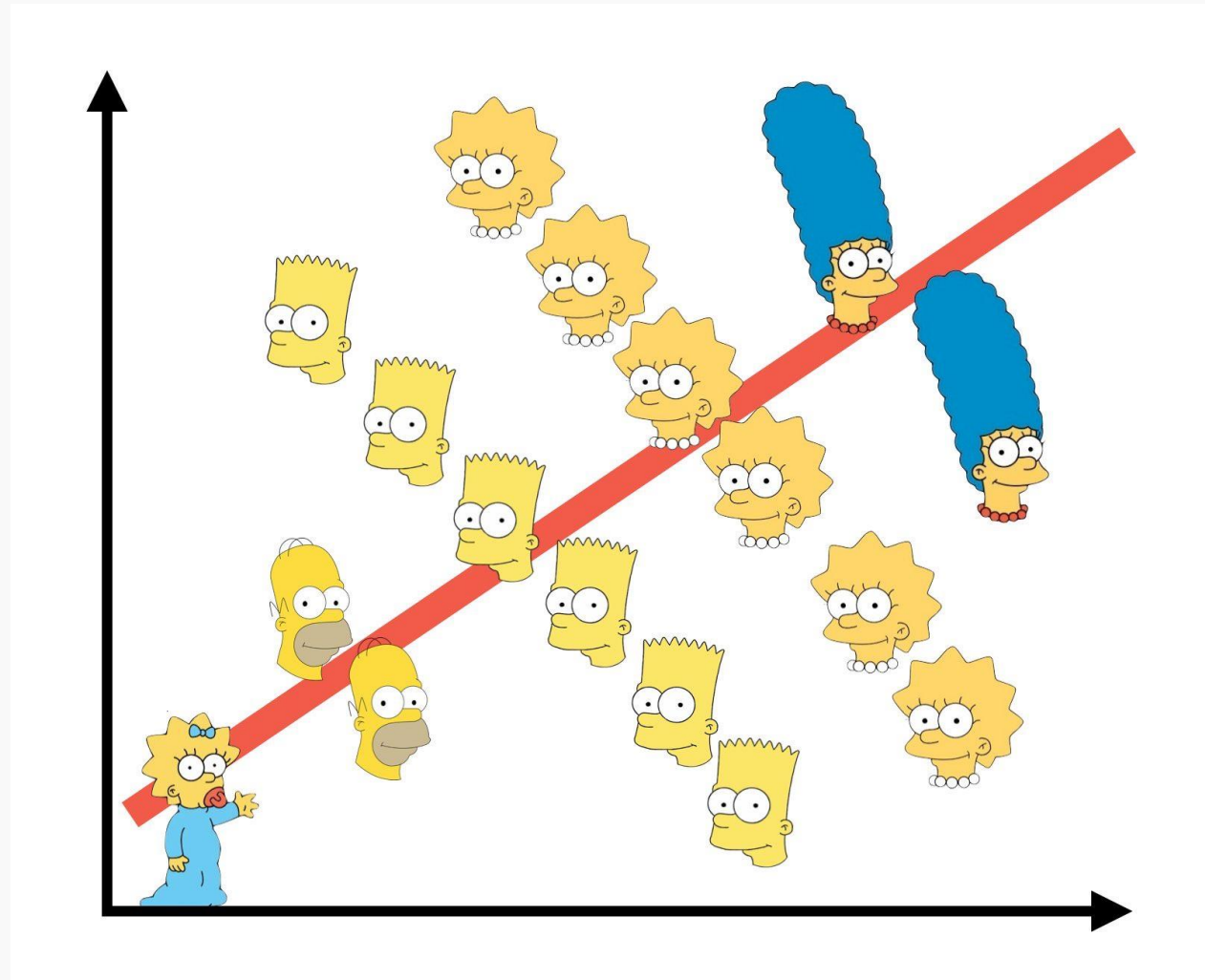
Mortality Rate Table

Treatment	Condition Severity		Total
	Mild	Severe	
A (Raderzole)	17% 150/900	40% 40/100	19% 190/1000
B (Gumboxin)	14% 50/350	33% 50/150	20% 100/500

$$\mathbb{E}[Y|T, C = 0] \quad \mathbb{E}[Y|T, C = 1] \quad \mathbb{E}[Y|T]$$

Simpson's paradox

Test Scores



Chores (physical exertion that day)

Which treatment should we choose?

Options

- A. Raderzole
- B. Gumboxin
- C. It may still depend on the conditions

Which treatment should we choose?

Options

A. Raderzole

B. Gumboxin

C. It may still depend on the conditions

Simpson's paradox

Why might it still depend on the condition?

	Condition Severity		
Treatment	Mild	Severe	Total
A (Raderzole)	17% 150/900	40% 40/100	19% 190/1000
B (Gumboxin)	14% 50/350	33% 50/150	20% 100/500

$$\mathbb{E}[Y|T, C = 0] \quad \mathbb{E}[Y|T, C = 1] \quad \mathbb{E}[Y|T]$$

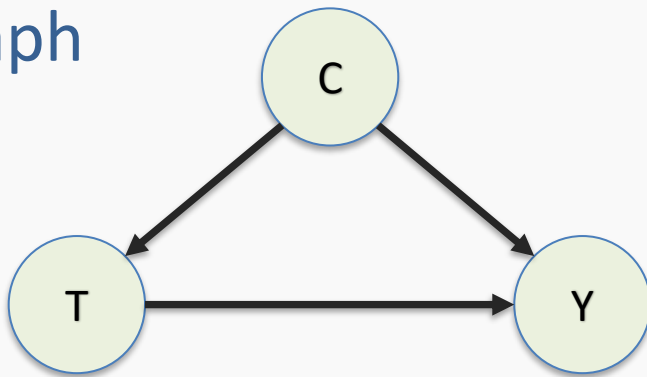
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Causal Structure

	Condition Severity		
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Causal Graph

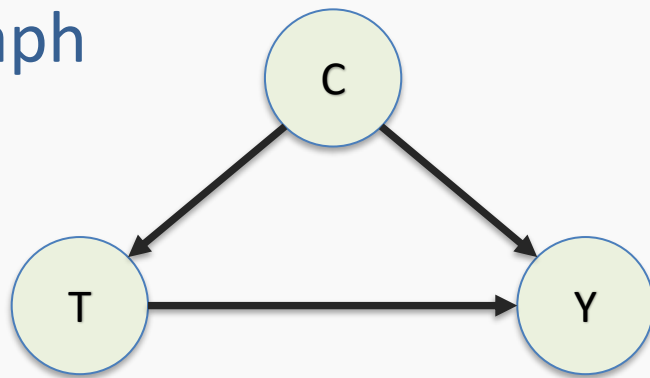


- Condition Severity is a cause of the treatment (leads to a different treatment).
- Condition Severity and Treatment are both causes of the outcome

Causal Structure

	Condition Severity		
Treatment	Mild	Severe	Total
A (Raderzole)	17% 150/900	40% 40/100	19% 190/1000
B (Gumboxin)	14% 50/350	33% 50/150	20% 100/500

Causal Graph



- Condition Severity is a cause of the treatment



Mild conditions



Visit doctor PPPP

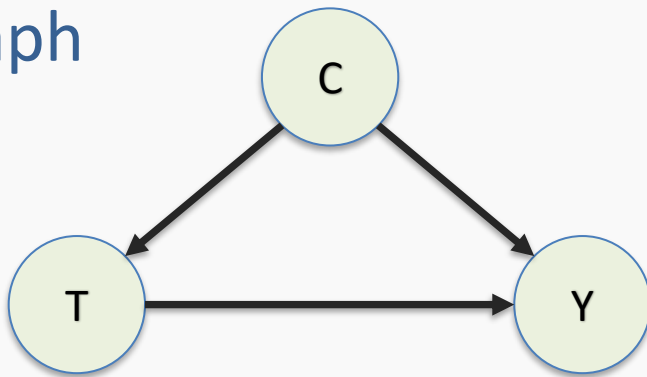


Treatment A (Raderzole) –
doctor wants to save
treatment B (Gumboxin) for
more severe conditions

Causal Structure

	Condition Severity		
Treatment	Mild	Severe	Total
A (Raderzole)	17% 150/ 900	40% 40/100	19% 190/1000
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Causal Graph



- Condition Severity is a cause of the treatment



Mild conditions



Visit doctor PPPP

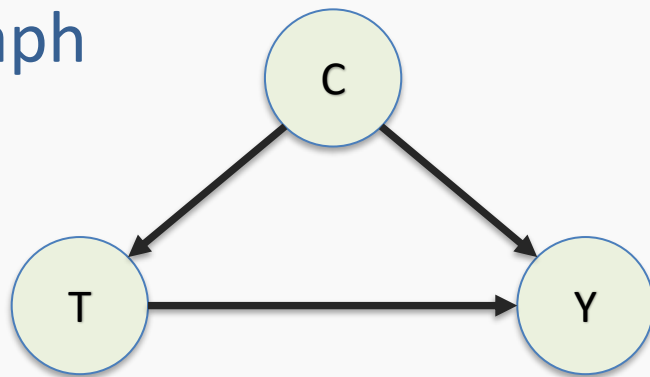


Treatment A (Raderzole) –
doctor wants to save
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Causal Structure

	Condition Severity		
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Causal Graph



- Condition Severity is a cause of the treatment



Severe conditions



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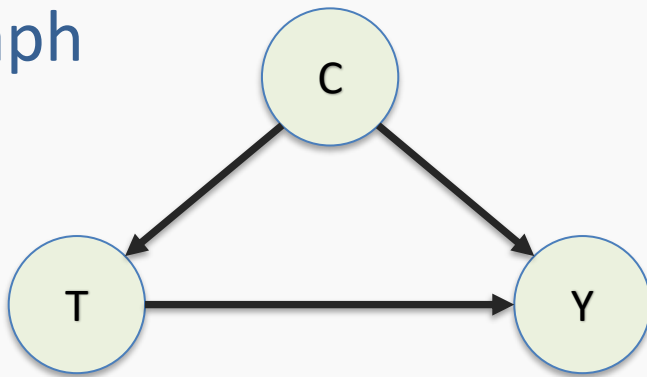
B

Treatment B (Gumboxin) –
doctor knows Gumboxin is
more effective

Causal Structure

	Condition Severity		
Treatment	Mild	Severe	Total
A (Raderzole)	17% 150/900	40% 40/ 100	19% 190/1000
B (Gumboxin)	14% 50/350	33% 50/ 150	20% 100/500

Causal Graph



- Condition Severity is a cause of the treatment



Severe conditions



Visit doctor PPPP



Treatment B (Gumboxin) –
doctor knows Gumboxin is
more effective

Which treatment should we choose?

Options

- A. Treatment A (Raderzole)
- B. Treatment B (Gumboxin)

Which treatment should we choose?

Options

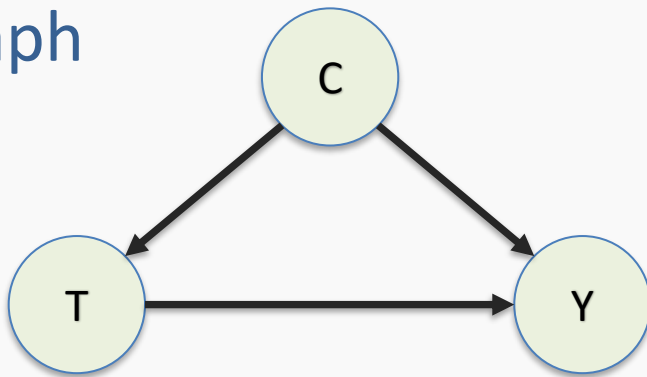
A. Treatment A (Raderzole)

B. Treatment B (Gumboxin) -- because the subgroups give lower mortality rate

Causal Structure

	Condition Severity		
Treatment	Mild	Severe	Total
A (Raderzole)	17% 150/900	40% 40/100	19% 190/1000
B (Gumboxin)	14% 50/350	33% 50/150	20% 100/500

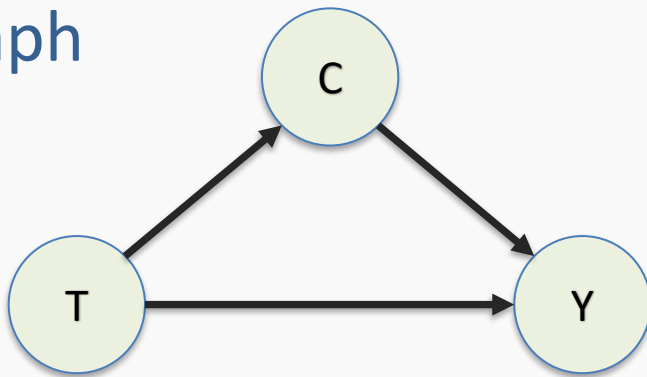
Causal Graph



Causal Structure

	Condition Severity		
Treatment	Mild	Severe	Total
A (Raderzole)	17% 150/900	40% 40/100	19% 190/1000
B (Gumboxin)	14% 50/350	33% 50/150	20% 100/500

Causal Graph



- Treatment is a cause of the Condition Severity



Visit doctor PPPP



B



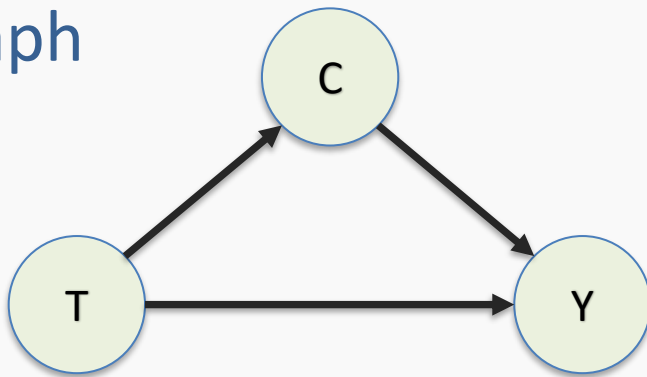
Treatment B (Gumboxin).
Takes long to get and while
waiting conditions become
severe

Conditions become severe

Causal Structure

	Condition		
Treatment	Mild	Severe	Total
A (Raderzole)	17% 150/900	40% 40/100	19% 190/1000
B (Gumboxin)	14% 50/350	33% 50/150	20% 100/500

Causal Graph



- Treatment is a cause of the Condition Severity



Visit doctor



Treatment A (Raderzole). No wait time



Conditions never becomes severe, remains mild

Treatment causes people to have mild or severe conditions

Which treatment should we choose?

Options

- A. Treatment A (Raderzole)
- B. Treatment B (Gumboxin)

Which treatment should we choose?

Options

- A. Treatment A (Raderzole) because B (Gumboxin) makes you have severe conditions
- B. Treatment B (Gumboxin)

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- **Correlation and causation**
- Causal Effects
- Randomized Control
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Correlation does not imply causation

Example:

People who sleep fully dressed wake up with headaches

Correlation does not imply causation

Example:

Sleeping fully dressed correlates with waking up with headaches

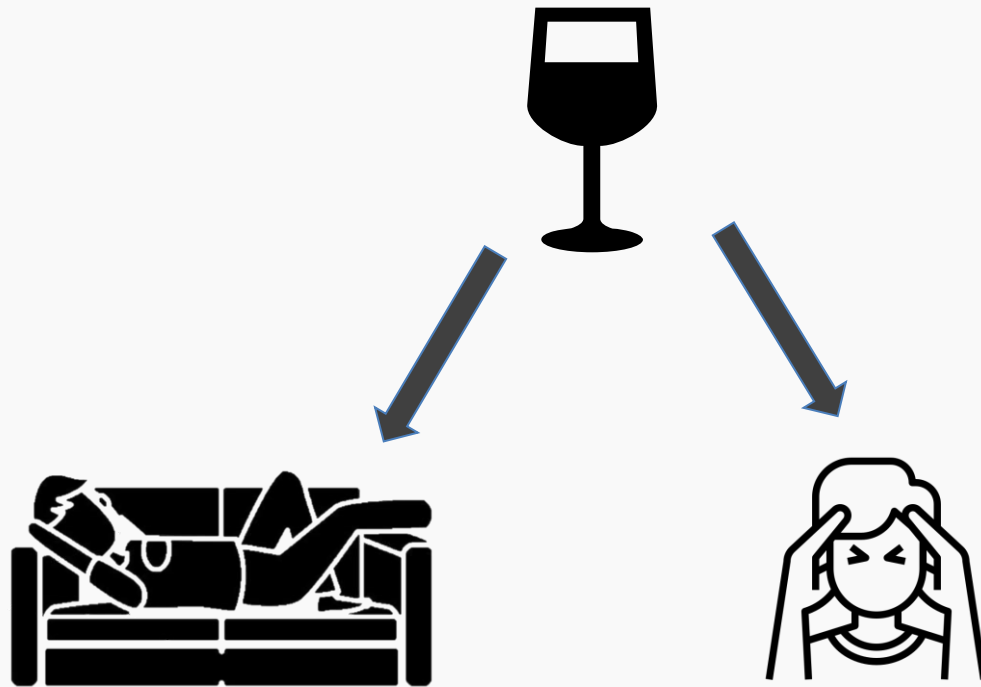


Correlation does not imply causation

Example:

Sleeping fully dressed correlates with waking up with headaches

Common cause: drinking the night before



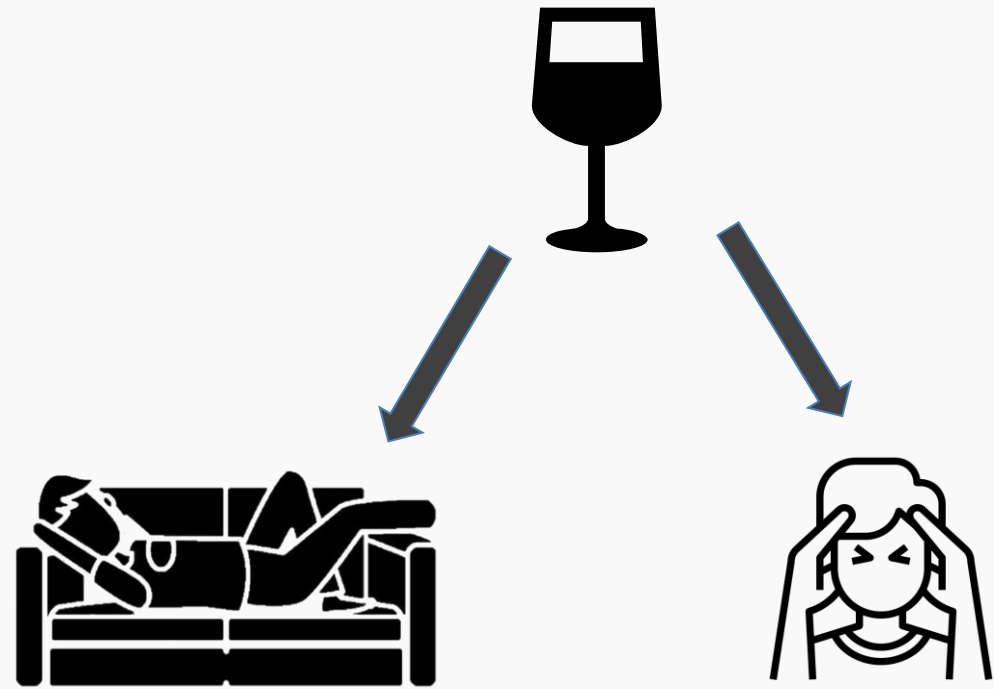
Example:

Sleeping fully dressed correlates with waking up with headaches

Common cause: drinking the night before

Two type of people in this world:

1. Dressed up sleepers
2. Non-dressed up sleepers



Sleeping fully dressed correlates with waking up with headaches

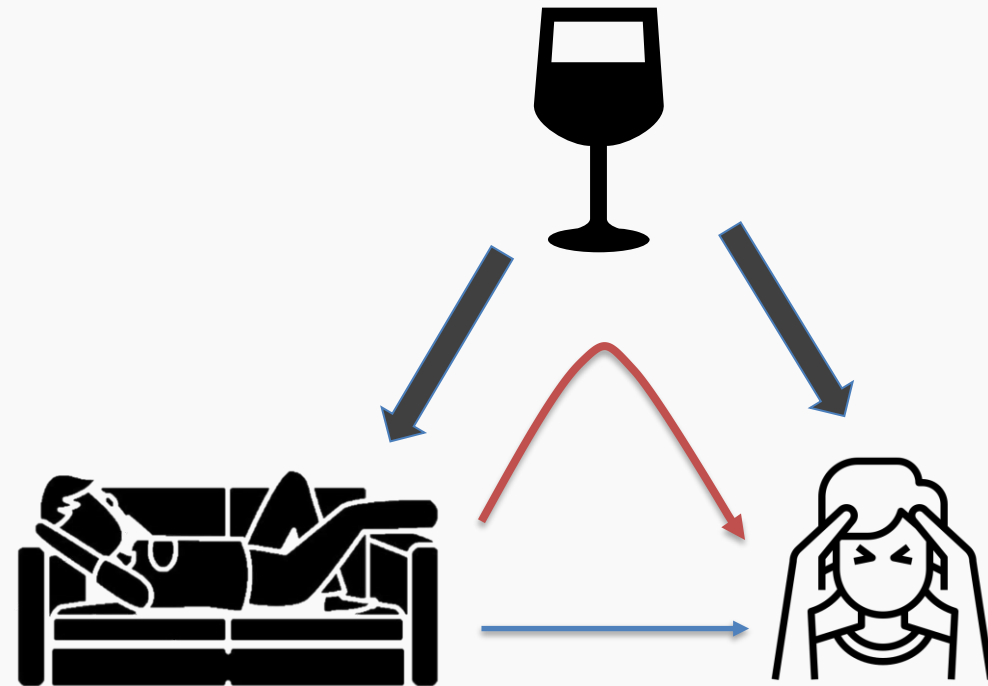
Common cause: drinking the night before

Two type of people in this world:

1. Dressed up sleepers
2. Non-dressed up sleepers

Confounding Association ->

Causal Association ->



What is association and what is correlation?

Options

- A. They are the same describing relationship of two variables
- B. Association is a statistical dependance where correlation is ML dependance
- C. Association is a statistical dependance and correlation a linear statistical dependence
- D. Association is a statistical dependance and correlation a linear statistical dependence but we used them interchangeably

What is association and what is correlation?

Options

- A. They are the same describing relationship of two variables
- B. Association is a statistical dependance where correlation is ML dependance
- C. Association is a statistical dependance and correlation a linear statistical dependence (aka, direction)
- D. Association is a statistical dependance and correlation a linear statistical dependence but we used them interchangeably

Sleeping fully dressed correlates with waking up with headaches

Common cause: drinking the night before

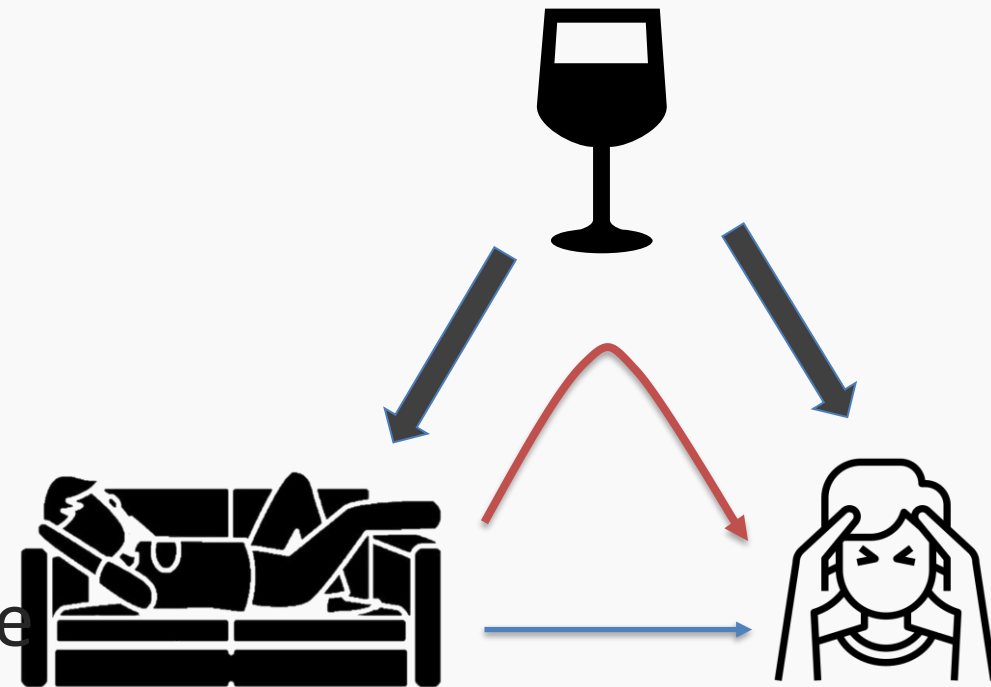
Two type of people in this world:

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Confounding Association ->

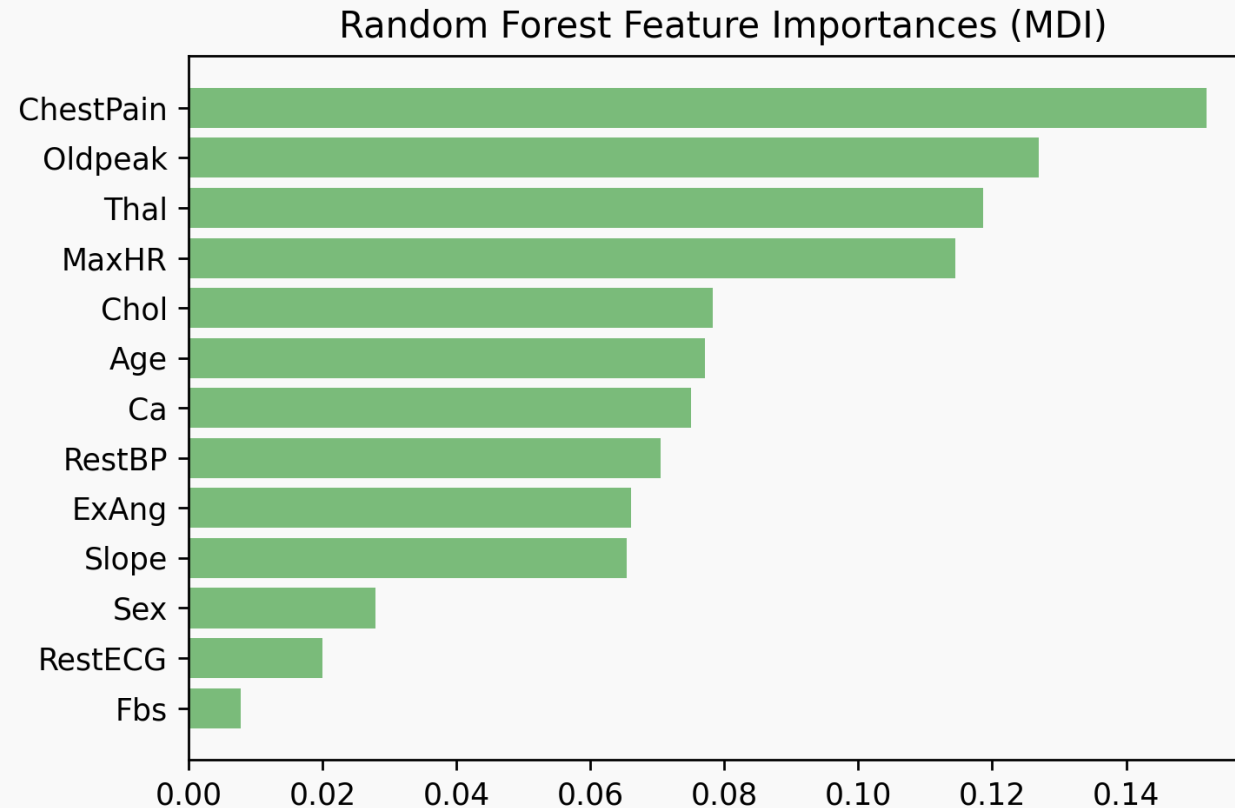
Causal Association ->

If we measure correlation, we see a mixture
of causal and confounding



Correlation = causation is something we do all the time

Sometimes because we have in our minds, **cognitive bias**



Correlation = causation is something we do all the time

Sometimes because it is convenient:

If correlation does not imply causation, then what does imply causation

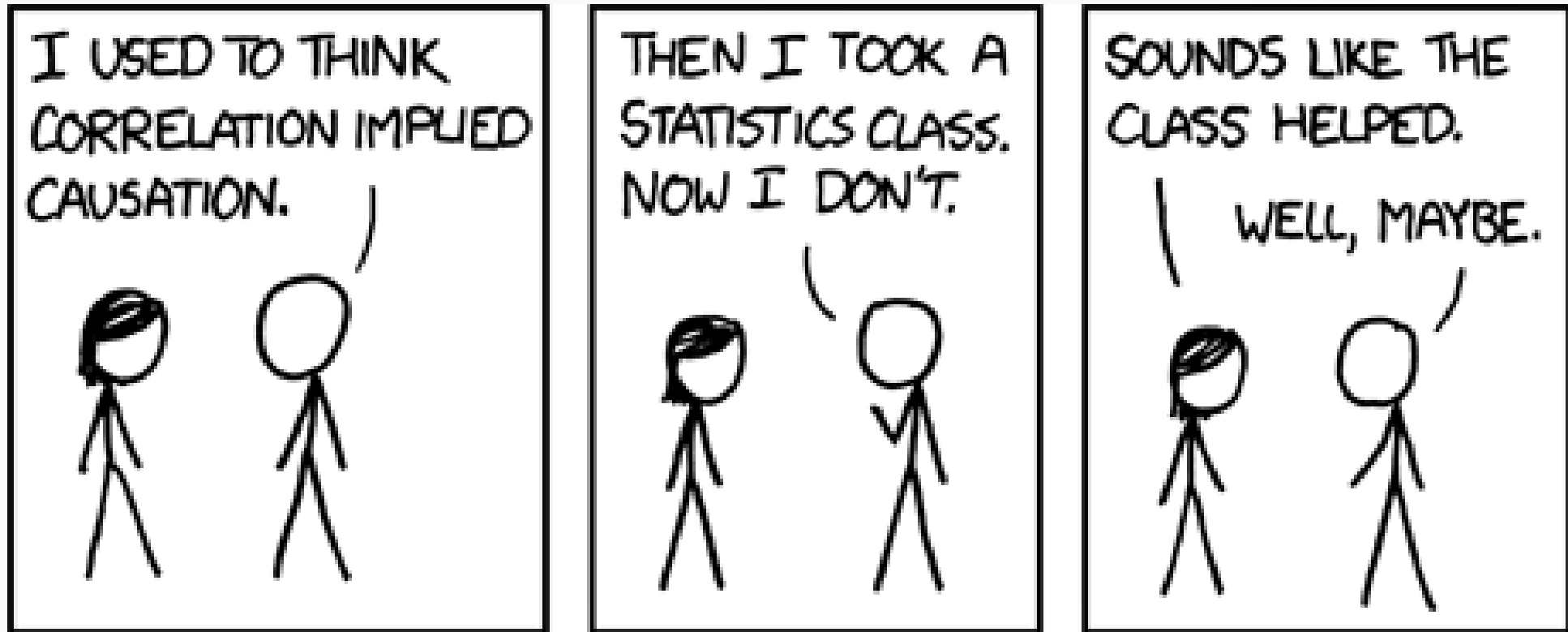
Options

- A. Anything with correlation above 0.9 is a causation
- B. Any non-linear correlation
- C. To know if something causes something else

If correlation does not imply causation, then what does imply causation

Options

- A. Anything with correlation above 0.9 is a causation
- B. Any non-linear correlation
- C. To know if something causes something else. We need to figure out, how to identify and measure it

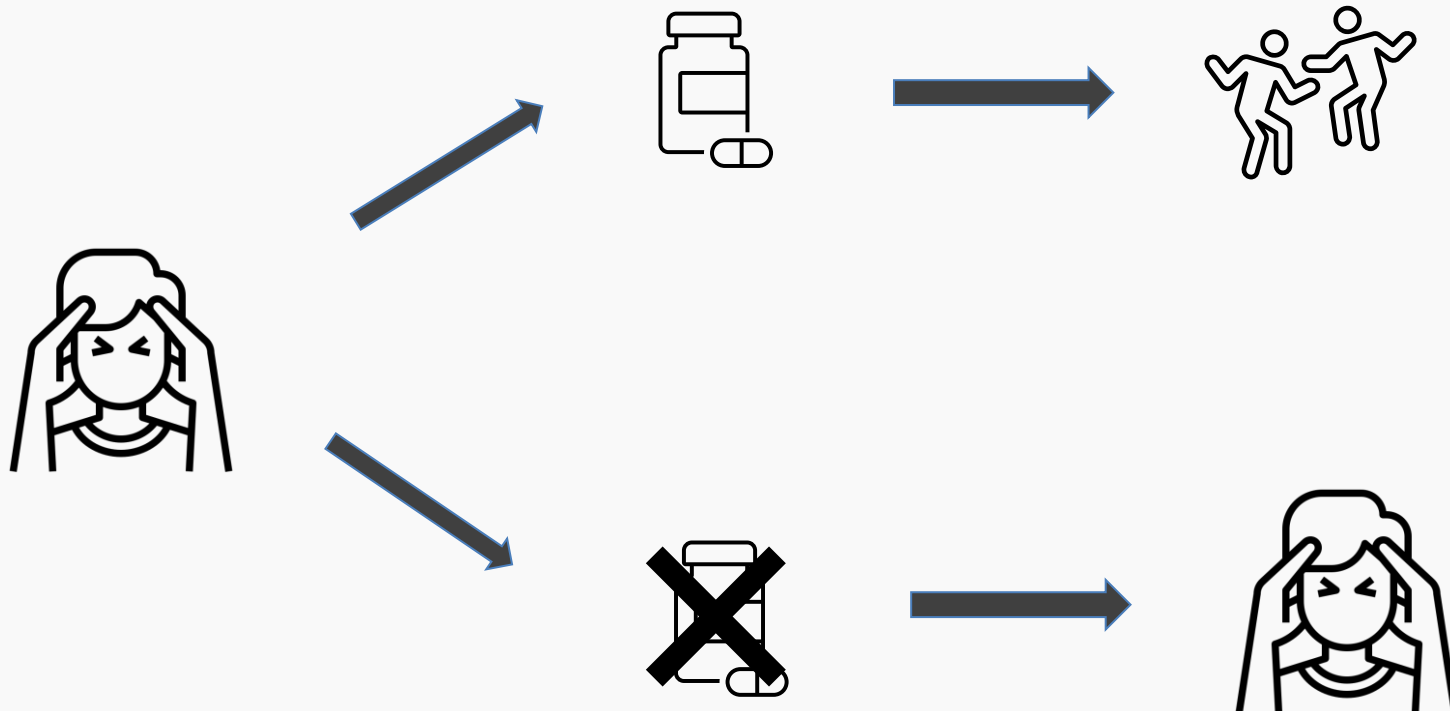


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- Causal Structure
- Correlation and causation
- **Causal Effects (formalism)**
- Randomized Control

Potential outcomes

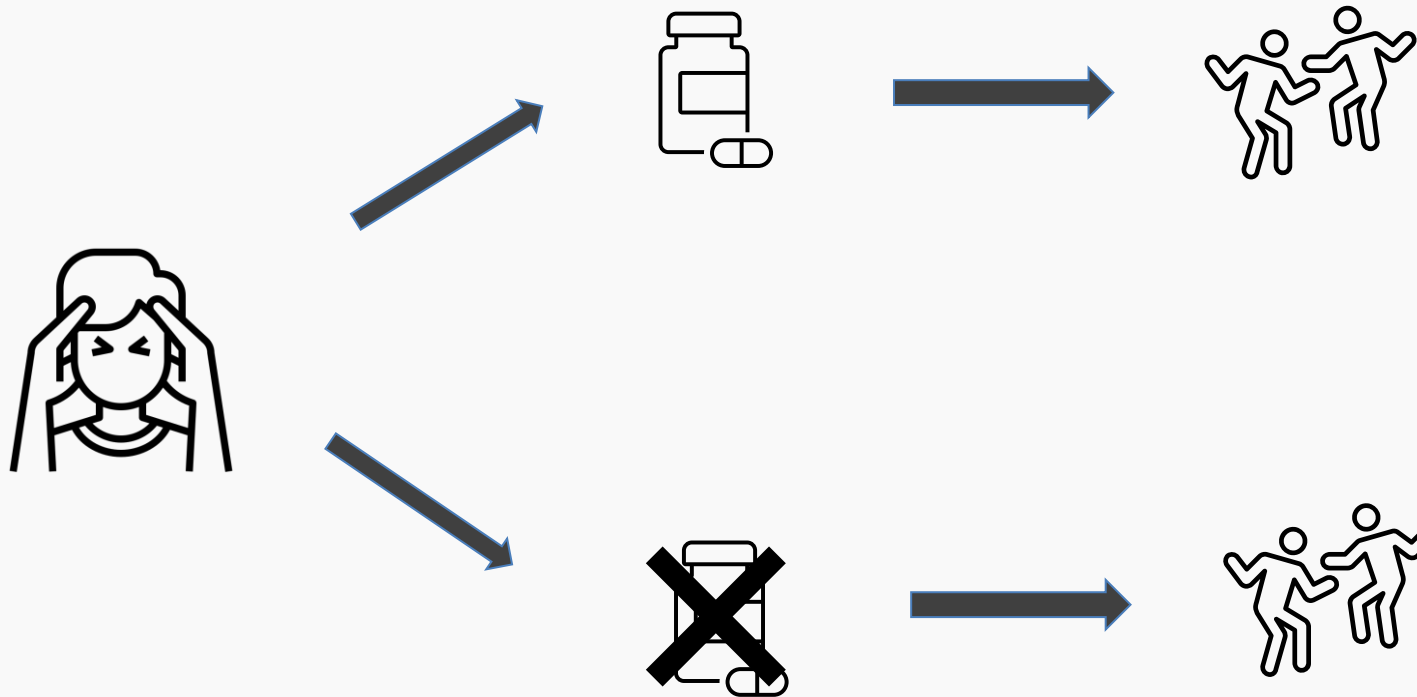
Inferring the effect of treatment on some outcome



Causal effect

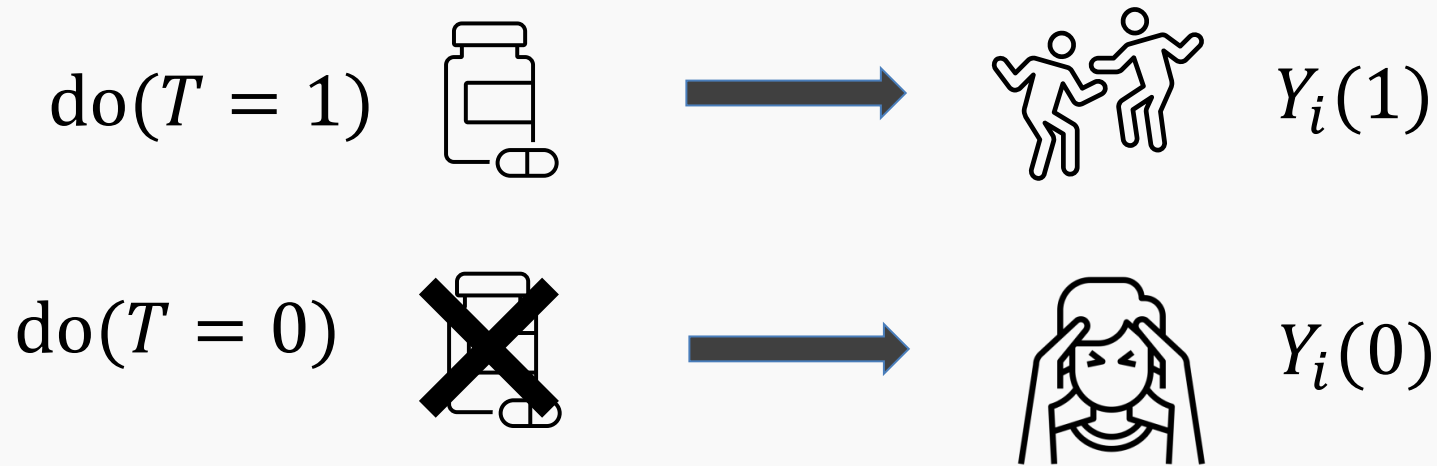
Potential outcomes

Inferring the effect of treatment on some outcome



No causal effect

Some notation

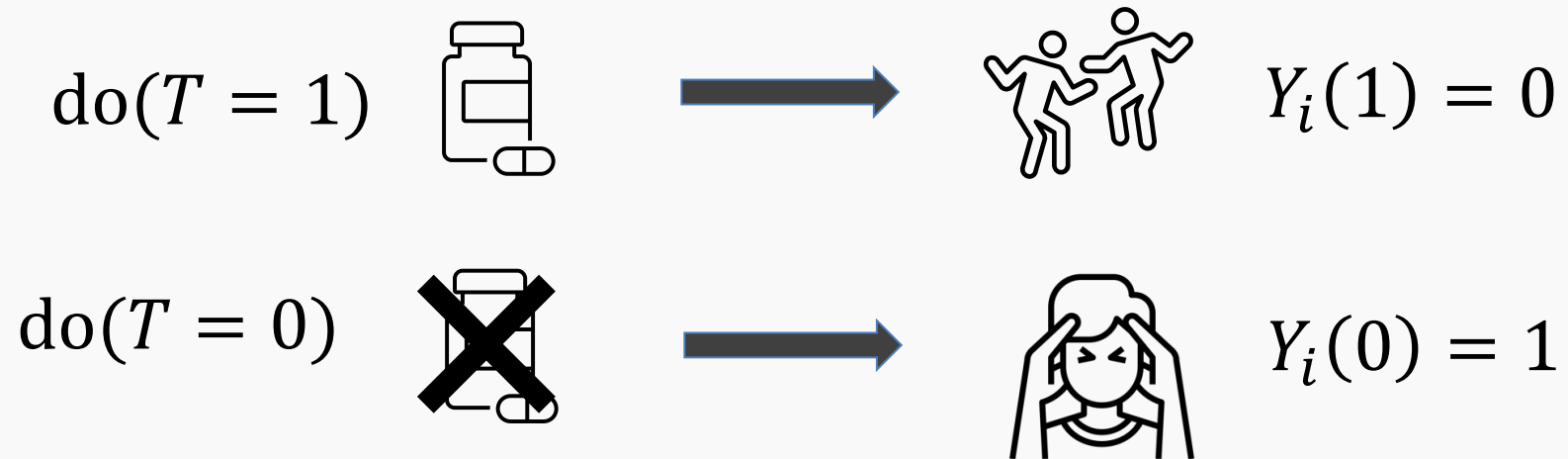


T : treatment
 Y : observed outcome
 i : denotes individual/observation
 $Y_i(T)$: potential outcome under treatment T

Causal effect

$$Y_i(1) - Y_i(0)$$

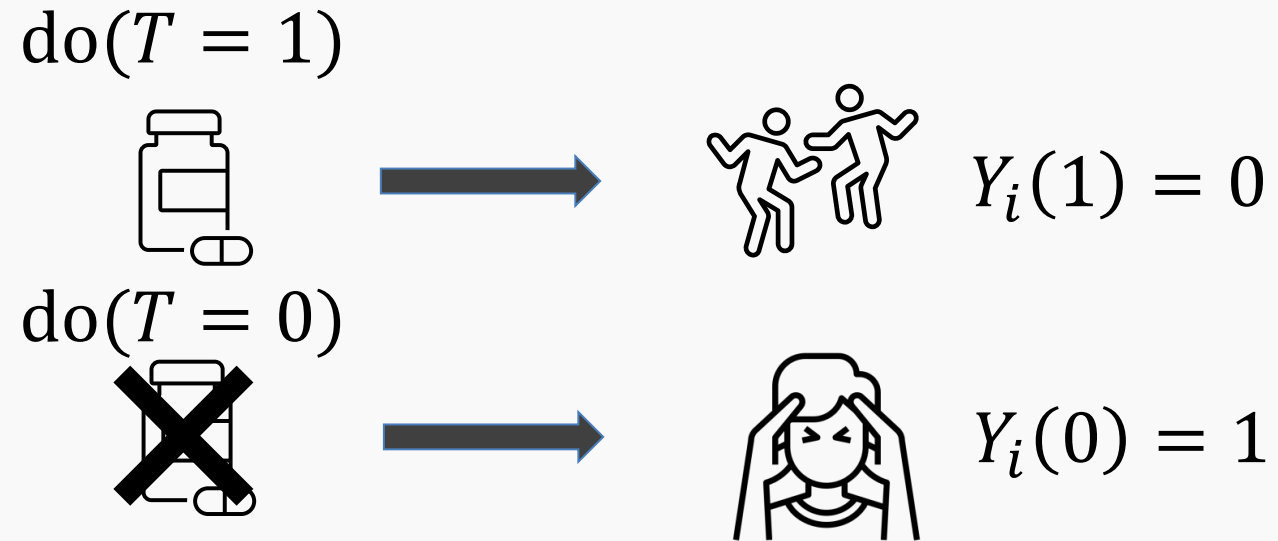
Fundamental problem of causal inference



Causal effect

$$Y_i(1) - Y_i(0) = -1$$

Fundamental problem of causal inference



Causal effect

$$Y_i(1) - Y_i(0) = -1$$

Fundamental problem of causal inference

Counterfactual

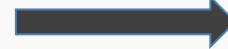
$\text{do}(T = 1)$



$$Y_i(1) = 0$$

Factual

$\text{do}(T = 0)$

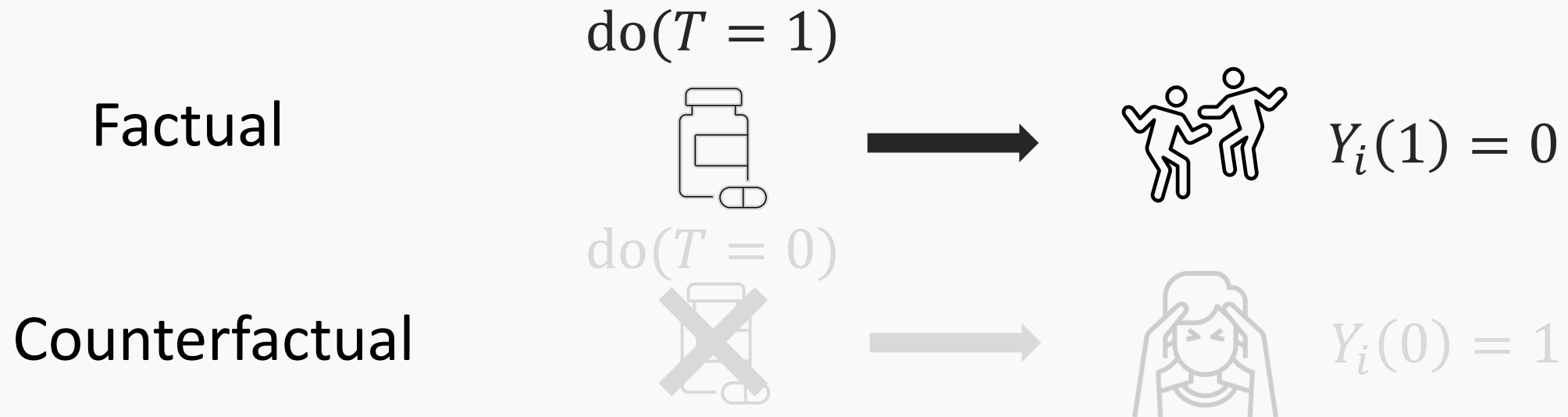


$$Y_i(0) = 1$$

Causal effect

$$Y_i(1) - Y_i(0) = -1$$

Fundamental problem of causal inference



Causal effect

$$Y_i(1) - Y_i(0) = -1$$

How do we avoid the fundamental problem of causal inference

Options

- A. We will do what we usually do: ignore all problems unless they affect my life
- B. Guess the counterfactual effect
- C. Infer the counterfactual effect using some imputation method
- D. Take the averages

How do we avoid the fundamental problem of causal inference

Options

- A. We will do what we usually do: ignore all problems unless they affect my life
- B. Guess the counterfactual effect
- C. Infer the counterfactual effect using some imputation method
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Average Treatment Effect

Individual treatment effect (ITE):

$$Y_i(1) - Y_i(0)$$

Average treatment effect (ATE):

$$\begin{aligned}\mathbb{E}[Y_i(1) - Y_i(0)] &= \mathbb{E}[Y(1)] - \mathbb{E}[Y(0)] \\ &\neq \mathbb{E}[Y|T = 1] - \mathbb{E}[Y|T = 0]\end{aligned}$$

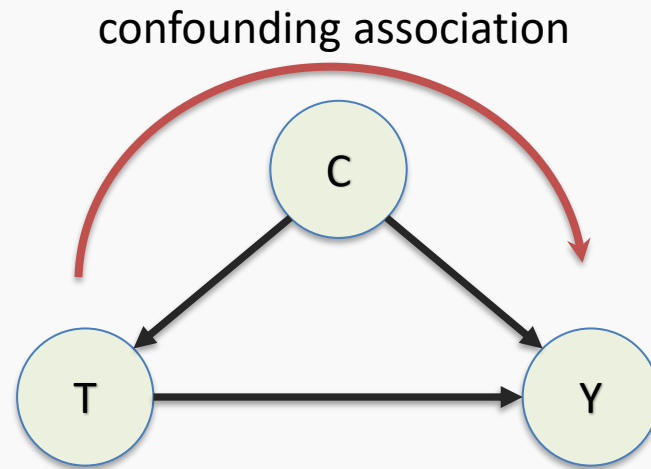
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Average Treatment Effect

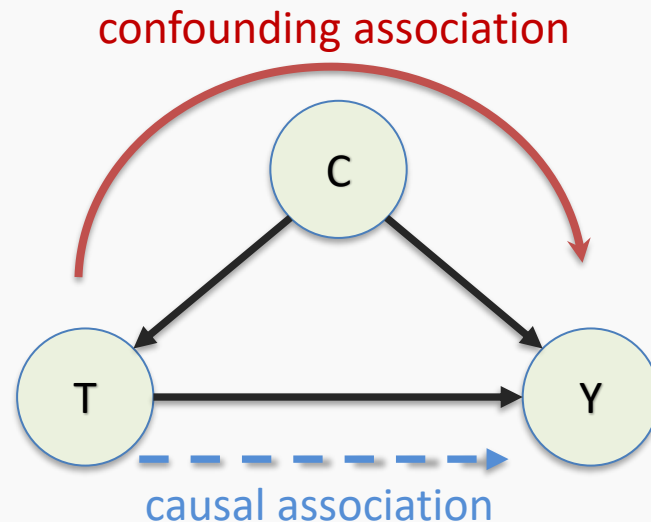
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Mix of
confounding
and causal



Necessary Assumptions

- In order to measure the causal effect of treatment (T) on the outcome (Y), we need to make a major assumption:
- **Stable Unit Treatment Values Assumption (SUTVA)**: the response/outcome of a particular subject depends only on the treatment to which they were assigned, not the treatments of others around them.
- How can this be violated?
 - If there are any **spillover effects** between subjects (sometimes called **interference**): if one subject's treatment affects another subject's response (or treatment). Think: individuals within a household.
 - If the treatment is not consistent (sometimes called **hidden variations of treatments**): if a subject's treatment *level* may be different the second time around. Think: a bad batch of pills.

Note: this is different than heterogeneous treatment *effects*.

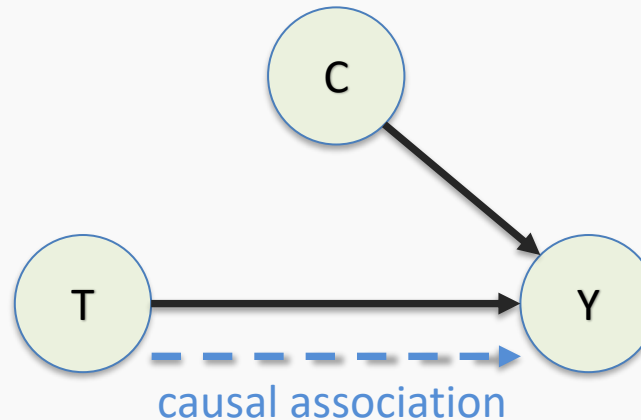
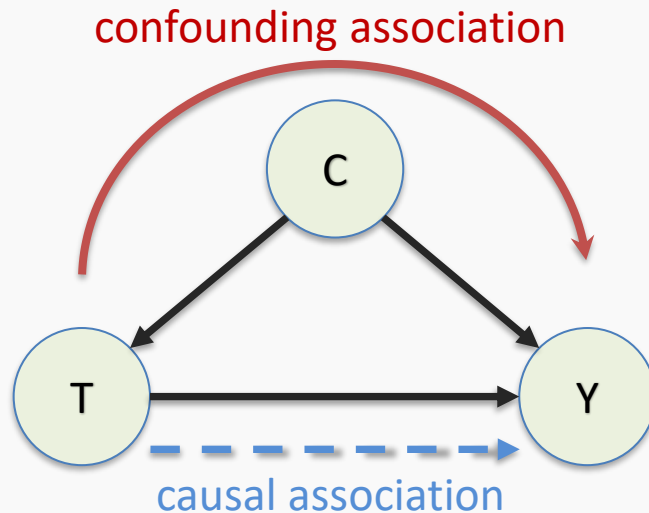
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- **Randomized Control**
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Randomized control trials (RCT)

Average treatment effect (ATE) with confounding:

$$\mathbb{E}[Y(1)] - \mathbb{E}[Y(0)] \neq \mathbb{E}[Y|T = 1] - \mathbb{E}[Y|T = 0]$$



$$\mathbb{E}[Y(1)] - \mathbb{E}[Y(0)] = \mathbb{E}[Y|T = 1] - \mathbb{E}[Y|T = 0]$$

Randomized control trials (RCTs):

Randomize subjects into treatment group or control group

1. Treatment does not have any causal parents
2. The groups now are equal

Can we always do RCTs?

Options

- A. Yes, it is easy, just randomize the treatments
- B. No, because sometimes we can not control the experiment
- C. No, because sometimes in not ethical
- D. No, because sometimes it is not feasible
- E. No, because sometimes it is not possible
- F. RCTs are too expensive

Can we always do RCTs?

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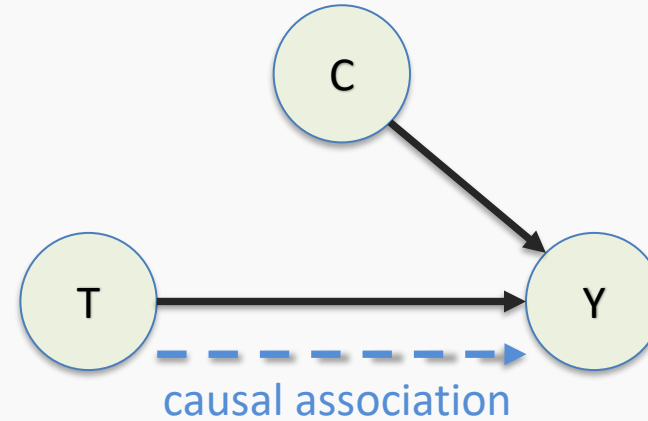
- Introduction
- Simpson's Paradox
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- Causal Effects
- Randomized Control
- **Adjusting for Confounders**

Observational Studies

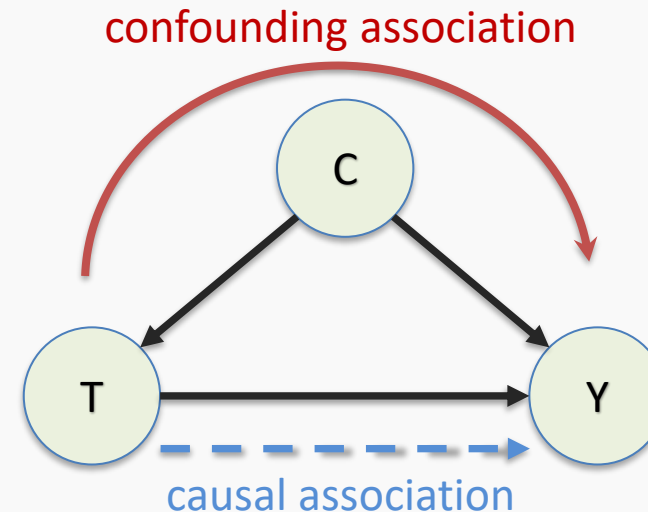
Can't always perform RCT

- Can't randomize
- Ethical
- Not feasible
- Not possible
- RCTs are expensive and difficult to set

Ideal



Observational studies



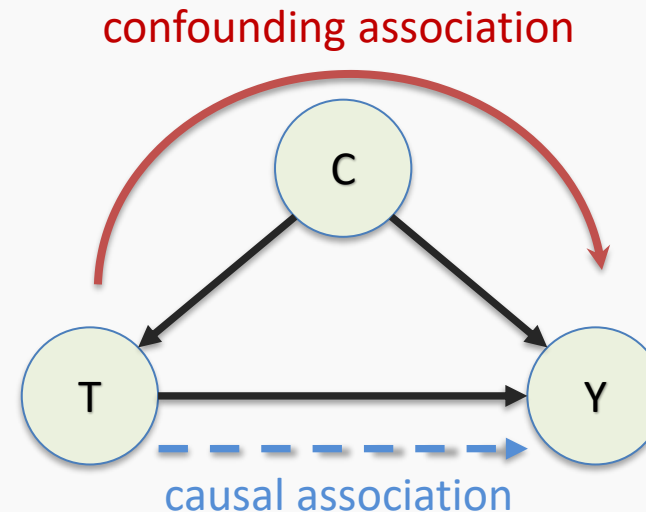
Measuring a Causal Effect in Observational Studies

Adjust for cofounders. Or adjust for a variable W . If W is the right variable: in this case $W = C$ (condition severity).

$$\mathbb{E}[Y(t)|W = w] = \mathbb{E}[Y|t, w]$$

No causal quantities in it

Observational studies



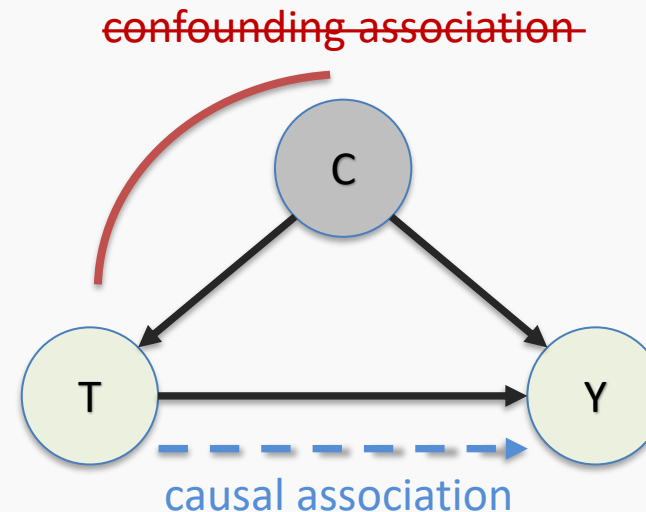
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No causal quantities in it

Observational studies



Measuring a Causal Effect in Observational Studies

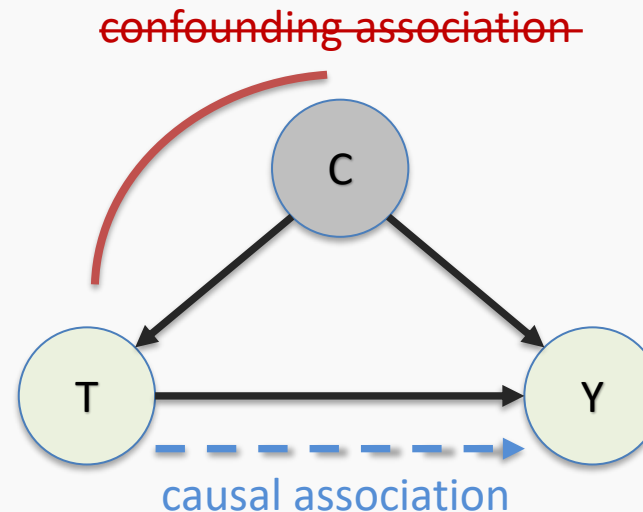
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$$\mathbb{E}[Y(t)|W = w] = \mathbb{E}[Y|t, w]$$

$$\mathbb{E}[Y(t)] = \mathbb{E}_w[Y|t, W]$$

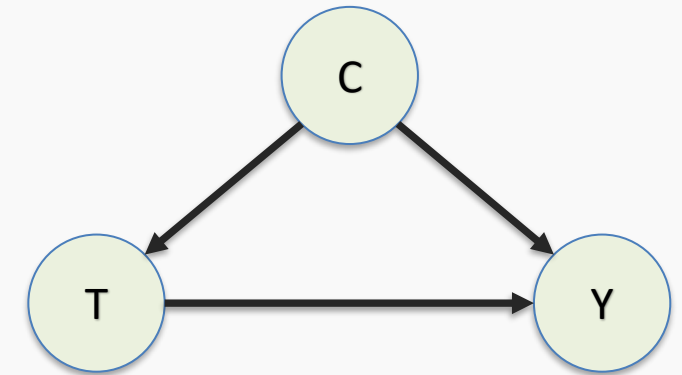
Marginalize
over W

Observational
studies



Example of Adjustment on PyCA-109a

$$\mathbb{E}[Y(t)] = \mathbb{E}_C[Y|t, C]$$

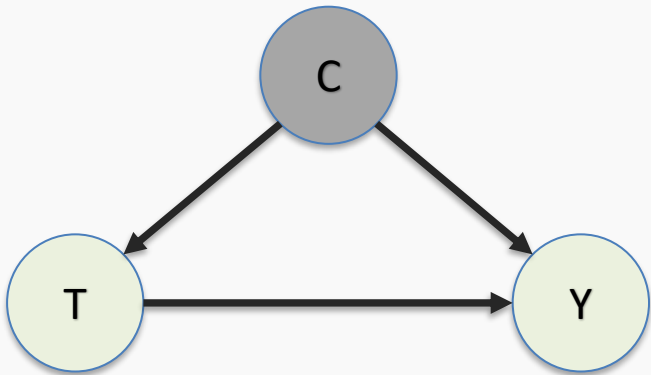


	Condition		
Treatment	Mild	Severe	Total
A	17% 150/900	40% 40/100	19% 190/1000
B	14% 50/350	33% 50/150	20% 100/500

$$\mathbb{E}[Y|T, C = 0] \quad \mathbb{E}[Y|T, C = 1] \quad \mathbb{E}[Y|T]$$

Example of Adjustment on PyCA-109a

$$\mathbb{E}[Y(t)] = \mathbb{E}_C[Y|t, C]$$



	Condition		
Treatment	Mild	Severe	Total
A	17% 150/900	40% 40/100	19% 190/1000
B	14% 50/350	33% 50/150	20% 100/500

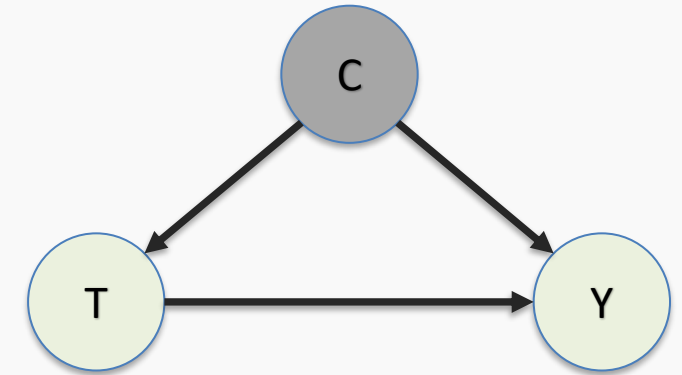
$\mathbb{E}[Y|T, C = 0]$

$\mathbb{E}[Y|T, C = 1]$

$\mathbb{E}[Y|T]$

Example of Adjustment on PyCA-109a

$$\mathbb{E}[Y(t)] = \mathbb{E}_C[Y|t, C] = \sum_c \mathbb{E}_C[Y|t, c] P(c)$$



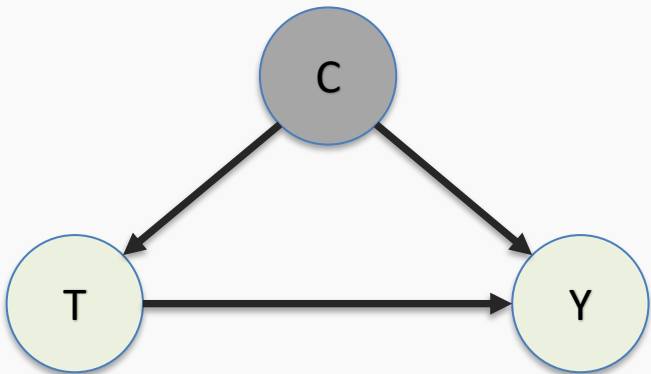
	Condition		
Treatment	Mild	Severe	Total
A	17% 150/900	40% 40/100	19% 190/1000
B	14% 50/350	33% 50/150	20% 100/500

$$\mathbb{E}[Y|T, C = 0] \quad \mathbb{E}[Y|T, C = 1] \quad \mathbb{E}[Y|T]$$

Example of Adjustment on PyCA-109a

$$\mathbb{E}[Y(t)] = \mathbb{E}_C[Y|t, C] = \sum_c \mathbb{E}_C[Y|t, c] P(c)$$

$$\mathbb{E}[Y|T = A, C = 0] = 0.17$$



	Condition			
Treatment	Mild	Severe	Total	Causal
A	17% 150/900	40% 40/100	19% 190/1000	
B	14% 50/350	33% 50/150	20% 100/500	

$\mathbb{E}[Y|T, C = 0]$

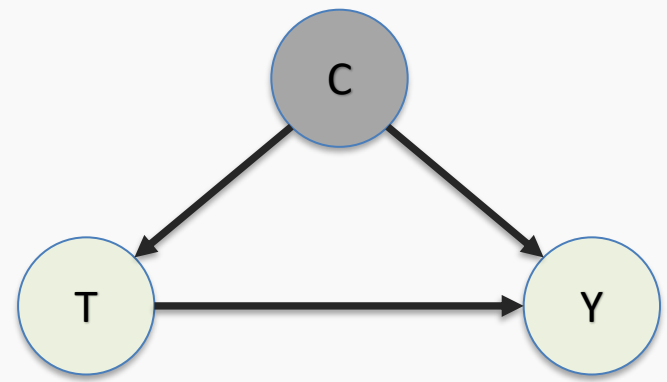
$\mathbb{E}[Y|T, C = 1]$

$\mathbb{E}[Y|T]$

$\mathbb{E}[Y|do(T)]$

Example of Adjustment on PyCA-109a

$$\mathbb{E}[Y(t)] = \mathbb{E}_C[Y|t, C] = \sum_c \mathbb{E}_C[Y|t, c] P(c)$$



	Condition			
Treatment	Mild	Severe	Total	Causal
A	17% 150/900	40% 40/100	19% 190/1000	22.5%
B	14% 50/350	33% 50/150	20% 100/500	17.2%

$\mathbb{E}[Y|T, C = 0]$

$\mathbb{E}[Y|T, C = 1]$

$\mathbb{E}[Y|T]$

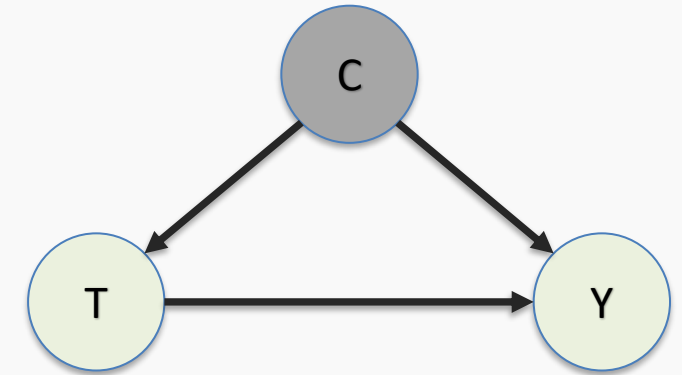
$\mathbb{E}[Y|do(T)]$

$$\frac{1250}{1500} (0.17) + \frac{250}{1500} (0.4) = 0.208$$

$$\frac{1250}{1500} (0.14) + \frac{250}{1500} (0.33) = 0.172$$

Example of Adjustment on PyCA-109a

$$\mathbb{E}[Y(t)] = \mathbb{E}_C[Y|t, C] = \sum_c P(c) \mathbb{E}_C[Y|t, c]$$



Treatment	Condition Severity		Total	Causal, adjusted for C
	Mild	Severe		
A	17% 150/900	40% 40/100	19% 190/1000	20.83%
B	14% 50/350	33% 50/150	20% 100/500	17.17%

$\mathbb{E}[Y|T, C = 0]$

$\mathbb{E}[Y|T, C = 1]$

$\mathbb{E}[Y|T]$

$\mathbb{E}_C[Y|T]$

$P(c)$ ***rebalances*** the confounder across treatments.

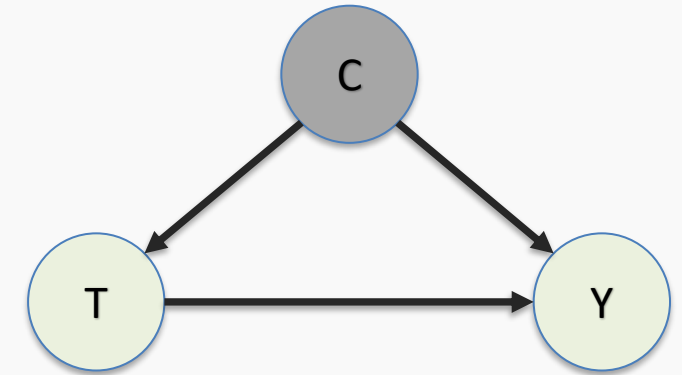
$$\frac{1250}{1500} (0.17) + \frac{250}{1500} (0.4) = 0.2083$$

$$\frac{1250}{1500} (0.14) + \frac{250}{1500} (0.33) = 0.1717$$

Example of Adjustment on PyCA-109a

Average treatment effect (ATE):

$$\mathbb{E}[Y_i(1) - Y_i(0)] = \mathbb{E}[Y(1)] - \mathbb{E}[Y(0)]$$



Treatment	Condition Severity		Total	Causal, adjusted for C
	Mild	Severe		
A	17% 150/900	40% 40/100	19% 190/1000	20.83%
B	14% 50/350	33% 50/150	20% 100/500	17.17%

$$\mathbb{E}[Y|T, C = 0]$$

$$\mathbb{E}[Y|T, C = 1]$$

$$\mathbb{E}[Y|T]$$

$$\mathbb{E}_C[Y|T]$$

$P(c)$ ***rebalances*** the confounder across treatments.

$$\frac{1250}{1500} (0.17) + \frac{250}{1500} (0.4) = 0.2083$$

$$\frac{1250}{1500} (0.14) + \frac{250}{1500} (0.33) = 0.1717$$

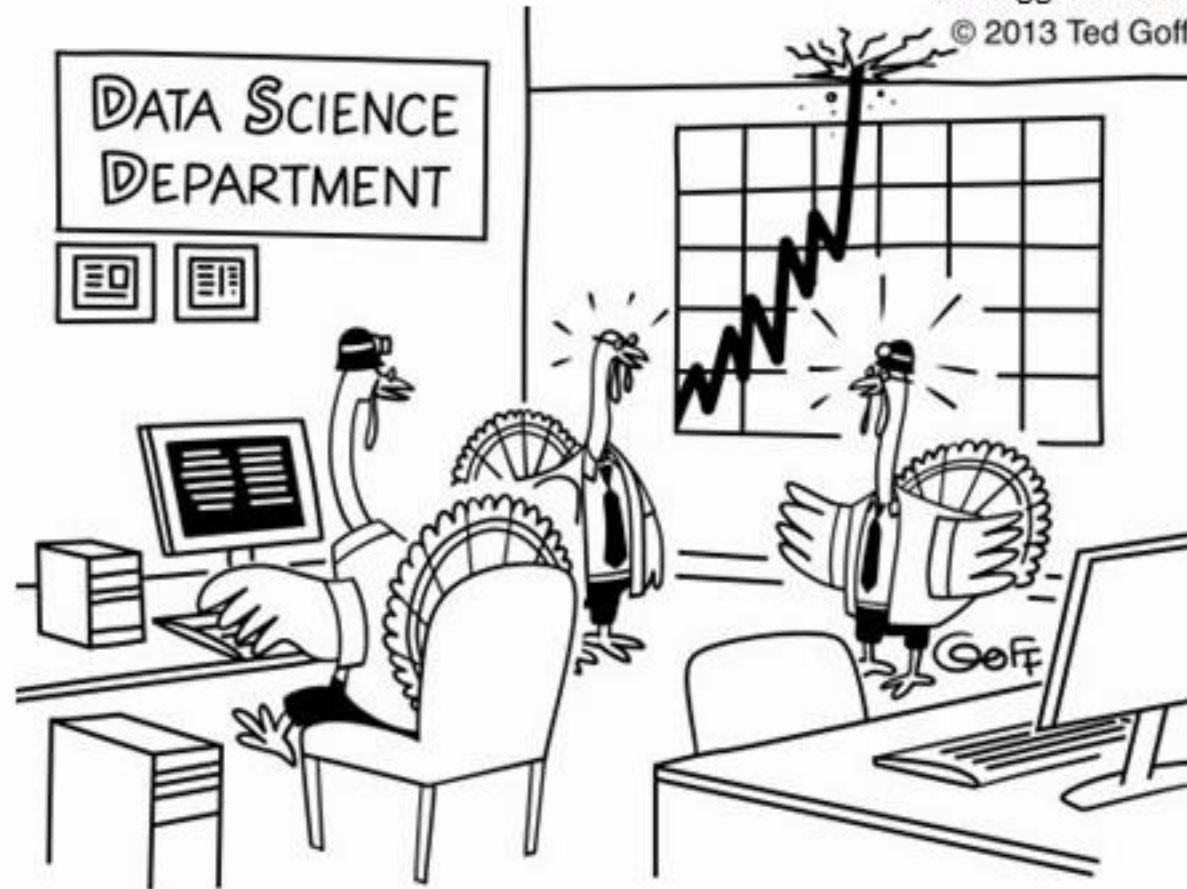
AND THIS IS JUST THE BEGINNING!!!
(and overly simplified)

- Synthetic control
- Regression discontinuity
- Instrumental variables
- Propensity scores
- Etc.

Assessing Causal Relationships – Next Time

- There are 2 main approaches to assessing causal associations between variables (we have hinted at them already)
 1. Using careful study design: we could perform **experiments** (called **A-B Testing** in the world of Data Science).
 2. Using careful analysis of observational data: we perform **observational causal inference** and take extra care in creating the counterfactual.
- **Key issue:** we may never be able to recreate the correct counterfactual when using observational data.
- **Note:** causal inference of experimental data is often very simple – a simple *t*-test or randomization test. Causal inference of observational data is very difficult – we will learn a few approaches next time.

KDnuggets cartoon
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**"I don't like the look of this.
Searches for gravy and turkey stuffing
are going through the roof!"**