SNR estimation for Constant envelope signals in AWGN channel

YE Pei-jun AN Jian-ping
Department of E&E
Beijing Institute of Technology
Beijing 100081, China
yepeijun@bit.edu.cn

Abstract—**This paper presents a way of SNR estimation based on envelopes of the signals. To analyze the properties of the algorithm, we used MATLAB to simulate MPSK signals and implemented the algorithm in hardware. The results indicate that the algorithm estimates the signals accurately if provided with enough data. The complexity is low enough for a hardware-implementation for real-time estimation to be feasible.**

Keywords- *SNR estimation, AWGN channel, MPSK modulation, constant envelope signals*

I. Introduction

SNR(Signal-to-Noise Ratio) is an important parameter in communication. It is especially useful for digital communication.

For example, in the area of adaptive equalization, the ability of the BER (Bit Error Ratio) improvement of the adaptive equalizer in common use depends on the SNR of the input signal. The performance of the equalizers with the SNR is shown in Fig 1.1.

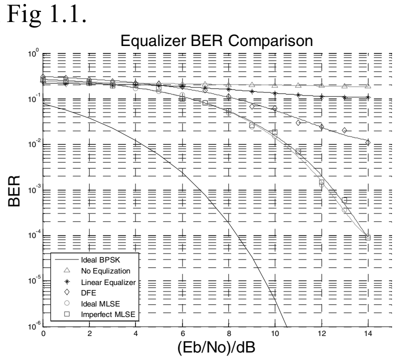

Fig 1.1 The ability of BER improvement for different equalizer

From Fig 1.1 we can deduce that, for an SNR lower than 5dB all of the adaptive equalizers cannot improve the performance of the BER. They may even become worse. So when the SNR is too low, the equalization is useless. The system is wasting resources. Estimating the SNR accurately can avoid invalid calculations that reduce the power of the system.

Similar problems occurred for other applications of adaptive ratio adjustment, adaptive modulation, and others.

A lot of work has been done in the area of SNR estimation. In [1], Shin introduces a method to estimate QPSK signals with training sequences in the AWGN channel. In [2], Beaulieu presents four equations to estimate the SNR of QPSK signals, and compares the characteristic of each equation. In [3], Matzner uses a method based on the second and forth order moments of the observed noisy signal. In [4], Pauluzzi selects BPSK and 8PSK modulation signals in AWGN channels to compare SNR estimators. The estimators include the estimator that has been used for the training sequence or decision-feedback sequence to construct a likelihood function. The estimator used the relationship between the second and forth order moments of the signal and the noise. In [5], HuaXU used the estimator introduced in [1], and used a transitional variable to estimate the SNR accurately.

In the papers introduced above, the methods in [1] and [2] are easy to realize, but they are inaccurate for SNRs lower than 4dB,and cannot be used for 8PSK signals (look at the simulation behind). In [3] and [4] the second and forth order moments are used to estimate the SNR, but the load of calculation is too heavy to perform the estimation in real time. The estimator introduced in [5] is a useful estimator, but it cannot be used for 8PSK and QPSK signal, only for π/4-QPSK signals(look at the simulation behind).

This paper describes a way of analyzing the envelope of a signal with an estimator based the assumption that the envelope of the signal is a constant. This estimator can be implemented in hardware.

Overview of the rest of the paper:

(1) Use MPSK signals to create system model.

(2) The SNR estimation methods introduced in [1]-[5].

(3) The SNR estimation method introduced in this paper.

(4) Result of the simulation and the comparison to other methods.

(5) Hardware implementation.

(6) Conclusion.

II. System Model

Phase-Switch-Key（PSK）modulation signals can be represented by,

$$s_{MPSK}(t) = Ae^{j(2\pi f_c t+\theta_m)} \qquad (2.1)$$

where $\theta_m=2\pi m/M$（m=0，1，2，⋯,M-1）is the possible phase of the carrier, A is the amplitude fo the signal.

Assuming the signal to be undistorted during transmission over the AWGN channel, the input signal of the receiver can be represented by,

$$r(t) = s(t) + n(t) \qquad (2.2)$$

where $n(t)$ is a complex AWGN with zero mean and variance $2\sigma^2$. The signal weight and the noise weight are independent.

Assuming that we have the synchronization of carrier and bit, the residual error of the output of the equalizer is small enough. Thus it is similar to the condition of AWGN channel.

The received signal is sampled by an ADC. To get I and

Q of the base-band signal, we used DDC (Digital Down Converter) technology. The output of the DDC is I(n) and Q(n).

$$\begin{aligned} I(n) &= s_I(n)+n_I(n) \\ &= A(n)\cos(\theta_m(n))+n_I(n) \\ Q(n) &= s_Q(n)+n_Q(n) \\ &= A(n)\sin(\theta_m(n))+n_Q(n) \end{aligned} \tag{2.3}$$

where n_I(n) and n_Q(n) is the zero mean AWGN whose variance is σ2.

So the SNR needed to be estimated is defined as

$$SNR = \frac{s_I^2+s_Q^2}{E(n_I^2+n_Q^2)} = \frac{A^2}{2\sigma^2} \triangleq \lambda \tag{2.4}$$

III. The primary Methods of SNR estimation

Method 1 is based on the sign of the received signal.

In the [1], the author considered that ideally, for the base-band signal I(n) and Q(n) should be 1or -1. So he calculated the mean and variance according to the sign of the sampled signal, and estimated the SNR from the equation (3.1),

$$\hat{\lambda} = \frac{1}{2}\left(\sum \frac{(mean_+)^2}{Var_+} + \sum \frac{(mean_-)^2}{Var_-}\right) \tag{3.1}$$

To improve the accuracy, he calculated many times and used the mean as the result.

Method 2 is based on the fact that the projections of the received signal on the I or Q axis are equal for this constellation.

In [2], the author exploited that ideally, the projection of the received π/4-QPSK signal on the I or Q axis are equal for this constellation. He presented four estimators: two of them are based on receiver statistics directly related to SNR while the other two are based on receiver statistics inversely related to SNR.

The author compared the four estimators by simulating. He found the estimators below to be the best.

$$\hat{\lambda} = L\left(\sum_{i=1}^{L} \frac{\left(|I(i)|-|Q(i)|\right)^2}{I^2(i)+Q^2(i)}\right)^{-1} \tag{3.2}$$

Method 3 is based on the fact that the SNR of I(Q) of the received signals are equal.

In [5], the author used the characteristic that the SNR of I or Q were equal, to deal with the real part and imaginary part independently. He used second order statistics and data fitting to get an accurate SNR-estimator for QPSK signals. He used the transitional variable z,

$$z = E\left[y^2_{k\_I/Q}\right] / E(y_{k\_I/Q})^2 \tag{3.3}$$

and got the function between z and SNR(λ)

$$z = \frac{1+\lambda}{\left\{\sqrt{\frac{2}{\pi}}e^{-\frac{\lambda}{2}} + \sqrt{\lambda}\left[erf\left(\frac{\lambda}{2}\right)\right]\right\}^2} \tag{3.4}$$

We can construct a function to estimate the SNR in a limited range by multinomial fitting.

$$\begin{aligned} \lambda \;=\; 10^4\times\big(&-0.41292958452235\times z^5 \\ &+ 2.66418532072905\times z^4 \\ &+ 6.86724072350538\times z^3 \\ &+ 8.84039993634297\times z^2 \\ &+ 5.68658561155135\times z \\ &+ 1.464045795143920\big) \end{aligned} \tag{3.5}$$

IV Estimator based on the envelope of the signal

The basic idea of the estimator is that for AWGN channels we can consider the variance of the envelope of the signal as the power of the noise, and consider the square of the mean of the envelope of the signal as the power of the signal. The function of the estimator is

$$\hat{\lambda} = \frac{E[a(n)]^2}{Var[a(n)]} \tag{4.1}$$

where $a(n)$ is the instantaneous envelope of the signal.

Aimed at complex baseband signals, we defined the envelope of the signal as

$$\begin{aligned} \xi(n) &= I^2(n)+Q^2(n) \\ &= A^2(n)+n_I^2(n)+n_Q^2(n) \\ &+2A(n)[\cos(\theta_m(n))n_I(n) \\ &+\sin(\theta_m(n))n_Q(n)] \end{aligned} \tag{4.2}$$

To aid the analysis, we define k(n) and h^2(n),

$$\begin{aligned} k(n) &= \cos(\theta_m(n))n_I(n) \\ &+\sin(\theta_m(n))n_Q(n) \end{aligned} \tag{4.3}$$

$$h^2(n) = n_I^2(n)+n_Q^2(n) \tag{4.4}$$

So the envelop can be written as

$$\xi(n) = A^2(n)+2A(n)k(n)+h^2(n) \tag{4.5}$$

Because the signal and noise are irrelevant, the mean of the envelope "μ" is

$$\begin{aligned} \mu &= E[\xi(n)] \\ &= E[A^2(n)]+E[2A(n)k(n)]+E[h^2(n)] \\ &= A^2+2\sigma^2 \end{aligned} \tag{4.6}$$

The variance of the envelope is

$$\begin{aligned} \sigma_\xi^2 &= E[(\xi(n)-\mu)^2] = E[\xi^2(n)]-\mu^2 \\ &= A^4+8\sigma^2A^2+8\sigma^4-(A^2+2\sigma^2) \\ &= 4\sigma^2(A^2+\sigma^2) \end{aligned} \tag{4.7}$$

With

$$z = \sigma_\xi^2/\mu^2 \tag{4.8}$$

we get

$$z = \frac{4\sigma^2A^2+A^4}{(A^2+2\sigma^2)^2} = \frac{\lambda^2+4\lambda+2}{(\lambda+1)^2}-1 \tag{4.9}$$

Rearranging the function, we can get

$$1-z = (\frac{\lambda}{\lambda+1})^2 \tag{4.10}$$

And from that we can get

$$\lambda = \frac{1}{\sqrt{\frac{1}{1-z}-1}} \qquad (4.11)$$

From (4.8) (4.11), we can estimate the SNR of the received signal by simply calculating its mean and variance. If the unit of the SNR is dB, then

$$\hat{\lambda} = -10\lg\left(\sqrt{\frac{1}{1-z}}-1\right)\ \text{(dB)} \qquad (4.12)$$

Explanations-1: The method described in this paper uses the received signal's characteristic of a constant envelope. So, it can be used for estimating SNR of other constant envelope signals like MFSK.

Explanations-2: In the course of analysis, we saw that this paper's method is insensitive to errors of synchronization of bit or carrier. In the actual condition, due to the Doppler effect and others, the error of synchronization is unavoidable. Even in this condition, this paper's method is effective.

V. Simulation

Simulating method, Monte Carlo analysis, repeat times is 500.

Simulating object, BPSK、QPSK、8PSK signal.

Simulating condition, AWGN channel.

Simulating range,E_b/N_0 [-2,16]dB。

(1) The Mean and Variance of the estimator are used in this paper's method to estimate differently modulated signals.

Fig 5.1 shows the mean of the SNR estimation by differently modulated signals. The length is 500.

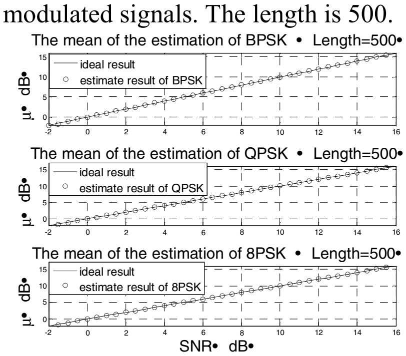

Fig 5.1 The mean of the SNR estimation of differently modulated signals

From Fig 5.1, we can see that the result of estimation is almost not related to the mode of modulation. In all simulations, the result of the estimation is close to the actual value.

Fig 5.2 shows the variance of BPSK, QPSK and 8PSK signals of different sample length.

From Fig 5.2, we can conclude that by increasing the sample length, the performance of the estimation is improved, especially for low SNRs. For SNRs higher than 0dB, the variance of the estimation is less than 1dB.

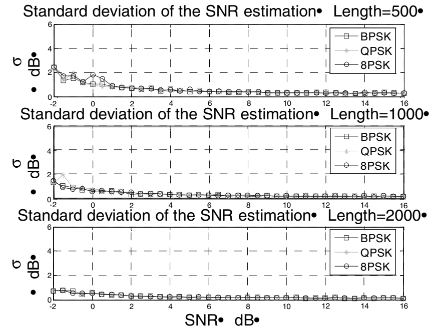

Fig 5.2 The variance of the SNR estimation of differently modulated signals

(2) Comparison of the proposed method to methods 1,2, and 3.

Fig 5.3 shows the mean characteristic of the different estimators. The sample length is 1000.

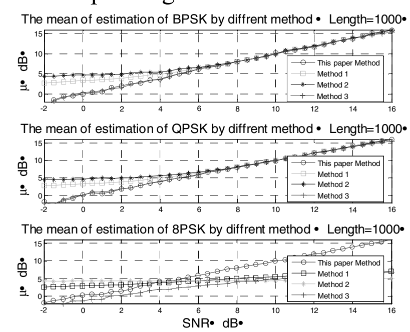

Fig 5.3 The mean of the SNR estimation for different algorithms

Fig 5.4 shows the variance characteristic of the different estimators. The sample length is 1000.

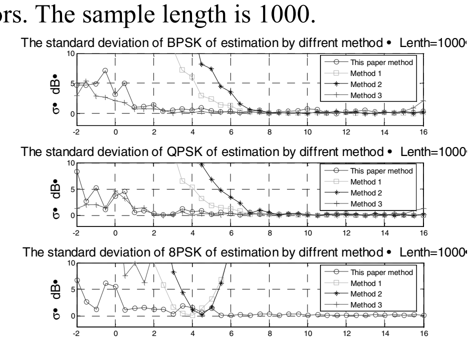

Fig 5.4 The variance of the SNR estimation for different algorithms

From Fig 5.3 and Fig 5.4, we can conclude that ,

a) The proposed method is a good estimator, because the estimated value is close to the actual value. On the other hand, this method can be used with all of the BPSK, QPSK, 8PSK signals. Thirdly, the estimator displays a good performance for very low SNRs.

b) For BPSK and QPSK signals, methods 1 and 2 are not suited for SNRs lower than 6dB.

c) Methods 1,2, and 3 cannot estimate the SNR of 8PSK signals.

VI. Hardware-implementation

This SNR estimation unit is part of a software radio. Fig 6.1 shows the relational part of the system.

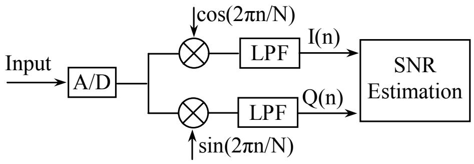

Fig 6.1 Relational part of the system

The input analog signal is converted to a digital signal. Digital Down Convert (DDC) technology is employed to get the I and Q signals. The procedure is

(1) Sample the signal and store enough samples.

(2) Calculate the envelope $\xi(n)$ of the samples.

(3) Calculate the mean μ and variance σ_{ξ}^{2}

$$\mu = \bar{\xi} = \sum_{i=1}^{Length} \xi(i) \qquad (6.1)$$

$$\sigma_{\xi}^{2} = \frac{\sum_{i=1}^{Length} (\xi(i) - \xi)^2}{Length} \approx \frac{\sum_{i=1}^{Length} (\xi(i) - \bar{\xi})^2}{Length} \qquad (6.2)$$

(4) Calculate the transitional variable z.

(5) Lookup the estimated value of the SNR in the table

The core device of the system is a TMS320C6701, which is a Floating-Point Digital Signal Processor produced by Texas Instruments Incorporated, whose process ability is 1 GFLOPS. Considering the actual work environment, this system can realize SNR estimation in real-time.

VII. Conclusion

This paper presented an SNR estimation method particularly applicable to constant envelope signals like MPSK signals in AWGN channels. The results of simulations indicated that the estimation's performance is good for a proper number of samples. This method is easy to implement due to its simple arithmetics and low operation time.

[1] Dong-Joon Shin, Wonjin Sung,In-Kyung Kim. Simple SNR estimation methods for QPSK modulated short bursts [A]. Proc IEEE GLOBECOM 2001[C]. 2001. 3644-3647.

[2] Norman C. Beaulie,Andrew S. Toms,David R.Pauluzzi. Comparion of Four SNR Estimator for QPSK[J]. IEEE communications letters,2000,4（2）, 43-45.

[3] Rolf Matzner,Ferdinand Enlberger. An SNR Estimation Algorithm Using Fourth-Order Moments[A]. Proceedings of the 1994 IEEE Symposium on Information Theory[C]. Trodheim. 1994.119.

[4] David R. Pauluzzi,Norman C. Beaulieu. A Comparision of SNR Estimation Techniques For the AWGN Channel[J]. IEEE Transactions on Communications, 2000,48（10）:1681-1691.

[5] XU Hua-, FAN Long-fei, ZHENG Hui. A precise SNR estimation algorithm for QPSK signals.[J]. journal of china comunicaiton, 2004,25(2):55-60.