STAT 2131:
Applied Statistical Methods I
HW #1
Due Tuesday 11am, September 14th

1. A researcher is interested in evaluating the efficacy of a new treatment over conventional treatment in improving the quality of life among depression patients. Suppose she randomly assigns 50 patients to the new treatment group and the other 50 patients to the conventional treatment group. It is reasonable to assume that the quality of life for the new treatment group and the conventional treatment group both follow a normal distribution, with means μ_1 and μ_2, respectively. Without loss of generality, the standard deviations of the two groups are assumed to be 1. The researcher expects to see a difference in the means $\Delta = \mu_1 - \mu_2 = 0.6$. She plans to employ a simple one-sided test for the two groups comparison at the significance level $\alpha = 0.05$. What is the power of detecting such a difference based on the sample she has?

power = prob of correctly rejected h0 when h1 correct

2. Suppose that washing behavior after using the bathroom is independent from person to person. At a meeting of infectious disease scientists, a researcher who surveyed on people in public restrooms reported that 138 out of 180 men washed up after using the bathroom, while 129 out of 150 women did.

   (a) State the null and alternative hypotheses to carry out a hypothesis test to see whether the population proportion of washing hands after using the bathroom is different between men and women.

   (b) Calculate the test statistic for the hypothesis test in part (a) and find the p-value.

   (c) Interpret your results.

H0: p1 = p2
Ha: p1 != p2

test stat is z score for 2 proportion (pooled) test
z=-2.152
p val is 0.032 => reject

3. A food company is developing a new packaging design for a granola bar. To help with a marketing strategy, the company was interested in testing whether the appeal of the new packaging design was related to a person's age. The company recruited 62 volunteers and asked each of them to rate the new and old design with a score out of 100 (a higher score means more appealing). The sample mean and sample standard deviation (in the parenthesis) of the scores are summarized in the following table.

	New design	Old design	Difference between new and old	Sample size
Adults	89.78 (11.05)	86.02 (11.46)	3.76 (3.2)	37
Children	86.93 (12.56)	82.11 (18.63)	4.82 (3.5)	25
Combined	88.63 (11.46)	84.44 (14.09)	4.19 (3.3)	62

Answer the following questions by conducting an appropriate statistical hypothesis test.

(a) Do the data for the new design suggest that children have higher rating for the new design than adults?

(b) Do the combined data suggest that the new design is more appealing to the old design?

(c) Does the improvement in the appeal of the design more prominent in children than in adults?

4. (Use computer) Set $x_i = i/100$ for $i = 1, \ldots, 50$, generate $y_i = 1 + 2x_i + \epsilon$ with $\epsilon \sim N(0, 1)$ for $i = 1, \ldots, 50$. Then fit a simple linear regression model to obtain $\hat{\beta}_1$ through MLE, as well as a 95% t confidence interval for β_1. Repeat that 100 times.

   (a) Derive the sampling distribution of $\hat{\beta}_1$ in this setting (start with the formula of $\hat{\beta}_1$).

   (b) Show a histogram of the estimated β_1s. What is the sample mean and variance of $\hat{\beta}_1$?

   (c) What is the empirical coverage of the confidence interval for β_1?

   (d) Do (b) and (c) agree with the theoretical properties of MLE and the confidence intervals?