

- Calculating the EVSI using non-parametric regression
 - ▶ Sampling future datasets
 - ▶ Summary Statistics
 - ▶ Using non-parametric regression
- Calculating the EVSI in R.
- Calculating the EVSI in SAVI

References

Strong et al. (2015) *Estimating the Expected Value of Sample Information Using the Probabilistic Sensitivity Analysis Sample: A Fast, Nonparametric Regression-Based Method.*

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- As we did with the expression for EVPPI, we can rewrite EVSI as

$$\begin{aligned}\text{EVSI} &= E_{\mathbf{X}} \left[\max_t E_{\theta|\mathbf{X}} \{ \text{NB}(d, \boldsymbol{\theta}) \} \right] - \max_t E_{\theta} \{ \text{NB}_t(\boldsymbol{\theta}) \} \\ &= E_{\mathbf{X}} \left[\max_t E_{\theta|\mathbf{X}} \{ \text{NB}(d, \boldsymbol{\theta}) \} \right] - \max_t E_{\mathbf{X}} [E_{\theta|\mathbf{X}} \{ \text{NB}(d, \boldsymbol{\theta}) \}]\end{aligned}$$

- This is helpful when it comes to Monte Carlo estimation

- The probability sensitivity analysis (PSA) sample
 - ▶ Samples from $p(\theta)$ and corresponding net benefits

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- Note that any single sample \mathbf{x} is a sample dataset from the proposed study.
- Each sample \mathbf{x} may be a vector of values

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$E_{\boldsymbol{\theta}|\mathbf{X}} \{ \text{NB}(d, \boldsymbol{\theta}) \}$ potentially difficult

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Regression equation

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- Where $g_t(\mathbf{X})$ is some unknown *smooth* function of \mathbf{X}
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- If \mathbf{X} is vector, summarise $S(\mathbf{X})$ and regress on $S(\mathbf{X})$

$$\begin{array}{ccccccc}
 \theta_1^{(1)} & \dots & \theta_p^{(1)} & \text{NB}_1^{(1)} & \dots & \text{NB}_T^{(1)} & \mathbf{x}^{(1)} \\
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 \theta_1^{(1)} & \dots & \theta_p^{(1)} & \text{NB}_1^{(1)} & \dots & \text{NB}_T^{(1)} & \boldsymbol{x}^{(1)} & S(\boldsymbol{x}^{(1)}) \\
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 \end{array}$$

- For each dataset calculate scalar summary statistic $S(\mathbf{x}^{(i)})$
- Regress net benefits $NB_t(\boldsymbol{\theta}^{(i)})$ on $S(\mathbf{x}^{(i)})$

¹Strong M, Oakley JE, Brennan A, Breeze P. Estimating the Expected Value of Sample Information using the Probabilistic Sensitivity Analysis Sample. A Fast Non-Parametric Regression Based Method. *Medical Decision Making*. 2015;**35**(5):570-583

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- Repeat for each $t \in \{1, \dots, T\}$

- Extract fitted values $\hat{g}_t\{S(\mathbf{x}^{(i)})\}$
- These are estimates of $E_{\boldsymbol{\theta}|\mathbf{X}^{(i)}}\{\text{NB}_t(\boldsymbol{\theta})\}$

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- EVSI estimator is now just¹

$$\frac{1}{N} \sum_{i=1}^N \max_t \hat{g}_t\{S(\mathbf{x}^{(i)})\} - \max_t \frac{1}{N} \sum_{i=1}^N \hat{g}_t\{S(\mathbf{x}^{(i)})\}$$

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```
library(mgcv)
model <- gam(INB ~ s(Sx))
g.hat <- fitted(model)
evsi <- mean(pmax(0, g.hat)) - max(0, mean(g.hat))
```

Sample sizes			Coefficient of variation	Computation time (s)
Outer	Inner	Total		
<i>Two-level Monte Carlo method</i>				
10^4	10^4	10^8	1.9%	4,456
10^5	10^4	10^9	0.6%	43,303
10^6	10^4	10^{10}	0.2%	424,686
<i>Non-parametric regression method</i>				
10^4	-	10^4	2.4%	0.1
10^5	-	10^5	0.8%	0.7
10^6	-	10^6	0.2%	8

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50,000 times speed up!

Example in SAVI

SAVI - Sheffield Accelerate...

University of Sheffield

SAVI - Sheffield Accelerated Value of Information

NIHR

Release version 2.0.10 (2015-09-24)
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Home About your model Import files Check upload PSA Results EVPI EVPPI single parameters EVPPI groups Report About us

What SAVI does

Using **only** PSA results from your model

In a matter of seconds from the SAVI online application you can generate:

1. Standardised assessment of uncertainty (C-E planes and CEACs)
2. Overall EVPI per patient, per jurisdiction per year and over your decision relevance horizon
3. Expected Value of Perfect Parameter Information (EVPPI) for single and groups of parameters

For individual-level simulation models you only need to simulate a small number of individuals per PSA sample. See the "About your model" tab.

Disclaimer: This application is based on peer-reviewed statistical approximation methods. It comes with no warranty and should be utilised at the user's own risk (see [here](#)). The [underlying code](#) is made available under the [BSD 3-clause license](#).

For more information on the method see [Mark Strong's website](#) or [this paper](#).

The SAVI process has 4 steps (using the TABS from left to right)

Step 1: Save PSA input parameters, costs and effects as separate .csv files

Sign up for SAVI news and updates

Send a blank email to savi@sheffield.ac.uk

We won't share your email address with anyone.

Also, you can now follow SAVI on Twitter. The SAVI team tweet regular updates and new features.

[Follow @SheffieldSAVI](#)

News

SAVI is now available as an R package, allowing you to run SAVI directly on your own machine. You can download instructions [here](#).

Known issues

Sometimes SAVI will either not load, or will hang for a while. This is because SAVI can only deal with one set of computations at a time, even though SAVI allows multiple concurrent users. Be assured that SAVI keeps concurrent users' data and results separate.

The "Save session" and "Load previously saved session" facilities are temporarily out of action due to problems of backward compatibility with SAVI version 1.

The report that SAVI generates is not quite as polished as we would like. We are working on this.

New features and bug fixes

Fix for version 2.0.9

We have added a note on the EVPPI Groups tab to say that the GP method for calculating partial EVPI for groups of five or more parameters uses only the first 7,500 rows of the PSA.

- Fast and efficient method for EVSI
- Only the PSA sample required
- No need to run the model again
- 'Black box' Value of Information