PRE-LAB 4

3 Preparation

Please submit a **complete** preparation at the beginning of the lab session. Prior to the lab, you should make sure that your repulsive and attractive force functions return the expected values. In this lab, instead of having a pencil attached to the gripper, we will be using the gripper to grasp objects. The offset a_6 now corresponds to the length of the gripper which is approximately $a_6 = 156$mm.

1. Modify your motion planning algorithm from Lab 3 (developed for the PUMA 560 manipulator) to work for the KUKA manipulator. The main difference is the DH table. For better results, set $a_6 = 0$ when deriving the attractive and repulsive force functions. You can create a second DH table called `DH_forces` for this purpose. This shifts the point on the gripper where the attractive and repulsive forces act. However you must still use $a_6 = 156$ in your inverse kinematics computations. Otherwise the gripper will collide with the ground.
2. Write neatly on paper the expression of repulsive forces for the following two objects: 1) repulsion upward from the Workspace plane, which we assume to be parallel to the $x_0 - y_0$ plane, and have a z_0 value of 32 mm; 2) Repulsion from a cylinder of finite length. The bottom of the cylinder lies on the $x_0 - y_0$ plane and the height of the cylinder is a parameter h.
3. Modify your repulsive function to account for the two new objects described in point 2 above: the horizontal plane $z = 32$ mm and the finite length cylinder. Set $\eta = 1$ in the repulsive function.

Artificial Potential Field (APF) framework:
Eq(7.5) defines the repulsive potential as:

$$\boxed{U_{rep}(q) = \frac{1}{2}\eta\left(\frac{1}{\rho(q)} - \frac{1}{\rho_0}\right)^2} \quad \text{for } \rho(q) \le \rho_0$$

where: $\rho(q) = \|O_i(q) - b\|$ is the distance from robot's point $O_i(q)$ to closest obstacle point

ρ_0 – threshold distance beyond which repulsion is 0

η – positive scaling factor

The gradient of the repulsive potential determines the repulsive force:

$$\boxed{F_{rep}(q) = \eta\left(\frac{1}{\rho(q)} - \frac{1}{\rho_0}\right)\frac{1}{\rho^2(q)}\nabla\rho(q)}$$

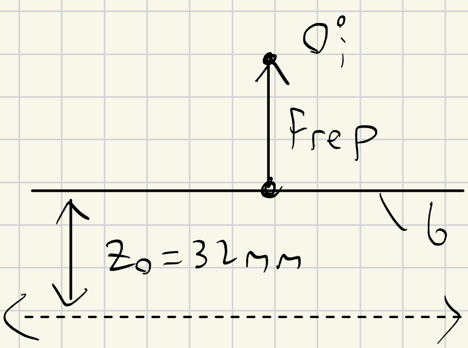
where $$\boxed{\nabla\rho(q) = \frac{O_i(q) - b}{\rho(q)}}$$

2. Expression for Repulsive forces.

1) Repulsion upwards from workspace plane: (Plane Case)
↳ // to $x_0 - y_0$ plane
↳ $z_0 = 32$mm

$$b = \begin{bmatrix} O_x \\ O_y \\ 32 \end{bmatrix} \quad O_i - b = \begin{bmatrix} 0 \\ 0 \\ O_z - 32 \end{bmatrix}$$

Case 1 & 2:



Note: O_i^0 can be above or below relative to workspace

For all cases
i. use $O_i - b$ to solve for $\rho(q)$
ii. substitute in $\rho(q)$ & $O_i - b$ into the following generalized Frep equation:

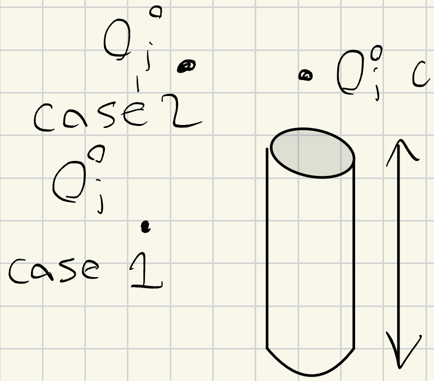
$$F_{rep}(q) \begin{cases} \eta\left(\frac{1}{\rho(q)} - \frac{1}{\rho_0}\right)\frac{O_i - b}{\rho^3(q)} & \text{if } \rho(q) \le d_0 \\ 0 & \text{if } \rho(q) > d_0 \end{cases}$$

* d_0 is the design parameter of the workspace plane.

case 3: if $O_z = 32$ *colliding with plane

$f_{rep} = 0$ according to design decision

Object 2: Cylindrical obstacle; cylinder of height h, centre at $c = (c_x, c_y)$ and axis is // to z_0 axis, of radius R



Case 1: if $0 \le z \le h$ & $\|O_{xy} - C_{xy}\| > R$
(point is within cylinder height)

Magnitude of distance to the cylinder's surface:
- need to subtract radius R again

$$\boxed{\|O_i(q) - b\| = \underbrace{\sqrt{(O_x - C_x)^2 + (O_y - C_y)^2}}_{\text{distance to cylinder center from DH frame of joint } i} - \underset{\uparrow \text{cylinder radius}}{R}}$$

position of center in cylinder

$$\begin{bmatrix} C_x \\ C_y \\ O_i(q)\cdot\hat{z} \end{bmatrix}$$

$$b = c + \overbrace{R \cdot \frac{O_i - C}{\|O_i - C\|}}^{\text{scaling direction vector by } R.}$$

$$\boxed{O_i(q) - b = \begin{bmatrix} O_x - C_x \\ O_y - C_y \\ 0 \end{bmatrix} - R \cdot \frac{1}{\sqrt{(O_x - C_x)^2 + (O_y - C_y)^2}} \begin{bmatrix} O_x - C_x \\ O_y - C_y \\ 0 \end{bmatrix}}$$

Case 2.

$O_z > h$ (point is above cylinder)

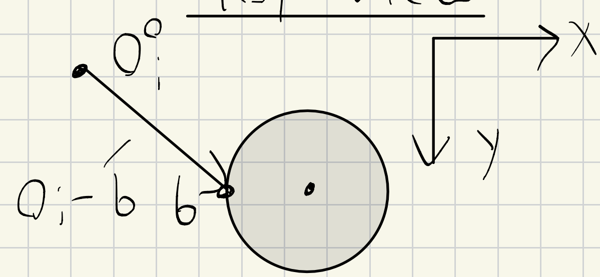
and

$\|O_{xy} - C_{xy}\| > R$

$$b = \begin{bmatrix} C_x \\ C_y \\ h \end{bmatrix} + R \frac{O_{xy} - C_{xy}}{\|O_{xy} - C_{xy}\|}$$

$$O_i - b = \begin{bmatrix} O_x - C_x \\ O_y - C_y \\ O_z - h \end{bmatrix} - R \frac{O_{xy} - C_{xy}}{\|O_{xy} - C_{xy}\|}$$

Top View



case 3: $O_z > h$ & $\|O_{xy} - C_{xy}\| \leq R$ * (directly above cylinder)

we can treat it the same as plane case

$$O_i - b = \begin{bmatrix} 0 \\ 0 \\ O_z - h \end{bmatrix}$$

case 4: if $O_z < 0$, then $h = 0$

use the same expressions from 2 & 3 and substitute h as equal to zero to obtain expressions for $O_i - b$

Case 5: $\|O_{xy} - C_{xy}\| < R$ and $0 < z < h$

* completely within the Cylinder our, then $F_{rep} = 0$ according to design decision

For all cases

i. use $O_i - b$ to solve for $\rho(q)$

ii. substitute in $\rho(q)$ & $O_i - b$ into the following generalized Frep equation:

$$F_{rep}(q) \begin{cases} \eta\left(\frac{1}{\rho(q)} - \frac{1}{\rho_0}\right)\frac{O_i - b}{\rho^3(q)} & \text{if } \rho(q) \leq d_0 \\ 0 & \text{if } \rho(q) > d_0 \end{cases}$$

* d_0 is the design parameter of the cylinder object