Summary

- Calculating the EVSI using non-parametric regression
  - Sampling future datasets
  - Summary Statistics
  - Using non-parametric regression
- Calculating the EVSI in `R`.
- Calculating the EVSI in `SAVI`

References

Strong et al. (2015) *Estimating the Expected Value of Sample Information Using the Probabilistic Sensitivity Analysis Sample: A Fast, Nonparametric Regression-Based Method.*

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Expected Value of Sample Information (EVSI)

- As we did with the expression for EVPPI, we can rewrite EVSI as

$$\begin{aligned}\mathsf{EVSI} &= \mathsf{E}_{\boldsymbol{X}}\left[\max_t \mathsf{E}_{\boldsymbol{\theta}|\boldsymbol{X}}\{\mathsf{NB}(d,\boldsymbol{\theta})\}\right] - \max_t \mathsf{E}_{\boldsymbol{\theta}}\{\mathsf{NB}_t(\boldsymbol{\theta})\} \\ &= \mathsf{E}_{\boldsymbol{X}}\left[\max_t \mathsf{E}_{\boldsymbol{\theta}|\boldsymbol{X}}\{\mathsf{NB}(d,\boldsymbol{\theta})\}\right] - \max_t \mathsf{E}_{\boldsymbol{X}}\left[\mathsf{E}_{\boldsymbol{\theta}|\boldsymbol{X}}\{\mathsf{NB}(d,\boldsymbol{\theta})\}\right]\end{aligned}$$

- This is helpful when it comes to Monte Carlo estimation

The PSA sample

- The probability sensitivity analysis (PSA) sample
  - Samples from $p(\boldsymbol{\theta})$ and corresponding net benefits

$$
\begin{array}{cccccc}
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- Each sample $\boldsymbol{x}$ may be a vector of values

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Expected Value of Sample Information (EVSI)

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$\mathsf{E}_{\boldsymbol{\theta}|\boldsymbol{X}}\{\text{NB}(d,\boldsymbol{\theta})\}$ potentially difficult

Treat as regression problem

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- Write the following

Regression equation

$$\begin{aligned} \mathsf{NB}_t(\boldsymbol{\theta}) &= \mathsf{E}_{\boldsymbol{\theta}|\boldsymbol{X}} \{ \mathsf{NB}_t(\boldsymbol{\theta}) \} + \varepsilon \\ &= g_t(\boldsymbol{X}) + \varepsilon \end{aligned}$$

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- If $\boldsymbol{X}$ is vector, summarise $S(\boldsymbol{X})$ and regress on $S(\boldsymbol{X})$

PSA plus datasets

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Treat as regression problem

- For each dataset calculate scalar summary statistic $S(\boldsymbol{x}^{(i)})$
- Regress net benefits $\mathrm{NB}_t(\boldsymbol{\theta}^{(i)})$ on $S(\boldsymbol{x}^{(i)})$

[1] Strong M, Oakley JE, Brennan A, Breeze P. Estimating the Expected Value of Sample Information using the Probabilistic Sensitivity Analysis Sample. A Fast Non-Parametric Regression Based Method. *Medical Decision Making*. 2015;**35(5)**:570-583

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- Extract fitted values $\hat{g}_t\{S(\boldsymbol{x}^{(i)})\}$
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- EVSI estimator is now just[1]

$$\frac{1}{N}\sum_{i=1}^{N}\max_t \hat{g}_t\{S(\boldsymbol{x}^{(i)})\} - \max_t \frac{1}{N}\sum_{i=1}^{N}\hat{g}_t\{S(\boldsymbol{x}^{(i)})\}$$

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Example R code

```
library(mgcv)
model <- gam(INB ~ s(Sx))
g.hat <- fitted(model)
evsi <- mean(pmax(0, g.hat)) - max(0, mean(g.hat))
```

Empirical case study results

Sample sizes			Coefficient of variation	Computation time (s)
Outer	Inner	Total		
Two-level Monte Carlo method				
10^4	10^4	10^8	1.9%	4,456
10^5	10^4	10^9	0.6%	43,303
10^6	10^4	10^{10}	0.2%	424,686
Non-parametric regression method				
10^4	-	10^4	2.4%	0.1
10^5	-	10^5	0.8%	0.7
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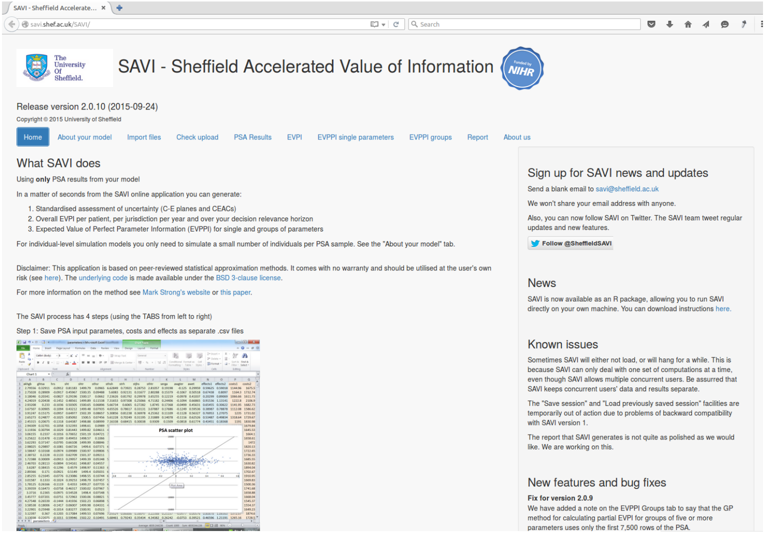
50,000 times speed up!

Example in SAVI



Summary

- Fast and efficient method for EVSI
- Only the PSA sample required
- No need to run the model again
- ‘Black box’ Value of Information