1. Figure 3 depicts the robot in a generic configuration away from the rest position. Print out this page and draw on the figure coordinate frames $o_1x_1y_1z_1$, $o_2x_2y_2z_2$, $o_3x_3y_3z_3$, $o_4x_4y_4z_4$ and $o_5x_5y_5z_5$ according to the DH convention, and write the DH table of the robot. In particular, choose the x_1 axis parallel to the x_0 axis, $x_1 = x_0$ (this is done for convenience).

   If your DH frame assignment is correct, you should find that when $\theta_1 = \theta_2 = \theta_3 = 0$, link 2 is parallel to the ground, and link 3 is vertical, pointing downward. A schematic diagram of the KUKA robot arm in the HOME configuration is illustrated in Figure 4 which you should verify corresponds to $(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6) = (0, \pi/2, 0, 0, \pi/2, 0)$.

Side View Check:

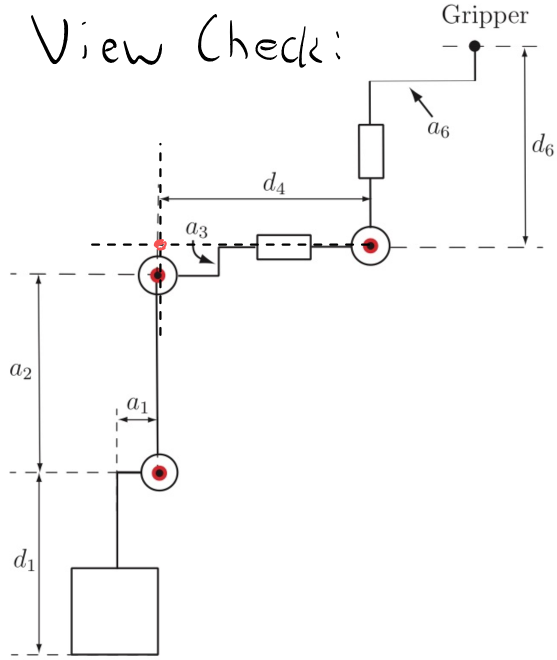
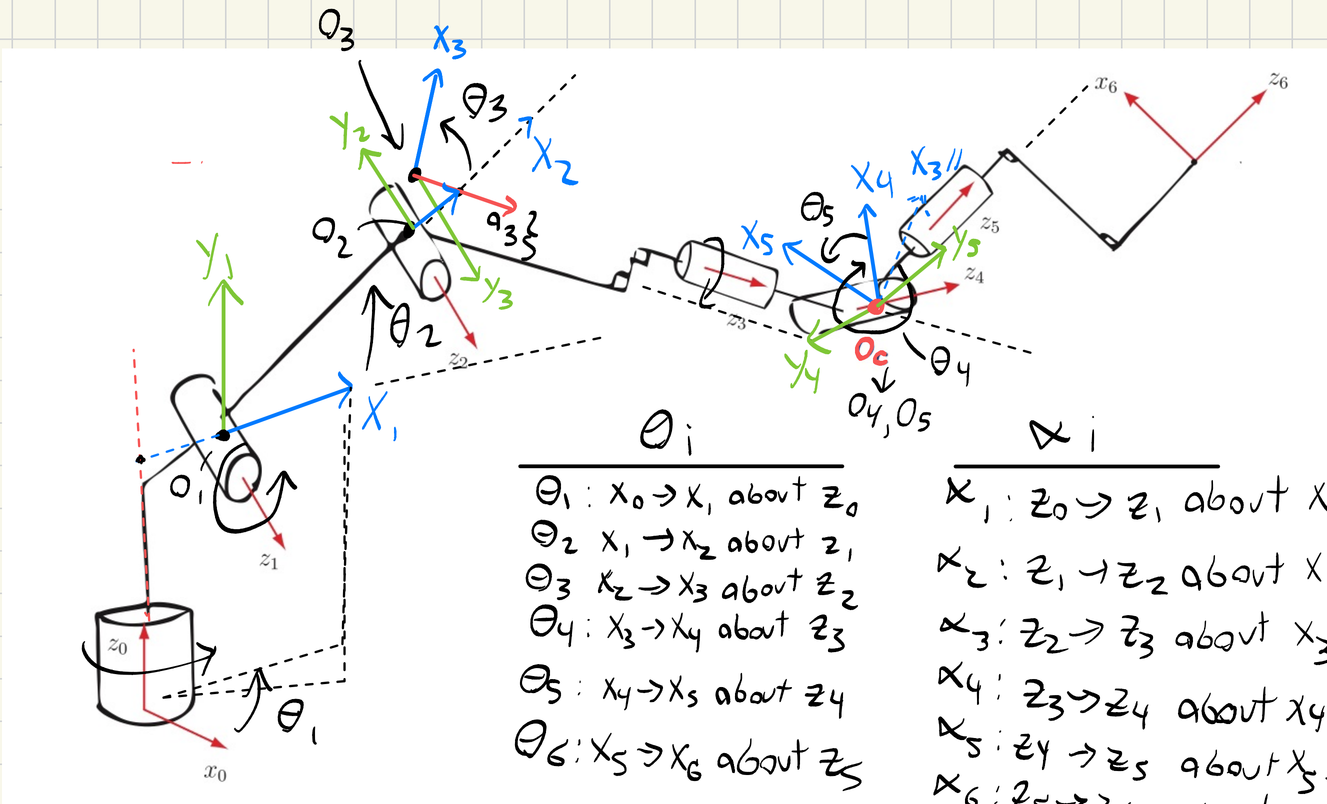


Figure 4: The KUKA robotic arm schematic diagram in the HOME configuration



θ_i
θ_1 : $x_0 \to x_1$ about z_0
θ_2 $x_1 \to x_2$ about z_1
θ_3 $x_2 \to x_3$ about z_2
θ_4 : $x_3 \to x_4$ about z_3
θ_5 : $x_4 \to x_5$ about z_4
θ_6 : $x_5 \to x_6$ about z_5

α_i
α_1 : $z_0 \to z_1$ about $x_1 = \pi/2$
α_2 : $z_1 \to z_2$ about $x_2 = 0$
α_3 : $z_2 \to z_3$ about $x_3 = \pi/2$
α_4 : $z_3 \to z_4$ about $x_4 = -\pi/2$
α_5 : $z_4 \to z_5$ about $x_5 = \pi/2$
α_6 : $z_5 \to z_6$ about $x_6 = 0$

Figure 3: Schematic diagram of KUKA robot arm not at rest.

✓ Check: $\theta_1, \theta_2, \theta_3 = 0$ that $\theta_2 \parallel$ GND & link 3 points down

DH Table:

Links	a_i	α_i	d_i	θ_i
1	a_1	$\pi/2$	d_1	θ_1
2	a_2	0	0	θ_2
3	a_3	$\pi/2$	0	θ_3
4	0	$-\pi/2$	d_4	θ_4
5	0	$\pi/2$	0	θ_5
6	$-a_6$	0	d_6	θ_6

Given: d_i 4 a_i (offsets)

	d_1	400 mm
	a_1	25 mm
	a_2	315 mm
	a_3	35 mm
	d_4	365 mm
−	a_6	296.23 mm
+	d_6	161.44 mm

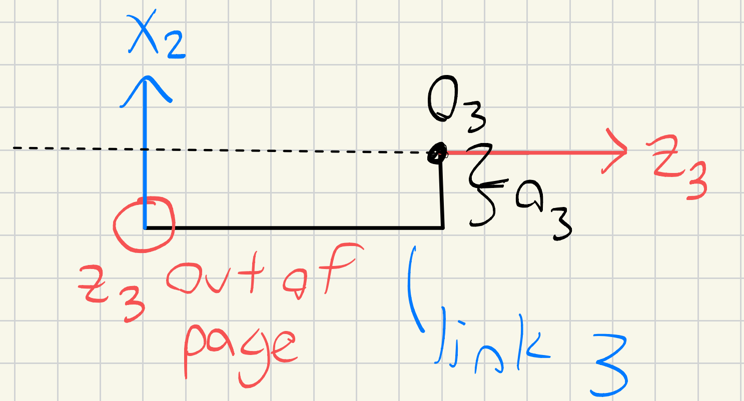
θ_i shown in diagram

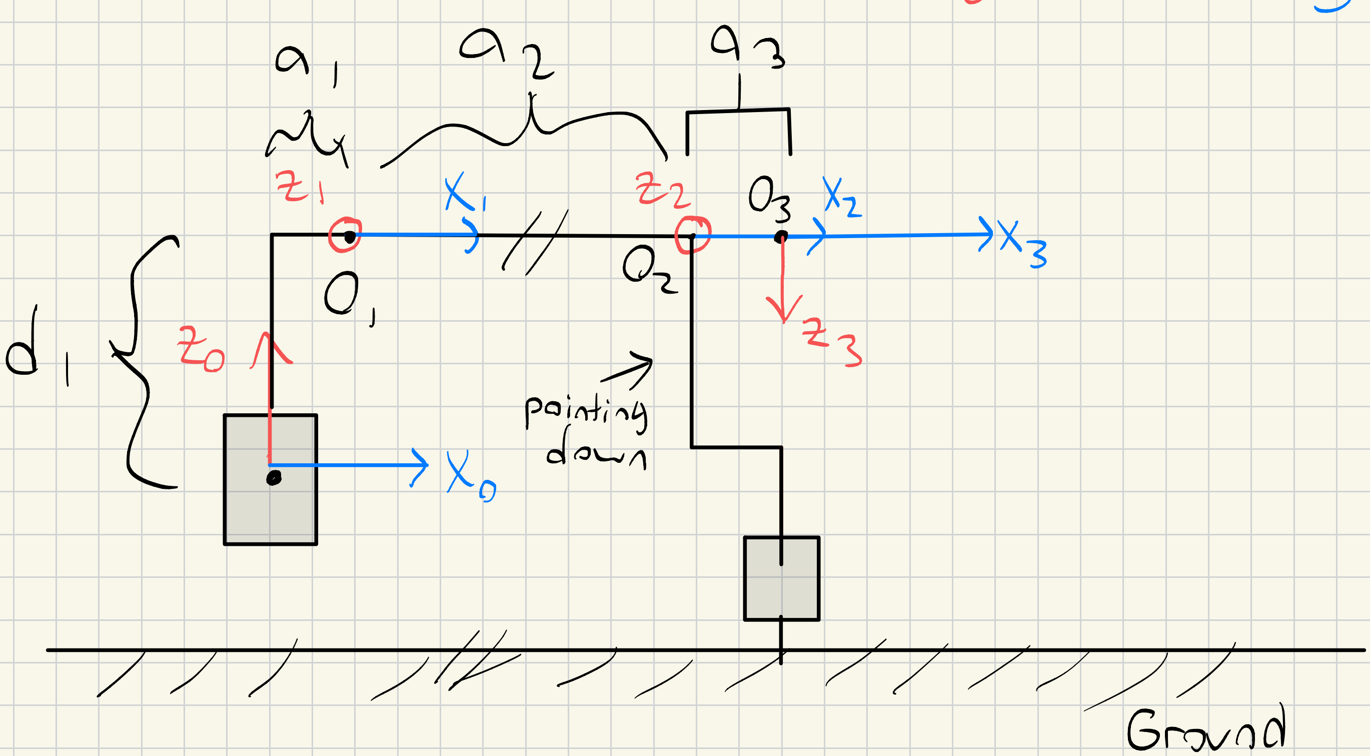
α_i shown in diagram

Check:

$\theta_1 = \theta_2 = \theta_3 = 0$

Side view:





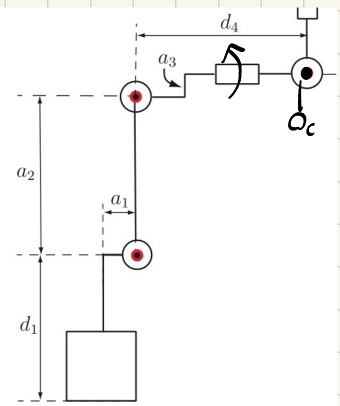
2. Derive the inverse kinematics of the robot. Specifically, given $R_6^0(\theta_1, \ldots, \theta_6) = R_d$ and $o_6^0(\theta_1, \ldots, \theta_6) = o_d^0$, find $(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6)$. You will find two solutions: elbow up and elbow down. Find the elbow up solution. Write your derivations neatly on paper. As in lab 1, one can solve the inverse kinematics problem by the technique of kinematic decoupling in which the problem is divided in two parts: inverse position and inverse orientation.

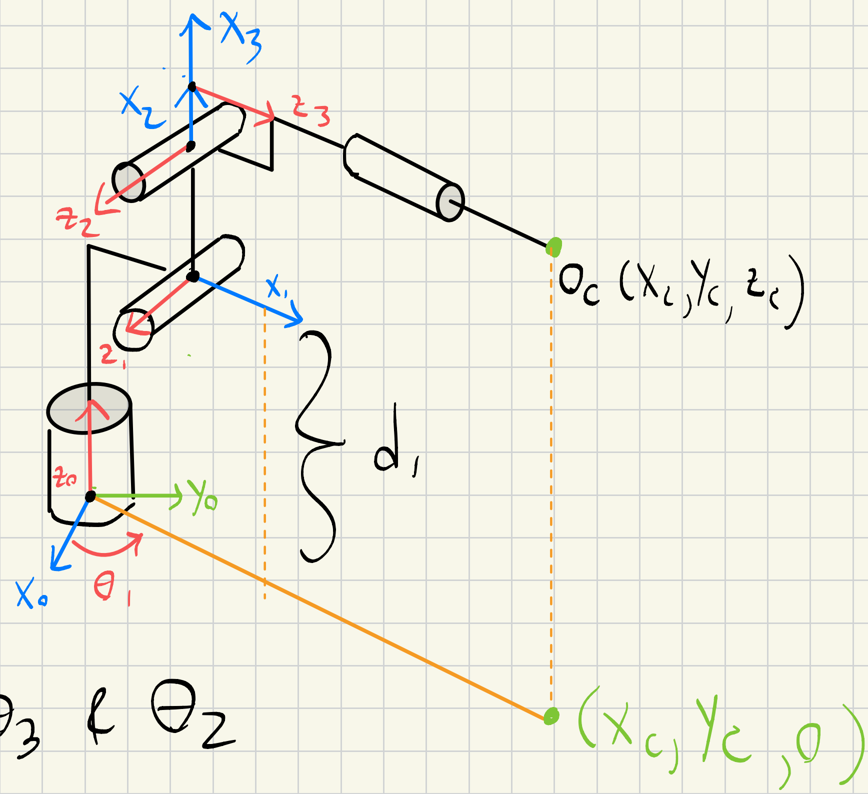
   - The position of the wrist centre o_c is shown in Figure 2. First find $(\theta_1, \theta_2, \theta_3)$ such that $o_c^0(\theta_1, \theta_2, \theta_3) = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = o_d^0 - R_d \begin{bmatrix} -a_6 \\ 0 \\ d_6 \end{bmatrix}$.
     ← +ve due to sign convention from side view
     ← -ve due to sign convention from side view
   - Then solve the equation

$$R_6^3(\theta_4, \theta_5, \theta_6) = (R_3^0)^\top R_d. \text{ for } \theta_4, \theta_5, \theta_6$$

Inverse Position Kinematics: solving for $\theta_1, \theta_2, \theta_3$ through geometry.

Given:

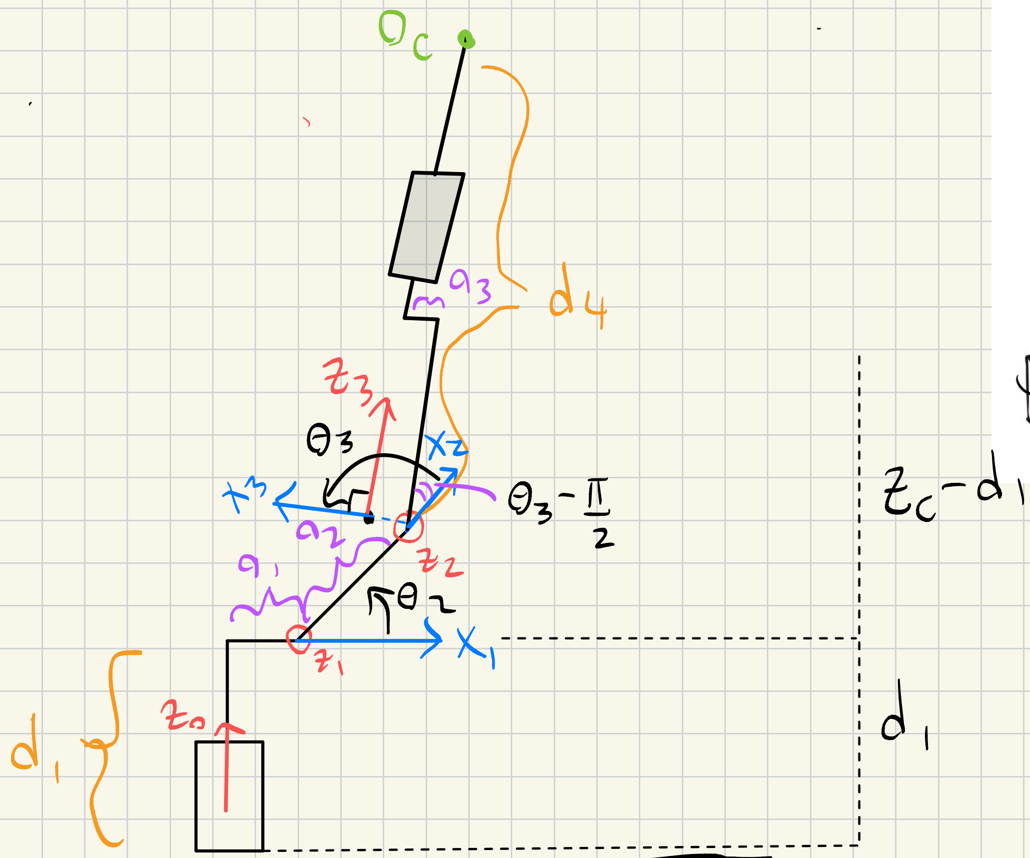


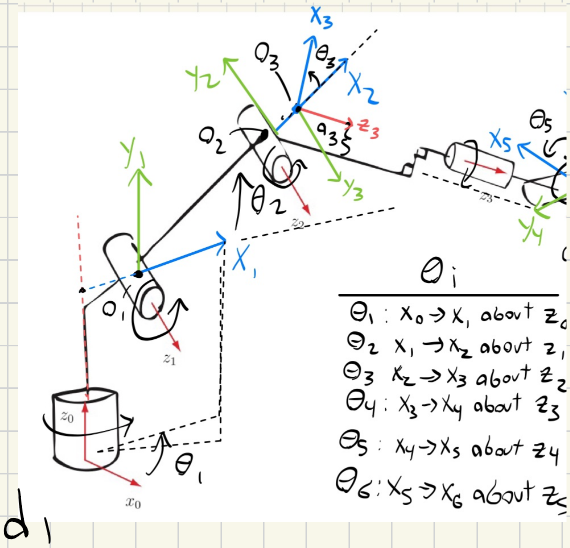


$$\boxed{\theta_1 = \text{atan2}(y_c, x_c)}$$

Side View: Derive θ_3 & θ_2

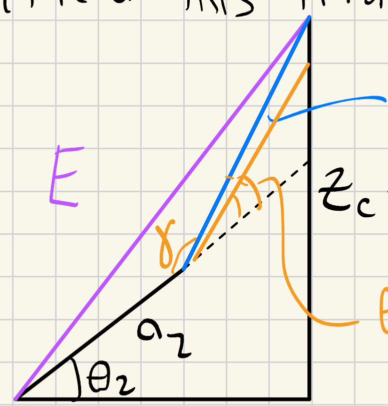
IKP Side View:





$$r_1 = \sqrt{x_c^2 + y_c^2}$$

Formed this triangle from θ_2 and above:



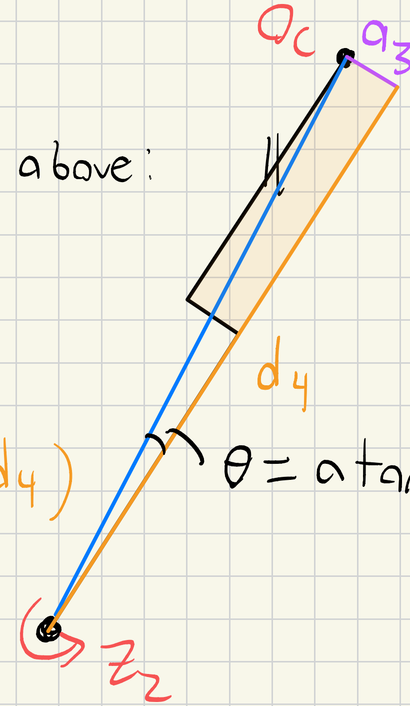
$$F = \sqrt{a_3^2 + d_4^2}$$

$$z_c - d_1 = s$$

$$\theta_3 - \frac{\pi}{2} + \operatorname{atan2}(a_3, d_4)$$

$$r = r_1 - a_1$$

Link 3/4 fixed angle



$$\theta = \operatorname{atan2}(a_3, d_4)$$

Using cosine law to solve for γ:

$$r^2 + s^2 = a_2^2 + a_3^2 + d_4^2 - 2a_2\sqrt{a_3^2 + d_4^2}\cos\left(\pi - \theta_3 + \frac{\pi}{2} - \operatorname{atan2}(a_3, a_4)\right)$$

$$\downarrow$$

Simplifying simplification:

simplify $\pi - \theta_3 + \frac{\pi}{2} - \text{atan2}(a_3, d_4) = \frac{3\pi}{2} - \theta_3 - \text{atan2}(a_3, d_4)$

Next we know $\cos\left(\frac{3\pi}{2} - X\right) = -\sin(X)$

$\therefore$ let $X = \theta_3 + \text{atan}(a_3, d_4)$

$$r^2 + s^2 = a_2^2 + a_3^2 + d_4^2 + 2a_2\sqrt{a_3^2 + d_4^2}\,\sin(\theta_3 + \text{atan2}(a_3, d_4))$$

$$\sin(\theta_3 + \text{atan2}(a_3, d_4)) = \boxed{\frac{(r^2 + s^2) - (a_2^2 + a_3^2 + d_4^2)}{2a_1\sqrt{a_3^2 + d_4^2}} = D}$$

corresponding cosine term is:

$$\cos(\theta_3 + \text{atan2}(a_3, d_4)) = \pm\sqrt{1 - \sin^2(\theta_3 + \text{atan2}(a_3, d_4))}$$

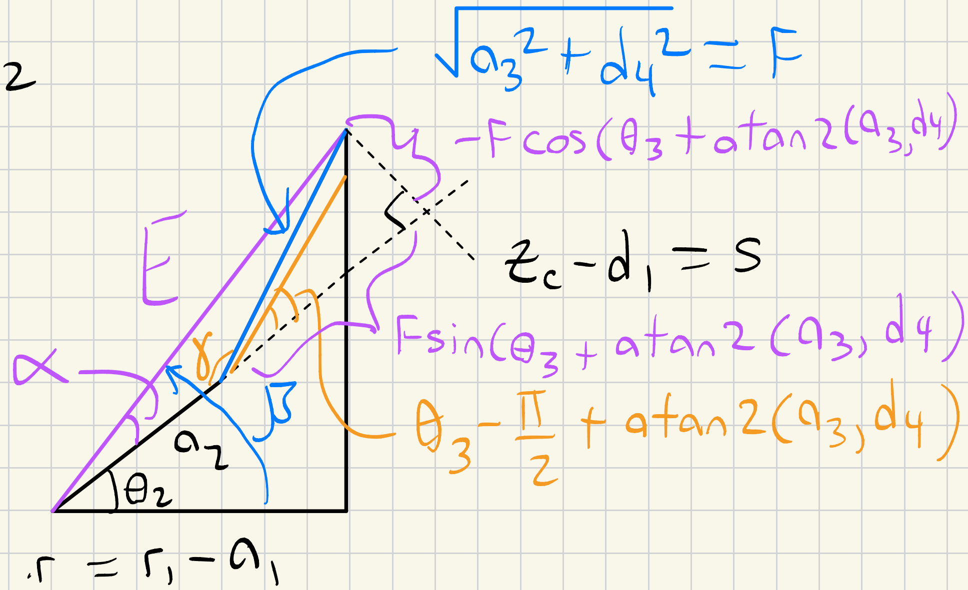
using atan 2 propetry $\theta_3 = \text{atan2}(\sin\theta_3, \cos\theta_3)$

$$\theta_3 = \text{atan2}\left(D, \pm\sqrt{1 - D^2}\right) - \text{atan2}(a_3, d_4)$$

for elbow up solution

$$\boxed{\theta_3 = \text{atan2}\left(D, +\sqrt{1 - D^2}\right) - \text{atan2}(a_3, d_4)}$$

Solve for θ_2



$\beta = \text{atan2}(s, r)$

$\alpha = \text{atan2}(-F\cos(\theta_3 + \text{atan2}(a_3, d_4)), a_2 + F\sin(\theta_3 + \text{atan2}(a_3, d_4)))$

$$\boxed{\theta_2 = \beta - \alpha}$$

Part 2) Inverse orientation kinematics, we can leverage that we have a spherical wrist that has zyz euler angles: to determine $\theta_4, \theta_5, \theta_6$ expressions.

desired rotation matrix

Knowing $R_6^0 = R_3^0 R_6^3 = R_d$; $R_6^3(q_4, q_5, q_6) = [R_3^0(q_1, q_2, q_3)]^T R_d$

From lecture we derived:

$$R_6^3 = \begin{bmatrix} * & * & c_{\theta_4} s_{\theta_5} \\ * & * & s_{\theta_4} s_{\theta_5} \\ -s_{\theta_5} c_{\theta_6} & s_{\theta_5} s_{\theta_6} & c_{\theta_5} \end{bmatrix}$$

where we proved:

$\theta_4 = \text{atan2}(m_{23}, m_{13})$ or $\theta_4 = \text{atan2}(-m_{23}, -m_{13})$

$\theta_5 = \text{atan2}(\sqrt{1 - m_{33}^2}, m_{33})$ or $\theta_5 = \text{atan2}(-\sqrt{1 - m_{33}^2}, m_{33})$

$\theta_6 = \text{atan2}(m_{32}, -m_{31})$ or $\theta_6 = \text{atan2}(-m_{32}, m_{31})$

Using this set of $\theta_4, \theta_5, \theta_6$