The Geography of Assortative Matching

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This paper investigates why assortative matching between workers and firms is stronger in large cities than in small cities. I build a search and matching model with heterogeneous workers and firms to study the effect of labor market composition and labor market frictions on labor market sorting. I calibrate the model to match salient moments of the matched employer-employee data from Germany. I find that labor market efficiency plays a major role in explaining differences in assortative matching across cities. Moreover, its effect is amplified by a more disperse workers productivity distribution, since there are higher returns from matching with similar types for both workers and firms. Using the calibrated version of the model, I show that around 5% of GDP gap observed between large and small cities can be explained by differences in assortative matching. Overall, the paper stresses the importance of studying local labor market frictions and workers productivity distribution together to understand why the allocation between heterogeneous workers and firms vary between cities, and the resulting implications for spatial inequality.

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1. Introduction

Many countries around the world exhibit strong and persistent regional inequalities. The disparity in economic performance is particularly significant between large and small cities. In fact, large urban areas typically experience higher GDP per capita and higher wages compared to their smaller counterparts. While the gap has usually been associated to the concentration of more productive workers and firms into metropolitan areas, a more efficient allocation of resources can potentially amplify regional disparities. Recently, it has been documented that the allocation of workers and jobs is more efficient in large cities compared to small cities, as they feature a higher degree of assortative matching (Dauth et al. 2022). However, little is known about the underlying forces that determine the joint allocation of workers to jobs in local labor markets.

The primary goal of this paper is to study why assortative matching between workers and firms varies with city size. Studying labor market sorting in this context is important, since inefficient sorting between types -i.e. the wrong person for the wrong job - results in a misallocation of resources that eventually hinders local productivity and wages. First, I confirm the existing evidence of a positive relationship between assortative matching and city size and then I investigate its causes. Motivated by the empirical analysis, I focus on two main different channels that affect assortative matching in equilibrium: the distribution of workers and firms productivity within city, and local labor market frictions. Building on the canonical model of assortative matching with search frictions, I show that an increase in the dispersion of workers productivity distribution and lower search frictions can both determine a stronger labor market sorting in equilibrium. Then, I work out a quantitative version of the model to evaluate the relevance of these two channels in explaining differences in assortative matching across cities of different dimension. Lastly, I use the calibrated model to quantify the impact of assortative matching on the output and wage gap between large and small cities observed in the data.

In the first part of the paper, I use data from the German Federal Agency to motivate the focus of the paper. By exploiting information from German matched employer-employee data, I run a AKM (Abowd, Kramarz, and Margolis 1999) regression to obtain a measure of assortative matching for each local labor market - i.e. *cities* - in West Germany, which I compute as the correlation between workers and firm fixed effects. I document a positive and statistically significant relationship between labor market sorting and city size, confirming the evidence documented by Dauth et al. (2022). Furthermore, I show

how the distributions of workers and firms fixed effects differ with city size.In large cities, the workers fixed effect distributions tend to be more spread out than in smaller cities, with a higher standard deviation and wider percentile ranges. I regard this as evidence that the distribution of workers productivity in large cities is more dispersed. At the same time, I find that the dispersion of the firms fixed effect distributions is almost invariant across local labor markets of different dimensions.

Next, I use information about the unemployment spells records contained in the administrative data to document the extent of differences in local labor market frictions. First, I show a significant geographic variation in the job finding rate. In particular, it displays a positive relationship with city size, meaning that is easier for unemployed workers to find a job in large cities than in small cities. To further test the hypothesis that labor markets are more efficient in large cities, I regress a job-finding status dummy on various individual characteristics and the size of the labor market where the unemployed worker resides. The results show a positive and statistically significant coefficient for labor market size: in particular, by doubling the size of the labor market the probability of transition towards employment increases of 1.1% . I interpret this empirical findings as further evidence supporting the hypothesis that workers and firms in large and dense cities benefit from thick labor markets (Moretti and Yi 2024). This fact, combined with a declining trend in the job separation rate with respect to city size, indicates that the unemployment rate is lower in large cities, which is consitent with recent evidence about Germany (Kuhn, Manovskii, and Qiu 2021).

Motivated by these facts, I develop a framework to study how the degree of assortative matching between workers and firms varies with respect to worker composition and labor market frictions. To do this, I build on Shimer and Smith (2000) and propose a model in which heterogeneous workers and firms produce in an economy where the labor market is frictional . Workers can be employed or unemployed, while firms can be producing or vacant. Crucially, complementarity between worker and firm productivity in the production function entails positive assortative matching (PAM) in equilibrium (Shimer and Smith 2000). Next, I show a positive relationship between PAM and the two objects of interest: labor market frictions and the dispersion of the productivity distribution of workers. I assume that the aggregate matching function is affected by a parameter that positively influences the amount of meetings between unemployed workers and vacant jobs in the economy, i.e. a *matching efficiency* parameter. Moreover, I assume that workers and firms productivity follow a log-normal distribution (Lopes de Melo 2018). A higher matching efficiency implies a higher probability of meeting with the optimal partner in the labor market, therefore making both parts more selective in matching in the labor market. At the same time, an increase in the dispersion of workers productivity changes the incentives to match with individuals at the extremes of the distribution: while firms are less willing to match with workers at the bottom of the distribution who became less productive, high quality workers find less profitable to match with average firms; consequently, matches are more selective in equilibrium and hence, assortative matching increases.

Finally, I calibrate a quantitative version of the model to match salient labor market moments of large and small cities in West Germany. The exercise aims at quantifying the role of workers composition and labor market frictions in determining differences in labor market sorting. First, I distinguish between city-specific parameters and general parameters that are common across locations. The two key city-specific parameters are the matching efficiency parameter and the standard deviation of the workers productivity distribution. The first is informed by the differences in the job-finding rate across cities, while the second is informed by the differences in the standard deviation of worker fixed effects obtained from the data. Crucially, I leave the correlation between worker and firm type in the small city untargeted, to understand how much variation the model is able to reproduce. The calibrated version of the model is able to tightly match the sorting patterns between workers and firms observed in the data. While the correlation between workers and firms types in large cities is perfectly matched, the model generates a correlation between workers and firms productivity in small cities of 0.088 vs 0.109 in the data. Finally, I perform counterfactual exercises to understand how much each component accounts for differences in sorting. While I find a small role for the dispersion of the worker productivity distribution, matching efficiency alone can explain around a third of the differences in PAM generated by the model. Also, I show that when I increase the matching efficiency and the dispersion of the productivity distribution at the same time, the model explains 2/3 of the differences in PAM generated by the model.

Next, I test whether the model is able to explain the differences in economic performance between cities and what is the role of assortative matching in generating such gap. Overall, I find that the model generates around 16.9 % of the GDP gap between large and small cities observed in the data, while it does not generate variation in the average wage. While assigning the same degree of sorting as in the small city to the large city, I find that the higher degree of sorting in the large city can explain up to 4.8% of the output gap between large and small city observed in the data. While I find that the increase in matching efficiency is beneficial towards the economy and increases aggregate output through higher employment and higher sorting, higher dispersion has a different impact. In fact, total output decreases, while within city inequality increases. However, the increase in sorting contrasts the negative effect on output: in fact, without reallocation forces, total output per match would be 0.2% lower.

Overall, my results show new insights about differences in the labor market allocation across cities. A more diverse composition of workers in large cities is not enough to explain the higher degree of assortative matching. However, a higher matching efficiency strengthens the degree of sorting by increasing the number of interactions in the labor market and it amplifies the returns to sorting when there is higher dispersion.

Related Literature. This paper contributes to multiple strands of literature. The urban economic literature has largely documented the existence of agglomeration forces and their benefits for local labor markets (Duranton and Puga 2020) only recently it has focused on quantifying its potential impact on matching between workers and firms. Dauth et al. (2022) shows a positive relationship between assortative matching and market size in West Germany. Bleakley and Lin (2012) investigate the effects of thick labor market on occupation and industry switching for young workers. Moretti and Yi (2024) shows that the probability of re-employment is higher in large labor markets, finding also a higher average quality of the match. From a theoretical perspective, Papageorgiou (2022) proposes a model where a larger a number of occupations in big cities lead to better matches for workers, providing a microfoundation for agglomeration economies. The main focus of this project is rather exploring how a more efficient labor market in the large city affecta the equilibrium allocation between heterogeneous workers and firms. Differences in the labor market composition across cities have already been documented by the literature. Eeckhout, Pinheiro, and Schmidheiny (2014) shows that the distribution of workers skills is more dispersed in US large cities and develop a thoery that rationalizes this fact. Orefice and Peri (2024) studies the impact of immigration on positive assortative assortative matching in local labor markets, highlighting the role of higher screening activity by firms. My work puts emphasis on the implications of higher dispersion of workers skills in a context where the labor market is frictional.

From a methodological point of view, the paper contributes the growing body of research that uses search and matching frameworks to explore spatial inequality. My contribution is to study how local matching efficiency and within-city heterogeneity influence the sorting between workers and firms. The theoretical framework relates to the literature that studies assortative matching in the context of search frictions, started with the seminal paper by Shimer and Smith (2000), who introduced random search in a frictionless two-sided matching model proposed by Becker (1973). [1]. I bring the theory to a spatial context to disentangle the sources of assortative matching and to study the impact of sorting on aggregate outcomes [2] . Heise and Porzio (2022) examine how local labor market and spatial frictions jointly hinder the efficient allocation of workers to firms. More similar to my context, Martellini (2022) develops a spatial equilibrium model with search frictions and human capital accumulation through learning to quantify the productivity advantages of large cities. Consistently with my results, and opposite to previous studies (Baum-Snow and Pavan 2012), the paper finds significant lower search frictions in large cities.

A fast-growing literature examines the causes of regional inequality in Germany. Heise and Porzio (2022) study how spatial and local labor market frictions jointly affect the wage and output gap between East and West Germany. Lindenlaub, Oh, and Peters (2022) and Mann (2023) investigate what is the impact of firm sorting across space on wage inequality in the context of West Germany. Kuhn, Manovskii, and Qiu (2021) finds that the significant geographic dispersion in job separation rate contributes to the persistent dispersion in unemployment rate across local labor markets. Closest to my work is Dauth et al. (2022), who study the role sorting between and within cities for wage inequality in West Germany. I provide with a structural framework to study the reasons for higher assortative matching in cities, focusing in particular on worker heterogeneity and local labor market frictions and in deriving aggregate implications for spatial inequalities between cities.

OUTLINE. The rest of the paper is structured as follows. Section 2 describes the data and reports facts about local labor markets in Germany, Section 3 introduces the theoretical framework, Section 4 shows the results of the quantitative analysis and Section 5 concludes.

2. Empirical Analysis

In this section I describe the empirical evidence that motivates the theoretical model and that finally informs the quantitative analysis. In section 2.1 I introduce the main

[1]See Chade, Eeckhout, and Smith (2017) for a comprehensive review of the theoretical literature

[2]Lacava (2023) uses a version of the model with two regions to study the impact of sorting on regional migration in the context of Italy.

data sources used for the analysis. Section 2.2 illustrates the empirical evidence about assortative matching between workers and firms in German cities, while section 2.3 foces on the dispersion in the local labor market flows.

2.1. Data

The principal dataset used for the analysis is the longitudinal version of the Linked-Employer-Employee-Data of the IAB (LIAB), linked with the Establishment History Panel (BHP). The LIAB data is a random sample obtained from the Integrated Employment Biographies (IEB) of the IAB, which includes all the employment episodes of individuals that were subject to social security and all the unemployment episodes associated to unemployment benefits offered by the social security [3] . Between the 2010 and 2017, which is the time period I consider, the LIAB reports information about 1.6 million individuals. Each observation in the dataset is either an unemployment spell or an employment spell with the exact begin data and end date.[4] This dataset is linked to the BHP, that is a 50% random sample of all the establishments in Germany with at least one employee liable to social security on June 30 of a year. Importantly, each establishment reports the district in which it is located. In the appendix A.1 I discuss in more detail how the final sample is obtained.

2.2. Assortative Matching in Cities: Evidence from AKM regression

Assortative matching and city-size. From the original spell-level data I obtain an annual panel that I use for the main regression. To do that, I consider the employment episode recorded at date June 30th for every year between 2010 and 2017. When more than one employment episode is reported, I include only the one that reports the highest pay. As reported daily wages are top-coded, I use the methodology developed by Card, Heining, and Kline (2013) in order to impute the wages in the upper-tail of the distribution. I exploit the information in the BHP to define a local labor market: a worker belongs to a local labor market if its employer is located in that area. While the for every establishment is recorded its administrative area, I use a crosswalk in order to map those district to commuting zone, or local labor markets. Finally, I map those district to 171 local labor markets in West Germany. Next, I document the relationship between the degree of assortative matching between workers and firms and the city-size. I closely

[3]See Ruf et al. (2021) for more details.

[4]The LIAB only contains unemployment spells registered in the LeH, which contains unemployment episodes where individuals were receiving benefits according to Social Code Book III

follow the approach by Dauth et al. (2022), which estimates an AKM regression (Abowd, Kramarz, and Margolis (1999) to obtain workers and firms fixed effect. I estimate the following model:

$$\log wage_{it} = \mu_i + \psi_{j(i,t)} + X_{it}\beta + \epsilon_{it}$$

where μ_i denotes the worker fixed effect, $\psi_{j(i,t)}$ denotes the firm fixed effect and X_{it} contains time-varying individual characteristics - skill-specific cubic age profile and year dummies as in Card, Heining, and Kline (2013). I focus on the largest connected set [5], which represents almost 95% of the initial sample [6] In order to measure assortative matching in the labor market we compute the correlation between worker and firm fixed effect, namely $corr(\mu_i, \psi_j)$, in each local labor market. Figure 1A shows the degree of assortative matching at commuting-zone level in West Germany. The intense red color indicates a high degree of assortative matching, while the intense blue color indicates a low degree. Shades indicates values in the middle as indicated in the legend. The figure shows substantial geographical dispersion in assortative matching, with the difference between highest level and the lowest level being equal to 0.7. To further understand the relationship with city-size, I regress the measure of assortative matching against (log) population. I find a coefficient of 0.059 (se =0.01), meaning that by doubling city-size the strength of assortative matching increases by 5.9%. The positive significant relationship between assortative matching and city size is consistent with previous findings in the urban literature [7]. As it is well-known in the literature related to the AKM methodology, the variance of firm fixed-effect can be upward biased - due to the limited mobility of workers across firms. This can eventually negatively affect the estimate of the covariance, therefore implying a lower measure of assortative matching. Moreover, this phenomenon can severly affect the estimates of sorting for smaller cities, where a lower amount of movements is observed. To control for this I use the methodology developed by Bonhomme, Lamadon, and Manresa (2019) and I cluster firms into 20 classes according to their log-wage distribution. I still obtain a positive relationship between assortative matching and city size (See Appendix A.3).

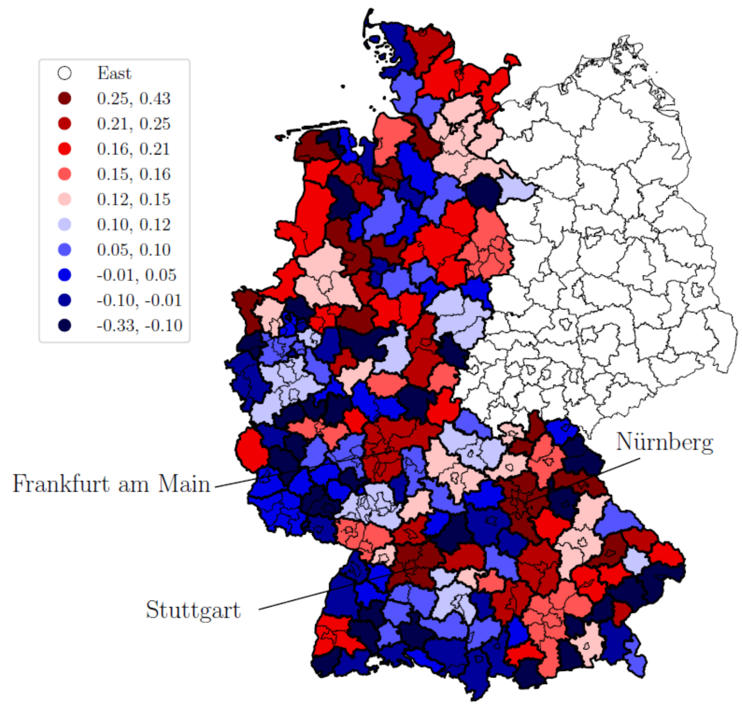
[5] largest set of firms that are connected by at least one mover

[6] For more details see table A2

[7] Using matched employer-employee data for Germany from BeH and using a similar methodology, Dauth et al. (2022) finds a coefficient of 0.061. Pérez, Meléndez, and Nuno-Ledesma (2023) finds an estimate between 0.014 and 0.045 for Mexico, while Hong (2024) finds 0.005 for Canada.

FIGURE 1. Dispersion of assortative matching across local labor markets and relationship with city-size

A. PAM in local labor markets



B. PAM vs city-size

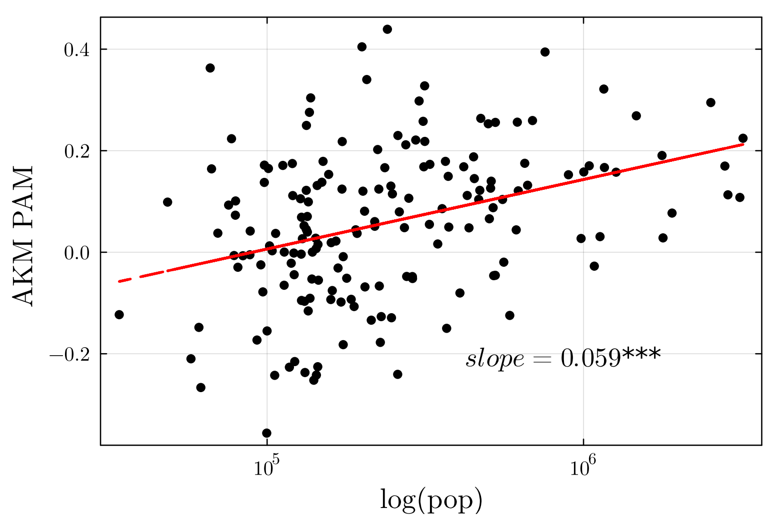


Figure 1A displays PAM for each local labor market (commuting zones) in West Germany between 2010 and 2017. Local labor markets are defined over *kreise* (administrative units in Germany), which represent the basic geographic unit in the map. The measure of PAM is computed as the correlation between worker and firms fixed effect obtained from the AKM regression within each local labor market. In Figure 1B each dot is a local labor market. I plot PAM against (log)population for each local labor market. (* p < 0.10, ** p< 0.05, *** p< 0.01)

Workers and Firms Type Distributions and City-Size. Next I show how the distributions of workers and firms fixed effect recovered from the AKM regression vary across local labor markets. In particular, I will focus on measures of dispersion such as standard deviation and distance between 90th and 10th percentile. In order to do this, I consider the distributions of workers and firms fixed effect in each single commuting zone. In figure 2A I plot the standard deviation of the local distribution of worker fixed effect, while in figure 2B I plot the difference between the 90th percentile and the 10th percentile of the local distribution of worker fixed effect. In both cases the relationship with city-size is positive and statistically significant, implying a more disperse productivity distribution of workers in larger cities. This fact is consistent with Eeckhout, Pinheiro, and Schmidheiny (2014), whose motivating evidence lies on the fact that workers distribution in large cities is more disperse than in small cities [8].
For what concerns the local distribution of firm fixed effect, I obtain a flat relationship with city-size for both the standard deviation and for the difference between the 90th and the 10th percentile. In fact, while I obtain a positive coefficient, this is not statistically significant (see figure 3A and 3B). [9]

FIGURE 2. Standard deviation and p90-p10 for workers fixed effect distribution

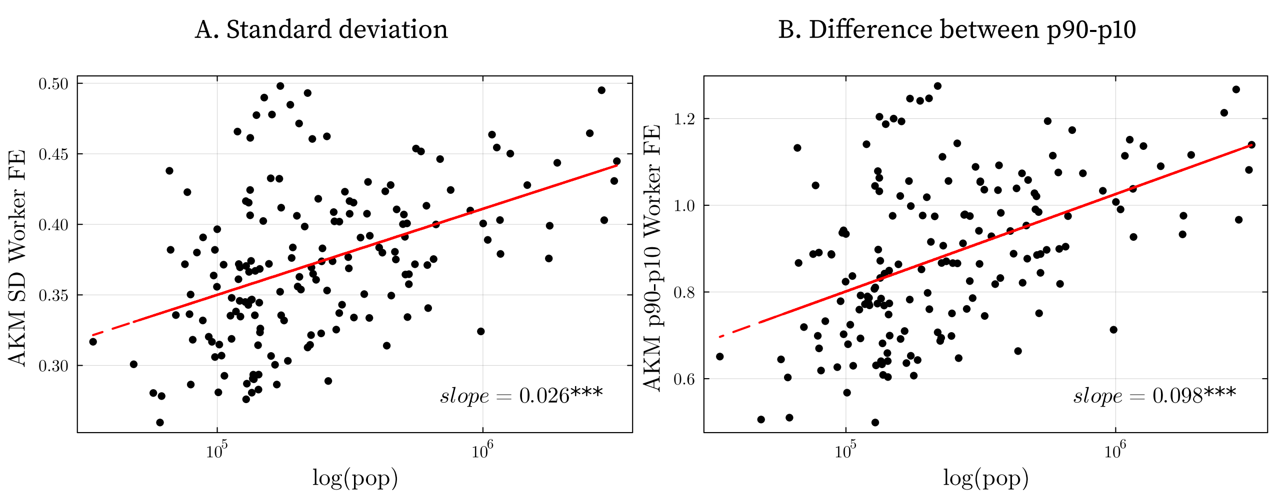


Figure 2A plots the standard deviation for worker fixed effect against log-population for each local labor market. 2B plot the difference between the 90th percentile and 10th percentile of the worker FE distribution of each LLM. (* $p < 0.10$, ** $p< 0.05$, *** $p< 0.01$)

[8]I also find a statistically significant relationship between the average worker fixed effect and city-size, confirming pre-existing evidence in the literature, see A1.

[9]While the distribution of firm fixed effect might show similar dispersion across city-size, they indeed differ in terms of mean, consistent also with findings from Dauth et al. (2022). For more details, see A3.

FIGURE 3. Standard deviation and p90-p10 for firms fixed effect distribution

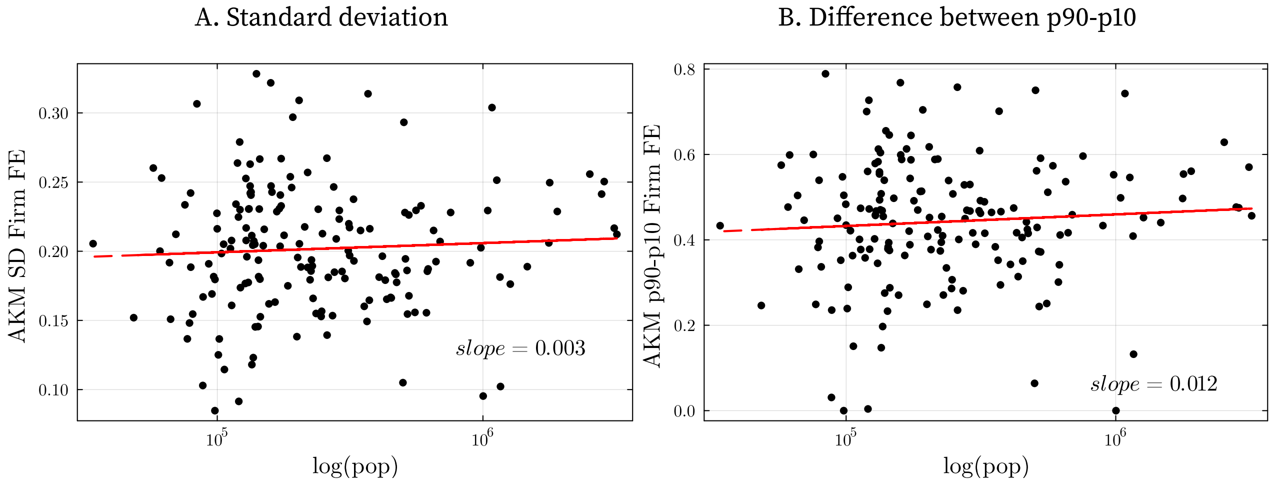


Figure 3A plots the standard deviation of firm fixed effects against log-population for each local labor market. 3B plot the difference between the 90th percentile and 10th percentile of the firm FE distribution of each LLM. (* p < 0.10, ** p< 0.05, *** p< 0.01)

2.3. Local labor market frictions

JOB FINDING RATE AND JOB SEPARATION RATE DISPERSION ACROSS CITY-SIZE. In this section I document pervasive differences in labor market frictions across local labor markets. The main objective is to assess to what extent labor markets in larger cities benefit from higher efficiency. I start by constructing monthly labor market transitions between 2010 and 2017 using unemployment spells contained in the LIAB data. In particular, I define the local job finding rate as the share of workers that move from unemployment to employment within the same local labor market [10], while I define the local separation rate as the share of workers transition from employment to unemployment. In Figure 4A I plot the job finding rate across German local labor markets. The graphs shows a positive relationship: by doubling city size the transition rate increases by 0.6 % . To understand whether the pattern is driven by individual characteristics, I construct a job-finding dummy variable that takes value 1 in case of a transition, while 0 in the case worker remained unemployed, and I regress it on a series of individual characteristics and (log) city size. I find that relationship between job finding probability and city size is positive and statistically significant, even after controlling for all individual characteristics – by doubling city size the probability of

[10] Unemployment benefits recipients reports the location of their residence at the beginning of the unemployment spell. I use that location as their local labor market at $t-1$. Alternatively, one can impute the last location where the individual was employed. Use this definition I still obtain the same qualitative pattern

finding a job increases by 1.1% (see table A4) . This evidence suggests that workers in big cities benefit from larger labor markets, as they experience higher probability of finding a job when unemployed.

FIGURE 4. Job Separation Rates and Job Finding Rates across German local labor markets

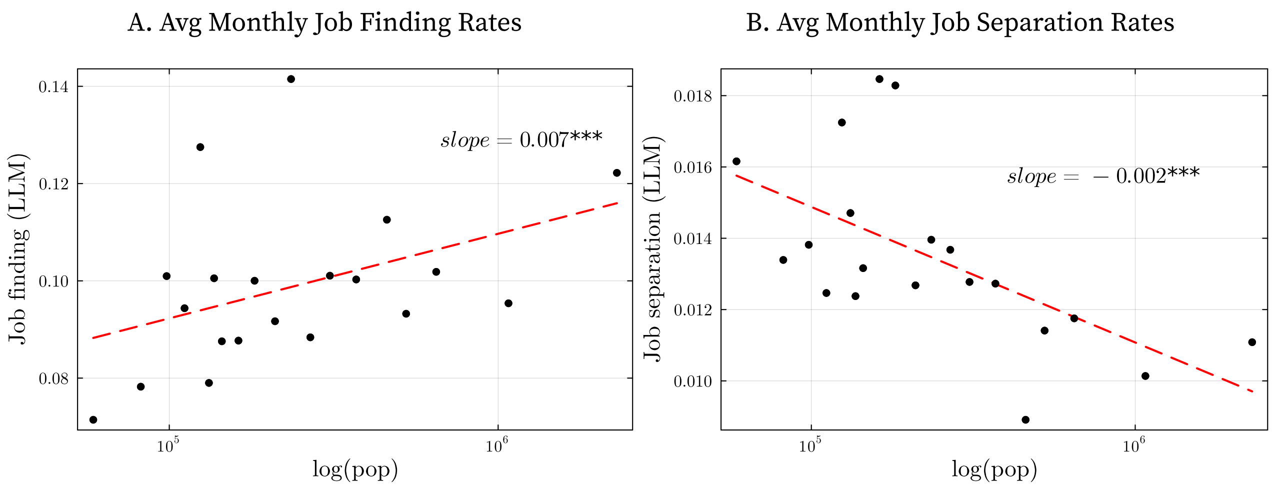
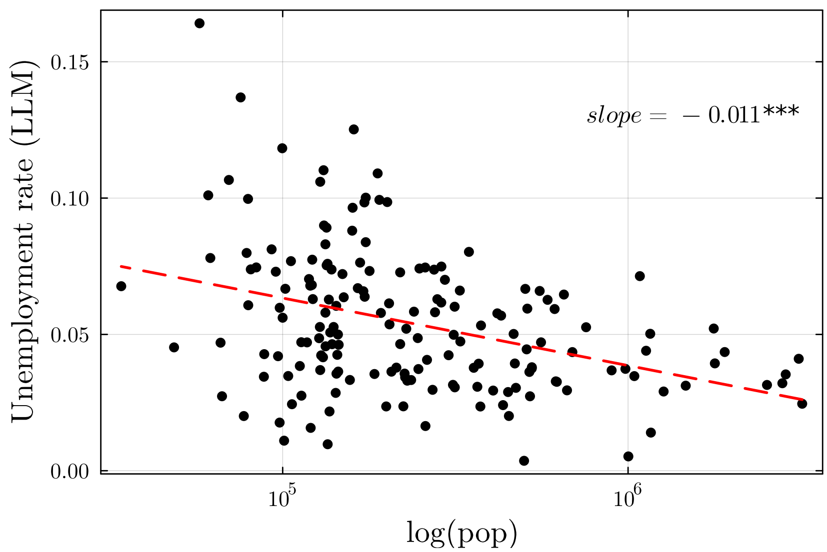


Figure 4A displays a binscatter of the average monthly job-finding rate against log(population) across West Germany local labor markets. Figure 4B displays a binscatter of the average monthly separation rate against (log)population across West Germany local labor markets.The red line displays a linear fit and the slope reports the OLS coefficient. (* p < 0.10, ** p< 0.05, *** p< 0.01)

To further assess the role of local labor market frictions, I further analyse the geographic variation in the separation rate. Figure 4B shows a negative relationship between the separation rate and city size. This fact, combined with the pattern for the job finding rate previously observed, implies a negative relationship between unemployment and city-size (Figure). [11]. The evidence about the dispersion in regional labor market flows exposed in this section is consistent with recent evidence about the German context. In fact, while Kuhn, Manovskii, and Qiu (2021) reports that both dispersion in job finding rate and job separation rate matters to explain unemployment differences, they also show that the unemployment rate is negatively related to regional productivity, consistent with my evidence. Also, while they particularly focus on the role of separation rate in explaining disparities in local labor market outcomes, my study emphasizes the importance of dispersion in job finding rate for labor market sorting, interpreting it as evidence that local labor market in large cities are more efficient.

[11]As for the job finding rate probability, I run a linear probability model for unemployment in the spirit of Kline and Moretti (2013) . I find a negative relationship between the probability of being unemployed and city size (See Table A5 in the Appendix)

FIGURE 5. Average monthly unemployment rate across German local labor markets



The graph shows the average monthly unemployment rate in cities in West Germany between 2010 and 2017. Black dots represent local labor markets. The dashed line represent the linear fit of unemployment rate on (log) population. The slope reports the coefficient of (log)population obtained from the OLS regression. Stars indicate the significance level of the coefficient (* p < 0.10, ** p< 0.05, *** p< 0.01)

3. Theoretical Framework

Motivated by the empirical evidence, I propose a search and matching model with heterogeneous firms and workers and log-supermodular production function, which is based on the seminal work of Shimer and Smith (2000). Similarly to Hagedorn, Law, and Manovskii (2017), I endogenize the meeting rate in the economy by assuming a constant returns to scale matching function that combines vacant firms and unemployed workers.

3.1. Environment and timing

Time is continuous. All agents are infinitely-lived and discount future payoffs at rate $r \in (0, 1)$. There is a unit mass of heterogenous workers that can be either employed or unemployed. Workers differ in terms of productivity which I denote as $x \in [\underline{x}, \overline{x}]$, with density $l(x)$. There is also a unit mass of heterogeneous firms which can either be matched with workers or vacant. Firms are heterogeneous in terms of productivity denoted by $y \in [\underline{y}, \overline{y}]$, with density $g(y)$. After matching, workers and firms produce final output through a production function $f(x, y)$. The production function is increasing in both argument, $f_x > 0$ and $f_y > 0$, and is be log-supermodular. This is crucial for obtaining an equilibrium in which the allocation of workers and firms displays positive assortative matching. [12]The output produced by a single match is eventually divided into wage for the worker, $w(x, y)$ and profit for the firm $\pi(x, y)$. While unemployed workers and vacant firms can form new matches by meeting in the labor market, existing matches can be destroyed at an exogenous rate δ.

3.2. Distributions and Aggregates

The measure of producing matches in the economy is denoted by $h(x, y)$. Therefore, the following relationship holds:

$$l(x) = \int_y h(x, y)dy + u(x) \quad \forall x$$

That is, the total amount of workers of type x in the economy are either matched with firms y or unemployed. At the same time:

$$g(y) = \int_x h(x, y)dx + v(y) \quad \forall y$$

The total mass of firms y in the economy is equal to the total mass of firms matched and the total mass of firms that are vacant.

By integrating the densities previously described we can obtain the measure of aggregate employment, $E = \int_x \int_y h(x, y)dxdy$, of unemployment, $U = \int_x u(x)dx$, of operating firms $P = \int_x \int_y h(x, y)dydx$, and of vacant firms $V = \int_y v(y)dy$.

3.3. Frictional Labor Market

Unmatched workers and firms engage in a frictional labor market. We assume that search is *random*. There is a matching function $M : [0, 1] \times [0, 1] \to [0, \min(U, V)]$ that combines the total number of vacancies V and unemployed workers U. It is increasing in both arguments and it features constant returns to scale. Furthermore, I assume that the total number of matches in the economy is obtained by multiplying the matching function by a term χ, which can be interpreted as the matching efficiency. Eventually, I

[12]More specifically, $\frac{\partial log f(x,y)}{\partial x \partial y} > 0$. See Shimer and Smith (2000) a formal proof.

follow the notation of Lise, Meghir, and Robin (2016) and define the meeting rates k as:

$$\kappa = \frac{\chi M(U, V)}{UV}$$

In particular, an unemployed individual meets a vacancy of type y at rate $\kappa v(y)$, while a vacant firm meet an individual of type x at rate $\kappa u(x)dx$. Notice that not all the meetings between workers and firms will result in matches: agents might not find profitable to match and therefore they will rather continue to search.

3.4. Definition of Values and Surplus Sharing

Let $W_u(x)$ denote the value of unemployment for worker x and $\Pi_p(y)$ the value of a vacancy for firm y; $W_e(x, y, w(x, y))$ the value of employment of worker x at firm y and $\Pi_p(x, y, w(x, y))$ the value of a firm of type y matched with worker x. The total surplus produced by a match between worker x and firm y is:

$$S(x, y) \equiv W_e(x, y, w(x, y)) + \Pi_p(x, y, w(x, y)) - W_u(x) - \Pi_v(y) \tag{1}$$

Following Shimer and Smith (2000), we assume that wages are determined through Nash bargaining. Workers obtained a share $\alpha \in (0, 1)$ of the total surplus produced by the match, while firms obtain a share $(1 - \alpha)$ [13]. The value of employment for a worker can be expressed as:

$$rW_e(x, y, w(x, y)) = w(x, y) + \delta\,(W_u(x) - W_e(x, y)) \tag{2}$$

Similarly, the value of a producing firm can be written as:

$$r\Pi_p(x, y, w(x, y)) = f(x, y) - w(x, y) + \delta\left(\Pi_v(y) - \Pi_p(x, y)\right) \tag{3}$$

Both agents receive an instantaneous payoff and the continuation value of match, which of course takes into account the possibility of separation. The optimal wage rate

[13]While Shimer and Smith (2000) assume that $\alpha = 0.5$ in their seminal work, this assumption can be relaxed as discussed by Hagedorn, Law, and Manovskii (2017). See appendix for a solution of the Nash bargaining problem.

that is going to solve the Nash bargaining problem is satisfying the following condition:

$$(1-\alpha)(W_e(x,y)-W_u(x)) = \alpha(\Pi_p(x,y)-\Pi_v(y))$$

This implies that, conditional on being matched, workers obtain $W_e(x,y) = W_u(x) + \alpha S(x,y)$, while firms obtain $\Pi_p(x,y) = \Pi_v(y) + (1-\alpha)S(x,y)$.

3.5. Optimal strategies for workers and firms

Using the definition of value of employment and production in equations 2 and 3, and by plugging them into equation 1 one can obtain:

$$(r+\delta)S(x,y) = f(x,y) - rW_u(x) - r\Pi_v(y) \tag{4}$$

This expression resumes the decisions of matching by workers and firms. Both sides will decide to match whenever the output produced from the match is larger than the total value of being unmatched, that is when $S(x,y) \geq 0$. In that case surplus will be positive and both sides will receive a payoff larger than their reservation value.

Next, using the result in equation 4 we can characterize the Bellman equation for unemployed workers and firms. The value of unemployment for worker x, $W_u(x)$, satisfies the following Bellman equation:

$$rW_u(x) = b(x) + \kappa\alpha \int S(x,y)^+ v(y)dy \tag{5}$$

where I define the operator $\mathcal{F}^+ = \max\{\mathcal{F}, 0\}$ to indicate the decision of matching. Unemployed individuals receive an unemployment benefits $b(x)$, and flow value from matching with a firm y from the distribution of vacancy $v(y)$. This depends on the rate k at which she is going to meet a job, but also from the expected value from matches which implies non-negative share α of surplus $S(x,y)$.

Similarly, the value of vacant jobs is given by:

$$r\Pi_v(y) = -c(y) + \kappa(1-\alpha) \int S(x,y)^+ u(x)dx \tag{6}$$

Firms with open vacancies pay a fixed per period cost c. Also, they receive the expected value from matching with an unemployed worker which delivers positive surplus $S(x,y)$.

3.6. Stationary Search Equilibrium

In a steady-state search equilibrium values $W_u(x), \Pi_v(y), W_e(x, y, w(x, y)), \Pi_p(x, y, w(x, y))$, are such that the Bellman equations are solved. The density $h(x, y)$ describes the optimal distribution of matches in the economy. Equilibrium wages $w(x, y)$ are such that the equation 1 is satisfied and formed match generates positive surplus. I define $\mathcal{B}$ the equilibrium set of pairs (x, y) that yields positive surplus in equilibrium: $\mathcal{B} \equiv \{(x, y) : S(x, y) > 0\}$. In steady-state for every couple of worker x and firm y in the matching set $h(x, y)$, inflow into unemployment must be equal to the outflow from unemployment:

$$\forall (x, y) \in \mathcal{B} \quad \delta h(x, y) = (1 - \delta)\kappa u(x) v(y) \tag{7}$$

When integrating over the matching set $\mathcal{B}$, one can obtain the relationship for aggregate densities:

$$\int_{\mathcal{B}} \delta h(x, y) = \int_{\mathcal{B}} (1 - \delta)\kappa u(x) v(y) dx dy$$

Equation (7), together with the relationship between distribution of workers $l(x)$ and firms $g(y)$, determines the equilibrium densities of unmatched workers and firms, $u(x)$ and $v(y)$. The labor market tightness parameter k is eventually determined by the matching function M and the aggregate densities of unmatched agents, U and V.

Equilibrium wages can be written as:

$$w(x, y) = \alpha(r + \delta)S(x, y) + rW_u(x)$$

Intuitively, by matching with firms, workers obtain a share of the produced surplus plus the discounted value of unemployment.

3.7. Assortative matching according to theory

Consistent with the equilibrium above described, the model is able to generate a pattern of assortative matching between workers and firms. I measure the degree of assortative matching in the economy as the correlation between workers and firms type in equilibrium. First, I define the joint density of matched workers and firms $\phi(x, y) = \frac{h(x,y)}{E}$.Therefore, the unconditional density of workers x is $\phi(x) = \int_y \phi(x, y) dy$ with CDF

$\Phi(x)$, and the unconditional density of firms y can be written as $\phi(y) = \int_x \phi(x, y)dx$, with CDF $\Phi(y)$. Also, the density of firm y conditional on worker x is $\phi_x(y) = \frac{\phi(x,y)}{\int_{y'} \phi(x,y')dy'}$, with CDF $\Phi_x(y)$. Hence, the correlation between the worker types x and firm types y can be written as:

$$corr(x, y) = \frac{\int \int (x - \hat{x})(y - \hat{y})d\Phi_x(y)d\Phi(x)}{\hat{\sigma_x}\hat{\sigma_y}}$$

where $\tilde{\sigma_x}$ is the standard deviation of worker types in equilibrium:

$$\hat{\sigma}_x = \sqrt{\int_x (x - \hat{x})^2 d\Phi(x)}$$

and similarly, $\tilde{\sigma_y}$ is the standard deviation of firm types in equilibrium:

$$\hat{\sigma}_y = \sqrt{\int_y (y - \hat{y})^2 d\Phi(y)}$$

The terms $\hat{x} = \int xd\Phi(x)$ and $\hat{y} = \int yd\Phi(y)$ denote the average worker and firm types.

Indeed, the closer $corr(x, y)$ is to 1, the stronger is the degree positive assortative matching between workers and firms, while the closer to 0, the weaker.

3.8. Changes in Assortative Matching: the role of distribution and matching efficiency

Next, I show how the equilibrium assortative matching pattern depends on the model fundamentals. In particular, in light of the empirical findings described in section 2 , I show how the strengh of sorting varies with respect to efficiency of meeting process in the labor market, namely *matching efficiency*, and with respect to the dispersion of workers distribution. Since the steady state equilibrium can only be computed numerically, the comparative statics exercises are obtained through numerical simulations. In order to do this, I need to assume functional forms for the production function, the matching function and the distribution of workers and firms. Following the literature, I assume a CES production function $f(x, y) = (x^\rho + y^\rho)^{\frac{1}{\rho}}$. The CES parameter ρ is fundamental to for determining the *sign* of sorting between workers and firms in the economy. As I am interested in cases in which assortative matching is positive I assume that $\rho < 1$, such that the log-supermodularity property is guaranteed. The aggregate matching

TABLE 1. Baseline parametrization for the numerical simulation

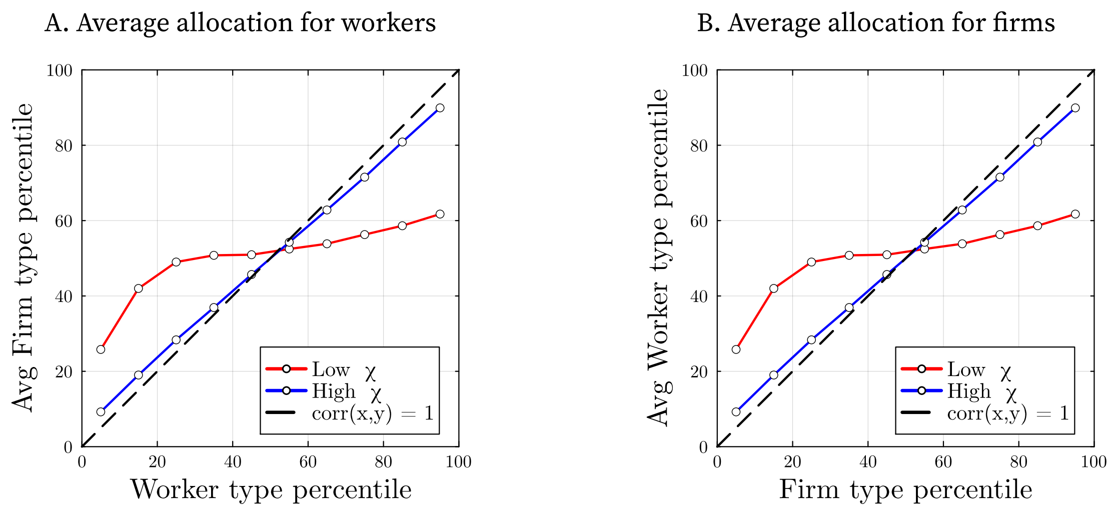
Parameter	Symbol	Value
Discount rate	r	0.05
Vacancy cost	$c(y)$	0.0
Unemployment benefit	$b(x)$	0.0
Separation rate	δ	0.008
Workers bargaining power	α	0.5
CRS matching function	β	0.5
Matching efficiency	χ	0.3
CES parameter	ρ	−2.0
Mean Normal dist. (workers)	μ_x	0.0
St. Dev Normal dist. (workers)	σ_x	1.0
Mean Normal dist. (firms)	μ_y	0.0
St. Dev Normal dist. (firms)	σ_y	1.0

The table report the values of the parameters used for the baseline numerical exercise.

function determining the meeting rate is given by $N = \chi U^{\beta} V^{1-\beta}$ where χ is the *matching efficiency* parameter and β is the constant returns to scale parameter. Finally I assume lognormal distribution characeterized by the parameters μ and σ, for both workers and firms distribution type. The baseline parametrization for the baseline numerical exercise is reported in Table 1.

CHANGES IN MATCHING EFFICIENCY. In Figure 6 I show how the equilibrium labor market sorting between workers and firms changes given changes in the matching efficiency parameter. In 6A the blue line indicates the average percentile of the firm allocated for any worker percentile for high value of χ, while the red line indicates the allocation for a low value of χ. The dashed line indicates the case of perfect sorting between workers and firms. In 6B I show a specular plot for firms. From the figure it is evident that the blue line is closer to the dashed line than the red line, indicating that a higher matching efficiency implies a stronger assortative matching between workers and firms in equilibrium. Intuitively, a higher matching efficiency makes sure that unemployed workers and vacant firms meet in the labor market at a higher frequency, increasing the probability of meeting their "own" types. Both workers and firms experience an increase in their option value of waiting, since finding a profitable match is easier. This eventually results in an increase in labor market sorting in equilibrium.

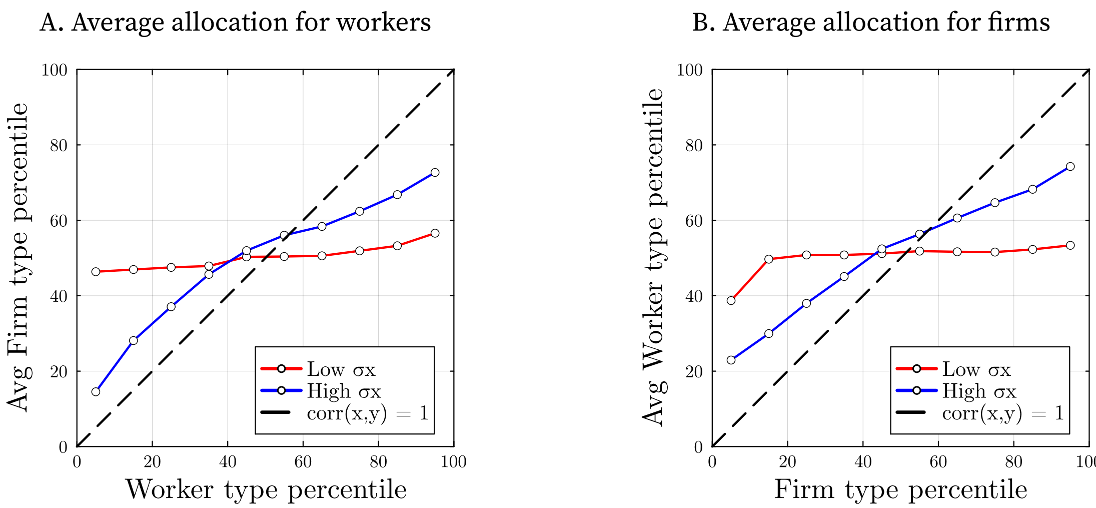
FIGURE 6. Changes in assortative matching between workers and firms for different level of χ



In Figure 6A for every percentile of the worker distribution on the x axis, the blue and the red lines represent the average percentile of the allocation firms in equilibrium for different values of χ. I discretize the productivity distribution into 50 types and in the graph I report the allocation for deciles starting from the 5th percentile. For example, the first white dot represents the average percentile of firm types allocated to the 5th percentile of workers productivity distribution. The blue line represents the allocation for a high value of χ, while the red line for a low value. The dashed line indicates the case in which the correlation between percentiles is perfect. The same logic applies for Figure 6B

CHANGES IN THE DISPERSION OF WORKER TYPE DISTRIBUTION. Next, I ask how the allocation changes given a change in the underlying distribution of workers productivity. In particular, I investigate how labor market sorting varies as the standard deviation of the worker productivity distribution changes. In order to do this, I fix the mean of the log-normal distribution and I let σ_x vary. In 7A I report the average allocation for workers, while in 7B I report the average allocation for firms. From both plots it is evident that a higher standard deviation leades to higher sorting in the labor market since the blue line, which represents the allocation for higher standard deviation, is closer to the dashed line than the red line. Intuitively, when the standard deviation is low there is little variation in worker types and therefore there is very little difference in the value of matches in the labor market; this eventually leads to an equilibrium where firms and workers are mismatched, and hence with little correlation between workers and firms types. On the other hand, a high standard deviation means a more differentiated variety of skills: because of complementarity forces firms obtain larger returns from matching with their ideal partner. This results in higher assortative matching between workers and firms in equilibrium.

FIGURE 7. Changes in assortative matching between workers and firms for different level of σ_x



In Figure 7A for every percentile of the worker distribution on the x axis, the blue and the red lines represent the average percentile of the allocation firms in equilibrium for different values of σ_x. I discretize the productivity distribution into 50 types and in the graph I report the allocation for deciles starting from the 5th percentile. For example, the first white dot represents the average percentile of firm types allocated to the 5th percentile of workers productivity distribution. The blue line represents the allocation for a high value of σ_x, while the red line for a low value. The dashed line indicates the case in which the correlation between percentiles is perfect. The same logic applies for Figure 7B

4. Quantitative Analysis

In this section I bring the theoretical model to the data to assess whether it is able to reproduce the geographic dispersion in assortative matching observed in the data. I will consider two cities, one representative of the small cities and another representative of the large city. Through the lense of the model, I will consider these as two separate economies and therefore my analysis treat them as two different steady-state. I exploit the variation in labor market frictions and worker composition observed across cities in the data to inform the quantitative versin model and to study how the two channels affect labor market sorting. In 4.1 I describe the functional form and the parametrization of the model, in section 4.2 I explain how parameters are linked to moments that are matched in the data and lastly in section 4.3 I discuss the results.

TABLE 2. Summary of functional forms

Object	Description	Functional form
$f(x, y)$	Production function	$(x^{\rho} + y^{\rho})^{\frac{1}{\rho}}$
$m(U, V)$	Matching efficiency	$\chi U^{\beta} V^{1-\beta}$
$b(x)$	Unemployment benefits	$\bar{b} f(x, y^{*}(x))$

The table reports the functional forms used for the calibration exercise

4.1. Parametrization and functional forms

I conduct a calibration exercise considering two cities, small and large. I define small and large according to the population distribution of local labor markets: the *small* ($l = small$) location includes all the local labor markets with population below the 33th percentile of the overall population distribution; the *large* ($l = large$) location includes all the local labor markets with population that is larger than the 66th percentile [14] . Crucially, the moments informing the parameters of the model in the calibration procedure are computed in the data according to the definition of location described above. While some of these parameters are obtained externally , the most relevant ones are going to be calibrated following a method of moments procedure. I set the unit of time to one month, so that empirical moments are constructed as monthly averages across years 2010-2017.

The functional forms for the quantitative version of the model are the same as those described in the previous section, however, some of the parameters are going to be location specific. In table 2 I summarize the functional forms used for the quantiative exercise. The CES production function takes form: $f(x, y) = (x^{\rho} + y^{\rho})^{\frac{1}{\rho}}$; furthermore, I assume a constant returns to scale matching function $M(U, V) = \chi U^{\beta} V^{1-\beta}$. Finally, I establish that unemployment benefits are a function of the ability of workers. Following Lopes de Melo (2018) I assume that $b(x) = \bar{b} f(x, y^{*}(x))$, where $y^{*}(x)$ denotes the frictionless allocation of workers to firms [15] . For what concerns the distribution of workers and firms I maintain the log-normal assumption already done in the previous section.

I now describe how parameters value are chosen for the quantitative exercise. The discount rate is set to 0.008, which results in an annual interest rate of 10%. I set the bargaining power parameter α to be equal to 0.5 as it is common in the literature, and

[14] See Table **??** for examples of cities included in each group

[15] The function $y^{*}(x)$ is such that: $\int_{x}^{1} l(x')dx' = \int_{y^{*}(x)}^{1} g(y')dy'$

TABLE 3. Summary of externally calibrated parameters

Parameters	Description	Source	Value	
r	Discount rate	Annual interest rate	0.008	
α	Worker bargaining power	Assumption	0.5	
$c(y)$	Vacancy cost	Assumption	0	
β	CRS Matching Function parameter	Petrongolo and Pissarides (2001)	0.5	
			Location	
Parameters	Description	Source	Small	Large
δ_l	Separation rate	EU rate	0.011	0.008
$\sigma_{y,l}$	LogNormal param.	AKM std(firm FE)	0.2	0.2

The table outlines the parameter values used for the baseline calibration. The top table summarizes the parameter that are general to both cities, while the bottom table refers to the parameter that are location specific. The parameters in the top table are chosen accordingly to what is common in the literature. The values for the separation rate are obtained from the data, while the parameters for the firm distribution are assumed and kept constant across cities.

the vacancy cost $c(y)$ is set to be equal to 0. The constant returns to scale parameter of the matching function β is set to 0.5. As far as it concerns the parameter that are location-specific but calibrated externally, the separation rate δ_j is set such that it matches the mothly job separation rate in each location j observed in the data. [16] For what concerns the distribution of firms productivity, I assume that the standard deviation is the same across locations and I plug the estimate for the standard deviation of firm fixed effect obtained from AKM regression, $\sigma_{y,l} = 0.2$. [17]

4.2. Calibration strategy: parameters and targeted moments

The rest of the parameters are calibrated through a method of moments procedure. In particular, for each location I calibrate a set of parameters θ_l such that the objective function $\mathcal{L}(\theta_l)$ is minimized:

[16] I recover δ as a Poisson rate, interpretating the EU rate in the data as the probability d an employed individual separates from the job, namely: $d = 1 - e^{-\delta}$.

[17] A two-way fixed effect regression on the equilibrium wages of the model is not able to recover the variation in firm-types because of non-monotonicities of wages in firm type. See Lopes de Melo (2018) for a discussion. Also, I assume that the $\mu_{y,l}$ parameter is $\mu_{y,l} = -\frac{\sigma^2_{y,l}}{2}$ so that the mean of the lognormal distribution is 1.

$$\mathcal{L}(\theta_l) = \sum_i^N \left(\frac{\hat{m_i} - m_i(\theta_l)}{\hat{m_i}} \right)^2 \tag{8}$$

where $\hat{m_i}$ denotes the empirical moment and $m_i(\theta_l)$ denotes the moment obtained from the model.

The moments are directly obtained by solving numerically for the stationary equilibrium given a set of parameter θ_l. In Table 4 I list all the internally calibrated parameters and their related moments.For the large city I will calibrate the following set of parameters: $\theta_{large} = \left\{\sigma_{x,\text{large}}, \chi_{\text{large}}, \rho, \bar{b}\right\}$, while for the small city I calibrate $\theta_{small} = \left\{\sigma_{x,small}, \chi_{small}\right\}$. Notice that while $corr(x, y)$ is calibrated for the large city, it is not targeted in the small city. Instead, I use parameter value for ρ obtained in the large city to calibrate the other parameters in the small city. This choice is motivated by the primary goal of the exercise, which is to test whether variation in matching efficiency and productivity dispersion can produce differences in assortative matching across locations.

Next, I discuss how each parameter is tightly linked to the associated moment in table 4. In figure A4 I show graphically how moments are related to parameters. The matching efficiency parameter is linked to the job-finding rate in the economy: a higher level of χ_j implies that it is easier for workers and firms to meet in the labor market and therefore making it easier to find a profitable partner to match with - this of course implies a higher job finding rate. The parameter $\sigma_{x,l}$ is chosen such that the standard deviation of the AKM worker fixed effect matches the standard deviation of the worker component of the following linear regression:

$$log w_{ij} = \alpha_i + \psi_j + \epsilon_{ij}$$

where w_{ij} is the equilibrium wage produced by the model, α_i denotes the worker type fixed effect, ψ_j denotes the firm type fixed effect, and ϵ_{ij} a residual. [18]. The elasticity of substitution parameter ρ is chosen such that the correlation between workers and firms productivity in the model matches the correlation obtained in the data. A lower value of ρ dictates a higher complementarity between factors of production -therefore

[18]For both locations I fix the mean of the log-normal to be 1 and I compute the value of $\mu_{x,l}$ accordingly, that is:$-\frac{\sigma^2_{x,l}}{2}$

more incentives to sort with similar types and a higher measured correlation between workers and firm productivity in equilibrium $corr(x, y)$. The parameter $\bar{b}$ is positively linked to the average replacement rate, since this is computed as the ratio between benefits and wages in the economy.

TABLE 4. Summary of parameters calibrated internally

Parameter	Description	Targ. Moment	Source
Large city			
χ_{large}	Matching efficiency	UE rate	LIAB-BHP
$\sigma_{x,large}$	Worker dist.	std(AKM Worker fe)	LIAB-BHP
$\frac{1}{1-\rho}$	CES parameter	AKM $corr(\alpha_i, \psi_j)$	LIAB-BHP
$\bar{b}$	Unemp. benefits	Avg. Repl. Rate	Lit.
Small city			
χ_{small}	Matching efficiency	UE rate	LIAB-BHP
$\sigma_{x,small}$	Worker dist.	std(AKM Worker fe)	LIAB-BHP

The table reports results from a summary of the calibrated parameter together with the moments they are associated with.

Table 5 reports the parameter values and the model fit, by including the value of the empirical moments and the counterpart in the model. The parameter χ is equal to 0.163 in the large city, while it is lower in the smaller city, 0.125. As a result, the meeting rate between unemployed workers and vacancies in the large city is around 2.063 higher than in the small city. The ρ parameter is calibrated to be -0.291, implying an elasticity of substituion between workers and firms productivity of 0.775. The shape parameter of the lognormal worker productivity distribution in the large city is calibrated to 0.614 and it is notable higher than in the small city, indicating a more disperse distribution. Turning to the untargeted moments, the most important is the correlation between worker and firm types in the small city. I find that it is equal to 0.088 while in the data is around 0.109, implying that quantitative model is able to explain slightly more than the observed differences in assortative matching between cities observed. I also compare other untargeted of interest to further validate the model. The model produces a standard deviation in log wages for the large city equal to 0.42 vs 0.5 in the data; while 0.36 in the small city vs 0.4 in the data. While the moments is not perfectly reproduced my the model, the qualitative pattern generated is consistent with the data. Also, these numbers are not surprising since the standard deviation of the worker fixed effect was explicitely targeted in our model. For what concerns the percentile rank difference p90-p10 for worker fixed effect, I find around 1.00 for the large city in the model and 0.89 for the small city, while I find 1.30 vs 1.0 in the data. Also in this case the model produces moments that are qualitatively consistent with the data. In figure 8A in the left panel I plot the average firm allocated to each worker type in the large city, and in the right panel the average worker allocated to each firm type. While the model is calibrated to match only the equilibrium correlation between worker and firm type, both figure shows that it matches fairly well the shape the of the average allocation for both agents estimated from the data. More interestingly, the model is able to replicate the almost absent presence of assortative matching in the small city. The average equilibrium allocation in fact almost overlap with the flat average allocation for workers and firms in the small city that is observed in the data (figure 9A and 9B).

TABLE 5. Targeted moments vs data moments

Parameter	Value	Moment	Model	Data
Targeted moments in the large city				
χ_{large}	0.163	UE rate	0.143	0.148
$\sigma_{x,large}$	0.614	StDev AKM worker FE	0.420	0.420
$\frac{1}{1-\rho}$	0.774	Corr(x,y)	0.252	0.252
$\bar{b}$	0.410	Avg. Repl. Rate	0.597	0.6
Targeted moments in the small city				
χ_{small}	0.125	UE rate	0.119	0.119
$\sigma_{x,small}$	0.538	StDev AKM worker FE	0.357	0.357

The table reports the value of each parameter, and report the difference between the moments produced by the model ($m_i(\theta_l)$) and the moment obtained from the data ($\hat{m}_i$)).

4.3. Counterfactuals: sources of assortative matching

This section examines the model's ability to replicate the gap in assortative matching between large and small cities as observed in the data. Specifically, I use the model calibrated for the small city to explore the quantitative impact of matching efficiency and the dispersion in worker productivity in driving differences in labor market sorting. In order to understand the role of matching efficiency, I recompute the equilibrium in the small city by setting the parameter χ equal to the value for the large city. As shown in Table 6 assortative matching increases by 39 % in the small city. The increase in matching efficiency intuitively reduces the extent of labor market frictions in the economy, and therefore workers and firms in the labor market have higher chances of meeting their ideal partner. This can also be seen from the lenses of equation 4: unproductive matches between workers and firms can't be sustained in equilibrium as the value of unemployment for workers and the value of vacancy for firms increase. As a consequence, matches in equilibrium are more selected. Next, I investigate what are the consequences of a change in the dispersion of the worker productivity distribution.

FIGURE 8. Average allocation for workers and firms in the large city

A. Avg allocation for workers percentile types

B. Avg allocation for firms percentile types

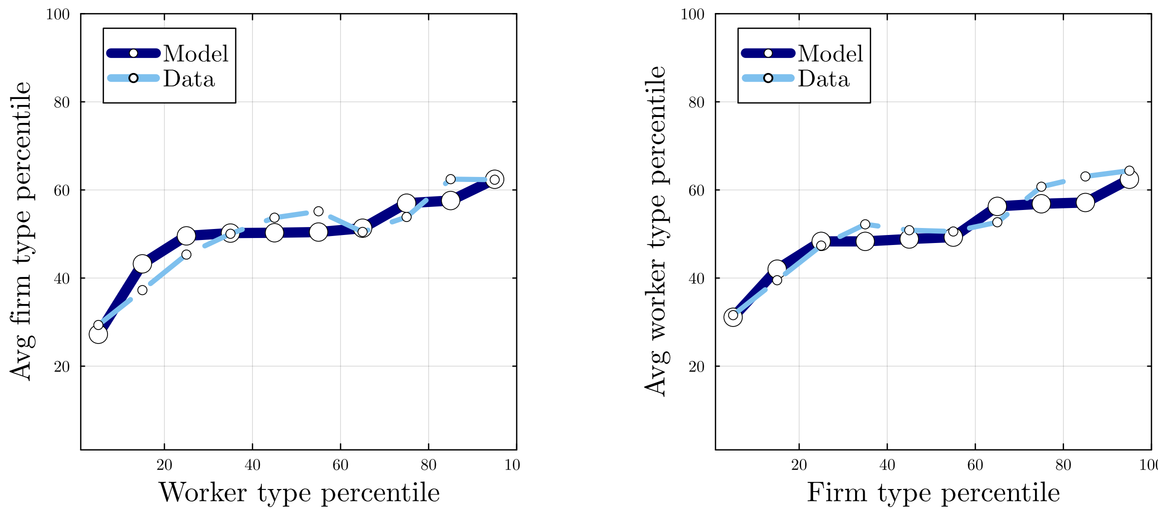


Figure 8A plots the average firm percentile allocated to each worker in the large city. The dotted dark blue line indicates the allocation produced by the model given the parameters obtained from the calibration, while the light blue line indicates the allocation obtained from the data. Similarly, Figure 8B plots the average worker percentile allocate to each firm percentile type.

FIGURE 9. Average allocation for workers and firms in the small city

A. Avg allocation for workers percentile types

B. Avg allocation for firms percentile types

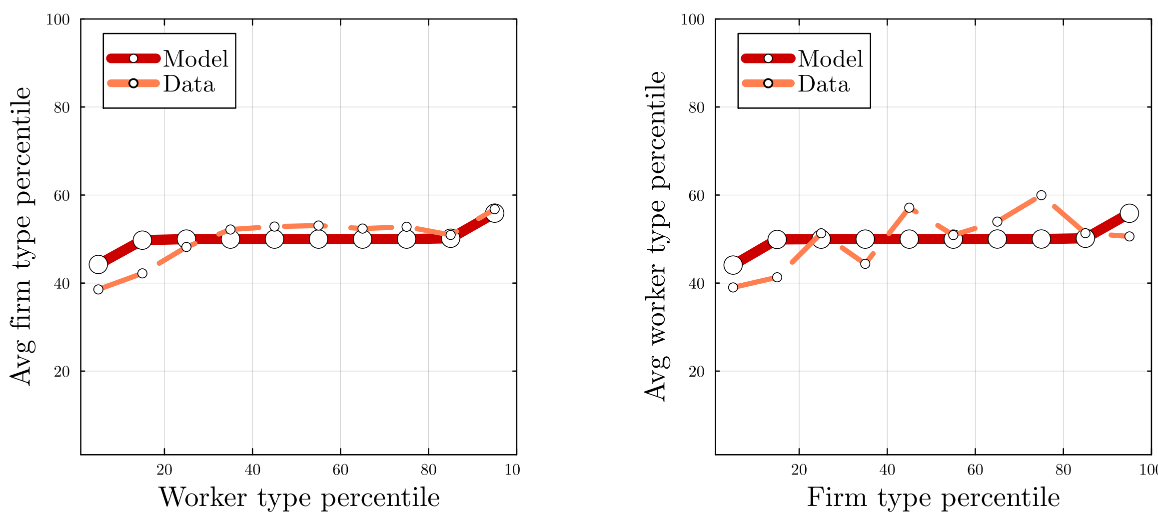


Figure 9A plots the average firm percentile allocated to each worker in the small city. The dotted dark red line indicates the allocation produced by the model given the parameters obtained from the calibration, while the light red line indicates the allocation obtained from the data. Similarly, Figure 8B plots the average worker percentile allocate to each firm percentile type.

When assigning the worker productivity distribution of the small city to the large city, assortative matching increases by 21%. The increase in the dispersion affects productivity of workers at the bottom and at the top of distribution, and therefore also the output they produce. While firms now find less profitable matches with low productivity workers as they would deliver less output, high productivity workers experience an increase in their marginal product and in their option value of unemployment at the same time, and therefore they are less willing to accept a mismatch with lower productive jobs. Finally, I study the case when both matching efficiency and dispersion increases to the same level as in the large city. The model is able to explain around 2/3 of the differences in assortative matching generate by the model. In this scenario, the effect of changes in the marginal products entailed by the change in the productivity distribution on labor market sorting are further amplified by the increase the matching efficiency. Not only there is higher differentiation between workers productivity but it is also relatively easier for both parts to meet in the labor market, which makes stronger the gains from complementarity and makes possible to sustain a higher degree of sorting in equilibrium. This set of results not only strengthens the fact that the matching efficiency is important to understand differences in labor market sorting, but it also highlights the role of distributions: when they interact with labor market frictions they can severely affect the equilibrium allocation.

4.4. Implications for spatial inequality

In this section I use the calibrated model to quantify the implications of differences in the degree of assortative matching for differences in economic performances between large and small cities. In the top panel of Table 7 I compare aggregate outcomes between cities. Overall, the model is able to explain 16.9 % of the GDP gap observed in the data, while it is not able to generate variation in average wages. Moreover, total welfare [19]

in the large city is slightly higher than in the small city (around 1%). Next, I investigate what is the impact of assortative matching on aggregate outcomes. I recompute the equilibrium in the small city by changing the distribution of workers productivity and by increasing the mass of each match so to close the gap in employment. The increase in the mass is proportional to the equilibrium one, so that the degree of sorting between

[19] Total welfare is defined as the sum of total output plus home production, namely:

$$\mathcal{W} = \int_x \int_y f(x,y)h(x,y)dydx + \int_x b(x)u(x)dx$$

TABLE 6. Counterfactuals: sources of assortative matching

	$corr(x, y)$	$\Delta corr(x, y)$	% Change
Baseline - Large	0.252	–	–
Baseline - Small	0.088	–	–
Contribution of individual parameters (Baseline - Small)			
$\chi = \chi_{large}$	0.152	0.064	21.3%
$\sigma_x = \sigma_{x,large}$	0.123	0.035	39.0%
Contribution of parameters together (Baseline - Small)			
$\chi = \chi_{large}$ and $\sigma_x = \sigma_{x,large}$	0.194	0.106	64.6%

The table reports results from a series of counterfactuals exercises. The first column reports the actual measure of sorting obtained; the second column reports the difference between baseline sorting and the counterfactual. The last column reports the share of the baseline difference explained by each component. In the first row I report the difference in PAM that I obtain from the baseline calibration between the large city and the small city. Then, I report differences due to individual parameters, that is, I set the baseline calibrated parameters of the small city equal to the calibrated parameters of the large city. In the last row I report counterfactuals obtained by changing two parameters at the same time.

workers and firms is not affected. I find that assortative matching can roughly explain up to 30 % of the difference in output generated by the model. Overall, it can explain up to 4.8 % of the differences in the GDP per capita observed in the data.

Next, I further investigate how matching efficiency and dispersion affect aggregate outcomes of cities, focusing in particular on the change in sorting between workers and firms that they imply. First, I increase the level of matching efficiency in the small city to the level of the large city and I recompute the equilibrium. Aggregate output increases by 2.2 % with respect to the baseline measure, while aggregate welfare increases by 2 %. While most of the aggregate gains can be accrued to the reduction in unemployment, better sorting has a very modest impact on output: the increase in assortative matching, in fact, can explain only 11 % of the increase in total output implied by the higher matching efficiency.

When increasing the dispersion of workers productivity, aggregate output decreases, as well as total welfare and total employment, while within-city inequality increases. The decline in output can be rationalized partly by the spread in productivity in the lower tail of workers productivity distribution. Notice however, that assortative matching has still a positive impact on output. In fact, without considering the reallocation forces, output per match would be 0.2 % lower.

TABLE 7. Differences in aggregate outcomes between large and small cities: model vs data

Variable	Description	Small	Large	Model (Large/Small)	Data (Large/Small)
Aggregate outcomes					
Y	Output	0.080	0.82	1.027	1.159
$\mathcal{W} = Y + \int_x b(x)u(x)dx$	Welfare	0.083	0.084		
$\overline{w(x,y)}$	Avg Wage	0.062	0.062	1.00	1.311

The table reports aggregate statistics computed in the model in the small and large city and compared them with the data.

TABLE 8. Effect of different channels on aggregate outcomes in the small city

Variable	Baseline - Small city	Matching efficiency	Dispersion
Aggregate outcomes			
Output	0.080	+2.28	-1.26
Employment	0.912	+2.03	-0.16
Welfare	0.083	+1.36	-1.47
Avg Wage	0.062	+0.77	-0.01
Distributional outcomes - Wages			
StDev(Log(Wage))	0.357	+0.05	+16.8

The table reports percentage changes in outcomes for large and small cities given changes in matching efficiency and dispersion. I use the small city as the baseline and change the value of parameters to the level of the large city.

5. Conclusion

This paper emphasizes the role of labor market frictions and workers heterogeneity to understand assortative matching in cities. I use a theoretical framework that encompasses both heterogenous workers and firms and labor market frictions to get insights about how the composition of workers and the extent of local labor market frictions affect assortative matching. Lastly, by calibrating the model with data from Germany, we obtain three key insights.

First, when improving the matching efficiency in the small city, the number of meetings in the economy increases and therefore both workers and firms become more selective in which matches to accept. As a consequence, assortative matching is stronger in equilibrium. Second, the composition of workers plays a minor role in affecting assortative matching in equilibrium; however, it can amplify the effect of matching efficiency. Third, the higher degree of assortative matching accounts for 5 % of the differences in GDP per capita between large and small cities.

Overall, the paper highlights the importance of studying local labor market frictions, by exploring their impact on assortative matching. Their effect could be further amplified in a context where regional labor markets are integrated, since they can affect the migration decision of workers. Higher assortative matching and more disperse workers distribution could then also arise because of workers geographical mobility. I regard this as a promising direction for future research.

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Appendix A. Appendix

A.1. Sample construction

The LIAB-BHP Data contains around 16 million observations. The number of unique persons is 1.6 million, while the number of unique firms is around 850k . Workers employment status in the sample can either be ‘1, Employees subject to social secturity’. ‘2, Trainees’, ‘3, marginal part-time workers’. ‘4. Employees in partial retirement’. ‘5 Interns and student trainees’. ‘6, Other employment status’ and ’11. Unemployment benefit’. Next, I am going to describe the restrictions applied to the dataset in order to obtain the final sample. First, I drop all the individual with age below 20 and above 60. Furthermore, I exclude female employees from the dataset. I also drop from the dataset all the employment spells related to part-time job relationships and all the individuals whose wage is below the marginal part-time income threshold (“Geringfuegigkeitsgrenzel”). Eventually, I drop all the workers who works in an establishment in East Germany, and all the spells that records an unemployment episode while resident in East Germany. Table A1 report basic statistics about the final sample.

TABLE A1. Spell-level data basic information

Varible	Obs.	Mean	St. Dev.
Real daily wage (imputed)	4769480	141.008	115.954
Real daily wage (not imputed)	4769480	119.355	49.144
Real daily wage (unemployment)	589559	25.529	17.337
Age	5359039	39.368	10.954
Employment (dummy)	5359040	.889	.312
College (dummy)	5359040	.302	.459

In the table I report basic information about the final sample used to perform the empirical analysis in section 2. After being restricted the dataset has 5359040 employment and unemployment records, while it counts 653625 unique individuals and 244449 unique establishments. Each row report a variable, the total number of observation, the mean and the standard deviation. Wages are deflated to 2015 euro. The first row displays the average daily wage for full-time workers, after the imputation process described in section 2.1. The second row displays the average daily wage before the imputation process. The third row reports real wages for unemployment benefits recipient. The fourth row shows statistics on the age of individuals. The fifth and sixth row inform about the share of people in employment and with a college degree in the sample.

A.2. AKM Sample Characteristics and Results

TABLE A2. Characteristics of the AKM sample

Set	Full Set	Connected Set
Full Set	✓	
Connected Set		✓
Sample counts (%)	100	95.54
Unique firms (1,000)	188	142
Share of full set (%)	100	75.6
Unique Workers (1,000)	580	539
Share of full set (%)	100	92.96

The table describes the sample used for the AKM regression. The whole sample counts more than 3 millions observation while the largest connected set is around 95 % of it.

TABLE A3. AKM Log-wage variance decomposition

Variable	Variance	Share explained (%)
Log wage	.2754948	100
Worker FE	.1745836	63.3709
Firm FE	.0463722	16.83232
Covariates	.0110731	4.019353
PAM	.0461446	16.74972
$cov(\alpha_i, xb)$	-.005251	-1.906034
$cov(\psi_{j(i,t)}, xb)$	.0025724	.9337403

The table reports the results obtained from the AKM regression. The second column indicates the variance while the third column reports the percentage share each variable accounts for.

FIGURE A1. Mean and p75-p25 for workers fixed effect distribution

A. Mean Worker FE

B. Difference between p75-p25

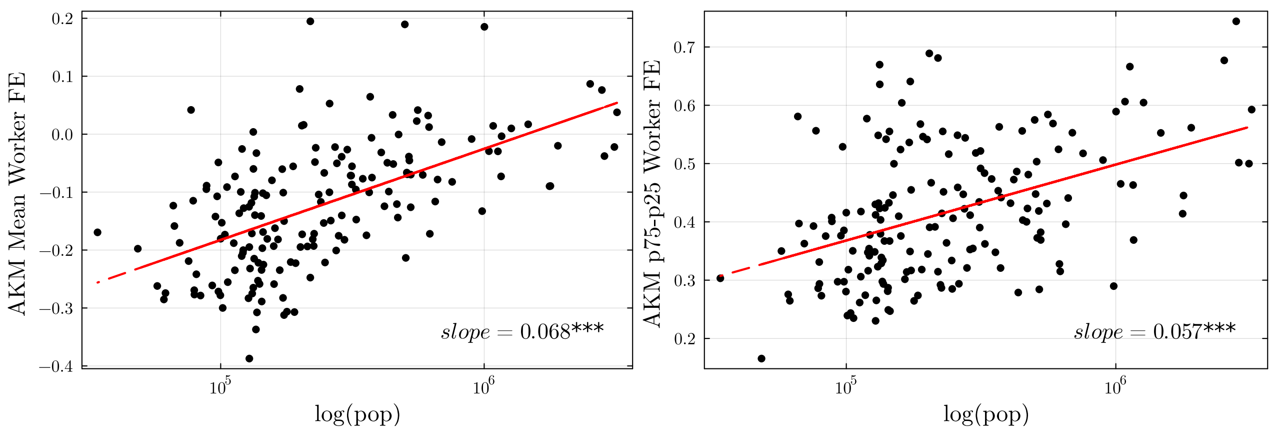

Figure 2A plots the mean for worker fixed effect against log-population for each local labor market. 2B plot the difference between the 75th percentile and 25th percentile of worker FE distribution of each LLM

FIGURE A2. Mean and p75-p25 for firms fixed effect distribution

A. Mean Firm FE

B. Difference between p75-p25

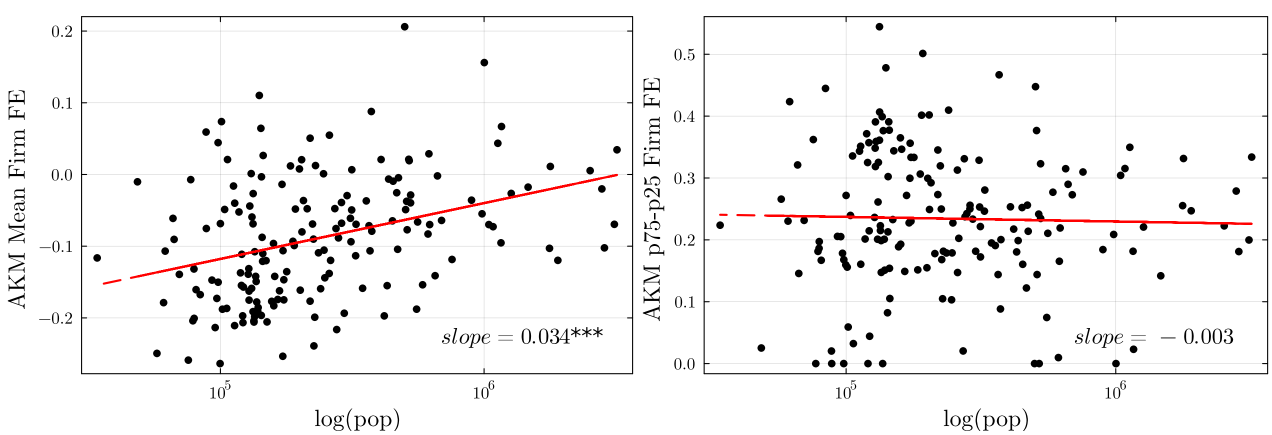

Figure 3A plots the mean for firm fixed effect against log-population for each local labor market. 3B plot the difference between the 75th percentile and 25th percentile of firm FE distribution of each LLM

FIGURE A3. Average allocation for workers and firms percentiles observed in the data

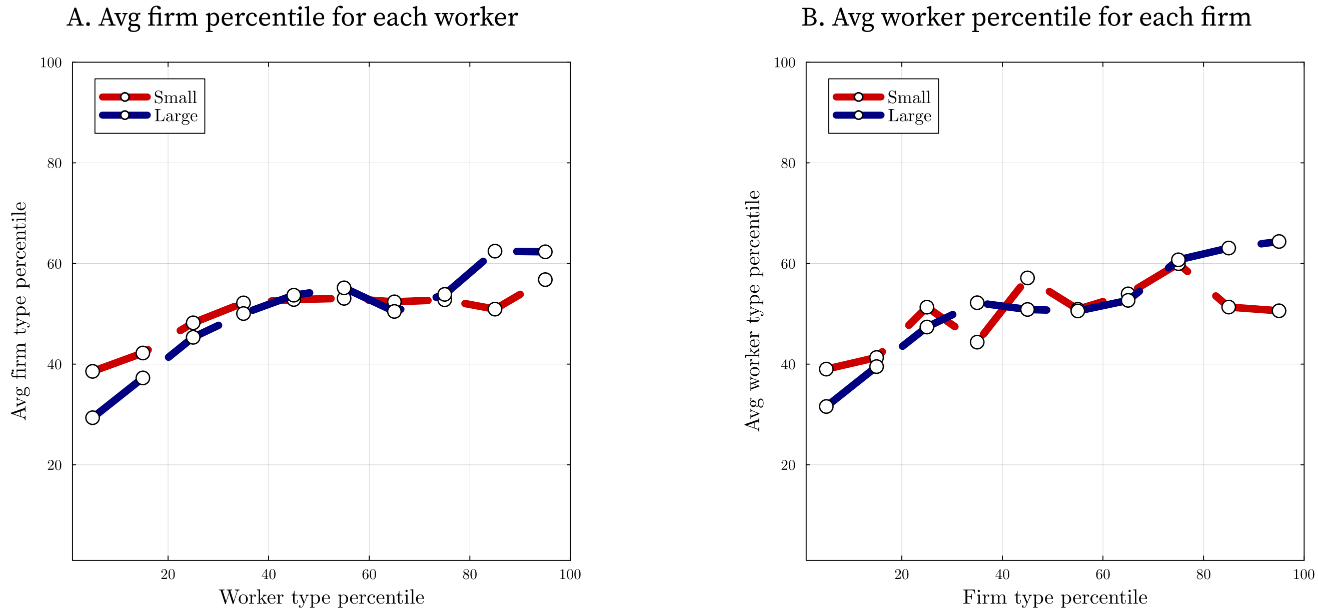

Figure A3A plot the average allocation for workers percentiles observed in the data. The blue line indicates the average allocation in the large city - which consider all the local labor market with population above the 66th percentile of distribution. The red line indicates the average allocation for small cities. Similarly, A3B plot the average allocation for firms percentiles. While the red line is flatter compared to red lines for both workers and firms, it implies a lower degree of assortative matching: there is very little variation in the average types allocated for either workers or firms in the small city.

A.3. Assortative Matching in Space: Robustness check

The AKM estimation suffer from the so-called limited mobility bias. In particular, the bias would positively affect the estimated variance of firm fixed effect and would therefore negatively effect the estimate of the covariance between workers and firms fixed effect, which is a proxy for assortative matching. In my particular case the lower degree of assortative matching observed for small cities can be the result the limited amount of workers transitions between workplaces. I follow Bonhomme, Lamadon, and Manresa (2019), who cluster firms according to their log-wage distribution using a kmeans algorithm [20]. First, I compute 20 quantiles of the log-wage distribution of firms and then I cluster them into 20 groups according to the similarity of the distribution. Then I run the following AKM specification, where the term $\psi_j(i,t)$ indicates the firm cluster.

[20]Bonhomme, Lamadon, and Manresa (2019) treats the worker effect as a random effect and further allows for interaction between workers and firm fixed effect. I follow Babet, Godechot, and Palladino (2022), which limit the analysis to the firm clustering step.

$$\log wage_{it} = \alpha_i + \psi_j(i,t) + \epsilon_{it}$$

When regressing the correlation between worker and firm fixed effect on log-population I obtain a coefficient of 0.028***.[21]. While correcting for the limited mobility bias in small cities, assortative matching betwee workers and firms is still significantly higher in large cities. Furthermore, when regression the measures of dispersion illustrated in section I obtain similar results: the distribution of worker fixed effect results more dispersed in large cities compared to small cities, while the dispersion of the firm fixed effect distribution does not show any significant variation.

A.4. Local labor market flows

DEFINING LOCAL JOB FINDING RATE AND JOB SEPARATION RATE . In this section I provide a description of how local labor market flows are constructed. I assume that workers make an unemployment-to-employment transition in a local labor market l if they were receiving unemployment benefits at time $t-1$ and resident in location l, and they are employed at time t at an establishment the same location l where they were residents. Therefore, the local job finding rate can be defined as:

$$UE_t^l = \frac{U_{t-1}^l \rightarrow E_t^l}{U_{t-1}^l}$$

Also, I assume that workers make an employment-to-unemployment transition if they were employed in an establishment a time $t-1$ in location l and they were unemployed at time t. The local job separation rate is defined as:

$$EU_t^l = \frac{E_{t-1}^l \rightarrow U_t}{E_{t-1}^l}$$

[21]Using a similar approach Dauth et al. (2022) find a coefficient of 0.038

TABLE A4. Job-finding probability across local labor markets

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
(log)Population	0.006***	0.009***	0.009***	0.009 ***	0.009***	0.011***
Nationality		X	X	X	X	X
Education			X	X	X	X
Age				X	X	X
Month					X	X
Year						X
R2	0.00	0.00	0.003	0.011	0.015	0.018
Obs	1166576	1165404	1088923	1088923	1088923	808129

The table reports the estimated coefficients for a regression with job-finding rate dummy as a dependent variable against (log)population. Models on columns include different combination of regressors (controls) reported on rows. * p < 0.10, ** p< 0.05, *** p< 0.01)

TABLE A5. Unemployment probability across local labor markets

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
(log)Population	-0.002***	-0.002***	-0.002***	-0.001 ***	-0.001***	-0.001***
Nationality		X	X	X	X	X
Education			X	X	X	X
Age				X	X	X
Month					X	X
Year						X
R2	0.000	0.002	0.004	0.008	0.009	0.009
Obs	32.8 Million	32.8 Million	32.5 Million	32.5 Million	32.5 Million	32.5 Million

The table reports the estimated coefficients for a regression with unemployment status ad a dependent variable against (log)population. Models on columns include different combination of regressors (controls) reported on rows. * p < 0.10, ** p< 0.05, *** p< 0.01)

A.5. Calibration strategy

In table **??** I report basic information about local labor markets in West Germany included in my dataset. As I split them between tercile of the population distribution, I report the first five cities and the last five cities for each tercile.

In figure A4 I show how the moments obtained from the steady-state equilibrium of the model are sensititive to calibrated parameters and therefore informed by them in the method of moments procedure.

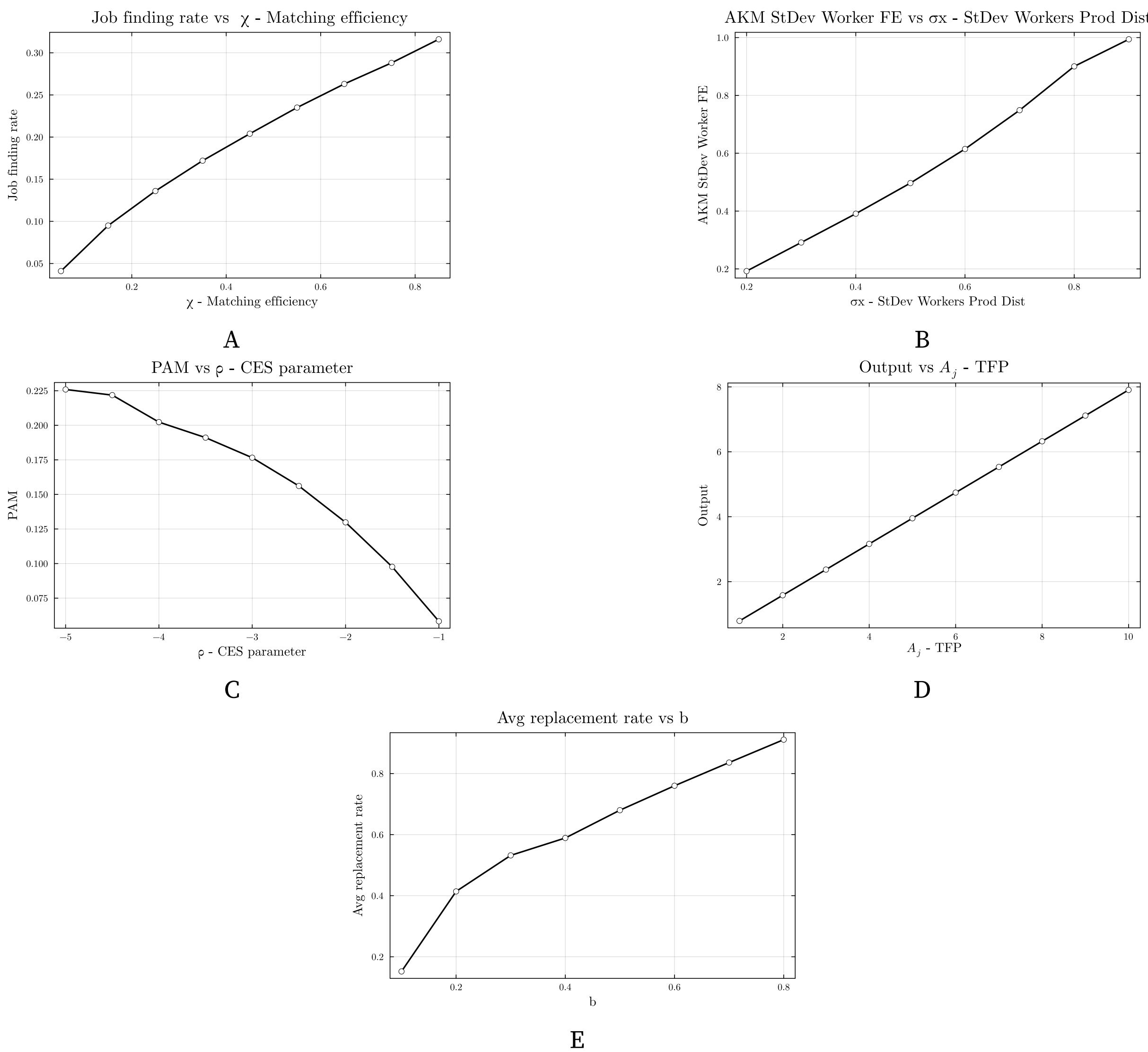

FIGURE A4. Moments vs Parameters

The figure shows how sensible moments are with respect to the parameters of the model. In A I report how the job-finding rate produced by the model changes with parameter χ. In B I report how the standard deviation of worker fixed effect obtained from the AKM regression on equilibrium wages varies with the σ parameter of the lognormal distribution. Panel C reports how assortative matching varies with the CES parameter ρ and eventually Panel D reports how output varies with the TFP parameter A. Panel E shows how the average replacement rate varies with the parameter b.

TABLE A6. Population and GDP for LLM in West Germany (2010 - 2017)

LLM	Population	Tercile	Avg GDP	Avg Real GDP	Avg Real GDP pc
Frankfurt am Main	3192266	3	151322.0	152377.0	47733.0
Köln	3119585	3	125226.0	126221.0	40461.0
Hamburg	2860344	3	128054.0	128897.0	45064.0
München	2797005	3	160321.0	160680.0	57447.0
Stuttgart	2524853	3	124571.0	125457.0	49689.0
Lippe	346151	3	9365.0	9457.0	27320.0
Rosenheim	326875	3	10080.0	10151.0	31055.0
Pforzheim	325766	3	9996.0	10107.0	31027.0
Kleve	314676	3	7984.0	7988.0	25384.0
Bremerhaven	312776	3	7640.0	7713.0	24660.0
Soest	302298	2	9515.0	9613.0	31801.0
Baden-Baden	287823	2	11428.0	11502.0	39963.0
Siegen-Wittgenstein	274342	2	10018.0	10115.0	36869.0
Oberbergischer Kreis	271621	2	8660.0	8754.0	32228.0
Bodenseekreis	218885	2	9296.0	9268.0	42343.0
Herzogtum Lauenburg	200819	2	3946.0	3959.0	19713.0
Leer	172421	2	3945.0	3993.0	23157.0
Calw	160686	2	3990.0	4011.0	24962.0
Ebersberg	144562	2	3844.0	3849.0	26624.0
Verden	138507	2	3754.0	3788.0	27346.0
Donau-Ries	134986	1	5701.0	5742.0	42540.0
Günzburg	128436	1	4960.0	4953.0	38563.0
Coburg	127499	1	5368.0	5378.0	42183.0
Altötting	112116	1	4983.0	5034.0	44903.0
Dingolfing-Landau	98045	1	5689.0	5710.0	58243.0
Cochem-Zell	61735	1	1610.0	1619.0	26219.0
Vulkaneifel	60882	1	1707.0	1718.0	28222.0
Wittmund	57455	1	1278.0	1284.0	22341.0
Lüchow-Dannenberg	48472	1	1113.0	1121.0	23120.0
Zweibrücken	34091	1	1387.0	1398.0	41015.0