Q1. Coordinate Frame mark-up:

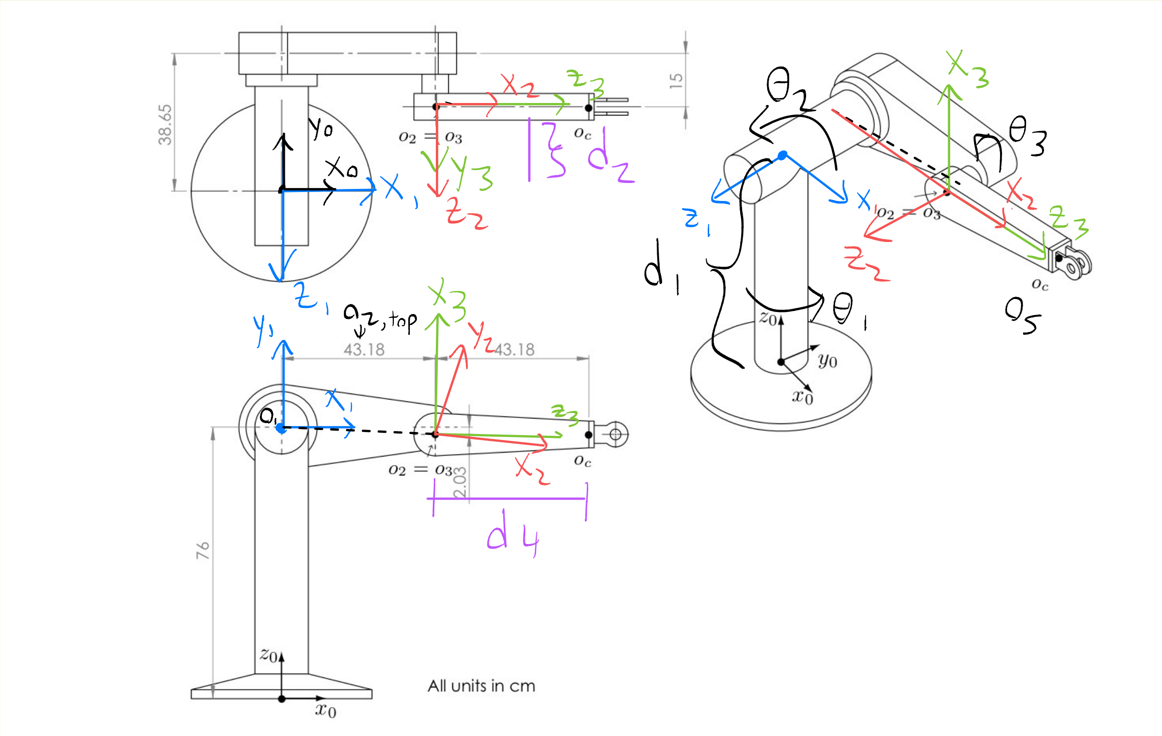


Figure 1: Schematics of the PUMA560 robot

Q2. DH Table

Link	a_i	α_i	d_i	θ_i
1	0	$\pi/2$	d_1	θ_1
2	a_2	0	d_2	θ_2
3	0	$\pi/2$	0	θ_3
4	0	$-\pi/2$	d_4	θ_4
5	0	$\pi/2$	0	θ_5
6	0	0	d_6	θ_6

Variable def'n:

$a_2 = \sqrt{43.18^2 + 2.03^2} = 43.23\text{ cm}$

$d_1 = 76\text{ cm}$

$d_2 = -(38.65 - 15\text{ cm}) = -23.65\text{ cm}$

$d_4 = 43.18\text{ cm}$

$d_6 = 20\text{ cm}$

Q3. Inverse Kinematics

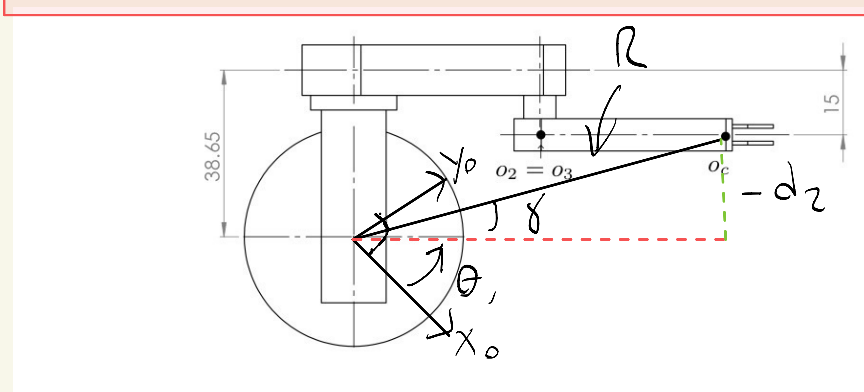
Using kinematic decoupling as a solving technique

Wrist position O_c is dependent on joint angles $(\theta_1, \theta_2, \theta_3)$

Part 1. Inverse position kinematics

$O_c^0 = O_c^0(q_1, q_2, q_3)$; Find $O_3^0(q_1, q_2, q_3)$

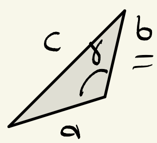
$$\boxed{\theta_1 = \measuredangle x_0, R - \gamma = \text{atan2}(y_c, x_c) - \text{atan2}\left(-d_2, \sqrt{x_c^2 + y_c^2 - d_2^2}\right)}$$



θ_3 derivation:

using cosine rule, we know:

$$^2 = a^2 + b^2 - 2ab\cos(\gamma)$$



where,

$a = a_2$, $b = d_4$, $\phi = \gamma$, $c = PR$

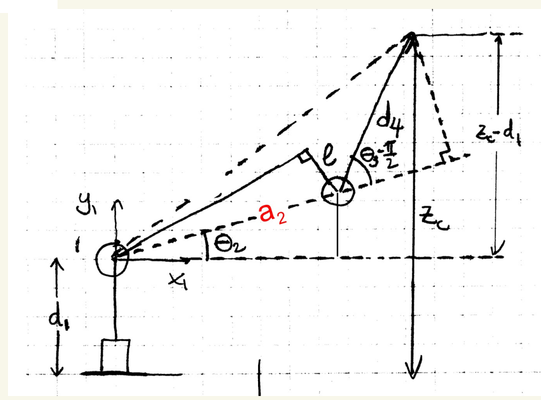
substituting a_2, d_4, ϕ, c and rearranging:

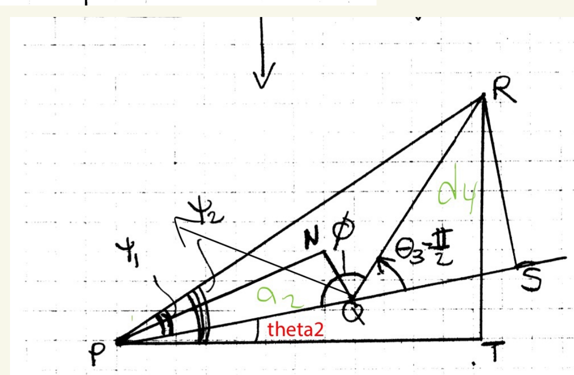
$$\therefore \sin(\theta_3) = \frac{a_2^2 + d_4^2 - (r^2 - s^2)}{2a_2 d_4} =: D$$

where $r = PT = \sqrt{x_c^2 + y_c^2 - d_2^2}$

$$D = \frac{(z_c - d_1)^2 + x_c^2 + y_c^2 - d_2^2 - a_2^2 - d_4^2}{2a_2 d_4}$$

$$\boxed{\theta_3 = \text{atan2}\left(D, \pm\sqrt{1 - D^2}\right)}$$





Note: $\overline{RT} = z_c - d_1$

$\overline{PT} = \sqrt{x_c^2 + y_c^2 - d_2^2}$,

$\overline{PR} = \sqrt{(z_c - d_1)^2 + x_c^2 + y_c^2 - d_2^2}$

$\overline{PQ} = a_2$

; $\overline{QR} = d_4$

θ_2 derivation

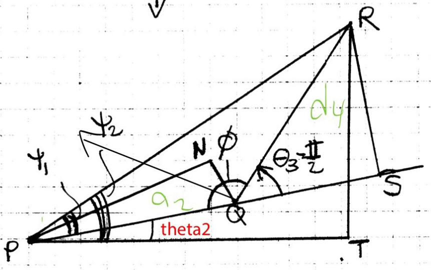
$\Psi_1 = \text{atan2}(RS, PS)$

$PS = a_2 + d_4\cos(\theta_3 - \pi/2) \Rightarrow a_2 + d_4\sin(\theta_3)$

$RS = d_4\sin(\theta_3 - \pi/2) = -d_4\cos(\theta_3)$

$\Psi_2 = \text{atan2}(RT, PT)$

$$\boxed{\theta_2 = \text{atan2}(RT, PT) - \text{atan2}(RS, PS)}$$



Note: $\overline{RT} = z_c - d_1$

$\overline{PT} = \sqrt{x_c^2 + y_c^2 - d_2^2}$

$\overline{PR} = \sqrt{(z_c - d_1)^2 + x_c^2 + y_c^2 - d_2^2}$

$\overline{PQ} = a_2$; $\overline{QR} = d_4$

Part 2) Inverse orientation kinematics, we can leverage that we have a spherical wrist that has zyz euler angles: to determine $\theta_4, \theta_5, \theta_6$ expressions.

Knowing $R_6^0 = R_3^0 R_6^3 = \underset{\text{desired rotation matrix}}{R_d}$; $R_6^3(q_4, q_5, q_6) = [R_3^0(q_1, q_2, q_3)]^T R_d$

From lecture we derived:

$$R_6^3 = \begin{bmatrix} * & * & c_{\theta_4} s_{\theta_5} \\ * & * & s_{\theta_4} s_{\theta_5} \\ -s_{\theta_5} c_{\theta_6} & s_{\theta_5} s_{\theta_6} & c_{\theta_5} \end{bmatrix}$$

where we proved:

$$\boxed{\begin{aligned} \theta_4 &= \text{atan2}(M_{23}, M_{13}) \quad \text{or} \quad \theta_4 = \text{atan2}(-M_{23}, -M_{13}) \\ \theta_5 &= \text{atan2}\left(\sqrt{1 - M_{33}^2}, M_{33}\right) \quad \text{or} \quad \theta_5 = \text{atan2}\left(-\sqrt{1 - M_{33}^2}, M_{33}\right) \\ \theta_6 &= \text{atan2}(M_{32}, -M_{31}) \quad \text{or} \quad \theta_6 = \text{atan2}(-M_{32}, M_{31}) \end{aligned}}$$