

1. Figure 3 depicts the robot in a generic configuration away from the rest position. Print out this page and draw on the figure coordinate frames $o_1x_1y_1z_1$, $o_2x_2y_2z_2$, $o_3x_3y_3z_3$, $o_4x_4y_4z_4$ and $o_5x_5y_5z_5$ according to the DH convention, and write the DH table of the robot. In particular, choose the x_1 axis parallel to the x_0 axis, $x_1 = x_0$ (this is done for convenience).

If your DH frame assignment is correct, you should find that when $\theta_1 = \theta_2 = \theta_3 = 0$, link 2 is parallel to the ground, and link 3 is vertical, pointing downward. A schematic diagram of the KUKA robot arm in the HOME configuration is illustrated in Figure 4 which you should verify corresponds to $(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6) = (0, \pi/2, 0, 0, \pi/2, 0)$.

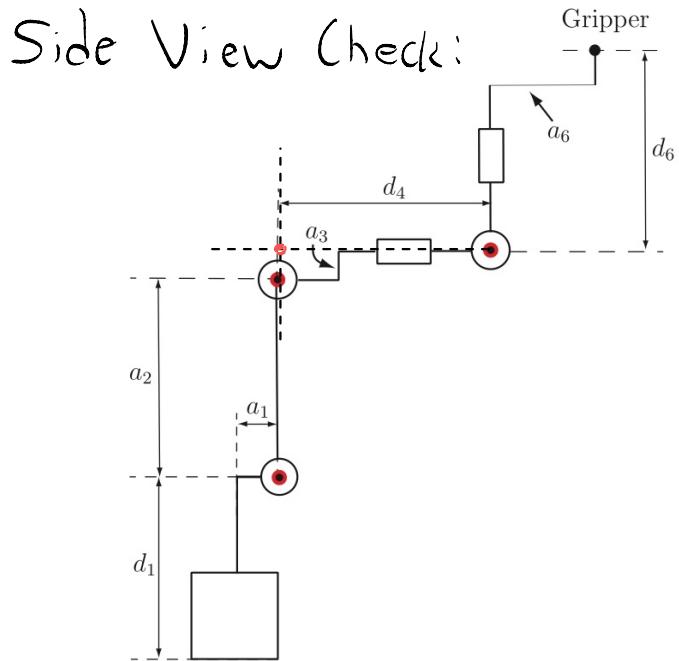


Figure 4: The KUKA robotic arm schematic diagram in the HOME configuration

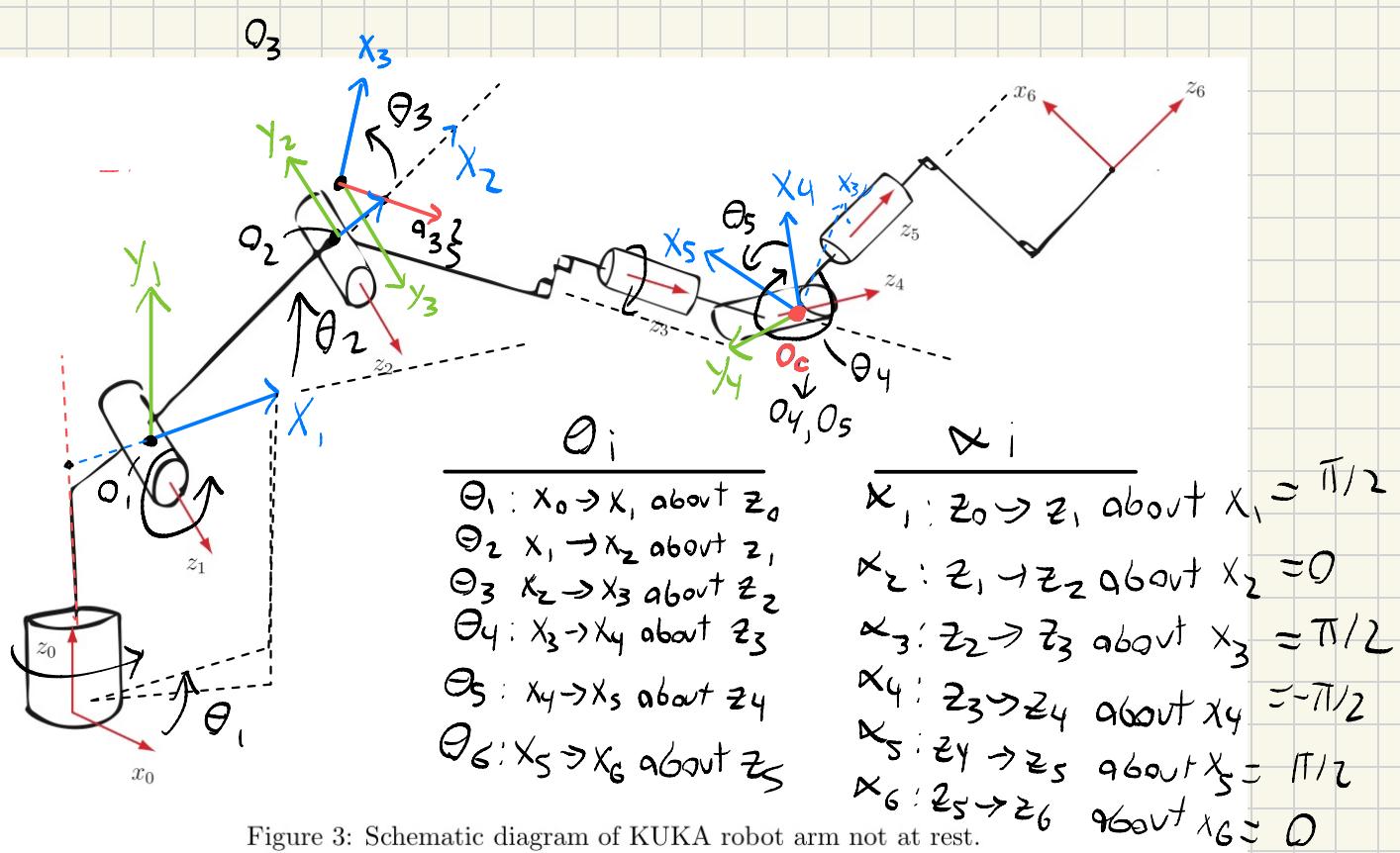


Figure 3: Schematic diagram of KUKA robot arm not at rest.

Check: $\theta_1, \theta_2, \theta_3 = 0$ that $\theta_2 \parallel \text{GND}$ & link 3 points down

DH Table:

Links	a_i	α_i	d_i	θ_i
1	$a_1 \pi/2$	d_1	θ_1	
2	$a_2 0$	0	θ_2	
3	$a_3 \pi/2$	0	θ_3	
4	0 $-\pi/2$	d_4	θ_4	
5	0 $\pi/2$	0	θ_5	
6	$a_6 0$	d_6	θ_6	

Given: d_i & a_i (offsets)

d_1	400 mm
a_1	25 mm
a_2	315 mm
a_3	35 mm
d_4	365 mm
a_6	296.23 mm
d_6	161.44 mm

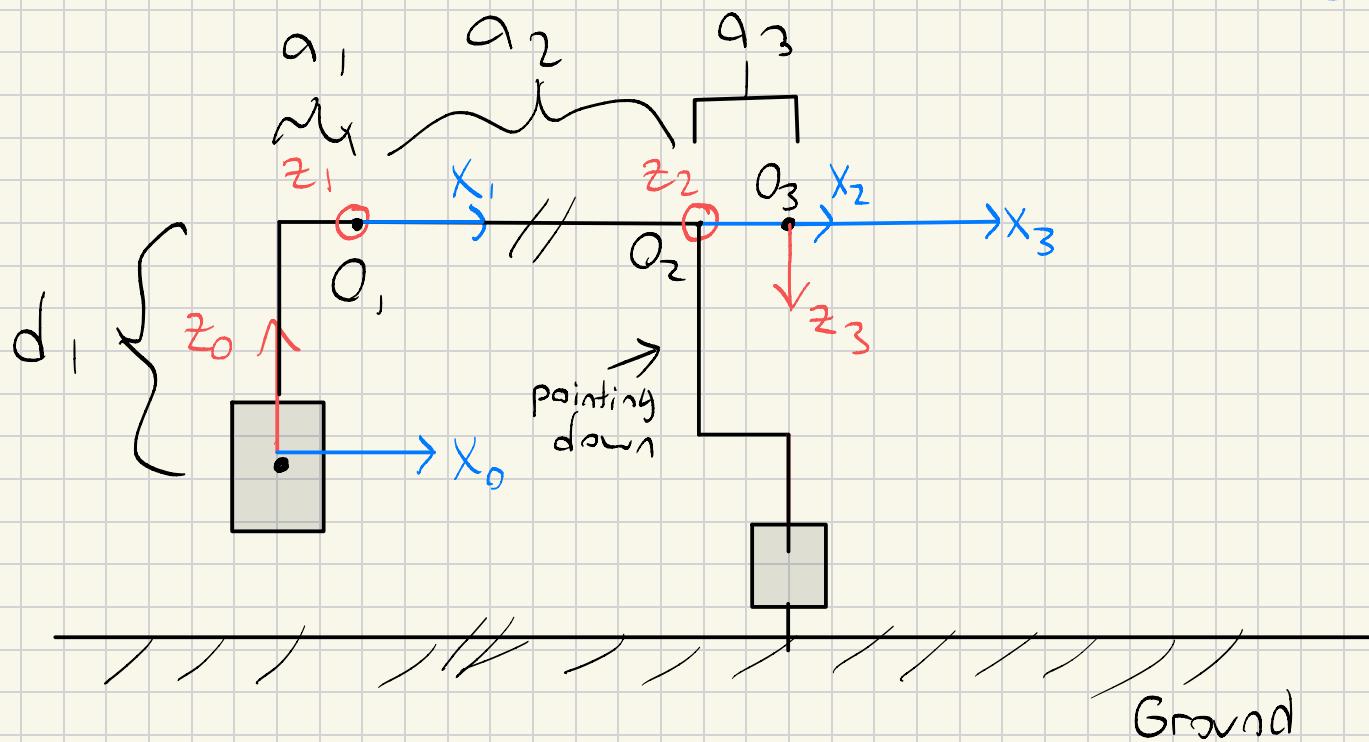
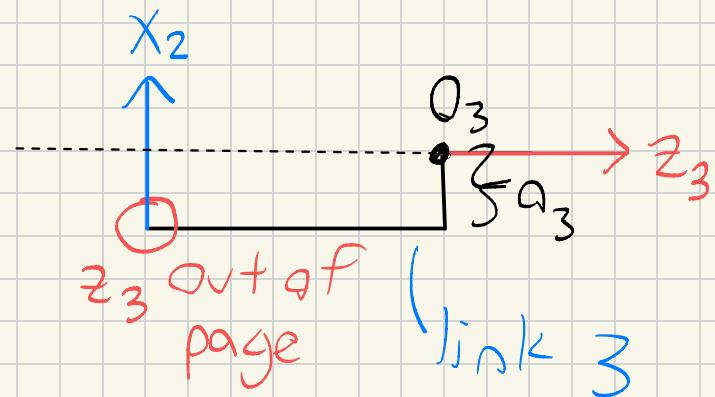
θ_i shown in diagram

α_i shown in diagram

check:

$$\theta_1 = \theta_2 = \theta_3 = 0$$

Side view:

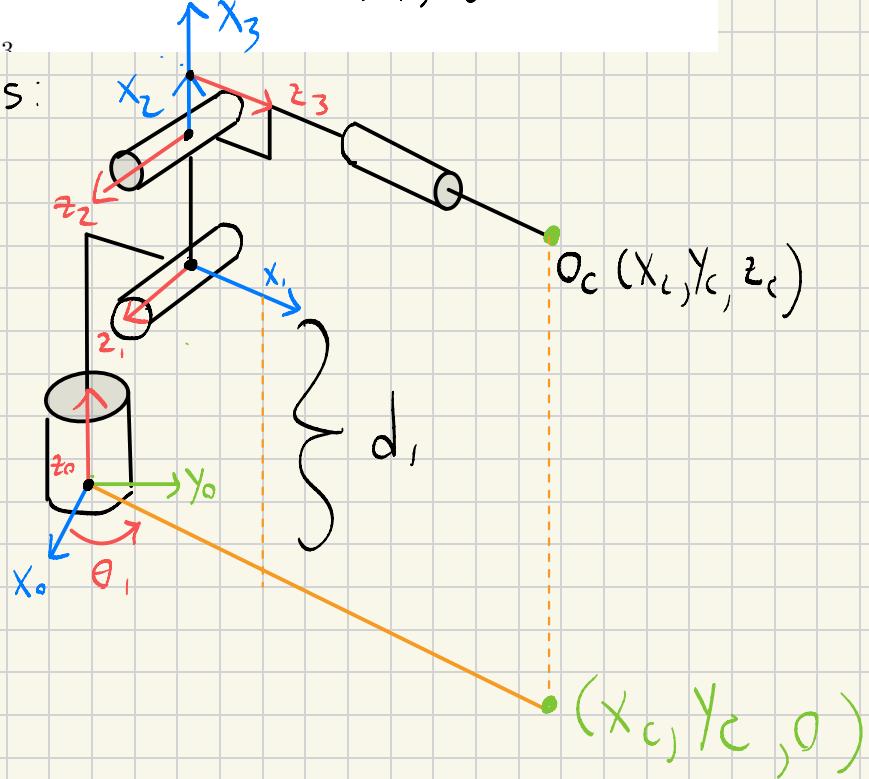
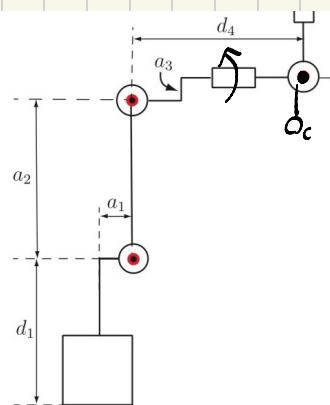


2. Derive the inverse kinematics of the robot. Specifically, given $R_6^0(\theta_1, \dots, \theta_6) = R_d$ and $o_6^0(\theta_1, \dots, \theta_6) = o_d^0$, find $(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6)$. You will find two solutions: elbow up and elbow down. Find the elbow up solution. Write your derivations neatly on paper. As in lab 1, one can solve the inverse kinematics problem by the technique of kinematic decoupling in which the problem is divided in two parts: inverse position and inverse orientation.

- The position of the wrist centre o_c is shown in Figure 2. First find $(\theta_1, \theta_2, \theta_3)$ such that $o_c^0(\theta_1, \theta_2, \theta_3) = [x_c \ y_c \ z_c]^T = o_d^0 - R_d \begin{bmatrix} -a_6 \\ 0 \\ d_6 \end{bmatrix}$.
- Then solve the equation

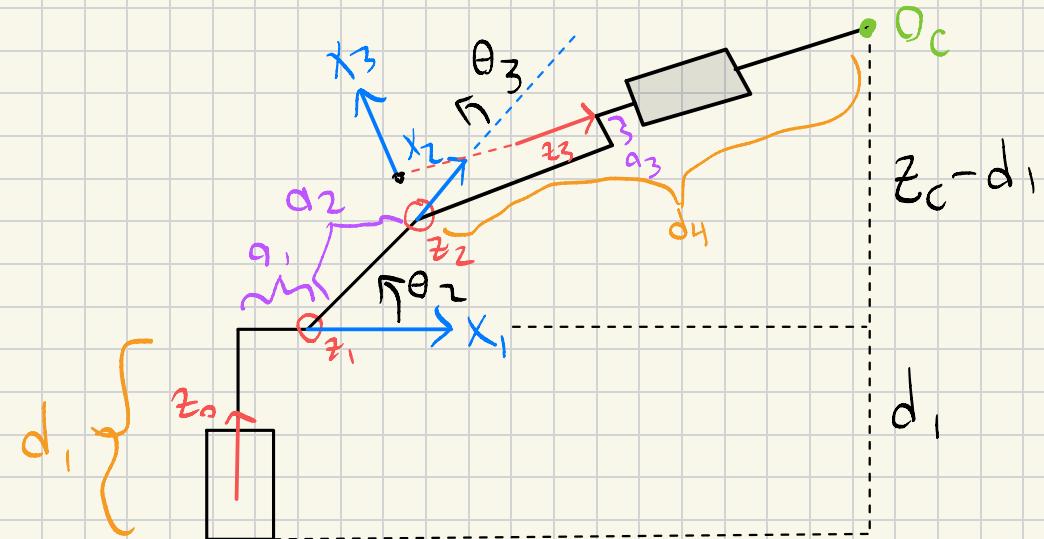
$$R_6^3(\theta_4, \theta_5, \theta_6) = (R_3^0)^T R_d \text{ for } \theta_4, \theta_5, \theta_6$$

Inverse Position Kinematics:



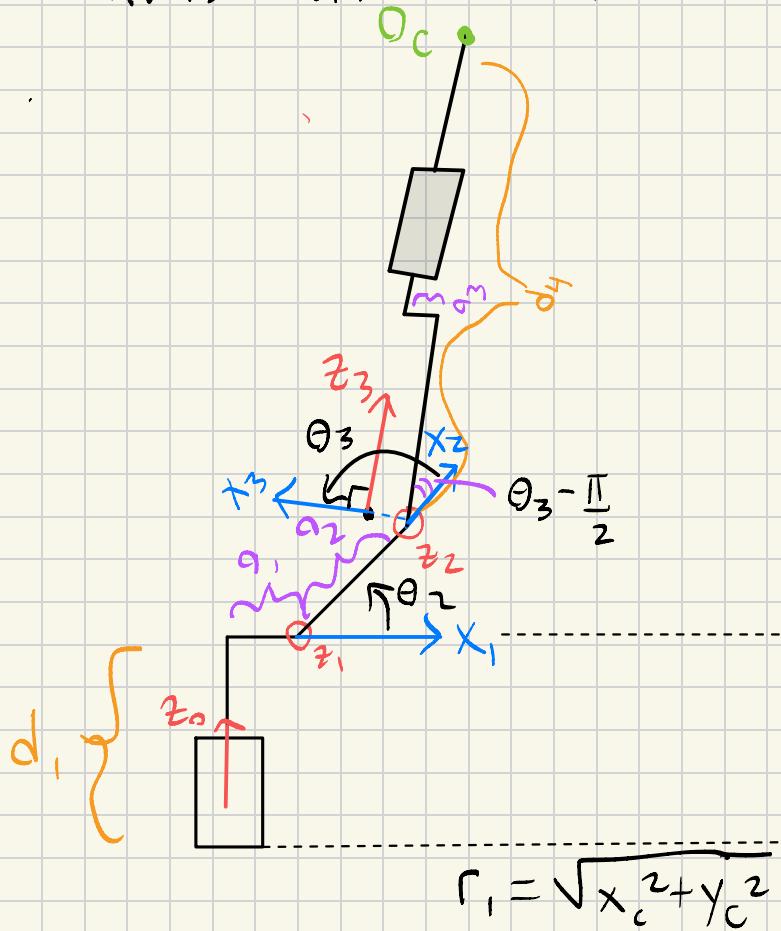
$$\theta_1 = \text{atan2}(y_c, x_c)$$

Side View Trig: Derive θ_3 & θ_2

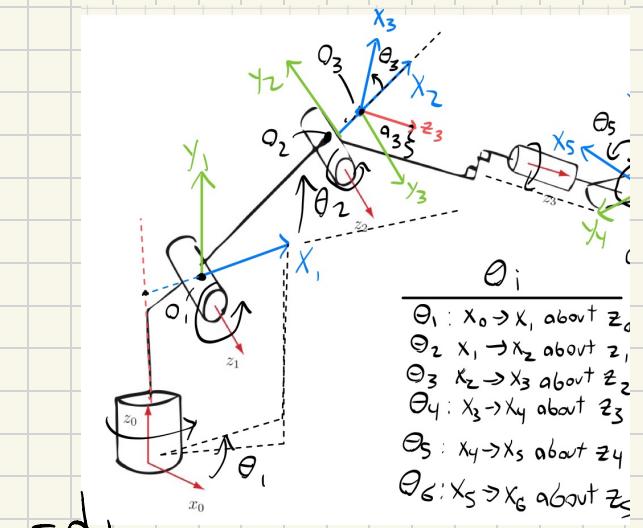
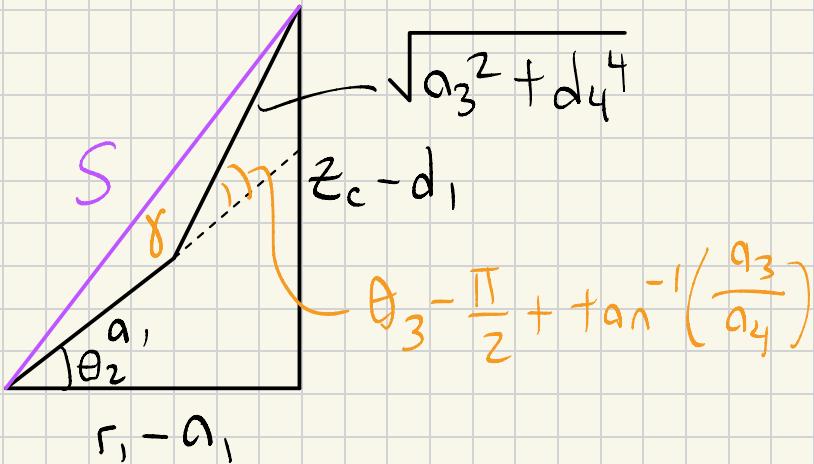


$$r_1 = \sqrt{x_c^2 + y_c^2}$$

Inverse Orientation Kinematics

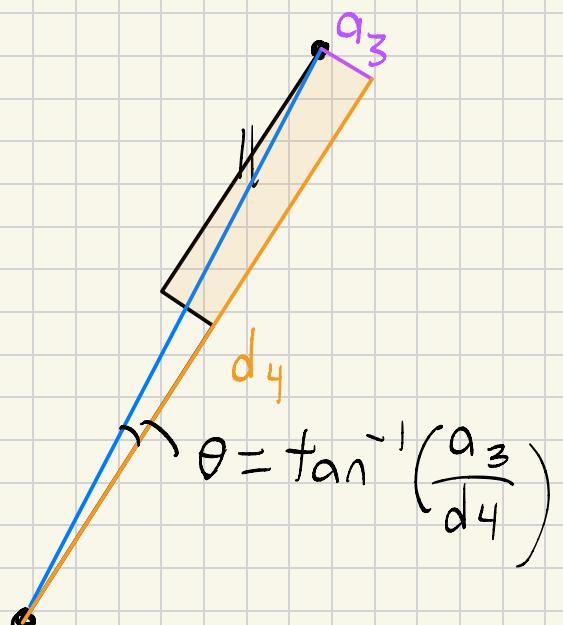


$$r_1 = \sqrt{x_c^2 + y_c^2}$$



- θ_i
- $\theta_1: x_0 \rightarrow x_1$ about z_0
 - $\theta_2: x_1 \rightarrow x_2$ about z_1
 - $\theta_3: x_2 \rightarrow x_3$ about z_2
 - $\theta_4: x_3 \rightarrow x_4$ about z_3
 - $\theta_5: x_4 \rightarrow x_5$ about z_4
 - $\theta_6: x_5 \rightarrow x_6$ about z_5

Link 3/4 fixed angle



$$\theta = \tan^{-1}\left(\frac{a_3}{d_4}\right)$$