

# hw3

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## Homework 3

(a)

The model I am assuming is

$$\text{logit}(\pi_i) = \beta_0 + \beta_1 \log_2(X_i)$$

where  $\pi_i = Y_i/m$  is the mortality rate.

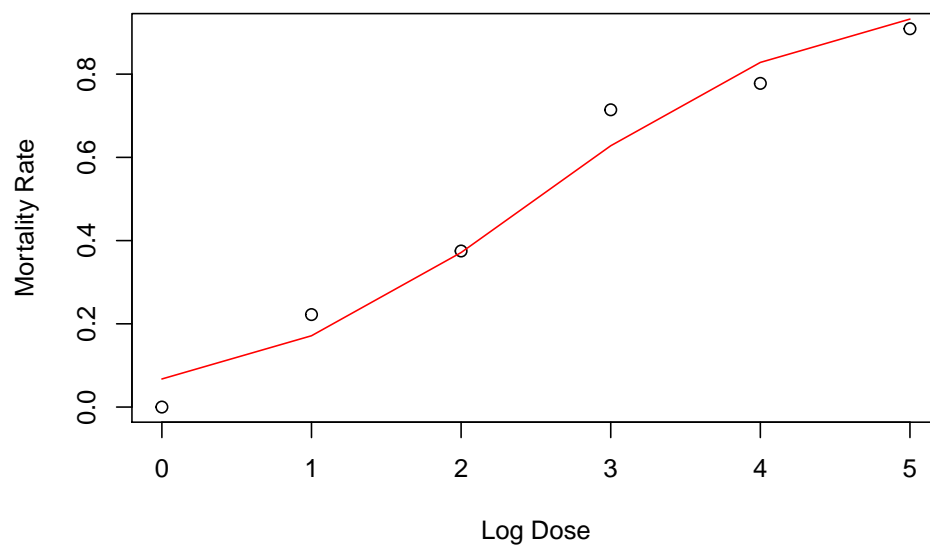
$\beta_0$  is log odds of  $Y_i = 1$  when  $\log_2(X_i) = 0$ .

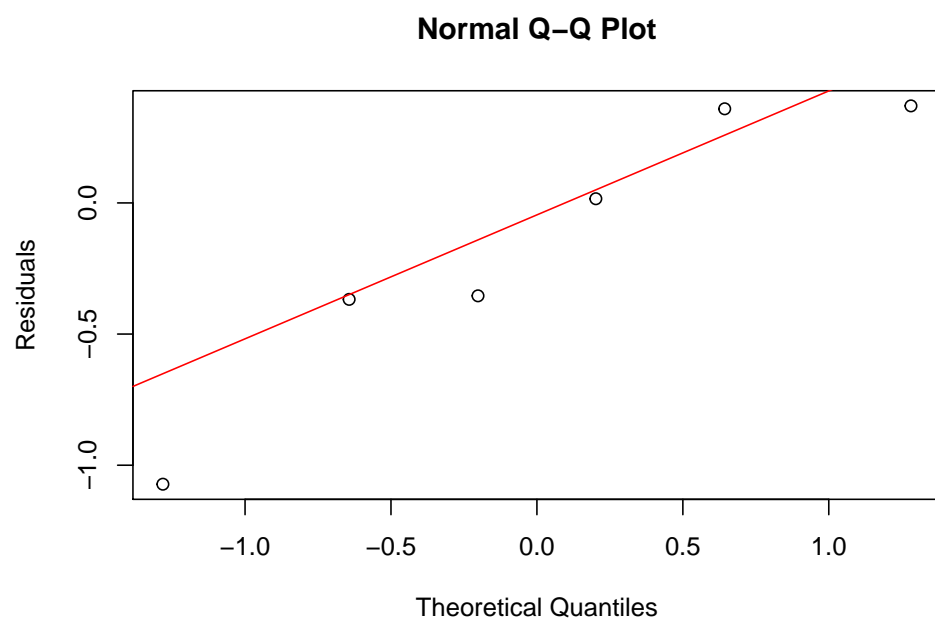
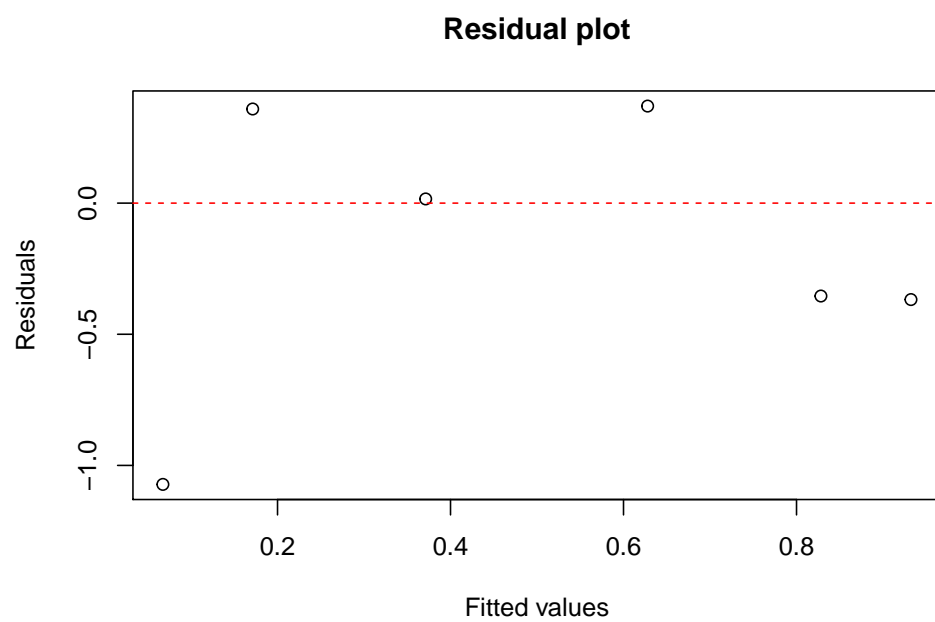
$\beta_1$  is the log odds ratio between log dose  $X_{i+1}$  and log dose  $X_i$ .

$Y_i|X_i$  are assumed to be independent.

(b)

After fitting the model using quasibinomial family with logit link, the plot of the fitted mortality rate and raw mortality fractions  $y/m$  against log dose is obtained below. Looking at the fit of the model, we could see that it fits quite well. I checked residual plot and qq-plot and the constant variances and normality assumptions are both satisfied as well.





c) (i)  $\hat{\gamma} = LD_{50} \Rightarrow \text{logit}(\pi) = \hat{\beta}_0 + \hat{\beta}_1 \log_2(\hat{\gamma}) = 0$   
 $\Rightarrow \hat{\gamma} = 2^{(-\hat{\beta}_0 / \hat{\beta}_1)} = 2^{(-\frac{2.626}{1.050})} \approx 5.66$

An estimate for  $\gamma$  is 5.66

(ii) We want to test null hypothesis

$$H_0: \log_2(\gamma) = \frac{-\beta_0}{\beta_1} = 4$$

$$\Rightarrow \beta_0 = -4\beta_1$$

Our restricted model is  $\text{logit}(\pi_i) = -4\beta_1 + \beta_1 \log_2(X_i)$

$$\Rightarrow \text{logit}(\pi_i) = \beta_1 (\log_2(X_i) - 4)$$

After fitting this model, we obtain coefficient estimate

$$\hat{\beta}_1 = 0.5528$$

After doing likelihood ratio test, the likelihood ratio statistic is 1.32 with p-value of 0.25.

We fail to reject  $H_0$ . If  $H_0$  is true,  $LR(4) \stackrel{n \rightarrow \infty}{H_0} \sim \chi^2_1$

(c)

(iii) I didn't understand what "plot the restricted log likelihood against the hypothesized value of  $\log_2(\gamma)$ " is, because restricted log likelihood is one number, and the hypothesized value of  $\log_2(\gamma)$  is 4 (null hypothesis in the previous part), so if I were to plot it it would be just a single point in the plane, so I did not understand how that would help with constructing a confidence interval.

I used some built in function to generate the 95% confidence interval for the parameter of the restricted model to be (0.083, 1.14), so the 95% confidence interval for  $\gamma$  would be (1.06, 2.20). This also doesn't really make sense because we fail to reject that  $\gamma = 4$  earlier, but 4 is outside of this confidence interval.

$$d) \quad (i) \quad \hat{\sigma}^2 = \frac{1}{|I|-2} \sum_{x \in I} \frac{\{y(x) - m(x)\hat{\pi}(x)\}^2}{m(x)\hat{\pi}(x)\{1 - \hat{\pi}(x)\}}$$

$$\Rightarrow \hat{\sigma}^2 (|I|-2) = \sum_{x \in I} \frac{\{y(x) - m(x)\hat{\pi}(x)\}^2}{m(x)\hat{\pi}(x)\{1 - \hat{\pi}(x)\}}$$

$$= \sum_{x \in I} \frac{\left\{ \frac{y(x)}{m(x)} - \hat{\pi}(x) \right\}^2 \cdot m^2(x)}{m(x)\hat{\pi}(x)\{1 - \hat{\pi}(x)\}}$$

$$= m(x) \sum_{x \in I} \frac{\left\{ \frac{y(x)}{m(x)} - \hat{\pi}(x) \right\}^2}{\hat{\pi}(x)\{1 - \hat{\pi}(x)\}} = \chi^2$$

(ii) Asymptotically,  $\chi^2 \sim \chi_{c-p}^2$

$$\Rightarrow \chi^2 \sim \text{Gamma} \left( \frac{c-p}{2}, \frac{1}{2} \right)$$

$$\Rightarrow \frac{1}{(|I|-2)} \chi^2 \sim \text{Gamma} \left( \frac{c-p}{2}, \frac{1}{2(|I|-2)} \right)$$

$$\Rightarrow \hat{\sigma}^2 \underset{m \rightarrow \infty}{\sim} \text{Gamma} \left( \frac{c-p}{2}, \frac{1}{2(|I|-2)} \right)$$

In the data we have,  $m$  is quite small, therefore the sample size may be too small for us to assume the asymptotic distribution is correct.

(iii) If  $\hat{\sigma}^2$  is large,  $\chi^2$  will also be large

$$\Rightarrow \hat{\pi}(x)\{1 - \hat{\pi}(x)\} = \widehat{\text{Var}}(\bar{y}(x)) \text{ is small}$$

(iv) If  $\hat{\sigma}^2$  is large,  $\chi^2$  will also be large

$\Rightarrow$  We should reject the model

But we did not reject in (c), so that means the CI is still too wide

If  $\hat{\sigma}^2$  is small, the CI would be too narrow

(v) Using R, I found  $\chi^2 = 0.077$  with a  $p$ -value of 0.999

With this Pearson statistic and this  $p$ -value, we fail to reject the model