



دانشکده مهندسی کامپیوتر
هوش مصنوعی و سیستم‌های خبره

تمرین تشریحی پنجم ۱

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Reinforcement Learning

Imagine an unknown game which has only two states {A,B} and in each state the agent has two actions to choose from: {Up,Down}. Suppose a game agent chooses actions according to some policy π and generates the following sequence of actions and rewards in the unknown game:

t	s_t	a_t	s_{t+1}	r_t
0	A	Down	B	2
1	B	Down	B	-4
2	B	Up	B	0
3	B	Up	A	3
4	A	Up	A	-1

Unless specified otherwise, assume a discount factor $\gamma = 0.5$ and a learning rate $\alpha = 0.5$

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Recall the update function of Q-learning is:

$$Q(s_t, a_t) \leftarrow (1 - \alpha)Q(s_t, a_t) + \alpha(r_t + \gamma \max_{a'} Q(s_{t+1}, a'))$$

Assume that all Q-values initialized as 0. What are the following Q-values learned by running Q-learning with the above experience sequence?

$$Q(A, \text{Down}) = \underline{1}, \quad Q(B, \text{Up}) = \underline{1.75}$$

Your Solution:

$$\begin{aligned}
 Q(A, \text{down}) &\leftarrow (1-0.5) \overbrace{Q(A, \text{down})}^0 + 0.5(2 + 0.5 \times 0) = 1 \\
 Q(B, \text{down}) &\leftarrow 0.5 \overbrace{Q(B, \text{down})}^0 + 0.5(-4 + 0.5 \times 0) = -2 \\
 Q(B, \text{up}) &\leftarrow 0.5 \overbrace{Q(B, \text{up})}^0 + 0.5(0 + 0.5 \times 0) = 0 \\
 Q(B, \text{up}) &\leftarrow 0.5 \overbrace{Q(B, \text{up})}^0 + 0.5(3 + 0.5 \times \max[\underbrace{Q(A, \text{up})}_0, \underbrace{Q(A, \text{down})}_1]) = \frac{3.5}{2} = 1.75
 \end{aligned}$$

	A	B	C
up		1.75	
down	1	-2	

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In model-based reinforcement learning, we first estimate the transition function $T(s,a,s')$ and the reward function $R(s,a,s')$. Fill in the following estimates of T and R , estimated from the experience above. Write "n/a" if not applicable or undefined.

$$\hat{T}(A, U_p, A) = \underline{\hspace{2cm}}, \quad \hat{T}(A, U_p, B) = \underline{\hspace{2cm}}, \quad \hat{T}(B, U_p, A) = \underline{\hspace{2cm}}, \quad \hat{T}(B, U_p, B) = \underline{\hspace{2cm}}$$

$$\hat{R}(A, U_p, A) = \underline{\hspace{2cm}}, \quad \hat{R}(A, U_p, B) = \underline{\hspace{2cm}}, \quad \hat{R}(B, U_p, A) = \underline{\hspace{2cm}}, \quad \hat{R}(B, U_p, B) = \underline{\hspace{2cm}}$$

Your Solution:

$$\begin{array}{llll} T(A, U_p, A) = 1 & T(A, U_p, B) = 0 & T(B, U_p, A) = \frac{1}{2} & T(B, U_p, B) = \frac{1}{2} \\ R(A, U_p, A) = -1 & R(A, U_p, B) = \text{n/a} & R(B, U_p, A) = 3 & R(B, U_p, B) = 0 \end{array}$$

طبق جدول بالا در 50٪ موارد با گرفتن U_p به A می‌رسیم

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To decouple this question from the previous one, assume we had a **different experience** and ended up with the following estimates of the transition and reward functions:

s	a	s'	$\hat{T}(s, a, s')$	$\hat{R}(s, a, s')$
A	Up	A	1	10
A	Down	A	0.5	2
A	Down	B	0.5	2
B	Up	A	1	-5
B	Down	B	1	8

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Give the optimal policy $\hat{\pi}^*(s)$ and $\hat{V}^*(s)$ for the MDP with transition function \hat{T} and reward function \hat{R} .

Hint: for any $x \in \mathbb{R}$, $|x| < 1$, we have $1 + x + x^2 + x^3 + x^4 + \dots = 1/(1 - x)$

$$\hat{\pi}^*(A) = \underline{\text{up}}, \quad \hat{\pi}^*(B) = \underline{\text{down}} \quad \hat{V}^*(A) = \underline{20}, \quad \hat{V}^*(B) = \underline{16}.$$

Your Solution:

$$1.3) \quad V_{k+1}^{\pi} \leftarrow \sum_{s'} T(s, \pi(s), s') [R(s, \pi(s), s') + \gamma V_k^{\pi}(s')]$$

$$A \xrightarrow{\text{up}} 1 [10 + 0.5 \hat{V}_k^{\pi}] = 10$$

$$\xrightarrow{\text{down}} 0.5 [2 + 0.5 \hat{V}_k^{\pi}(A)] + 0.5 [2 + 0.5 \hat{V}_k^{\pi}(B)] = 2$$

$$\text{argmax} [(2, \text{down}), (10, \text{up})] = \underline{\text{up}}$$

$$\hat{V}_A^* = \sum_{i=0}^{\infty} (0.5)^i 10 = \frac{10}{1-0.5} = \underline{20}$$

$$B \xrightarrow{\text{up}} 1 [-5 + 0.5 \hat{V}_k^{\pi}(A)] = -5$$

$$\xrightarrow{\text{down}} 1 [8 + 0.5 \hat{V}_k^{\pi}(B)] = 8$$

$$\sim \text{argmax} [(-5, \text{up}), (8, \text{down})] = \underline{\text{down}}$$

$$\hat{V}_B^* = \sum_{i=0}^{\infty} (0.5)^i 8 = \frac{8}{1-0.5} = \underline{16}$$

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If we repeatedly feed this new experience sequence through our Q-learning algorithm, what values will it converge to? Assume the learning rate α_t is properly chosen so that convergence is guaranteed.

- 1) the values found above, \hat{V}^*
- 2) the optimal values, V^*
- 3) neither \hat{V}^* nor V^*
- 4) not enough information to determine

Explain your answer in less than 2lines:

جواب گزینه ۱: \hat{V}^* نه بکت Q-learning، مقادیر نرونا به optimal value converge نمی‌کنند.
چون به جای بر اساس تجربه در موارد آموخته شده مناسب می‌شود. اما به مقادیر آموخته شده converge نمی‌کند.

Policy Evaluation ۲

In this question, you will be working in an MDP with states S , actions A , discount factor γ , transition function T and reward function R .

We have some fixed policy $\pi : S \rightarrow A$, which returns an action $a = \pi(s)$ for each state $s \in S$. We want to learn the Q function $Q^\pi(s, a)$ for this policy: the expected discounted reward from taking action a in state s and then continuing to act according to π :

$$Q^\pi(s, a) = \sum_{s'} T(s, a, s') [R(s, a, s') + \gamma Q^\pi(s', \pi(s'))]$$

The policy π will not change while running any of the algorithms below.

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Can we guarantee anything about how the values Q^π compare to the values Q^* for an optimal policy π^* ?

- 1) $Q^\pi(s, a) \leq Q^*(s, a)$ for all s, a
- 2) $Q^\pi(s, a) = Q^*(s, a)$ for all s, a
- 3) $Q^\pi(s, a) \geq Q^*(s, a)$ for all s, a
- 4) None of the above are guaranteed

Explain your answer in less than 2 lines:

(2.۱) جواب گزیده! چون مقادیر Q براساس learning بدست می‌آید، مقادیر آن حتماً کمتر و یا مساوی با Q^* است

۲.۲

Suppose T and R are *unknown*. You will develop sample-based methods to estimate Q^π . You obtain a series of *samples* $(s_1, a_1, r_1), (s_2, a_2, r_2), \dots, (s_T, a_T, r_T)$ from acting according to this policy (where $a_t = \pi(s_t)$, for all t).

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Recall the update equation for the Temporal Difference algorithm, performed on each sample in sequence:

$$V(s_t) \leftarrow (1 - \alpha)V(s_t) + \alpha(r_t + \gamma V(s_{t+1}))$$

which approximates the expected discounted reward $V^\pi(s)$ for following policy π from each state s , for a learning rate α .

Fill in the blank below to create a similar update equation which will approximate Q^π using the samples. You can use any of the terms $Q, s_t, s_{t+1}, a_t, a_{t+1}, r_t, r_{t+1}, Y, \alpha, \pi$ in your equation, as well as Σ and \max with any index variables (i.e. you could write \max_a , or Σ_a and then use a somewhere else), but no other terms.

$$Q(s_t, a_t) \leftarrow (1 - \alpha)Q(s_t, a_t) + \alpha [\text{ }]$$

Explain your answer in less than 2 lines:

$$Q(s_t, a_t) \leftarrow (1 - \alpha)Q(s_t, a_t) + \alpha [r_t + \gamma Q(s_{t+1}, a_{t+1})]$$

مسئله با ۵ است.

۲.۲.۲

Now, we will approximate Q^π using a linear function: $Q(s,a) = \sum_{i=1}^d w_i f_i(s,a)$ for weights w_1, \dots, w_d and feature functions $f_1(s,a), \dots, f_d(s,a)$.

To decouple this part from the previous part, use Q_{samp} for the value in the blank in part (2.2.1) (i.e. $Q(s_t, a_t) \leftarrow (1 - \alpha) Q(s_t, a_t) + \alpha Q_{\text{samp}}$).

Which of the following is the correct sample-based update for each w_i ?

- 1) $w_i \leftarrow w_i + \alpha [Q(s_t, a_t) - Q_{\text{samp}}]$
- 2) $w_i \leftarrow w_i - \alpha [Q(s_t, a_t) - Q_{\text{samp}}]$
- 3) $w_i \leftarrow w_i + \alpha [Q(s_t, a_t) - Q_{\text{samp}}] f_i(s_t, a_t)$
- 4) $w_i \leftarrow w_i - \alpha [Q(s_t, a_t) - Q_{\text{samp}}] f_i(s_t, a_t)$
- 5) $w_i \leftarrow w_i + \alpha [Q(s_t, a_t) - Q_{\text{samp}}] w_i$
- 6) $w_i \leftarrow w_i - \alpha [Q(s_t, a_t) - Q_{\text{samp}}] w_i$

Explain your answer in less than 2 lines:

2.2.2 جواب گزینه 4؛ در بدست آوردن approximate Q، باید difference را در f_i ضرب کنیم تا بتوانیم جاهایی که تا به حال مشاهده نشده اند نیز تجربه بدست بیاوریم

۳.۲.۲

The algorithms in the previous parts (part 2.2.1 and 2.2.2) are:

1) model-based 2) model-free

Explain your answer in less than 2 lines:

3.2.2 model-free هستند؛ دانش سوال‌ها بر اساس learn و کسب تجربه به مقادیر Q و V دسترسی پیدا کردیم اما

model-based، مدل از ابتدا به ما داده شده