The Interaction of Financial Frictions and Uncertainty Shocks: Implications for the Cyclicality of Excess Returns

Giacomo Cattelan

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Abstract

This paper presents new evidence on how the countercyclicality of excess returns is driven by the interaction between the financial sector's balance sheet conditions and uncertainty shocks. Using a nonlinear specification of the local projection method to estimate impulse response functions, I find that the effects of shocks to various volatility indices—both on excess returns and real economic variables—are amplified when the financial sector's balance sheet has weakened prior to the shock. These empirical findings are replicated by a macro-finance general equilibrium model that incorporates a financial sector subject to an occasionally binding constraint as in Gertler and Karadi (2011). The model introduces a novel source of uncertainty, modeled as a stochastic component affecting the total external funding available to financial intermediaries, consistent with real-world observations. When this "financial uncertainty" increases, it raises the likelihood that intermediaries' financial constraints will bind, triggering precautionary deleveraging. This, in turn, leads to a surge in excess returns and a decline in economic activity, effects that grow in magnitude as the economy moves closer to the constraint.

1 Introduction

Periods of financial turmoil, often characterized by sharp increases in risk premia, are typically accompanied by slowdowns in economic activity. This paper argues that a key factor behind the countercyclical behavior of excess returns during such periods is the interplay between the capitalization of the financial system and the time-varying uncertainty of financial markets. Financial intermediaries, driven by a combination of regulatory requirements and market pressures, are especially focused on maintaining appropriate net worth-to-asset ratios. In fact, these financial constraints are considered by many to have been a critical driver of the 2008 financial crisis. Therefore, faced with the risk of a sharp decline in asset values relative to their net worth, intermediaries engage in precautionary behavior when making investment decisions. Specifically, they hedge against balance sheet risks by reducing exposure to risky assets, particularly when market volatility increases. This uncertainty-induced precautionary deleveraging reduces demand for such assets, leading to higher risk premia and lower real investment, which in turn exacerbates the contraction in economic output.

In this paper, I present novel empirical evidence showing that financial uncertainty shocks, as identified by Bloom (2009) and Ludvigson et al. (2021), exert stronger effects when the financial sector's aggregate balance sheet is in a weakened state in the period leading to the shock itself. To estimate these effects, I use a local projection method developed by Jordà (2005), which allows for a nonlinear analysis of impulse response functions. These responses are conditional on the financial sector's balance sheet quality, measured by the year-on-year growth of equity-capital ratio. Periods of financial distress are defined as those in which this growth is negative. The results of the analysis show that when financial intermediaries are distressed prior to an uncertainty shock, stock excess returns rise on average by an additional 2 basis points compared to the case of healthy financial sector balance sheet, and this effect tends to be more persistent over time. Furthermore, the corporate bond premium, as calculated by Gilchrist and Zakrajšek (2012), increases by 4 basis points in periods of distress, whereas it remains relatively stable during periods of stronger capitalization. Finally, real economic activity, as measured by the index of industrial production, declines more significantly when the financial sector is under stress, with output falling by an additional 0.1 percentage points *vis-à-vis* the same shock hitting when the financial system is stable.

To rationalize these findings, I extend the standard Real Business Cycle (RBC) model by incorporating a financial sector that is subject to frictions *à la* Gertler and Karadi (2011). On top of the common macroeconomic uncertainty around Total Factor Productivity (TFP), the model introduces a novel source of financial uncertainty: shocks to the external funding available to financial intermediaries. These funding shocks divert a portion of household savings from (into) bank deposits into (from) government bonds.

This distinction between financial and macroeconomic uncertainty is crucial, as financial uncertainty refers to shocks that primarily affect the financial sector, without altering the overall endowment of the household. The model shows that an increase in financial uncertainty causes intermediaries to engage in precautionary deleveraging, leading to declines in investment and output while simultaneously raising excess returns on capital. These effects are magnified when the financial sector is already near its constraint, highlighting the critical role of intermediaries' precautionary behavior in amplifying the economic consequences of uncertainty shocks.

The model is solved globally, allowing for occasional binding of financial constraints. This generates differential responses to uncertainty shocks depending on the initial state of the economy. Importantly, the model replicates the observed precautionary behavior of financial intermediaries, who reduce their exposure to risky assets when uncertainty rises, particularly when their capitalization has been weakening before the shock. These dynamics can only be generated in the model when uncertainty shocks are financial in nature, as they directly affect intermediaries. Conversely, when uncertainty originates from macroeconomic sources—such as increased TFP volatility—households respond by increasing precautionary savings, which helps to offset the deleveraging pressures in the financial sector.

The distinction between financial and macroeconomic uncertainty is also reflected in the empirical component of the paper. Ludvigson et al. (2021) were among the first to emphasize this distinction, noting its importance for understanding the causal effects of uncertainty shocks. Their work highlights the fact that financial uncertainty often serves as a primary driver of business cycle fluctuations, while macroeconomic uncertainty tends to emerge in response to real economic shocks. For this reason, I employ their financial uncertainty index, U_F^{LMN}, as a key variable in my analysis. Additionally, I use the VIX, a widely recognized measure of financial uncertainty based on the dispersion of prices for SP500 futures, spliced with its predecessor, the VXO. Although the VIX is often interpreted as a proxy for macroeconomic uncertainty, it also reflects uncertainty surrounding the pricing of financial assets, where intermediaries are the marginal investors, as it has been empirically proven by He et al. (2017). As such, a substantial portion of the VIX's fluctuations can be attributed to financial uncertainty, making it a valuable metric for the analysis conducted in this paper.

By clearly separating financial uncertainty from macroeconomic uncertainty, this paper provides deeper insights into how financial sector fragility interacts with market volatility, shaping both asset prices and real economic activity. This approach not only advances the empirical understanding of the economic impact of uncertainty shocks but also offers a novel theoretical framework for modeling financial sector dynamics in periods of heightened uncertainty.

The rest of the paper is outlined as follows. Section 2 discusses data, methodology and results of the empirical analysis. Section 3 presents the model. Section 4 is dedicated to a quantitative exercise that computes theoretical impulse response functions using the model from Section 3. Section 5 concludes.

1.1 Related Literature

1.1.1 On Excess Return Cyclicality

The countercyclical nature of excess returns on stocks has been empirically documented since the seminal works of Fama and French (1989) and Ferson and Harvey (1991). Subsequent studies, including those by Lettau and Ludvigson (2001), Lettau and Ludvigson (2009), Backus et al. (2010), and Lustig and Verdelhan (2012), among others, have built on this phenomenon, demonstrating that strongly cyclical factors predict stock excess returns.[1] More recently, Nagel and Xu (2023) provided evidence that this countercyclical pattern is predominantly a feature of in-sample analysis of realized excess returns, while subjective risk premia—derived from surveys of individual investors—do not exhibit a similar cyclical behavior.

In theoretical asset pricing, this behavior is typically explained through models that generate a cyclical stochastic discount factor. For example, the habit formation model of Campbell and Cochrane (1999) and the model by Bansal and Yaron (2004), which introduces a combination of countercyclical risk aversion and a heteroskedastic endowment process, both account for the countercyclical movement in asset prices.

Other asset classes exhibit similar countercyclical patterns. Gilchrist and Zakrajšek (2012) document this behavior in corporate bond spreads through their measure, the so-called GZ spread. Likewise, Cochrane and Piazzesi (2005), Cochrane and Piazzesi (2008), and Ludvigson and Ng (2009) report similar findings for U.S. government bonds. Finally, Lettau et al. (2014) demonstrate the presence of countercyclical risk premia in commodities, sovereign bonds, and currency returns.

1.1.2 On Uncertainty Shocks

The asset pricing literature has long recognized time-varying volatility in economic fundamentals as a key driver of asset price fluctuations, as exemplified by the work of Bansal and Yaron (2004). However, the macroeconomic literature has been slower to incorporate heteroscedastic processes. Bloom (2009) was a pioneer in this area, introducing the study of uncertainty shocks and providing evidence that a shock to the volatility index (VXO) significantly depresses output and employment. Despite these insights, traditional real business cycle models have struggled to replicate the observed comovements of macroeconomic aggregates. Basu and Bundick (2017) demonstrated that, when these shocks

[1] More recent examples include Campbell et al. (2020).

are modeled as time-varying variance in productivity innovations, nominal frictions that dampen labor demand are necessary to reproduce the empirical findings. Various approaches to modeling uncertainty shocks have since emerged: Bloom (2009) and Bloom et al. (2018) interpret fluctuations in uncertainty as shocks to the cross-sectional variance of firms' productivity, while other studies, such as those by Fernandez-Villaverde et al. (2011), Fernandez-Villaverde et al. (2015), and Born and Pfeifer (2014), focus on policy uncertainty.

1.1.3 On Financial Intermediation

The role of frictional financial intermediation in macroeconomic dynamics has been widely discussed since Bernanke et al. (1999) introduced the financial accelerator theory, but gained significant prominence after the Great Financial Crisis. Early macro-financial models featuring a financial sector constrained by financial frictions, such as those by He and Krishnamurthy (2013) and Brunnermeier and Sannikov (2014), demonstrated the importance of financial intermediation in the transmission of aggregate shocks. He et al. (2017) provided empirical validation for these theories, showing that the capital-equity ratio of primary dealers—institutions that trade with the Federal Reserve during Open Market Operations—is a significant predictor of excess returns across various asset classes. In the macroeconomics literature, Gertler and Karadi (2011) and Gertler and Kiyotaki (2015) introduced constrained intermediaries in a model that accommodates both conventional and unconventional monetary policy interventions, which has since become the dominant framework for analyzing the impact of the financial sector on the broader economy.

1.1.4 On Uncertainty Shocks and Financial Frictions

The interaction between uncertainty shocks and financially constrained intermediaries has been explored theoretically by Christiano et al. (2014) and Cesa-Bianchi and Fernandez-Corugedo (2018). Both studies build on variants of the Bernanke et al. (1999) model, in which intermediaries lend to entrepreneurs whose quality is costly to verify. Their findings suggest that micro-dispersion, or cross-sectional volatility, accounts for a substantial proportion of business cycle variance, while total factor TFP volatility plays a more moderate role. Similarly, Gilchrist et al. (2014) investigate the impact of uncertainty shocks in a model with heterogeneous firms that borrow subject to a limited liability constraint, showing that agency frictions in the credit market are quantitatively significant for the transmission of uncertainty shocks. Arellano et al. (2019) propose a related framework where defaultable debt is used to finance wage payments rather than capital investment. In the context of international economics, Akinci et al. (2022) document and model the spillover effects of increased U.S. TFP volatility on other economies using a Gertler and

Karadi (2011) framework.

Other research assumes that the severity of financial frictions is time-varying and subject to heteroscedastic shocks. For instance, Fernandez-Villaverde and Guerrón-Quintana (2020) augment a standard Kiyotaki-Moore model with uncertainty shocks to the borrowing constraint. Chatterjee et al. (2020) identify shocks to the volatility of the time series of credit to the non-financial sector and, using local projections, find that uncertainty has a much larger impact during downturns. They explain their findings through a model where representative firms face collateral constraints when making hiring decisions. Finally, Fang and Liu (2021) globally solve a model with Gertler and Karadi (2011)-type frictions for international investors, incorporating shocks to the volatility of the time-varying parameter that regulates the tightness of financial constraints for intermediaries.

2 Empirical Analysis

This section explores how the countercyclical dynamics of excess returns become more pronounced following a structural uncertainty shock when the financial sector is undercapitalized. To investigate this, I employ the local projection method developed by Jordà (2005), which accommodates the possibility of nonlinear impulse responses in the variables of interest. These variables include excess returns on stocks, bonds, and mortgages, along with key macroeconomic aggregates such as consumption, investment, and industrial production. In addition, I analyze balance sheet quantities from the financial sector, including credit extended to non-financial firms, Treasury holdings, net worth, investment in private safe assets and mortgages. This latter category is particularly important for understanding the transmission mechanism of uncertainty shocks. Specifically, I will provide evidence of a significant portfolio reallocation effect following an uncertainty shock, where financial intermediaries reduce their exposure to the private sector—primarily by curtailing credit extension—and shift their investments toward safer assets, such as Treasuries.

2.1 Data

Excess returns on stocks are calculated using monthly returns on the SP500, sourced from the Center for Research in Security Prices (CRSP), and the monthly risk-free rate computed as a common risk factor by Fama and French (1993). Consistent with the literature, I assume a holding period of 10 years for stocks. This assumption offers two key advantages: it smooths out short-term noise and facilitates the comparison of stock excess returns with the spread on long-term assets. Using the one-month returns $R_{t,t+1}$, I compute the 10-year monthly returns on stocks and the corresponding 10-year monthly risk-free rate as follows:

$$R_{t,t+120} = \left[\prod_{j=0}^{119}(1 + R_{t+j,t+j+1}) - 1\right]^{\frac{1}{120}}.$$

The excess returns are then calculated as the difference between the 10-year stock returns and the 10-year risk-free rate:

$$R_t^{exc} = R_{t,t+120}^{SP500} - R_{t,t+120}^{f}.$$

The corporate bond credit spread (henceforth referred to as the GZ spread) is computed following Gilchrist and Zakrajšek (2012). These authors further decompose the GZ spread into two components: one explained by the distance to default of individual bonds, and the Excess Bond Premium (EBP), which is typically attributed by the literature to liquidity conditions and other financial factors in the bond market. Given the EBP's inherently financial nature, it will be the object of the empirical investigation together with the overall GZ spread.

The last excess return I will focus on is the mortgage spread. Mortgages are one of the most important items on the asset side of banks' balance sheet, hence, if the prediction of precautionary deleveraging is correct, a spike in the premium demanded over this type of asset will be observed. In line with much of the literature, this spread is computed as the difference between a 30-year fixed rate mortgage and the 10-year Treasury yield, both series being available on FRED.

Data on the balance sheets of financial intermediaries is sourced from the CRSP/COMPUSTAT merged database. Specifically, I collect the equity-to-capital ratio[2] for specific subclasses of intermediaries in order to construct a measure of the stability of the financial sector, which will be labelled η_t. The equity-to-capital ratio measures the proportion of the firm's own capital (common equity) relative to the total capital invested, which includes both shareholder and debtholder capital. To aggregate this measure across each class of financial intermediaries, I calculate a weighted average, with the weights based on each firm's total equity value as provided by the CRSP/COMPUSTAT database. Further details on the construction of η_t will be discussed in Section 2.2.

Data on credit extended to non-financial businesses by financial institutions (both depository and non-depository) is collected from BIS statistics.[3] Credit encompasses both loans and debt securities. I also gather data on safe assets held by the financial sector, distinguishing between public and private assets. Public safe assets refer to treasury securities,[4] while private safe assets are calculated as the sum of various types of debt

[2] The variable name in the CRSP/COMPUSTAT merged database is "equity_invcap."

[3] Time series code Q:US:P:A:M:USD .

[4] Sourced from the FRED series "Domestic Financial Sectors; Treasury Securities; Asset, Level."

securities issued by private financial institutions, as outlined by Gorton et al. (2012) and Almadani et al. (2020).[5]

Macroeconomic variables used in the analysis include Personal Consumption Expenditure (PCE), Gross Fixed Capital Formation (GFCF) as a proxy for investment, hours worked, and the index of industrial production. These aggregates are measured in dollar terms and converted into real quantities by adjusting for inflation using the Consumer Price Index (CPI). To focus on cyclical fluctuations, the data is HP-filtered ($\lambda = 1600$), and the results presented reflect percentage deviations of the cyclical component from the long-term trend.

Finally, as discussed in the Introduction, the uncertainty measures used in the empirical analysis are derived from the indexes developed by Bloom (2009) and Ludvigson et al. (2021). Specifically, I use the VXO index, spliced with realized volatility of SP500 returns, and the financial uncertainty index from Ludvigson et al. (2021). Shocks to these uncertainty indexes are identified following the methodologies of Bloom (2009) and Ludvigson et al. (2021).

2.2 Local Projections

Jordà (2005) defines the impulse response function for a vector of variables $\mathbf{y}$ at horizon h of a system hit by a shock $\mathbf{v}$ at time t as:

$$IRF(h, t, \mathbf{v}) = \mathbb{E}_t[\mathbf{y}_{t+h} \mid \epsilon_t = \mathbf{v}] - \mathbb{E}_t[\mathbf{y}_{t+h} \mid \epsilon_t = \mathbf{0}] \tag{1}$$

where the operator $\mathbb{E}_t$ is the expectation conditional on information available at time t. It is possible to generalize this definition to include impulse response functions conditional on the economy being in a given state upon the arrival of the shock, for example, in the context of this paper, the financial system being in distress. Let $\mathbf{x}_t$ represent the state of the economy. Hence, the impulse response functions conditional on the state being in a given region, i.e. $\mathbf{x}_t \in \mathcal{A}$, can be defined as:

$$IRF(h, t, \mathbf{v} \mid \mathbf{x}_t \in \mathcal{A}) = \mathbb{E}_t[\mathbf{y}_{t+h} \mid \epsilon_t = \mathbf{v}, \mathbf{x}_t \in \mathcal{A}] - \mathbb{E}_t[\mathbf{y}_{t+h} \mid \epsilon_t = \mathbf{0}, \mathbf{x}_t \in \mathcal{A}]. \tag{2}$$

Inspired by Equation 2, Proposition 1 follows.

Proposition 1. *Let $\{\epsilon_{t+h}\}_h$ be a sequence of i.i.d. vector valued shocks of dimension N_ϵ, and $\mathbf{x}_t \in \mathbb{R}^{N_\mathbf{x}}$ a vector of states. Let the collection of smooth operators $\left\{\Phi^h\right\}_h$, with*

[5] These include "Money Market Funds; Total Financial Assets," "Domestic Financial Sectors; Checkable Deposits and Currency," "Domestic Financial Sectors; Federal Funds and Security Repurchase Agreements," "Private Depository Institutions; Total Time and Savings Deposits," "Finance Companies; Commercial Paper," "Private Depository Institutions; Bankers' Acceptances," and "Domestic Financial Sectors; Total Miscellaneous Liabilities."

$\Phi^h \in C^\infty(\mathbb{R}^{N_\epsilon \times h + N_\mathbf{x}}, \mathbb{R}^{N_\mathbf{y}})$, *define the dynamics of the system* $\{\mathbf{y}_{t+h}\}_h \in \mathbb{R}^{N_\mathbf{y} \times \infty}$*:*

$$\mathbf{y}_{t+h} = \Phi^h(\epsilon_t, ..., \epsilon_{t+h}, \mathbf{x}_t). \tag{3}$$

Then:

$$\begin{aligned} IRF(h, t, \mathbf{v} \mid \mathbf{x}_t \in \mathcal{A}) =& \frac{\partial \Phi^h(\mathbf{0})}{\partial \epsilon_t} \cdot \mathbf{v} + \frac{\partial^2 \Phi^h(\mathbf{0})}{\partial \epsilon_t \partial \mathbf{x}_t} \cdot (\mathbb{E}[\mathbf{x}_t \mid \mathbf{x}_t \in \mathcal{A}] \otimes \mathbf{v}) \\ &+ \frac{1}{2} \frac{\partial^2 \Phi^h(\mathbf{0})}{\partial \epsilon_t^2} \cdot (\mathbf{v} \otimes \mathbf{v}) + \mathcal{R}(\mathbb{E}[\mathbf{x}_t \mid \mathbf{x}_t \in \mathcal{A}], \mathbf{v}). \end{aligned} \tag{4}$$

where $\mathcal{R}(\mathbb{E}[\mathbf{x}_t \mid \mathbf{x}_t \in \mathcal{A}], \mathbf{v}) = o(\|[\mathbb{E}[\mathbf{x}_t \mid \mathbf{x}_t \in \mathcal{A}], \mathbf{v}] \otimes [\mathbb{E}[\mathbf{x}_t \mid \mathbf{x}_t \in \mathcal{A}], \mathbf{v}]\|)$.

Proposition 1 and its Proof (available in Appendix A.1) suggest that, the estimates of the conditional impulse response functions can be retrieved from the following system of regressions:

$$y_{t+h} = \alpha_h + \beta_0^h \epsilon_t^{UNC} + \beta_1^h \epsilon_t^{UNC} \eta_{t-1} + \boldsymbol{\gamma}' \cdot \mathbf{z}_t + u_t \tag{5}$$

where y_{t+h} is on of the entries of $\mathbf{y}_{t+h}$, $\boldsymbol{\gamma}$ is a vector of coefficients and $\mathbf{z}_t$ is a vector of controls. Since the shocks have already been identified, they are orthogonal to the residuals, allowing for the use of OLS. The variable η_{t-1} is the proxy for financial sector stability mentioned above. I construct the aforementioned proxy of financial sector stability starting from the equity-to-capital ratio, focusing on intermediaries that serve as marginal investors in the specific asset class under analysis. For stocks, the relevant marginal investors are Primary Dealers—systemically important institutions that trade with the Federal Reserve during Open Market Operations, as shown by He et al. (2017). Since Primary Dealers are primarily large commercial and investment banks, I apply the same classification when analyzing the mortgage spread. For corporate bonds, I include all firms classified under the "Financial" sector according to the Global Industry Classification Standard (GICS),[6] capturing a broader range of financial intermediaries. Given the presence of a long-run downward dynamics in the raw data, I decompose the capitalization ratio κ_t in trend and cycle components, i.e. $\kappa_t = \kappa_t^{\text{trend}} + \kappa_t^{\text{cycle}}$, and then compute

$$\eta_t = \frac{\kappa_t - \kappa_t^{\text{trend}}}{\kappa_t^{\text{trend}}}$$

This adjustment results in a variable that fluctuates at business cycle frequency and allows me to evaluate whether financial intermediaries are under- or over-capitalized compared

[6]The GICS "Financial" sector includes three industry groups: Banks, Financial Services (excluding banking), and Insurance Companies.

to the trend.

Notice that, in the context of the current investigation, $\mathbf{x}_t = \eta_{t-1}$ and $\mathcal{A} = \{\eta_{t-1} < 0\}$, since it will be assumed that financial institutions are distressed if the equity-invested capital ratio is below trend in the period leading to the shock. Therefore, the conditional impulse responses to $\epsilon_t^{UNC} = v = 1$ s.d. of volatility index can be computed from the estimated coefficients as follows:

$$\widehat{IRF}(h, t, v \mid \eta_{t-1} < 0) = \left(\widehat{\beta_0^h} + \widehat{\beta_1^h}\underline{\eta}\right) v \tag{6}$$

$$\widehat{IRF}(h, t, v \mid \eta_{t-1} \geq 0) = \left(\widehat{\beta_0^h} + \widehat{\beta_1^h}\overline{\eta}\right) v. \tag{7}$$

where $\underline{\eta}$ and $\overline{\eta}$ are the time averages of η_{t-1} conditional on $\eta_{t-1} < 0$ and $\eta_{t-1} \geq 0$ respectively. Details on estimation and confidence intervals can be found in Appendix A.2.

The variables of interest in $\mathbf{y}$ have been described in the previous subsection: stocks, bonds and mortgages excess returns, credit from the financial sector to the non-financial sector, Treasuries held by the financial sector, consumption, investment, hours worked and industrial production. The vector of controls $\mathbf{z}$, on the other hand, is composed of lagged industrial production, CPI, FFR and term spread computed as the difference between the 10-years and 3-months real rate on government bonds.

2.3 Results

The impulse responses of excess returns to a one standard deviation shock to the VXO index are presented in Figure 1. The top panel shows the responses conditional on $\eta_{-1} \geq 0$, while the bottom panel captures the responses when $\eta_{-1} < 0$. The results indicate that, when the financial system's capitalization increased in the month before the shock, none of the four excess returns exhibit significant reactions. However, when the financial sector experienced a decline in capitalization during the preceding month, all four excess returns demonstrate notable increases. Specifically, the SP500 rises by 3 basis points (b.p.), and this effect remains significantly positive for 16 months. The excess bond premium (EBP) increases by 10 b.p., while the overall credit spread gradually rises, peaking at approximately 4 b.p. nine months after the shock. Finally, the mortgage spread increases over six months, reaching a peak of 5 b.p.

The real effects of the VXO shock are illustrated in Figure 2. Under the condition where $\eta_{-1} \geq 0$ (top row), the responses across all real variables are relatively muted. Investment declines by about 0.5 percentage points (p.p.) in the third quarter following the shock, gradually returning to baseline. Consumption declines modestly, reaching a trough of -0.1 p.p. by the third quarter, followed by a slight recovery. Hours worked decreases by 0.2 p.p. at its lowest point in the third quarter, while output contracts by

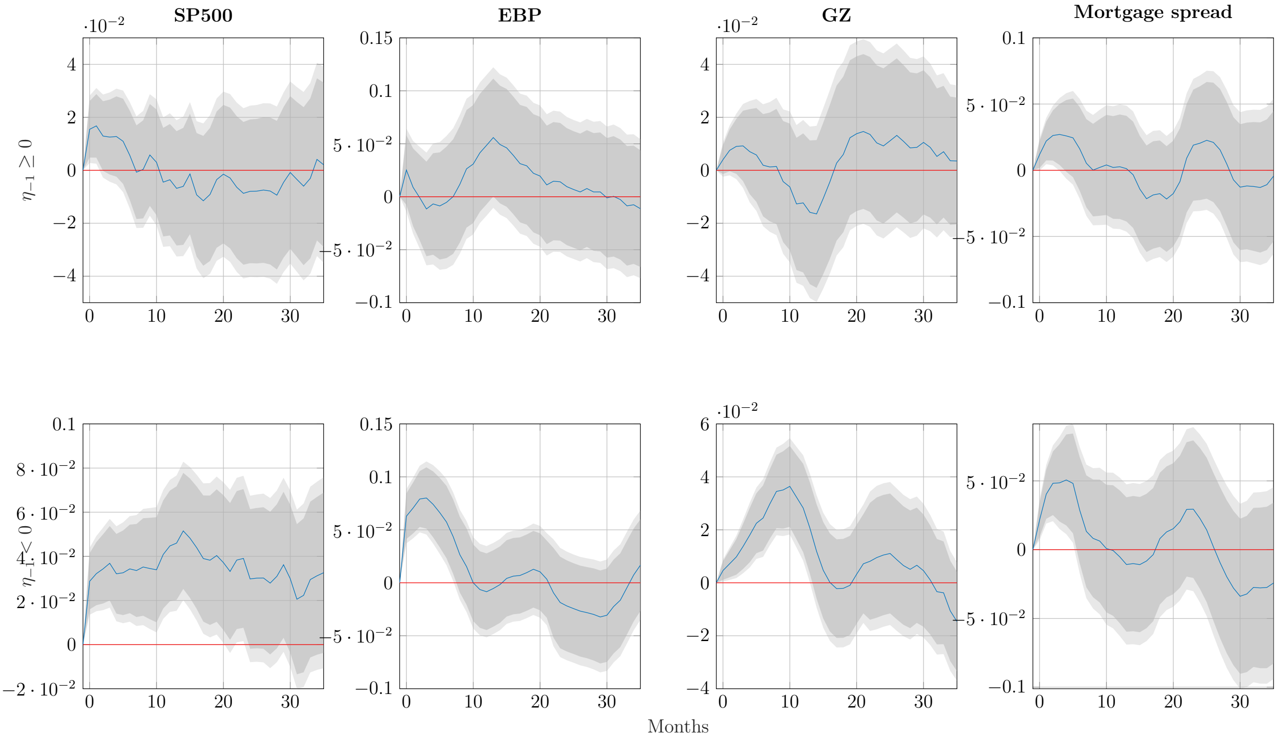

Figure 1: Impulse responses of excess returns to 1 s.d. shock to VXO. Dark grey: 90% confidence bands. Light grey: 95% confidence bands. Y-axis in percentage points.

0.3 p.p. in the second quarter. In contrast, when the financial sector's capitalization deteriorates prior to the shock ($\eta_{-1} < 0$, bottom row), the responses are significantly more pronounced. Investment shows a sharper initial decline of 0.8 p.p. by the second quarter, consumption decreases by more than 0.2 p.p., hours worked falls by 0.4 p.p., and output decreases by 0.25 p.p. Notably, only under the $\eta_{-1} < 0$ condition are these responses significant at both the 90% and 95% confidence levels, with larger magnitudes across all variables.

Figure 3 illustrates the impact of the uncertainty shock on the financial sector's balance sheet composition. When $\eta_{-1} \geq 0$, there is an insignificant decrease of 0.1 p.p. in credit to the non-financial sector, while Treasuries and net worth remain unresponsive. Debt instruments and mortgages decline by 0.2 p.p. and 0.3 p.p., respectively, but these changes are not statistically significant. In contrast, under the condition where $\eta_{-1} < 0$, the financial sector engages in a significant flight to liquidity, reducing its exposure to credit, debt instruments, and mortgages by 0.2, 0.3, and 0.4 p.p., respectively, while slightly increasing its holdings of Treasuries. Additionally, net worth declines by 0.2 p.p., further underscoring the stronger balance sheet adjustments under this scenario.

The impulse responses of key variables to a one standard deviation shock to the U_F^{LMN} index are presented in Figures 4, 5, and 6. The results align closely with those from the

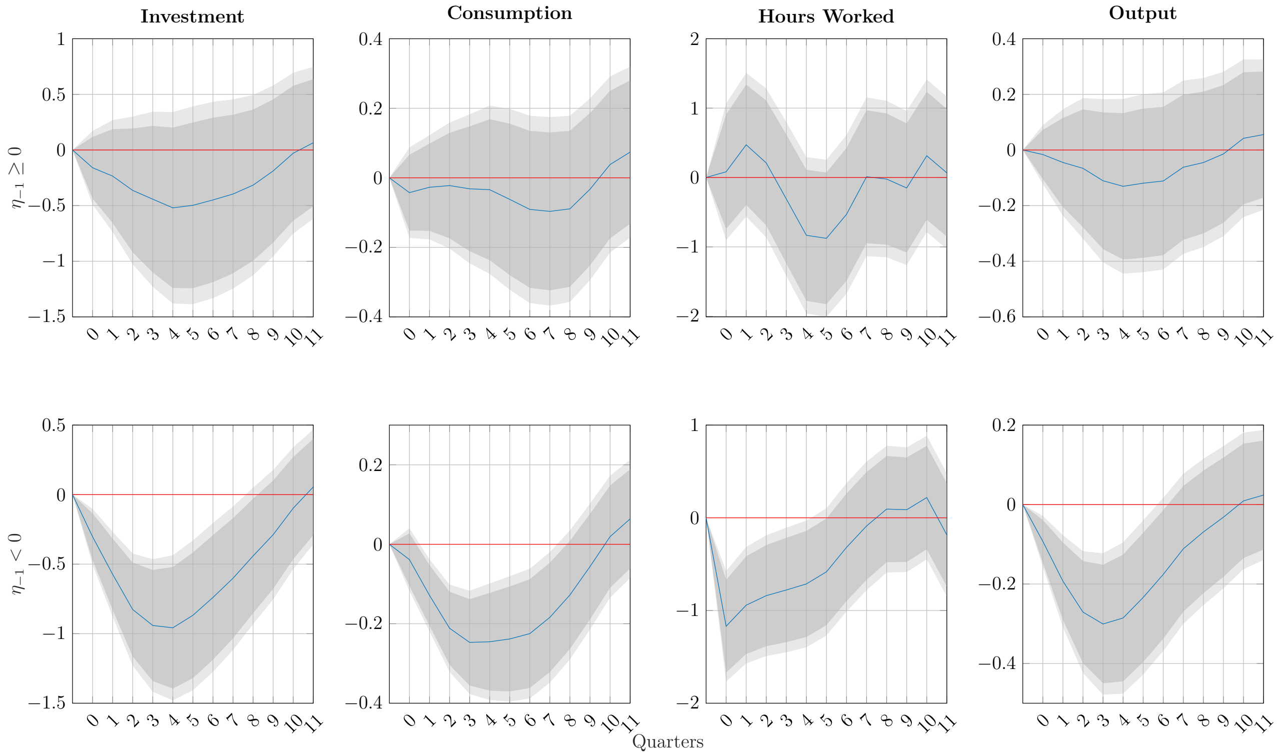

Figure 2: Impulse responses of real macro aggregates to 1 s.d. shock to VXO. Dark grey: 90% confidence bands. Light grey: 95% confidence bands. Y-axis in percentage points.

VXO shock, but with notable differences in the behavior of stock excess returns and real variables. Specifically, stock excess returns increase even when $\eta_{-1} \geq 0$, though the effect is 1 basis point smaller than when $\eta_{-1} < 0$. This increase is statistically significant at the 90% confidence level, though not at the 95% level. In contrast, under a VXO shock with $\eta_{-1} \geq 0$, excess stock returns remain close to zero and turn negative after 10 months. For macroeconomic aggregates, the responses to the two η_{-1} conditions exhibit larger differences compared to the VXO shock. Additionally, balance sheet adjustments suggest a stronger flight-to-safety dynamic under the U_F^{LMN} shock. The financial sector increases its holdings of Treasuries by 0.5 percentage points when $\eta_{-1} \geq 0$, and by 1 percentage point when $\eta_{-1} < 0$, while reducing mortgage provision by approximately 0.6 percentage points when $\eta_{-1} < 0$.

3 Model

The model follows closely Gertler and Karadi (2011). All the mathematical derivations are in Appendix B.

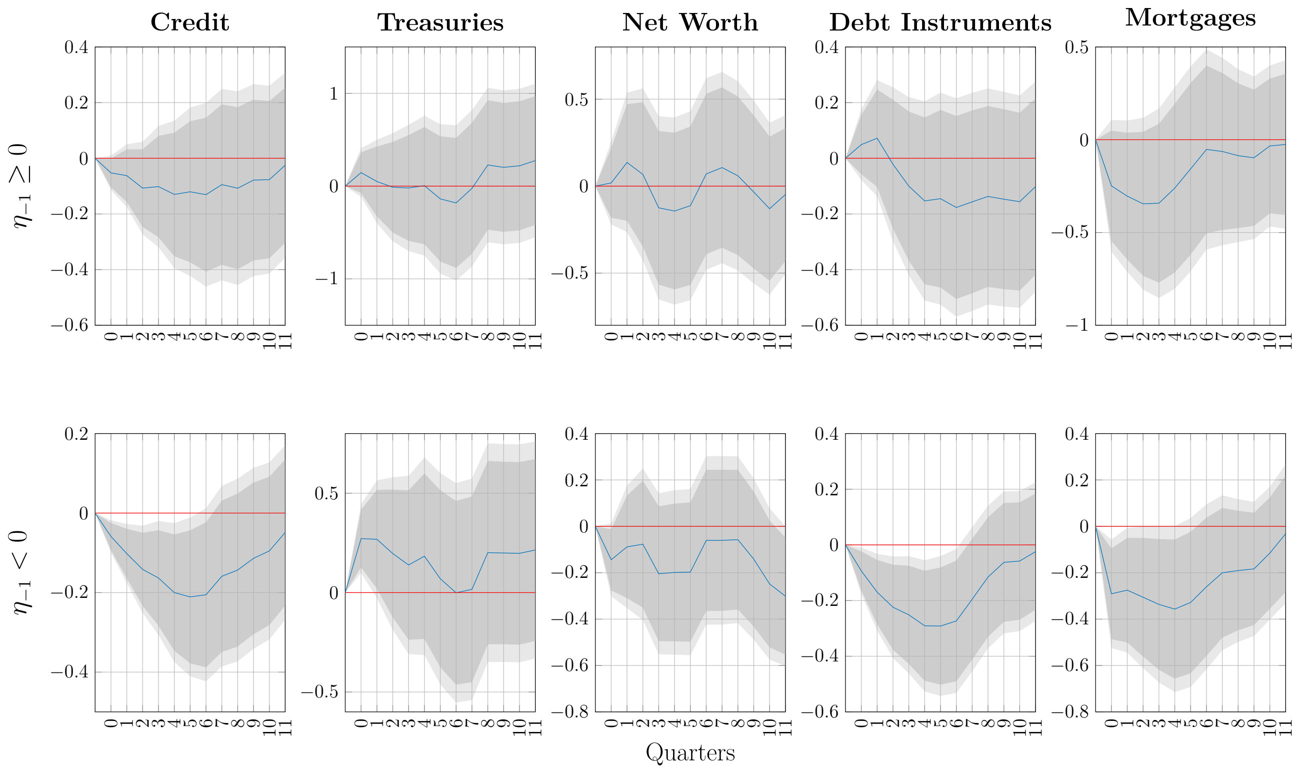

Figure 3: Impulse responses of the financial sector balance sheet composition to 1 s.d. shock to VXO. Dark grey: 90% confidence bands. Light grey: 95% confidence bands. Y-axis in percentage points.

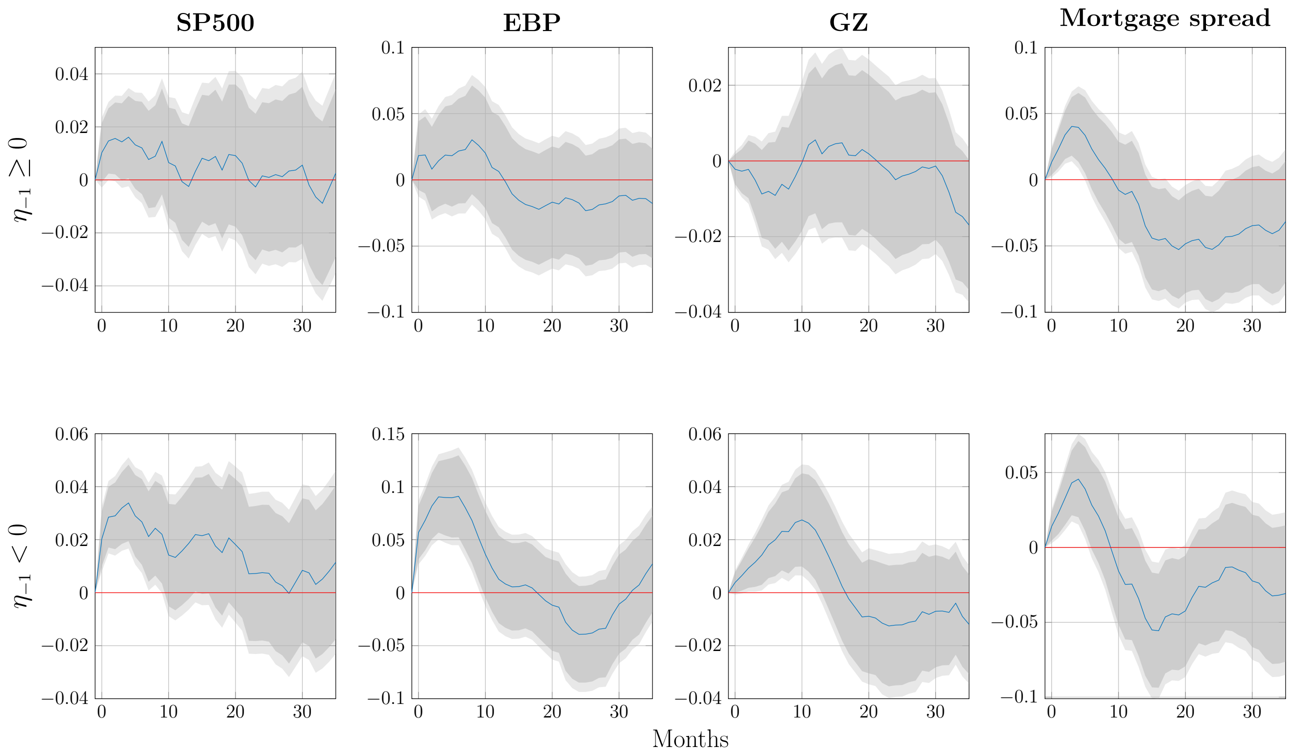

Figure 4: Impulse responses of excess returns to 1 s.d. shock to U_F^{LMN}. Dark grey: 90% confidence bands. Light grey: 95% confidence bands. Y-axis in percentage points.

3.1 Household

Each household chooses how much to consume, save and supply labor in every period. The household can invest in safe assets D_t which pay rate R_t in the next period. The household, therefore, solves

$$\max_{C_t, D_t, L_t} \mathbb{E}_0 \left[\sum_{t \geq 0} \beta^t \frac{(C_t - \chi L_t)^{1-\gamma}}{1-\gamma} \right]$$

subject to

$$C_t + D_t = W_t L_t + R_{t-1} D_{t-1} + T_t$$

where T_t are additional lump-sum transfers which will be discussed in detail later on.

The optimality conditions are:

$$\mathbb{E}_t[\Lambda_{t,t+1}] R_t = 1 \qquad \text{Euler Equation}$$

$$W_t = \chi \qquad \text{Labor Supply}$$

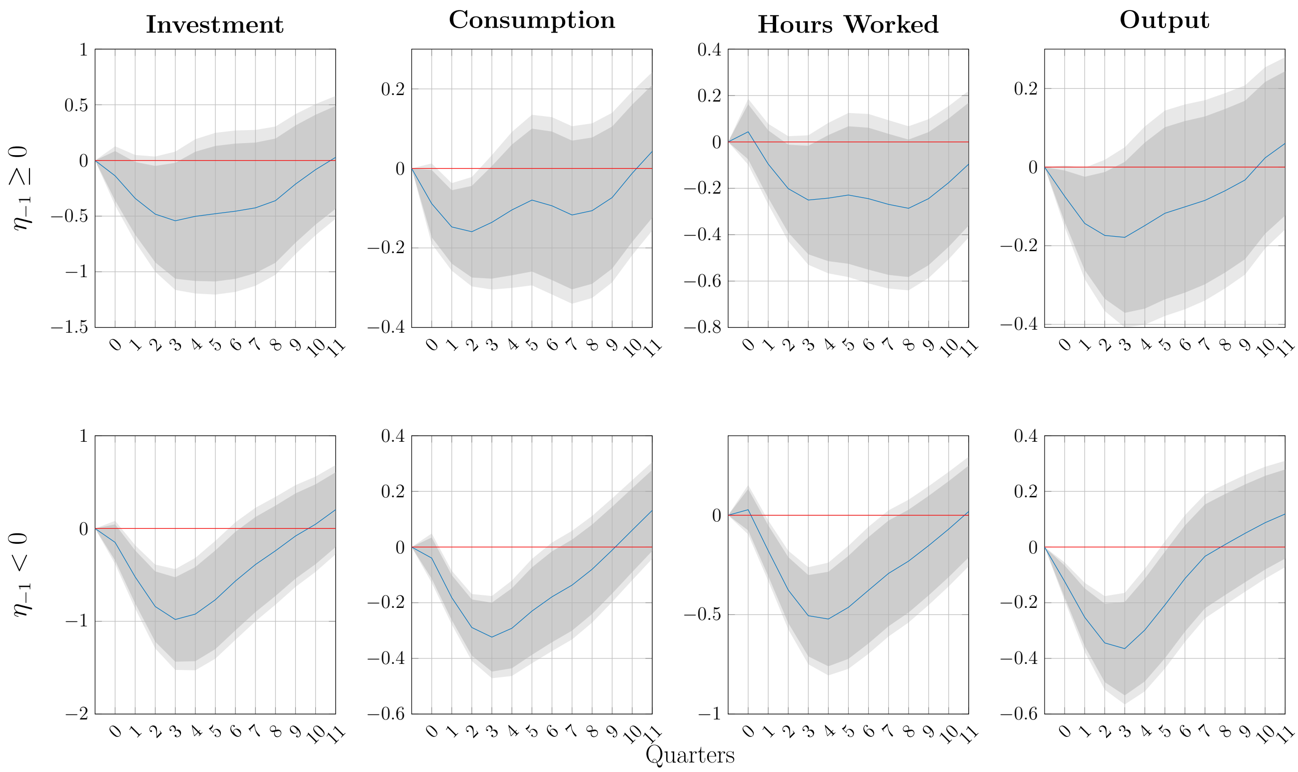

Figure 5: Impulse responses of real macro aggregates to 1 s.d. shock to U_F^{LMN}. Dark grey: 90% confidence bands. Light grey: 95% confidence bands. Y-axis in percentage points.

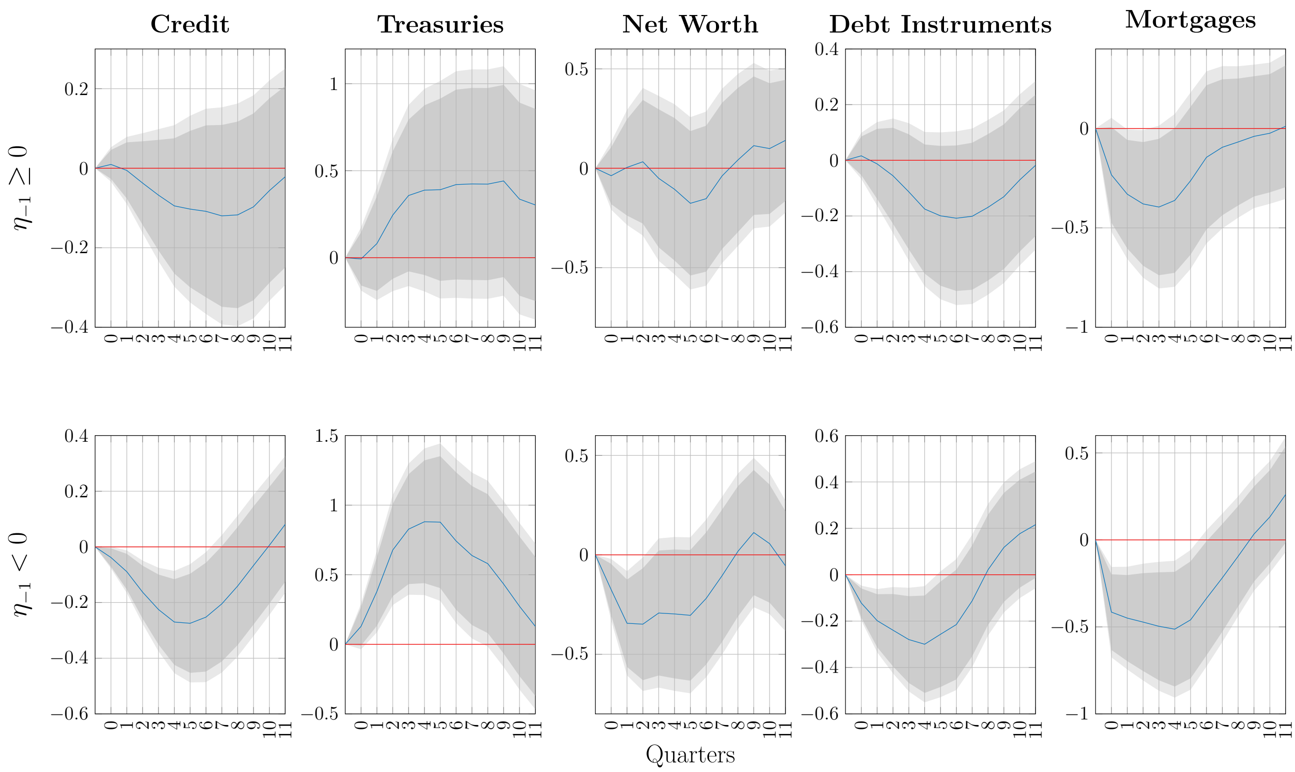

Figure 6: Impulse responses of the financial sector balance sheet composition to 1 s.d. shock to U_F^{LMN}. Dark grey: 90% confidence bands. Light grey: 95% confidence bands. Y-axis in percentage points.

where the stochastic discount factor is defined as:

$$\Lambda_{t,t+1} = \beta \left(\frac{C_{t+1} - \chi L_{t+1}}{C_t - \chi L_t} \right)^{-\gamma}.$$

3.2 Financial Intermediaries

Intermediaries buy capital K_{t+1} at a price Q_t, and operate a firm which produces the final good. Hence, the return on capital is $R_t^k = \frac{\Xi_t + Q_t K_{t+1}}{Q_{t-1} K_t}$, where Ξ_t represents the firm's profit. These operations are financed with net worth, N_t, and debt. We will refer to total debt services that intermediaries repay to the public as $\widetilde{D}_t = e^{\sigma_t^\nu \nu_t} R_t D_t$. The shock ν_t represents random disruptions to the short term debt markets, possibly coming from the presence of noise traders. In particular, I will assume that, upon realization of the shock ν_t, the household makes its deposits choice D_t and these noise traders divert $(1 - e^{\sigma_t^\nu \nu_t}) D_t$ to another saving instrument which will be modeled like a government bond paying the same interest as banks' deposits. In period $t+1$, the intermediary will have to repay p_t. I will assume that the shocks ν_t have time-varying standard deviation σ_t^ν representing our notion of financial uncertainty.

Hence, intermediaries balance sheet constraint is given by the following equation:

$$Q_t K_{t+1} = N_t + \widetilde{D}_t.$$

Net worth evolution of the individual banker:

$$N_{t+1} = R_{t+1}^k Q_t K_{t+1} - R_t \widetilde{D}_t.$$

Each intermediary faces a standard leverage constraint, which Gertler and Karadi (2011) micro-found as an ICC constraint for the intermediary not to default and abscond a fraction θ of its assets. In particular, such constraint takes the form:

$$V(N_t) \geq \theta Q_t K_{t+1}.$$

With probability $1 - \sigma$, the intermediary exits the market in the next period and consumes its net worth. Since the intermediary is owned by the household, it uses the household's stochastic discount factor when making intertemporal decisions. The problem of the individual intermediary is thus:

$$V(N_t) = \max_{K_{t+1},N_{t+1}} \mathbb{E}_t \left[\Lambda_{t,t+1}[(1-\sigma)N_{t+1} + \sigma V(N_{t+1})]\right] \tag{8}$$

$$\begin{aligned}
\text{s.t. } & N_{t+1} = R^k_{t+1} Q_t K_{t+1} - R_t \widetilde{D}_t \\
& Q_t K_{t+1} = N_t + \widetilde{D}_t \\
& V(N_t) \geq \theta Q_t K_{t+1}.
\end{aligned}$$

The optimality conditions for the intermediary are:

$$\mathbb{E}_t \left[\Lambda_{t,t+1}(1-\sigma+\sigma\psi_{t+1})(R^k_{t+1} - R_t)\right] = \mu_t \theta \tag{9}$$

where μ_t is a transformation of the Lagrange multiplier associated to the financial constraint and ψ_t is the marginal value of net worth for the intermediary, i.e. $\psi_t = V'_t(N_t)$. In fact, given the linear structure of the problem, we can write $V_t(N_t) = \psi_t N_t$. Furthermore, as shown in Bocola (2016), we can recover a closed form expression for ψ_t and μ_t (see Appendix B for the derivations):

$$\psi_t = \frac{\mathbb{E}_t[\Lambda_{t,t+1}(1-\sigma+\sigma\psi_{t+1})]R_t}{1-\mu_t} \tag{10}$$

$$\mu_t = \max\left\{1 - \frac{\mathbb{E}_t[\Lambda_{t,t+1}(1-\sigma+\sigma\psi_{t+1})]R_t N_t}{\theta Q_t K_{t+1}}, 0\right\}. \tag{11}$$

Notice that Equation 9 implies the following decomposition for the excess return on capital:

$$\frac{\mathbb{E}_t[R^k_{t+1}]}{R_t} = \underbrace{\frac{\theta\mu_t + \psi_t(1-\mu_t)}{1-\sigma+\sigma\mathbb{E}_t[\psi_{t+1}]}}_{Liquidity} \underbrace{-Cov_t(\Lambda_{t+1}, R^k_{t+1})}_{Risk} \underbrace{-\frac{\sigma Cov_t(\psi_{t+1}, \Lambda_{t+1}R^k_{t+1})}{1-\sigma+\sigma\mathbb{E}_t[\psi_{t+1}]}}_{Liquidity\ Risk}$$

At the beginning of every period, a fraction $1-\sigma$ new intermediaries are born, and their net worth is funded with a fraction ω of the proceedings of the sale of the assets of those who exited. Hence, the aggregate net worth of the economy evolves according to

$$N_{t+1} = \sigma(R^k_{t+1} Q_t K_{t+1} - R_t \widetilde{D}_t) + \omega Q_{t+1} K_{t+1}.$$

3.3 Production

Final good producer firms employ labor from the household and capital, and are subject to a stochastic TFP shocks. They also need to pay for the wage bill in advance and are subject to a collateral constraint. This means they are allowed to borrow up until a fraction ζ of the value of their assets. This constraint is similar to Chatterjee et al. (2020). Hence the problem becomes:

$$\begin{aligned}&\max_{L_t} e^{Z_t} K_t^{\alpha} L_t^{1-\alpha} - W_t L_t - Q_t I_t \\ &\text{s.t. } W_t L_t \leq \zeta Q_t K_t\end{aligned}$$

with the TFP law of motion following an AR(1) process: $Z_t = \rho_z Z_{t-1} + (1-\rho_z)\overline{z} + \sigma_z \varepsilon_t^z$. Optimality conditions imply

$$W_t = \max\left\{MPL_t, \frac{\zeta Q_t K_t}{L_t}\right\}.$$

Capital good producers are perfectly competitive and choose the optimal amount of investment according to the following program:

$$\begin{gathered}\max_{I_t} Q_t I_t - \Phi\left(\frac{I_t}{K_t}\right) K_t \\ \text{FOC: } Q_t = \Phi'\left(\frac{I_t}{K_t}\right).\end{gathered}$$

Hence, capital follows the law of motion:

$$K_{t+1} = (1-\delta)K_t + I_t.$$

3.4 Equilibrium

Debt market clearing: the government collects lump sum taxes (transfers in the case of positive balances) from the household to finance interest payment on the funds it receives from the noise traders:

$$(1 - e^{\sigma_{t-1}^{\nu}\nu_{t-1}})R_{t-1}D_{t-1} + T_t = (1 - e^{\sigma_t^{\nu}\nu_t})D_t.$$

Labor market clearing: we assume that a fraction of final good producers f are constrained, hence:

$$\chi = f\zeta\frac{Q_tK_t}{L_t} + (1-f)(1-\alpha)e^{Z_t}K_t^{\alpha}L_t^{-\alpha}.$$

This allows to pin down the expression for the return on capital:

$$R_t^k = \frac{\Xi_t + Q_tK_{t+1}}{Q_{t-1}K_t} = \frac{(\alpha + f(1-\alpha))\frac{Y_t}{K_t} + (1-\delta-f\zeta)Q_t}{Q_{t-1}}.$$

Note that if the fraction of constrained firms $f = 0$, we would recover the standard expression for the return on capital: $R_t^k = \frac{MPK_t + (1-\delta)Q_t}{Q_{t-1}}$.

Goods market equilibrium:

$$C_t + \Phi\left(\frac{I_t}{K_t}\right)K_t = Y_t.$$

4 Model Results

4.1 Solution Method

Given the highly non-linear nature of the problem at hand, I seek a global solution. Let's define the following variables: $c_t = \frac{C_t}{K_t}$, $i_t = \frac{I_t}{K_t}$, $y_t = \frac{Y_t}{K_t}$, $\ell_t = \frac{L_t}{K_t}$, $c_t = \frac{C_t}{K_t}$, $n_t = \frac{N_t}{K_{t+1}}$, $p_t = \frac{R_tD_t}{K_{t+1}}$ and $u_{c,t} = (c_t - \chi\ell_t)^{-\gamma}$. Let $X = (p, Z)$ be the states. Then the list of contemporaneous equations are (see Appendix B for full derivation):

$$\chi\ell = f\zeta Q + (1-f)(1-\alpha)e^{Z}\ell^{1-\alpha} \tag{12}$$
$$c + \Phi(i) = y \tag{13}$$
$$Q = \Phi'(i) \tag{14}$$
$$y = e^{Z}\ell^{1-\alpha}. \tag{15}$$

The evolution of net worth and the state p are

$$(1-\delta+i)n = \sigma[(1-\delta)Q + \alpha y - p] + \omega Q \tag{16}$$
$$p' = R(Q-n). \tag{17}$$

Lastly, the Euler equations to be satisfied are:

$$\beta\frac{(1-\delta+i)^{-\gamma}}{u_c}\mathbb{E}\left[u_c'\right]R = 1 \tag{18}$$

$$\beta\frac{(1-\delta+i)^{-\gamma}}{u_c}\mathbb{E}\left[u_c'(1-\sigma+\sigma\psi')((R^k)'-R)\right]=\mu\theta \tag{19}$$

$$\mu=\max\left\{0,1-\frac{1-\sigma+\sigma\beta(1-\delta+i)^{-\gamma}u_c^{-1}\mathbb{E}\left[u_c'\right]Rn}{\theta Q}\right\} \tag{20}$$

$$\psi=\max\left\{1-\sigma+\sigma\beta(1-\delta+i)^{-\gamma}u_c^{-1}\mathbb{E}\left[u_c'\psi'\right]R,\frac{\theta Q}{n}\right\}. \tag{21}$$

The algorithm adapts what is described in Wei Dou et al. (2023).

4.2 Calibration

Table 1 reports the parameter used in the numerical solution of the model. Household's discount factor, capital share, depreciation rate and TFP average and AR(1) parameter are standard in the literature on real business cycles. Household's CRRA parameter is $\gamma=1$, which is unusually low compared to the literature on time varying uncertainty. In fact, a high risk aversion is required to generate a meaningful dynamics in response to uncertainty shocks. However, higher values of γ would make the global solution computationally unfeasible, given that the model already features significant non-linearities in the form of occasionally binding constraints. Furthermore, the model still yields interesting results despite this limitation, eliminating the need for higher degrees of risk aversion. Parameters specific to financial intermediaries, like the survival rate σ, the transfer to newly born intermediaries ω and the tightness of the financial constraint θ are taken from the original paper from Gertler and Karadi (2011), as it is common in the literature.

Since the labor market described in the model are non-standard, the parameters featuring in the equation governing it are internally calibrated. In particular, the fraction of constrained firms, f, and the fraction of pledgeable assets, ζ, are selected so that the ratio of the ergodic variances of Q and ℓ in percent deviation from steady state matches the data. Household's disutility from labor, χ, is then computed to be consistent with so that the long run $\frac{L}{K}$ ratio and Q are consistent with the equilibrium condition of the labor market in steady state.

Lastly, the average financial uncertainty $\overline{\sigma^{\nu}}$ is calibrated as the standard deviation of total bank funding in percentage deviation from its trend. Data on bank funding comes from Bowman et al. (2020), who in turn took it from the Call Report and FR-Y9C. The persistence of the shock is instead calibrated to match the empirical IRF.

Table 1: Calibrated parameters

Variable	Parameter	Value	Source
RBC			**Standard**
Discount factor	β	0.995	
Risk aversion	γ	1	
Capital share	α	0.33	
Depreciation	δ	0.025	
Mean of TFP	$\overline{z}$	0	
Persistence of TFP	ρ_z	0.95	
S.d. of TFP innovations	σ_z	0.01	
Financial Sector			**GK (2011)**
Surviving bankers	σ	0.972	
Transfer to new bankers	ω	0.002	
Fraction of divertible assets	θ	0.381	
Calibrated Parameters			**Target Statistic**
Disutility of labor	χ	0.4352	$\text{g}Y/\text{g}L = 2.21$
Fraction of constrained firms	f	0.4233	$Var(\text{g}Q)/Var(\text{g}L) = 2.23$
Fraction of pledgeable assets	ζ	0.0933	$\text{g}Q/\text{g}L = 2.43$
Mean of σ_t^ν (s.d. of ν_t)	$\overline{\sigma^\nu}$	0.0179	$\frac{s.d.}{avg}$ g.r. bank funding
Persistence of σ_t^ν	ρ_σ	0.9	IRF persistence $R^{exc}_{SP500} = 0.98$

4.3 Impulse Responses

Impulse responses are generated first by simulating N economies for T periods to generate the stochastic steady state. To achieve this, I generate N sequences of shocks $(\varepsilon_t^z, \nu_t)_{t=1}^T$, I feed the shocks to the policy functions obtained after solving the model numerically and simulate the time paths. Let $(Y_{i,-1})_{i=1}^N$ be the vector of states achieved in the stochastic steady state. From each $Y_{i,-1}$ I then simulate the dynamics of the variables when the economy is hit by an MIT shock to σ^ν, i.e. $Y_{i,h}^{(1)} = f(X_{i,h}, \sigma_h^\nu)$ and when it is not hit, i.e. $Y_{i,h}^{(0)} = f(X_{i,h}, \overline{\sigma^\nu})$. The impulse responses are computed as the log-difference between the dynamics with the shock and the dynamics without the shock:

$$IRF(i,h) = \log Y_{i,h}^{(1)} - \log Y_{i,h}^{(0)}$$

I then compute the average response across initial state i conditional on whether the initial distance from the constraint, measured as $\frac{n_{-1}}{Q_{-1}}$, is above or below the median of the distribution.

Figure 7 shows the responses generated by this procedure, together with the average response, for output per unit of capital, excess return on capital and volatility of return on capital. The results displays a countercyclical dynamics for excess return: as output drops, excess return spikes. Furthermore, the effect is much amplified when the economy is closer to the binding constraint.

The recessionary effect of the uncertainty shock comes mostly through investment, as shown in Figure 8: the shock triggers the precautionary deleveraging on behalf of the intermediary, which means lower demand for investment and lower price of capital.

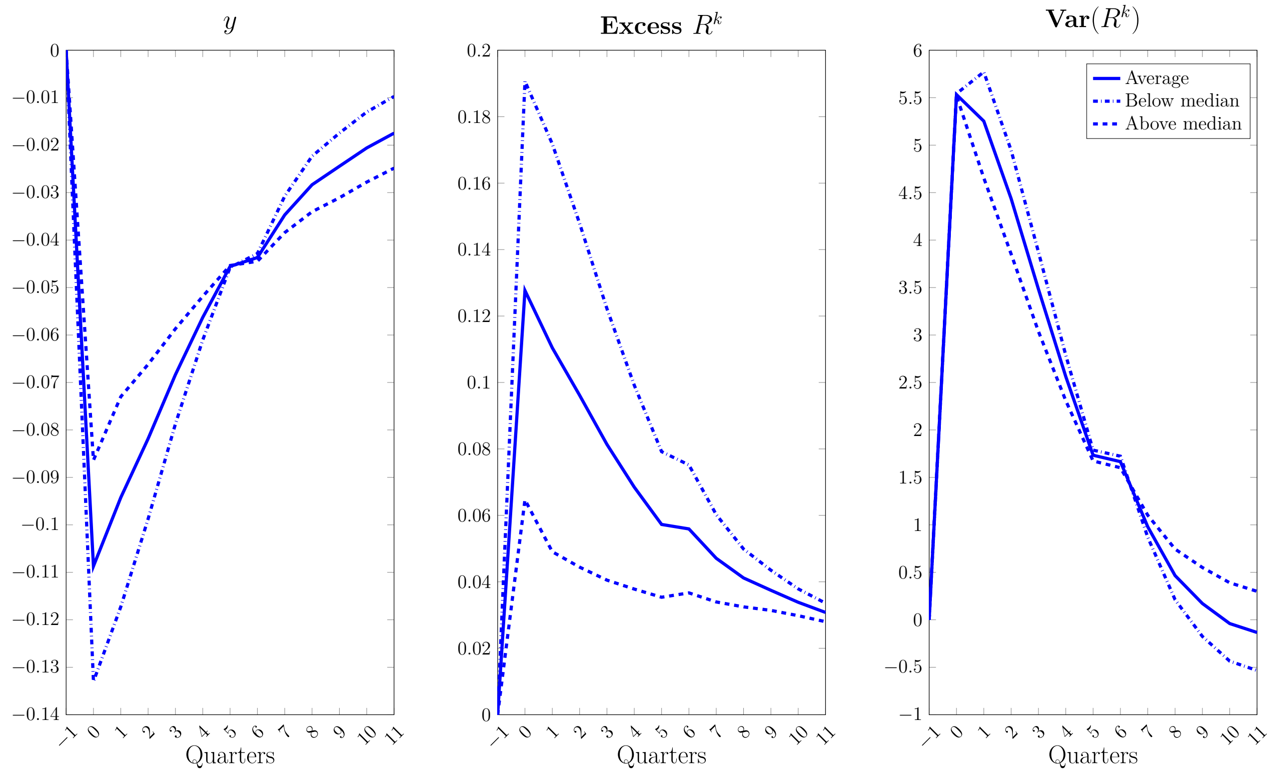

Figure 7: IRF to a 2-s.d. shock to σ_t^ν, conditional on the initial state of the economy.

This latter effect impacts the hiring decision of the final good producer, since lower asset value means lower demand for labor. The only variable whose pro-cyclical behavior is not reproduced by the model is consumption.

Figure 9 shows the model-generated responses plotted against the empirical IRF obtained in Section 2. The model manages to capture the sign and the order of magnitude of the movement of investment, price of capital and output, but it overestimates the movement of excess returns and predicts a countercyclical effect on consumption. The cause for these two failures might be common: since the real interest rate moves too much, nominal rigidities can take care of it.

5 Conclusion

This paper provides new insights into the critical interaction between the financial sector's balance sheet conditions and uncertainty shocks, particularly in how they drive the countercyclicality of excess returns. The analysis reveals that when financial intermediaries face weakened balance sheets, the effects of volatility shocks are magnified across both financial markets and the broader economy. Empirically, the results show that excess returns on risky assets increase significantly more in distressed conditions, while real economic variables such as investment, consumption, and output decline more sharply.

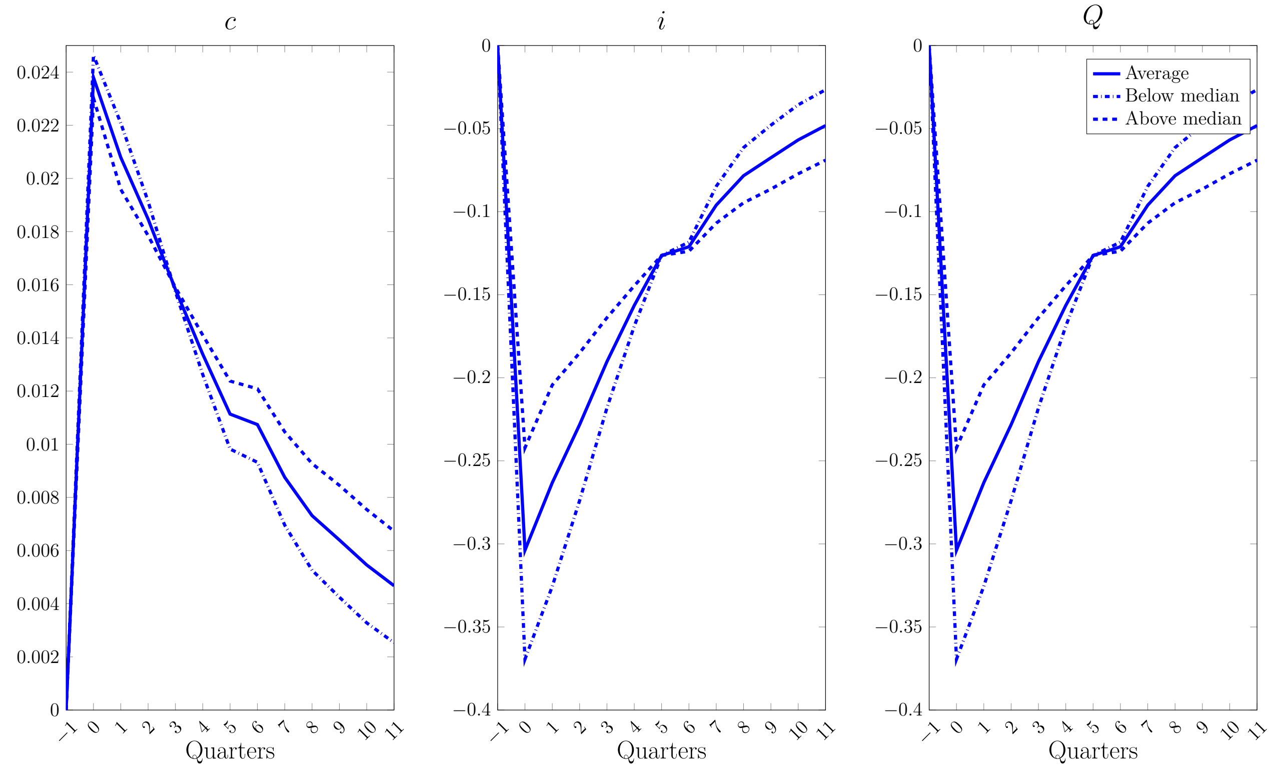

Figure 8: IRF to a 2-s.d. shock to σ_t^ν, conditional on the initial state of the economy.

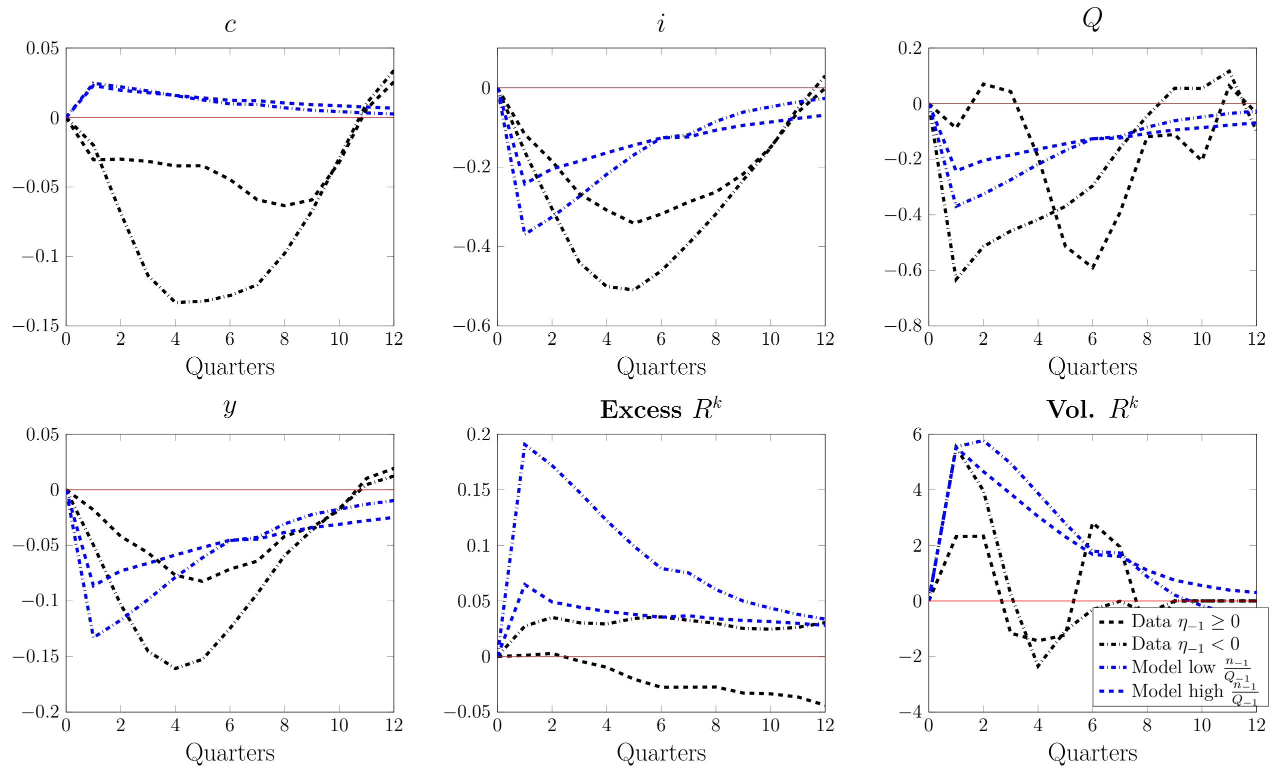

Figure 9: The empirical IRF have been rescaled to match the increase in volatility of asset returns.

This suggests that the financial sector's health is a crucial factor in determining the economy's resilience to uncertainty shocks.

A key novelty introduced in this paper is the application of a new empirical approach, leveraging a nonlinear specification of local projections to evaluate differential impulse responses. By conditioning the responses on the state of the financial sector's balance sheet, this approach allows for a more nuanced analysis of how uncertainty shocks propagate through the economy depending on the health of financial intermediaries. This empirical innovation enables a deeper understanding of the asymmetric effects of uncertainty on excess returns and real economic activity, particularly during periods of financial stress.

These empirical findings are replicated by a macro-finance model, which incorporates a new way to capture financial uncertainty. Specifically, the model distinguishes between the stability of the deposit base and the volatility of other liabilities, such as short-term wholesale funding. This differentiation reflects the reality that while deposits are relatively stable, other forms of bank liabilities are more prone to shocks, making them a crucial source of financial uncertainty. By introducing this stochastic component in the model, the paper captures the dynamics of financial intermediaries' balance sheet fragility and shows how increases in financial uncertainty can trigger precautionary deleveraging, thereby amplifying the impact of shocks on both excess returns and the real economy.

While the model and empirical results significantly enhance our understanding of these dynamics, there remain several promising directions for future research. One potential avenue is the introduction of nominal frictions into the model. Nominal rigidities, such as price stickiness or inflation dynamics, could provide deeper insights into how financial uncertainty interacts with the broader macroeconomic environment, particularly in relation to monetary policy. The role of inflation in shaping the response to financial shocks is especially relevant during periods of heightened uncertainty, where monetary authorities may need to balance financial stability with inflationary pressures. By incorporating these frictions, future research could offer a more comprehensive view of the interaction between financial and macroeconomic variables.

Another area worth exploring involves micro-founding the behavior of noise traders within this framework. Noise traders, whose trading decisions are not fully driven by fundamental information, may play a significant role in amplifying market volatility during times of financial distress. By understanding how these actors influence market dynamics, future models could better capture the complexities of financial market behavior, particularly the disconnect between asset prices and underlying economic fundamentals. This would also provide a more nuanced view of the interaction between rational intermediaries and less rational market participants, enhancing our understanding of asset pricing during periods of financial turbulence.

Additionally, a thorough examination of the policy responses to financial uncertainty shocks is necessary. Central banks and regulators face significant challenges in responding to such shocks, especially when financial intermediaries are the primary channel through which uncertainty impacts the broader economy. Future research could investigate the effectiveness of various macroprudential and monetary policies in mitigating the adverse effects of financial uncertainty. For example, policies aimed at stabilizing financial intermediaries' balance sheets or providing liquidity during periods of stress could help dampen the cyclical amplification of excess returns and real economic contractions. Exploring these policy tools within a theoretical and empirical framework would offer valuable guidance for policymakers seeking to enhance financial stability.

In conclusion, this paper highlights the crucial role of financial sector vulnerabilities in amplifying the effects of uncertainty shocks on both excess returns and real economic variables. The introduction of both a new empirical approach for evaluating differential responses through local projections and a novel model of financial uncertainty provides a better understanding of the mechanism through which precautionary behavior among intermediaries drives the countercyclicality of excess returns. As financial systems continue to evolve and become increasingly interconnected with global macroeconomic conditions, understanding these dynamics becomes even more critical. The suggested avenues for future research—ranging from the inclusion of nominal frictions and noise traders to an examination of policy responses—offer rich possibilities for further enhancing the understanding of financial uncertainty and its far-reaching effects on both markets and the real economy.

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A Local Projections

A.1 Proof of Proposition 1

Let's first introduce some notation. Let

$$\underset{N_{\mathbf{y}}\times kN_{\mathbf{x}}}{\frac{\partial^k\Phi}{\partial\mathbf{x}^k}} = \left[\operatorname{vec}\left[\frac{\partial^k\Phi_j}{\partial^{i_1}x_1...\partial^{i_{N_x}}x_{N_x}}\right]_{i_1+...+i_{N_x}=k}\right]_{j=1,..,N_{\mathbf{y}}}$$

denote the operator that yields all the k-th partial derivatives of Φ with respect to vector to all the components of vector $\mathbf{x}$, arranged in vector form. Furthermore, since the shocks $\epsilon_t, ..., \epsilon_{t+h}$ are i.i.d. and independent of the state $\mathbf{x}_t$, the following rules for computing expectations will be applied throughout the proof:

$$\mathbb{E}_t[\epsilon_{t+j}^n] = m_\epsilon(n)$$
$$\mathbb{E}_t[\epsilon_{t+j}^n \otimes \mathbf{x}_t \mid \mathbf{x}_t \in \mathcal{A}] = m_\epsilon(n) \otimes \mathbb{E}_t[\mathbf{x}_t \mid \mathbf{x}_t \in \mathcal{A}]$$

where $m_\epsilon(n)$ is the n-th moment of the distribution of ϵ, and $m_\epsilon(1) = \mathbf{0}$.

Fix $h > 0$. Since $\Phi^h \in C^\infty$, it can be expanded in its Taylor series around $\mathbf{0}$:

$$\begin{aligned}\mathbf{y}_{t+h} &= \Phi^h(\mathbf{0}) + \sum_{i=0}^{h}\frac{\partial\Phi^h(\mathbf{0})}{\partial\epsilon_{t+i}}\cdot\epsilon_{t+i} + \frac{\partial\Phi^h(\mathbf{0})}{\partial\mathbf{x}_t}\cdot\mathbf{x}_t + \\ &+\sum_{n\geq 2}\frac{1}{n!}\sum_{k=1}^{n}\sum_{i=0}^{h}\left[\frac{\partial\Phi^h(\mathbf{0})}{\partial{\mathbf{x}_t}^{n-k}\partial\epsilon_{t+i}^k}\cdot\left(\bigotimes^{n-k}\mathbf{x}_t\otimes\bigotimes^{k}\epsilon_{t+i}\right) + \frac{\partial\Phi^h(\mathbf{0})}{\partial\epsilon_{t+i}^{n-k}\partial{\mathbf{x}_t}^k}\cdot\left(\bigotimes^{n-k}\epsilon_{t+i}\otimes\bigotimes^{k}\mathbf{x}_t\right)\right]\end{aligned}$$

Taking expectations on both sides and assuming that shocks are uncorrelated, we have:

$$\begin{aligned}\mathbb{E}_t[\mathbf{y}_{t+h} \mid \epsilon_t = v, \mathbf{x}_t \in \mathcal{A}] &= \Phi^h(\mathbf{0}) + \frac{\partial\Phi^h(\mathbf{0})}{\partial\epsilon_t}\cdot\mathbf{v} + \frac{\partial\Phi^h(\mathbf{0})}{\partial\mathbf{x}_t}\cdot\mathbb{E}_t[\mathbf{x}_t \mid \mathbf{x}_t \in \mathcal{A}] \\ &+\sum_{n\geq 2}\frac{1}{n!}\left[\sum_{k=1}^{n}\sum_{j=1}^{h}\left(\frac{\partial^n\Phi^h(\mathbf{0})}{\partial\mathbf{x}_t^{n-k}\partial\epsilon_{t+j}^k}\cdot\left(\mathbb{E}_t\left[\bigotimes^{n-k}\mathbf{x}_t \mid \mathbf{x}_t \in \mathcal{A}\right]\otimes\bigotimes^{k}m_\epsilon(k)\right) + \right.\right. \\ &+\left.\frac{\partial^n\Phi^h(\mathbf{0})}{\partial\epsilon_{t+j}^{n-k}\partial\mathbf{x}_t^k}\cdot\left(\bigotimes^{n-k}m_\epsilon(n-k)\otimes\mathbb{E}_t\left[\bigotimes^{k}\mathbf{x}_t \mid \mathbf{x}_t \in \mathcal{A}\right]\right)\right) + \\ &+\sum_{k=1}^{n}\frac{\partial^n\Phi^h(\mathbf{0})}{\partial\mathbf{x}_t^{n-k}\partial\epsilon_t^k}\cdot\left(\mathbb{E}_t\left[\bigotimes^{n-k}\mathbf{x}_t \mid \mathbf{x}_t \in \mathcal{A}\right]\otimes\bigotimes^{k}\mathbf{v}\right) + \\ &+\left.\sum_{k=1}^{n-1}\frac{\partial^n\Phi^h(\mathbf{0})}{\partial\epsilon_t^{n-k}\partial\mathbf{x}_t^k}\cdot\left(\bigotimes^{n-k}\mathbf{v}\otimes\mathbb{E}_t\left[\bigotimes^{k}\mathbf{x}_t \mid \mathbf{x}_t \in \mathcal{A}\right]\right)\right]\end{aligned}$$

Of course, $\mathbb{E}_t[\mathbf{y}_{t+h} \mid \epsilon_t = \mathbf{0}, \mathbf{x}_t \in \mathcal{A}]$ can be computed in a similar fashion, substituting $\mathbf{v}$ by $\mathbf{0}$. Therefore, impulse response functions can be derived from Equation 2:

$$
\begin{aligned}
& IRF(h,t,v \mid \mathbf{x}_t \in \mathcal{A}) = \mathbb{E}_t[\mathbf{y}_{t+h} \mid \epsilon_t = v, \mathbf{x}_t \in \mathcal{A}] - \mathbb{E}_t[\mathbf{y}_{t+h} \mid \epsilon_t = \mathbf{0}, \mathbf{x}_t \in \mathcal{A}] \\
=& \frac{\partial \Phi^h(\mathbf{0})}{\partial \epsilon_t} \cdot \mathbf{v} + \sum_{n \geq 2} \frac{1}{n!} \left[\sum_{k=1}^{n} \frac{\partial^n \Phi^h(\mathbf{0})}{\partial \mathbf{x}_t^{n-k} \partial \epsilon_t^k} \cdot \left(\mathbb{E}_t \left[\bigotimes^{n-k} \mathbf{x}_t \mid \mathbf{x}_t \in \mathcal{A} \right] \otimes \bigotimes^{k} \mathbf{v} \right) + \right. \\
& \left. + \sum_{k=1}^{n-1} \frac{\partial^n \Phi^h(\mathbf{0})}{\partial \epsilon_t^{n-k} \partial \mathbf{x}_t^k} \cdot \left(\bigotimes^{n-k} \mathbf{v} \otimes \mathbb{E}_t \left[\bigotimes^{k} \mathbf{x}_t \mid \mathbf{x}_t \in \mathcal{A} \right] \right) \right] \\
=& \frac{\partial \Phi^h(\mathbf{0})}{\partial \epsilon_t} \cdot \mathbf{v} + \frac{\partial^2 \Phi^h(\mathbf{0})}{\partial \mathbf{x}_t \partial \epsilon_t} \cdot (\mathbb{E}_t[\mathbf{x}_t \mid \mathbf{x}_t \in \mathcal{A}] \otimes \mathbf{v}) + \frac{1}{2} \frac{\partial^2 \Phi^h(\mathbf{0})}{\partial \epsilon_t^2} \cdot (\mathbf{v} \otimes \mathbf{v}) + o(\| [\mathbb{E}_t[\mathbf{x}_t \mid \mathbf{x}_t \in \mathcal{A}], \mathbf{v}] \|)
\end{aligned}
$$

Since the event $(\mathbf{x}_t \in \mathcal{A})$ generates a σ-algebra which is a subset of the information set at time t, conditioning on both is equivalent to conditioning on their intersection, i.e. only the event itself:

$$
\mathbb{E}_t[\mathbf{x}_t \mid \mathbf{x}_t \in \mathcal{A}] = \mathbb{E}[\mathbf{x}_t \mid \sigma(\mathbf{x}_t \in \mathcal{A}) \cap \mathcal{F}_t] \overset{\sigma(\mathbf{x}_t \in \mathcal{A}) \subset \mathcal{F}_t}{=} \mathbb{E}[\mathbf{x}_t \mid \mathbf{x}_t \in \mathcal{A}]
$$

Hence, we get the expression in the statement of the Proposition.

A.2 Computation of IRFs and Standard Errors

Let $\widehat{\beta_0^h}$ and $\widehat{\beta_1^h}$ be the OLS estimators of the coefficients in Equation 5. This means that the variance of the estimators in Equation 6 can be computed as

$$
\begin{aligned}
Var\left(\widehat{IRF}(t,h,v \mid \eta_{t-1} < 0)\right) = \Big(& Var\left(\widehat{\beta_0^h}\right) + \mathbb{E}[\eta_{t-1} \mid \eta_{t-1} < 0]^2 Var\left(\widehat{\beta_1^h}\right) \\
& + 2Cov\left(\widehat{\beta_0^h}, \widehat{\beta_1^h}\right) \mathbb{E}[\eta_{t-1} \mid \eta_{t-1} < 0] \Big) v^2
\end{aligned}
$$

and analogously for $\widehat{IRF}(t,h,v \mid \eta_{t-1} \geq 0)$.

Hence the upper and lower bound for the α confidence interval are

$$
\widehat{IRF}(t,h,v \mid \eta_{t-1} < 0) \pm z_{\alpha/2} \sqrt{Var(\widehat{IRF}(t,h,v \mid \eta_{t-1} < 0))}
$$

where $z_{\alpha/2}$ is the $\alpha/2$-th percentile of the Normal CDF. In this case the Normal approximation must be used because it is not guaranteed that the sum of two random variables with Student-t distribution is not well behaved.

B Model Derivation

B.1 Intermediary

Consolidating the first and second constraints of Problem (8), subsitituting for N_{t+1}, and writing down the Lagrangian, the problem becomes

$$\begin{aligned}V_t(N_t) = \max_{K_{t+1}}(1+\varphi_t)\mathbb{E}_t\Lambda_{t,t+1}\big\{(1-\sigma)[(R^k_{t+1}-R_t)Q_tK_{t+1}+R_tN_t]\\ +\sigma V_{t+1}\big((R^k_{t+1}-R_t)Q_tK_{t+1}+R_tN_t\big)\big\}-\varphi_tQ_tK_{t+1}\end{aligned}$$

where φ_t is the Lagrange multiplier associated with the leverage constraint. First order conditions:

$$(1+\varphi_t)\mathbb{E}_t\left[\Lambda_{t,t+1}(1-\sigma+\sigma V'_{t+1}(N_{t+1}))(R^k_{t+1}-R_t)Q_t\right]-\varphi_t\theta Q_t = 0$$

Hence:

$$\mathbb{E}_t\left[\Lambda_{t,t+1}(1-\sigma+\sigma V'_{t+1}(N_{t+1}))(R^k_{t+1}-R_t)\right] = \frac{\varphi_t}{1+\varphi_t}\theta$$

Let $V_t(N_t) = \psi_tN_t$ and $\mu_t = \frac{\varphi_t}{1+\varphi_t}$ and obtain the FOC for the portfolio choice of the bank by combining the two equation above. Furthermore, we can rewrite the bank's Bellman equation as:

$$\begin{aligned}\psi_tN_t =&\mathbb{E}_t\left\{\Lambda_{t,t+1}(1-\sigma+\sigma\psi_{t+1})[(R^k_{t+1}-R_t)Q_tK_{t+1}+R_tN_t]\right\}\\ =&\mathbb{E}_t\left\{\Lambda_{t,t+1}(1-\sigma+\sigma\psi_{t+1})[(R^k_{t+1}-R_t)Q_tK_{t+1}]\right\}+\mathbb{E}_t[\Lambda_{t,t+1}(1-\sigma+\sigma\psi_{t+1})R_tN_t]\\ =&\theta\mu_tQ_tK_{t+1}+\mathbb{E}_t[\Lambda_{t,t+1}(1-\sigma+\sigma\psi_{t+1})]R_tN_t\\ =&\mu_t\psi_tN_t+\mathbb{E}_t[\Lambda_{t,t+1}(1-\sigma+\sigma\psi_{t+1})]R_tN_t\end{aligned}$$

Notice that the last equality holds always true: if the constraint binds, $\theta Q_tK_{t+1} = \psi_tN_t$, whereas if it doesn't bind $\mu_t = 0$ meaning that the first term disappears.

Cancelling out N_t and solving for ψ_t gives Equation 10. Furthermore we can solve for μ_t and get

$$\mu_t = \begin{cases} \frac{\psi_t N_t - \mathbb{E}_t[\Lambda_{t,t+1}(1-\sigma+\sigma\psi_{t+1})]R_t N_t}{\theta Q_t K_{t+1}} & \theta Q_t K_{t+1} = \psi_t N_t \\ 0 & \theta Q_t K_{t+1} < \psi_t N_t \end{cases}$$
$$= \begin{cases} 1 - \frac{\mathbb{E}_t[\Lambda_{t,t+1}(1-\sigma+\sigma\psi_{t+1})]R_t N_t}{\theta Q_t K_{t+1}} & \theta Q_t K_{t+1} = \psi_t N_t \\ 0 & \theta Q_t K_{t+1} < \psi_t N_t \end{cases}$$

Since $\frac{\mathbb{E}_t[\Lambda_{t,t+1}(1-\sigma+\sigma\psi_{t+1})]R_t N_t}{\theta Q_t K_{t+1}} = \frac{\mathbb{E}_t[\Lambda_{t,t+1}(1-\sigma+\sigma\psi_{t+1})]R_t N_t}{\psi_t N_t} < 1$ as long as the constraint binds, whereas $\frac{\mathbb{E}_t[\Lambda_{t,t+1}(1-\sigma+\sigma\psi_{t+1})]R_t N_t}{\theta Q_t K_{t+1}} = \frac{\psi_t N_t}{\theta Q_t K_{t+1}} > 1$ if it doesn't, we can rewrite the expression for the multiplier as Equation 11.

B.2 Asset Pricing

Let $\Psi_t = 1 - \sigma + \sigma\psi_t$. Using this notation and applying the definition of covariance, we can rewrite the left hand side of Equation 9 as follows:

$$\begin{aligned}
\mathbb{E}_t\left[\Lambda_{t+1}\Psi_{t+1}(R^k_{t+1} - R_t)\right] =& \mathbb{E}_t\left[\Psi_{t+1}\right]\mathbb{E}_t\left[\Lambda_{t+1}(R^k_{t+1} - R_t)\right] + Cov_t(\Psi_{t+1}, \Lambda_{t+1}(R^k_{t+1} - R_t)) \\
=& \mathbb{E}_t\left[\Psi_{t+1}\right]\mathbb{E}_t\left[\Lambda_{t+1}\right]\mathbb{E}_t[R^k_{t+1} - R_t] + \mathbb{E}_t\left[\Psi_{t+1}\right]Cov_t(\Lambda_{t+1}, R^k_{t+1} - R_t) \\
&+ Cov_t(\Psi_{t+1}, \Lambda_{t+1}(R^k_{t+1} - R_t)) \\
=& \mathbb{E}_t\left[\Psi_{t+1}\right]\frac{\mathbb{E}_t[R^k_{t+1} - R_t]}{R_t} + \mathbb{E}_t\left[\Psi_{t+1}\right]Cov_t(\Lambda_{t+1}, R^k_{t+1}) \\
&+ Cov_t(\Psi_{t+1}, \Lambda_{t+1}R^k_{t+1}) - Cov(\Psi_{t+1}, \Lambda_{t+1})R_t
\end{aligned}$$

Similarly, by definition of ψ_t we obtain the following:

$$\begin{aligned}
\Psi_t =& 1 - \sigma + \sigma\mu_t\psi_t + \sigma\mathbb{E}_t\left[\Lambda_{t+1}\Psi_{t+1}\right]R_t \\
=& 1 - \sigma + \sigma\mu_t\psi_t + \sigma\mathbb{E}_t\left[\Psi_{t+1}\right] + \sigma Cov_t(\Lambda_{t+1}, \Psi_{t+1})R_t
\end{aligned}$$

Hence, plugging these two expressions into the optimality condition for the representative financial intermediary (Equation 9), we can solve for the excess return on capital:

$$\begin{aligned}
\frac{\mathbb{E}_t[R^k_{t+1}]}{R_t} =& \frac{\theta\mu_t + \mathbb{E}_t[\Psi_{t+1}] + Cov(\Psi_{t+1}, \Lambda_{t+1})R_t}{\mathbb{E}_t[\Psi_{t+1}]} - Cov_t(\Lambda_{t+1}, R^k_{t+1}) - \frac{Cov_t(\Psi_{t+1}, \Lambda_{t+1}R^k_{t+1})}{\mathbb{E}_t[\Psi_{t+1}]} \\
=& \frac{\theta\mu_t + \frac{\Psi_t - 1 - \sigma}{\sigma} - \psi_t\mu_t}{\mathbb{E}_t[\Psi_{t+1}]} - Cov_t(\Lambda_{t+1}, R^k_{t+1}) - \frac{Cov_t(\Psi_{t+1}, \Lambda_{t+1}R^k_{t+1})}{\mathbb{E}_t[\Psi_{t+1}]} \\
=& \underbrace{\frac{\theta\mu_t + \psi_t(1 - \mu_t)}{1 - \sigma + \sigma\mathbb{E}_t[\psi_{t+1}]}}_{Liquidity} \underbrace{- Cov_t(\Lambda_{t+1}, R^k_{t+1})}_{Risk} \underbrace{- \frac{\sigma Cov_t(\psi_{t+1}, \Lambda_{t+1}R^k_{t+1})}{1 - \sigma + \sigma\mathbb{E}_t[\psi_{t+1}]}}_{Liquidity\ Risk}
\end{aligned}$$

B.3 Equilibrium

The total lump sum rebates to the household are the government transfers (taxes), the profit of the capital good producer, the net worth of exiting intermediaries (which will be labelled as N_t^{exit}) minus the net worth of the newly born intermediaries (which will be labelled as N_t^{new}). Hence:

$$\begin{aligned}
T_t &= \tilde{T}_t + Q_t I_t - \Phi\left(\frac{I_t}{K_t}\right) K_t + N_t^{exit} \\
&= (1 - e^{\sigma_t^\nu \nu_t}) D_t - (1 - e^{\sigma_{t-1}^\nu \nu_{t-1}}) R_{t-1} D_{t-1} + Q_t I_t - \Phi\left(\frac{I_t}{K_t}\right) K_t + N_t^{exit} - N_t^{new}
\end{aligned}$$

Furthermore, the total net worth in the economy at any given time is $N_t^{surv} + N_t^{new} = Q_t K_{t+1} - \frac{\tilde{D}_t}{R_t}$, whereas we can write

$$N_{t+1}^{surv} + N_{t+1}^{exit} = R_{t+1}^k Q_t K_{t+1} - \tilde{D}_t$$

Lastly, remember the formula for the return on capital:

$$\begin{aligned}
R_t^k &= \frac{\Xi_t + Q_t K_{t+1}}{Q_{t-1} K_t} \\
&= \frac{Y_t - W_t L_t - Q_t I_t + (1-\delta) Q_t K_t + Q_t I_t}{Q_{t-1} K_t} \\
&= \frac{Y_t - f\zeta Q_t K_t - (1-f)(1-\alpha) Y_t + (1-\delta) Q_t K_t}{Q_{t-1} K_t} \\
&= \frac{(\alpha + f(1-\alpha)) \frac{Y_t}{K_t} + (1 - \delta - f\zeta) Q_t}{Q_{t-1}}.
\end{aligned}$$

Hence, plugging the second equality above in the household budget constraint

$$\begin{aligned}
C_t + e^{\sigma_t^\nu \nu_t} D_t &= e^{\sigma_{t-1}^\nu \nu_{t-1}} R_{t-1} D_{t-1} + W_t L_t + Q_t I_t - \Phi\left(\frac{I_t}{K_t}\right) K_t + N_t^{exit} - N_t^{new} \\
C_t + Q_t K_{t+1} - N_t^{surv} - N_t^{new} &= R_t^k Q_{t-1} K_t - N_t^{surv} - N_t^{exit} + W_t L_t + Q_t I_t - \Phi\left(\frac{I_t}{K_t}\right) K_t \\
&\quad + N_t^{exit} - N_t^{new} \\
&= Y_t - W_t L_t - Q_t I_t + Q_t K_{t+1} - N_t^{surv} + W_t L_t + Q_t I_t - \Phi\left(\frac{I_t}{K_t}\right) K_t \\
&\quad - N_t^{new} \\
C_t + \Phi\left(\frac{I_t}{K_t}\right) K_t &= Y_t
\end{aligned}$$

B.4 Solution Method and Algorithm

1. Solve the unconstrained version of the model, which reduces to a standard RBC model with quadratic adjustment costs for investment;

2. Use the solution to the unconstrained model to solve the occasionally constrained versions of the model for different levels of θ, starting from very low and progressively increasing.

The solution of each model is achieved running a loop. We start with a guess $\vartheta_0 = (\ell_0, R_0, \psi_0)$ which is the solution to the previous model (either unconstrained or with a lower level of θ). Suppose that we have solved for $\vartheta_n = (\ell_n, R_n, \psi_n)$ at the end of the n-th iteration of the loop, then the $n+1$-th iteration reads:

1. Given a guess $\widetilde{\ell}_{n+1}$, compute the implied $(\widetilde{c}_{n+1}, \widetilde{i}_{n+1}, \widetilde{y}_{n+1}, \widetilde{Q}_{n+1}, \widetilde{u}_{cn+1})$ using the contemporaneous Equations 12 to 16;

2. Given (ℓ_n, R_n, ψ_n), compute $(c_n, i_n, y_n, Q_n, u_{cn}, n_n, p'_n)$ using the contemporaneous Equations 12 to 16;

3. Compute expectations for u'_{cn}, $u'_{cn}(1-\sigma+\sigma\psi'_n)$ and $u'_{cn}(1-\sigma+\sigma\psi'_n)(R^k_n)'$ using projection method (see Section B.4.1) and define

$$
\begin{aligned}
Esdf &= \mathbb{E}[u'_{cn}(1-\sigma+\sigma\psi'_n)] \\
ERk &= \mathbb{E}[u'_{cn}(1-\sigma+\sigma\psi'_n)(R^k)']
\end{aligned}
$$

4. Update ℓ_{n+1} solving the following nonlinear equation for $\widetilde{\ell}_{n+1}$:

$$
\left(\beta\frac{(1-\delta+\widetilde{i}_{n+1})^{-\gamma}}{\widetilde{u}_{cn+1}}(ERk - Esdf\cdot R_n)\right)^2 - (\widetilde{\mu}_{n+1}\theta)^2 = 0 \qquad (22)
$$

where all the tilde variables are functions of the unknown ℓ_{n+1}, and $\widetilde{\mu}_{n+1}$ is given by Equation 20. Notice that the above equation is the quadratic version of Equation 19, and the solution must be found for each point of the grid, hence requiring a nonlinear solver. There are multiple options, like "knitro" or the built-in "fsolve". In both cases, an initial guess is required, and we always start from ℓ_n;

5. Update R_{n+1} and ψ_{n+1} using Equations 18 and 21 respectively:

$$
\begin{aligned}
R_{n+1} &= \frac{(1-\delta+i_{n+1})^{\gamma} u_{cn+1}}{\mathbb{E}[u'_{cn}]} \\
\psi_{n+1} &= \max\left\{1-\sigma+\sigma\frac{\mathbb{E}[u'_{cn}\psi'_{n}]}{\mathbb{E}[u'_{cn}]}, \frac{\theta Q_{n+1}}{n_{n+1}}\right\}
\end{aligned}
$$

Repeat until $\|\log(\vartheta_{n+1}/\vartheta_n)\|_{\infty} < 10^{-8}$.

B.4.1 Projection Method

Expectations can be computed by approximating the integral in the following way. Each function f is approximated as a linear combination of basis functions evaluated on a grid $\mathbf{X}$, i.e. $f(\mathbf{X}) \approx \Phi(\mathbf{X}) \cdot \gamma_f$.

Suppose each realization of the grid in the next period will be $\mathbf{X}'_i = \Gamma(\mathbf{X}) + \epsilon_i$, given a vector of shocks ϵ_i and the law of motion of the states Γ. Hence we can compute the approximate value of the expectations of f as

$$
\mathbb{E}[f \mid \mathbf{X}] = \int f(\Gamma(\mathbf{X})+\epsilon)w(\epsilon)dx \approx \sum_i f(\Gamma(\mathbf{X})+\epsilon_i)w_i \approx \sum_i \Phi(\Gamma(\mathbf{X})+\epsilon_i)\cdot\gamma_f w_i = \Bigg[\sum_i \Phi(\Gamma(\mathbf{X})+\epsilon_i)
$$

Hence, once γ_f has been retrieved we can easily compute the expecattions of f.