

# Q1. Coordinate Frame mark-up

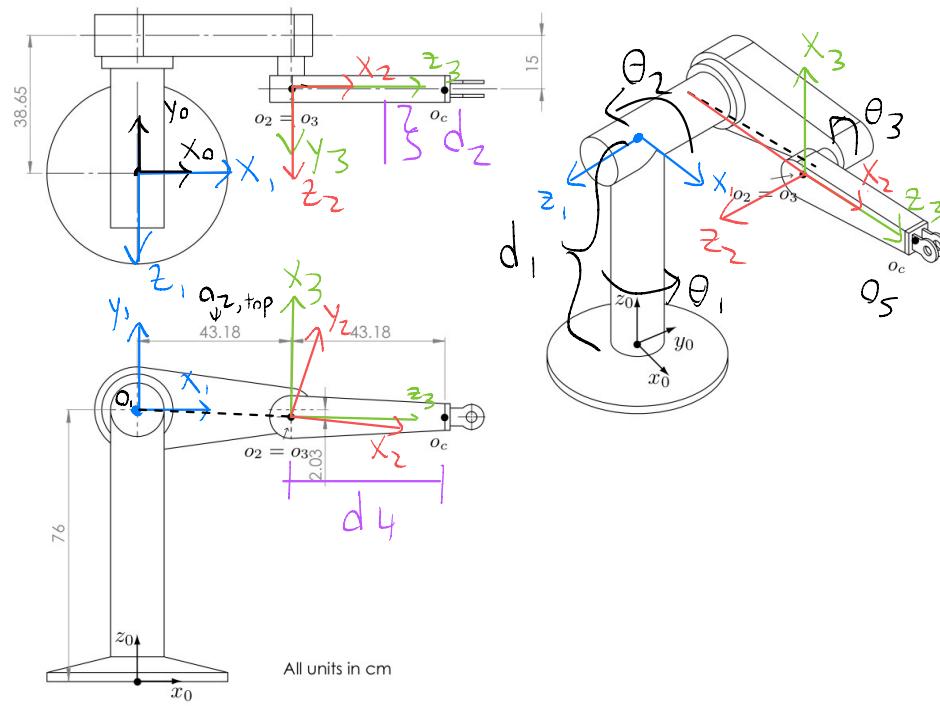


Figure 1: Schematics of the PUMA560 robot

## Q2. DH Table

Link	$a_i$	$\alpha_i$	$d_i$	$\theta_i$
1	0	$\pi/2$	$d_1$	$\theta_1$
2	$a_2$	0	$d_2$	$\theta_2$
3	0	$\pi/2$	0	$\theta_3$
4	0	$-\pi/2$	$d_4$	$\theta_4$
5	0	$\pi/2$	0	$\theta_5$
6	0	0	$d_6$	$\theta_6$

Variable def'n:

$$a_2 = \sqrt{43.18^2 + 2.03^2} = 43.23 \text{ cm}$$

$$d_1 = 76 \text{ cm}$$

$$d_2 = -(38.65 - 15 \text{ cm}) = -23.65 \text{ cm}$$

$$d_4 = 43.18 \text{ cm}$$

$$d_6 = 20 \text{ cm}$$

### Q3. Inverse kinematics:

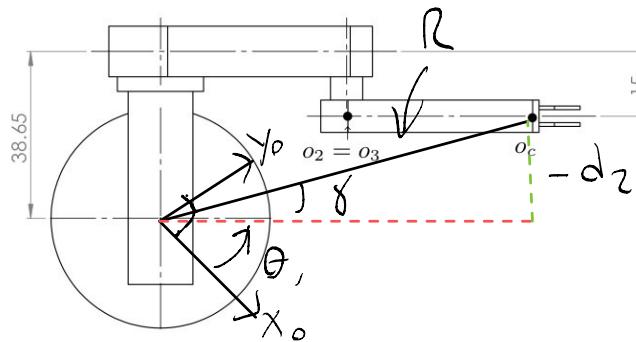
Using kinematic decoupling as a solving technique

Wrist position  $O_c$  is dependent on joint angles  $(\theta_1, \theta_2, \theta_3)$

Part 1. Inverse position kinematics

$$O_c^o = O_c^o(\theta_1, \theta_2, \theta_3); \text{ Find } O_3^o(\theta_1, \theta_2, \theta_3)$$

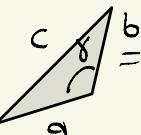
$$\theta_1 = \tan^{-1} \frac{y_o}{x_o}, \gamma = \tan^{-1} \frac{y_o}{x_o} - \tan^{-1} (23.65, 86.36)$$



$\theta_3$  derivation:

using cosine rule, we know:

$$c^2 = a^2 + b^2 - 2ab \cos(\gamma)$$



where,

$$a = a_2, b = d_4, \gamma = \phi, c = PR$$

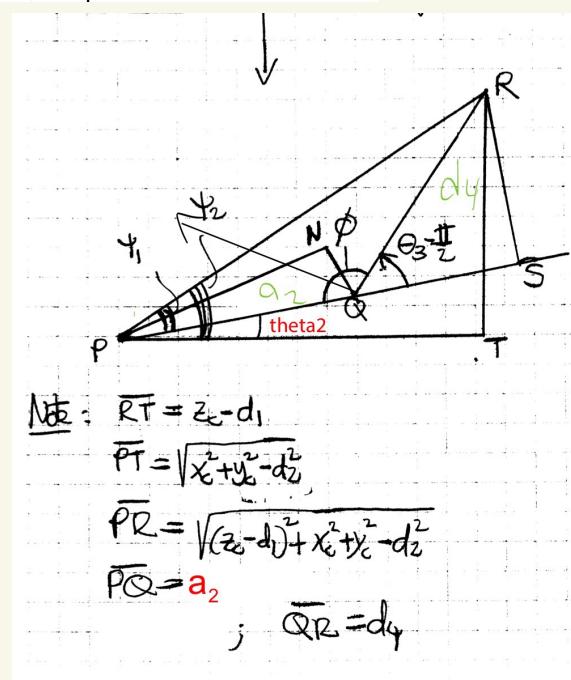
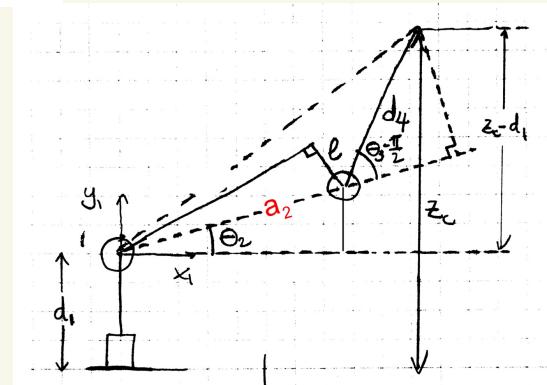
substituting  $a_2, d_4, \phi, c$  and rearranging:

$$\therefore \sin(\theta_3) = \frac{a_2^2 + d_4^2 - (r^2 - s^2)}{2a_2 d_4} = :D$$

$$\text{where } r = PT = \sqrt{x_c^2 + y_c^2 + d_2^2}$$

$$D = \frac{a_2^2 + d_4^2 - (z_c - d_1)^2 - x_c^2 - y_c^2 + d_2^2}{2a_2 d_4}$$

$$\theta_3 = \tan^{-1} (D, \pm \sqrt{1 - D^2})$$



$\theta_2$  derivation

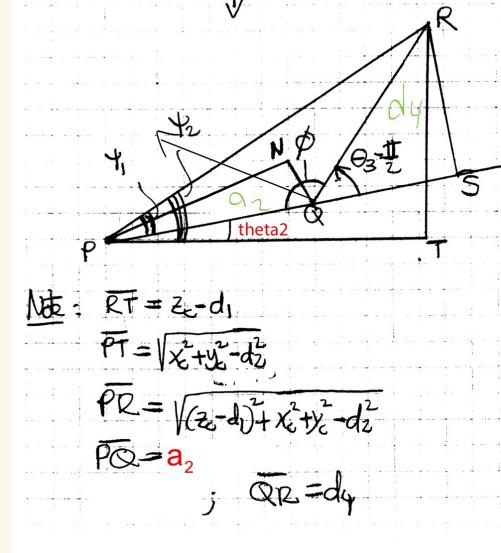
$$\Psi_1 = \text{atan}2(RS, PS)$$

$$PS = a_2 + d_4 \cos(\theta_3 - \pi/2) \Rightarrow a_2 + d_4 \sin(\theta_3)$$

$$RS = d_4 \sin(\theta_3 - \pi/2) = -d_4 \cos(\theta_3)$$

$$\Psi_2 = \text{atan}2(RT, PT)$$

$$\theta_2 = \text{atan}2(RT, PT) - \text{atan}2(RS, PS)$$



Part 2) Inverse orientation kinematics, we can leverage that we have a spherical wrist that has 2y2 euler angles to determine  $\theta_4, \theta_5, \theta_6$  expressions.

Knowing  $R_6^o = R_3^o R_6^3 = R_d$ ;  $R_6^3(q_4, q_5, q_6) = [R_3^o(\theta_1, \theta_2, \theta_3)]^T R_d$

From lecture we derived:

$$R_6^3 = \begin{bmatrix} * & * & \cos \theta_5 \sin \theta_5 \\ * & * & \sin \theta_5 \sin \theta_5 \\ -\sin \theta_5 \cos \theta_5 \sin \theta_5 \cos \theta_5 & \cos \theta_5 \end{bmatrix}$$

where we proved:

$$\theta_4 = \text{atan}2(M_{23}, M_{13}) \quad \text{or} \quad \theta_4 = \text{atan}2(-M_{23}, -M_{13})$$

$$\theta_5 = \text{atan}2(\sqrt{1 - M_{33}^2}, M_{33}) \quad \text{or} \quad \theta_5 = \text{atan}2(-\sqrt{1 - M_{33}^2}, M_{33})$$

$$\theta_6 = \text{atan}2(M_{32}, -M_{31}) \quad \text{or} \quad \theta_6 = \text{atan}2(-M_{32}, M_{31})$$