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The Containment Problem

a general introduction and the particular case for Steiner systems

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Associated primes

Definiton (Associated Prime)

Let M be an R-module. A prime ideal $\mathfrak{p} \subseteq R$ is an **associated prime** of M if there exists a non-zero element $a \in M$ such that $\mathfrak{p} = \mathrm{Ann}_R(a)$.

We define $\mathrm{Ass}_R(M)$ as the set of the associated primes of M.

For an ideal I we say that a prime is associated to I if it is associated to the R-module R/I.

Primary decomposition

We say that an ideal $\mathfrak{a} \subseteq R$ has a **primary decomposition** if there exists a finite set of primary ideal $\{\mathfrak{q}_1, ..., \mathfrak{q}_n\}$ such that:

$$\mathfrak{a} = \bigcap_{i=1}^{n} \mathfrak{q}_i$$