Output Gap Uncertainty and Fiscal Policy Adjustment in Real-Time in Emerging Economies

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Abstract

Uncertainty around real-time output gap measurement and revisions of the forecasts ex post has important implications for fiscal policy in planning and surveillance. This study uses successive vintages of the World Economic Outlook for emerging market (EM) economies during 1998-2022 to examine the reaction of discretionary fiscal policy to uncertain economic cycle in real-time. The findings show that EM tend to have persistently negative and significant higher volatile real-time output gap estimates compared to advanced economies (AE), and they are less responsive to output gap shocks. We rationalize these facts by implementing a New Keynesian DSGE model calibrated to match the behavior of an average EM. The results from the model suggest that when policy makers in EM are equally concerned about uncertainty around output gap estimation and fiscal implementation, fiscal policy is less counter-cyclical than the benchmark case with no uncertainty, entailing an efficiency loss for the purpose of output gap stabilization. On the other hand, when the concern is only about output gap uncertainty, the EM policy maker reacts more counter-cyclically but at a cost of a surge in public debt in the short term which stabilizes over the long term. This implies that, it would be optimal for the policy makers in EM to act more aggressively to stabilize the economy. We show that this can be achieved by adjusting the relative importance of output gap and debt stabilization in the policy maker's objective.

JEL Classification: C3, D8, E1, E6, H3, H6
Keywords: Fiscal policy, real-time output gap estimates, public debt

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1 Overview

Policy makers continue to rely on the estimates of output gap,[1] a measure of the deviation of the current output from the economy's productive potential, to provide an assessment of the cyclical position of the economy to guide policy formulation and implementation. A negative gap signals an economic downturn or initial stage of economic recovery while a positive gap indicates that the economy is on the upswing or overheating. However, output gap estimates are subject to considerable uncertainty owing to measurement errors, different models used in estimating output gap, update of model parameters over time based on new data releases, and data revisions ex post (Orphanides and van Norden (2002); Marcellino and Musso (2011); Ley and Misch (2013); Aastveit and Trovik (2014), and Borio, Disyatat, and Juselius (2017)).

Uncertainty surrounding the output gap estimates has important implications for fiscal policy in terms of fiscal planning (e.g., budgetary process) and when calibrating the appropriate scope of counter-cyclical fiscal policy to stabilize the economy (Hallet, Kattai, and Lewis (2007); IMF (2014) and Coibion, Gorodnichenko, and Ulate (2018)). Orphanides and van Norden (2002) observed that erroneous assumptions about the timeliness of data availability may lead to incorrect policy choices. For instance, policy advice based on underestimated output gap estimates envisaging a negative shock to the economy could prompt fiscal policy advice being too loose and recommending accommodative fiscal policy, while advice for fiscal consolidation or neutral fiscal stance would have been the preferred option (Type I error). This could have the unintended effects of running a pro-cyclical fiscal policy that might contribute to overheating of the economy and accumulation of public debt. On the other hand, overestimating the output gap could be interpreted as a positive shock to the economy, signaling to the authorities to pursue fiscal consolidation while looser fiscal policy would have been the preferred option, thus tipping the economy into recession (Type II error).

For fiscal policy, characterized by significant lags associated with policymaking, implementation and transmission, information available to policy makers at the time a decision is made may deviate significantly from the information available ex post. In studies of OECD countries, Beetsma and Giuliodori (2010) and Cimadomo (2012) found that while the fiscal stance appeared counter-cyclical when assessed based on real-time data, the fiscal policy was pro-cyclical when evaluated ex post. This divergence has important implications for optimal fiscal policy formulation and implementation. In budget preparation process, data on deficits and GDP available to policy makers at the time are likely to be

[1]Output gap is not directly observable as it is a function of potential output, a hypothetical construct, hence the need to rely on output gap estimates. The framework for estimating output gap has not radically changed in recent years. For an overview of the various output gap estimation methodologies and their implications see, for example, Cheremukhin (2013); Borio, Disyatat, and Juselius (2017) and Barkema, Gudmundsson, and Mrkaic (2020).

preliminary and, therefore, subject to many revisions in subsequent periods as newer and better information becomes available (Golinelli and Momigliano (2006); Ley and Misch (2013) and Kuusi (2018)). Acute fiscal implementation lag to correct for an adverse or favorable macroeconomic event also contributes to uncertainty surrounding output gap estimates. In some countries, tax measures depend on when tax laws take effect while for expenditure, it depends on the length it takes to disburse funds and fiscal transmission in the economy. Based on a study of OECD countries, Cimadomo (2012) noted that the overall information lag for policy makers can be roughly quantified to be around one year and a half, which implies the possibility of significant forecast errors and sub-optimal fiscal policy decisions. Policy makers are, therefore, left to take decisions under a substantial degree of uncertainty.

Since the seminal works of Orphanides (2001) and Orphanides and van Norden (2002), which documented large errors in real-time assessment of cyclical conditions in U.S. on monetary policy, there is a growing body of literature in the application of real-time data that captures the actual information available to policy makers at the time of decision making. Recent studies have expanded the literature to examine reliability of real-time output gap estimates in assessing economic cycle and as a basis of policy formulation and implementation (Orphanides and van Norden (2002); Cheremukhin (2013); Ley and Misch (2013); Grigoli et al. (2015); Kangur et al. (2019) and Barbarino et al. (2020)).

Forni and Momigliano (2004) were among the first to estimate a fiscal policy reaction function for the Euro area using real-time data and found counter-cyclical responses which do not show up when the same estimation is carried out with ex post data. On fiscal monitoring, Jonung and Larch (2006) investigate the effect of the role of errors in potential GDP forecasting, and find that for some Euro area countries, real-time assessments of fiscal position were over optimistic due to a systematic upward bias in government produced forecasts of potential output. Golinelli and Momigliano (2006) surveyed the empirical literature concerning the cyclicality of fiscal policies in the Euro area, and they find that the results are heavily affected by the data vintage used in the analysis of the fiscal policy reactions. Hallet, Kattai, and Lewis (2007) found that real-time estimates of the cyclically-adjusted budget balance are subject to significant revisions ex post, and that this lack of accuracy may explain why some fiscal slippages go unnoticed in real-time.

Most of the empirical studies on fiscal policy in EM have found overwhelming evidence that these countries pursue pro-cyclical fiscal policies, Gavin and Perotti (1997), Kaminsky, Reinhart, and Végh (2005), Talvi and Vegh (2005), Alesina, Campante, and Tabellini (2008), and Marioli and Vegh (2023). However, these studies are based on ex post data. A few recent studies have emerged applying real-time data analysis to EM. Ley and Misch (2013) implemented a static theoretical framework to examine the implications of output data revisions on the overall and structural fiscal balances for a group of countries, including EM. However, the paper does not explicitly model uncertainty inherent in output gap estimation.

The theoretical aspects of policy making under uncertainty have long been at the center of economic debate. Brainard (1967) showed in a very simple static framework that the uncertainty surrounding outcome variables and the sensitivity of such variables to the policy instrument drastically changes the optimal policy. More recently, policy design under uncertainty has been studied in the realm of dynamic models, with particular attention to optimal monetary policy in a New Keynesian framework. Specifically, Hansen and Sargent (1999) and Woodford (2010) introduced the concept of robustness in this class of models. A robust policy maker is uncertain about the probabilistic model that governs the data generating process (DGP), and takes into account all the models in a neighborhood of a reference model in terms of some statistical distance, usually the relative entropy between models. Since the policy maker is uncertainty-averse, the policy maker maximizes his/her objective function under the worst-case scenario that the set of models generates.[2] Real-time data can be thought of as data generated from a probabilistic model that has been misspecified, as it is different from the 'true' DGP. Furthermore, output gap estimation relies on the choice of a specific model, which again can be mispecified. Hence the real-time estimates that the policy maker relies on in making decision to stabilize output gap are subject to model uncertainty as there may be misspecification of the DGP and on the statistical model used to estimate the output gap.

The aim of this study is to assess the implication of output gap uncertainty on fiscal adjustment. Our paper bridges the real-time data literature in EM and the uncertainty literature by introducing a New Keynesian DSGE model in which the policy maker is uncertainty-averse and faces both output gap uncertainty and fiscal policy implementation uncertainty. We find evidence that fiscal policy is counter-cyclical in AE, and less strongly so in EM. Additionally, we calibrate a New Keynesian DSGE model to an average EM and solve the problem for the optimal fiscal reaction function. In this framework, when policy makers in EM are concerned about uncertainty around both output gap estimation and fiscal implementation, the resulting reaction function is less counter-cyclical than the benchmark case with no uncertainty. We posit that, the welfare losses deriving from the weaker fiscal response in EM can be counteracted by putting more weight on output gap stabilization motive in the policy maker's objective function. While this comes at the expense of a surge in public debt in the short run, however, we also find that, in the long run, a stronger recovery of output gap can stabilize public debt faster.

The rest of the paper is organized as follows. Section 2 outlines data methodology and provides real-time summary statistics and stylized facts for both AE and EM. Section 3 introduces the New Keynesian DSGE model which is calibrated using real-time data. Section 4 concludes.

[2]See also Maccheroni, Marinacci, and Rustichini (2006) and Hansen and Sargent (2008) for further discussion on the decision theoretical derivation and for the intuition.

2 Stylized Facts

2.1 Data and Methodology

We use data from the World Economic Outlook (WEO) Spring and Fall releases for the 1998-2022 period. Each vintage includes estimates of output gap and fiscal variables for the current year (real-time estimates) for 39 Advanced Economies (AE) and 73 Emerging Markets (EM), giving a total of 112 countries. We use the WEO database because it has the same release dates for a large number of countries. Since the timing of available information is crucial for real-time analysis, harmonizing the dates at which policy makers have access to information makes our results comparable across the countries.

We focus our analysis on output gap as an indicator of the economic cycle, and the cyclically adjusted primary balance (CAPB) as a percentage of potential GDP as a measure of fiscal stance, both produced in the WEO. It is common in the literature to interpret the CAPB as the discretionary component of fiscal policy, since it does not contain the automatic stabilizers or the interest expenses on public debt.

We interpret the Spring (S) vintage of a given year as the forecasts for that year, whereas the Fall (F) vintage is considered as the realized values (ex post). Let $x_{i,t|\tau+v}$ be the estimate of a generic variable x of country i for year t: the expression $\tau+v$ refers to the vintage of the data, with τ being the year and $v \in \{S, F\}$ the vintage release. If $\tau = t$ and $v = S$, the estimate is in real-time. Hence, policy makers use the figure $x_{i,t|t+S}$ as a forecast of that year to set their fiscal stance.

We then compute the real-time forecast error ($RTFE$) as the difference between estimates from Fall and Spring releases of a given year, and the final forecast error (FFE) as the difference between the final and the real-time estimate:

$$RTFE_{i,t} = x_{i,t|t+F} - x_{i,t|t+S} \tag{1}$$

$$FFE_{i,t} = x_{i,t|2022+F} - x_{i,t|t+S} \tag{2}$$

2.2 Summary Statistics

2.2.1 Output Gap

Table 1 summarizes the sample by income group. Both AE and EM have significantly negative real-time estimates of the output gap. This indicates the tendency of WEO to underestimate economic upswing in real-time. The final estimate confirms that output gap averaged below 0 for AE, although the magnitude is much smaller than its real-time counterpart. As for the EM, we cannot reject the hypothesis that output gap averages at 0. Real-time forecast errors for AE are significantly positive, meaning that the WEO persistently under-predicts the actual size of the output gap for this group. As for EM, real-time forecast errors are statistically 0. Finally, revisions using the latest vintage are significantly different from 0 and positive, particularly so for the EM, with the average magnitude of the final forecast error almost twice compared to AE.

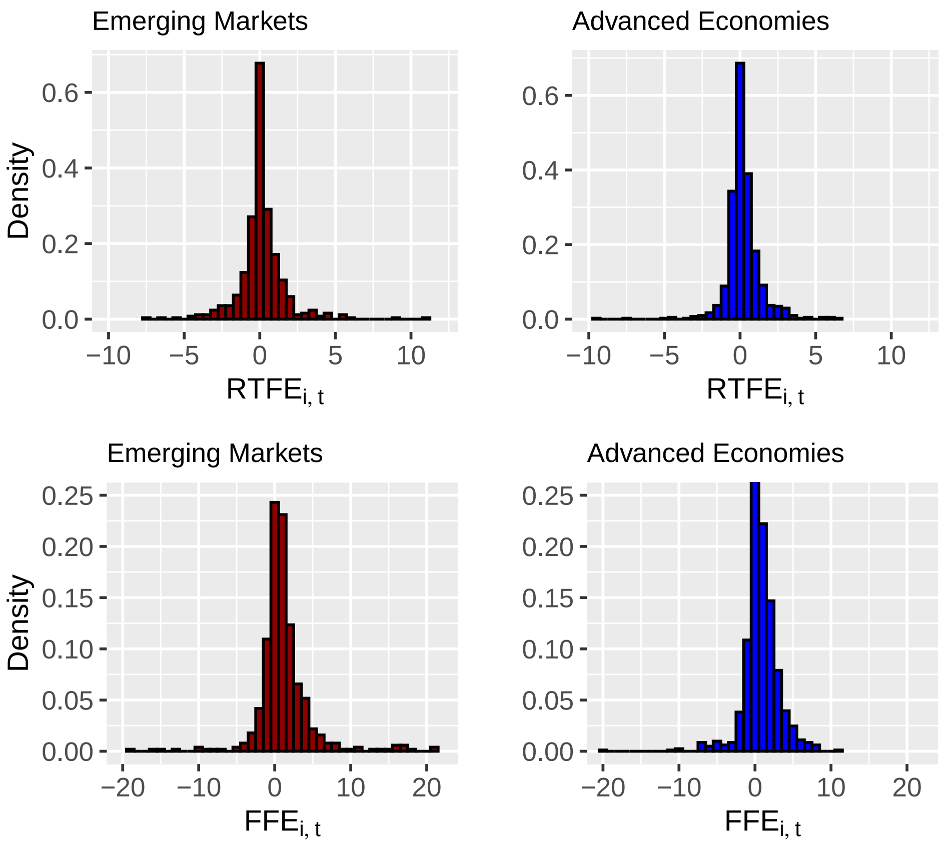
Table 1: **Summary statistics for output gap estimates and forecast errors by income group.**

	AE	EM	Variance ratio test
Real-time Spring Estimate			
Mean $\text{GAP}_{i,t\|t+S}$	-1.369	-1.636	
SD $\text{GAP}_{i,t\|t+S}$	2.385	3.240	
p-value H_0: $\text{GAP}_{i,t\|t+S} = 0$	0.000	0.000	
Real-time Fall Estimate			
Mean $\text{GAP}_{i,t\|t+F}$	-1.151	-1.520	
SD $\text{GAP}_{i,t\|t+F}$	2.259	3.459	
p-value H_0: $\text{GAP}_{i,t\|t+F} = 0$	0.000	0.000	
Final Estimate			
Mean $\text{GAP}_{i,t\|2022+F}$	-0.567	-0.470	
SD $\text{GAP}_{i,t\|2022+F}$	2.742	4.977	
p-value H_0: $\text{GAP}_{i,t\|2022+F} = 0$	0.000	0.035	
RTFE			
Mean $RTFE_{i,t}$	0.219	0.117	
SD $RTFE_{i,t}$	1.157	1.558	
p-value H_0: $RTFE_{i,t} = 0$	0.000	0.094	
$Var_{EM}/Var_{AE}(RTFE)$			1.815
p-value H_0: $\frac{Var_{EM}(RTFE)}{Var_{AE}(RTFE)} = 1$			0.000
FFE			
Mean $FFE_{i,t}$	0.803	1.166	
SD $FFE_{i,t}$	2.360	3.747	
p-value H_0: $FFE_{i,t} = 0$	0.000	0.000	
$Var_{EM}/Var_{AE}(FFE)$			2.521
p-value H_0: $\frac{Var_{EM}(FFE)}{Var_{AE}(FFE)} = 1$			0.000

The last column of Table 1 compares the variances of revisions for AE and EM, a proxy for uncertainty. As expected, revisions for EM are much more volatile than AE, indicating that output gap estimation in EM is more uncertain than in AE. This is evident in Figure 1, which shows the histogram of real-time and final forecast errors of output gap for EM and AE. We find that both real-time and final forecast errors for AE are concentrated around the mean value, with few observations on the tails. On the other hand, forecast errors for EM have fatter tails and exhibiting extreme values. This confirms that EM face higher uncertainty in their output gap estimation.

Figure 2 plots the series of output gap estimated in real-time, the estimates in the last vintage available and the estimates produced after 5 years for a selected group of EM. The discrepancy between the real-time estimates and the other two series is higher

Figure 1: **Histograms of output gap forecast errors by income group.**



at the beginning of the sample period, indicating that the final estimate is not yet stable. The final and the estimates 5 years after are very close to each other for most countries, perfectly overlapping in some cases, implying that 5 years is a long enough period for the revisions to stabilize.

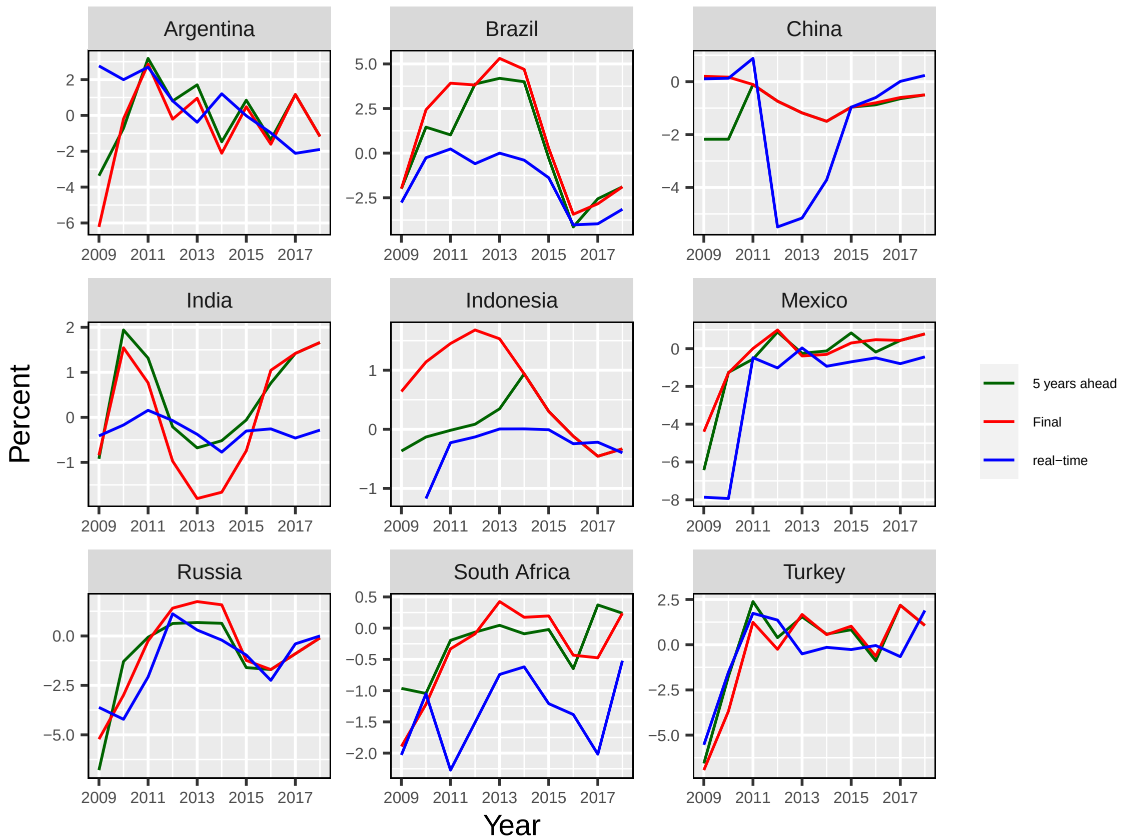
Next, we analyse the accuracy of real-time estimates by assessing the predictability of forecast errors. If output gaps were estimated under full information, a priori, the forecast errors should be the realization of a pure white noise from a measurement disturbance:

$$GAP_{i,t|t+F} = GAP_{i,t|t+S} + \nu_{i,t} \tag{3}$$

$$RTFE_{i,t} = \nu_{i,t} \sim WN(0, \sigma) \tag{4}$$

However, if instead the forecast errors were predictable, it would mean that the model used by the forecasters is biased and it would generate some persistence in the forecast errors themselves:

Figure 2: **Real-time, 5-years ahead and final estimates of output gap for selected EM.**



$$GAP_{i,t|t+F} = \alpha + \beta_0 GAP_{i,t|t+S} + \nu_{i,t} \tag{5}$$

$$RTFE_{i,t} = \alpha + \beta_1 GAP_{i,t|t+S} + \nu_{i,t} \tag{6}$$

with $\beta_0 = 1 \iff \beta_1 = 0$ under the hypothesis that output gap is estimated under the correct model. In general, if the forecasting model was correct and there was full information, forecast errors should not be predictable given information available when the forecast was formulated, including other possible predictors.

We assess forecast errors predictability by estimating the following regressions:

$$RTFE_{i,t} = \alpha + \mathbf{x}'_{i,t|t+S}\gamma + \nu_{i,t} \tag{7}$$

$$FFE_{i,t} = \alpha + \mathbf{x}'_{i,t|t+S}\gamma + \eta_{i,t} \tag{8}$$

The coefficients γ are jointly 0 under the hypothesis that the forecasters satisfy the Full Information Rational Expectations benchmark (FIRE). $\mathbf{x}_{i,t}$ is a vector of predictors containing $GAP_{i,t|t+S}$, as previously discussed. Since we are interested in the interplay between fiscal policy and output gap revisions, $\mathbf{x}_{i,t}$ includes also surplus and deficit ratios of GDP computed from the CAPB-to-GDP. We distinguish between the two variables because of the different economic interpretation: for instance, if the coefficient of surplus was significant positive (negative), forecasters would be over (under)-estimating the impact of a tight fiscal policy regime. On the other hand, a positive (negative) significant coefficient on deficit would imply under (over)-estimation of expansionary fiscal policy. To further check on statistical robustness of the forecast, we include the lagged forecast error, as the literature shows that this is the litmus test for FIRE (Coibion and Gorodnichenko (2015)). If forecast errors are autocorrelated, the forecaster is putting too much weight on its prior and does not update the predictive model fast enough as new data comes in. Lastly, we control for the log of GDP deflator as a proxy of macro-stability (many other macroeconomic variables are heavily correlated with the deflator, for instance., real interest rates, both short and long term).

The results are reported inTable 2. The only equation where the coefficients are jointly not significant is the one for $RTFE$ for EM, indicating that it is not possible, for this group of countries, to predict in advance whether the forecasts are incorrect in real-time. For all other regressions, the errors made are systematic and not just pure measurement noise, as they can be predicted using information available when the forecasts are released. This is purely a statistical feature, but it has strong policy implications, in particular in light of the negative sign of the coefficient. The lower the output gap estimate in real-time, the bigger the final forecast error. This means that the actual output gap is likely

Table 2: **Predictability of magnitudes of output gap forecats errors**

	Dependent variable:			
	$RTFE_{i,t}$		$FFE_{i,t}$	
	AE	EM	AE	EM
	(1)	(2)	(3)	(4)
Output Gap$_{i,t\|t+S}$	-0.320^{***}	-0.019	-0.501^{***}	-0.370^{***}
	(0.045)	(0.059)	(0.046)	(0.086)
Surplus-to-GDP$_{i,t\|t+S}$	-0.051	-0.104	0.259^{*}	0.175
	(0.137)	(0.260)	(0.147)	(0.372)
Deficit-to-GDP$_{i,t\|t+S}$	-0.017	0.066	-0.067	-0.440^{**}
	(0.057)	(0.129)	(0.060)	(0.187)
$RTFE_{i,t-1}$	-0.060	-0.054		
	(0.084)	(0.110)		
FFE$_{i,t-1}$			0.188^{**}	0.295^{***}
			(0.074)	(0.096)
log(Deflator$_{i,t\|t+S}$)	3.062^{**}	0.289	-0.093	-1.654
	(1.534)	(0.839)	(1.580)	(1.205)
Observations	165	186	165	186
R^2	0.403	0.010	0.572	0.149
Adjusted R^2	0.229	-0.290	0.448	-0.109
F Statistic	17.121^{***}	0.288	34.009^{***}	4.977^{***}

*p<0.1; **p<0.05; ***p<0.01

to be larger than the forecast, in particular if the forecast error is positive. This can be easily inferred from the formula $GAP_{i,t|2022+F} = GAP_{i,t|t+S} + FFE_{i,t}$. Hence, a policy maker that reacted strongly to a negative output gap forecast would end up making a Type I error.

Fiscal variables are found to be significant only in the long run, affecting final forecast errors. In particular, surplus has a positive impact for AE, which means that higher surplus leads to a higher output gap than what was expected by the forecasters. This implies that the negative impact of tighter fiscal policy is over-estimated, or the positive impact of fiscal consolidation is under-estimated. On the other hand, deficit is a negative predictor of final errors in EM, implying that forecasters over-estimated the positive impact of expansionary fiscal policy.

Lastly, final forecast errors are serially autocorrelated, indicating over-reliance on prior information rather than newly acquired data by the forecaster. Since autocorrelation is stronger for EM, we conclude that shocks are not easy to identify for this group, further hindering the effectiveness of fiscal policy response.

2.2.2 Cyclically Adjusted Primary Balance

Table 3: **Summary statistics for forecast errors of cyclically adjusted primary balance.**

	AE	EM	Variance Ratio test
RTFE			
Mean $RTFE$	-0.304	-0.283	
SD $RTFE$	1.482	1.367	
p-value H_0: $RTFE = 0$	0.000	0.000	
$Var_{EM}/Var_{AE}(RTFE)$			0.851
p-value $\frac{Var_{EM}(RTFE)}{Var_{AE}(RTFE)} = 1$			0.123
FFE			
Mean FFE	0.652	0.006	
SD FFE	1.508	1.637	
p-value H_0: $FFE = 0$	0.000	0.943	
$Var_{EM}/Var_{AE}(FFE)$			1.179
p-value $\frac{Var_{EM}(FFE)}{Var_{AE}(FFE)} = 1$			0.118

We perform similar analysis on the forecast errors and revisions of CAPB ratio to GDP. However, it must be noted that, while the errors for output gaps are due to statistical uncertainty and noisy information, errors of fiscal variables arise also due to discrepancies between the planning and the implementation phases of fiscal policy (Beetsma and Giuliodori (2010), Cimadomo (2012)). This leads to significant differences between the original budget plan of the government and what it is actually codified into the budget law. These differences are further amplified when the policies are implemented, due to political or administrative frictions.

Table 3 shows the summary statistics for forecast errors of CAPB. Both real-time and final forecast errors are significantly different from 0 for AE, whereas EM forecast errors are biased in real-time but not so in the long run. The last column of Table 3 tests whether the forecast errors are more volatile in one group of the other: we cannot reject the hypothesis that the two are equally uncertain.

Figure 3: **Histograms of fiscal forecast errors by income group.**

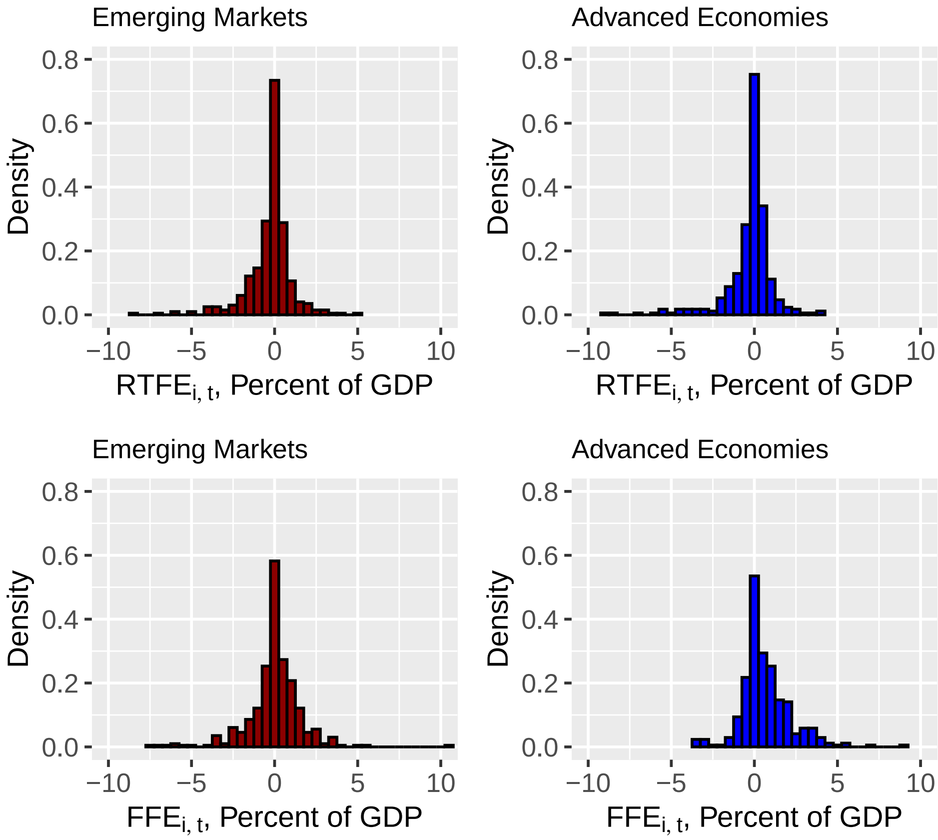


Figure 3 shows the histograms for fiscal forecast errors and revisions. For $RTFE$, the two groups are similar in terms of distributions. In particular, range and dispersion are similar, confirming the findings of Table 3: governments in both AE and EM face the same uncertainty when planning their fiscal budget. Unlike output gap, CAPB is a policy variable whose estimation does not rely on any statistical procedure on data whose measurement is highly uncertain, particularly so for EM. Hence the similarity in the distribution of forecast errors is plausible.

2.3 Real-time Fiscal Reaction Functions

2.3.1 Methodology

The goal of this section is to estimate a fiscal reaction function in real-time. The underlying intuition is that the government optimally chooses its fiscal stance based on the current estimate of the state variables of the economy, which depend on the policy makers' information set at the time of the decision. For this purpose the following regression is estimated:

$$CAPB_{i,t|t+S} = \beta_0 + \beta_{GAP} GAP_{i,t|t+S} + \beta_{DEBT} \Delta DEBT_{i,t-1|t+S} + \beta_{Elect} Elect_{i,t+1} + \epsilon_{i,t} \tag{9}$$

where β_{GAP} is our parameter of interest and captures the sensitivity of cyclically adjusted primary balance to output gap's deviations from its intended theoretical value of 0. Since CAPB is computed as revenues minus expenses, a counter-cyclical stabilization motive would imply $\beta_{GAP} > 0$.

Everything else equal, a government that is concerned with uncertainty of macroeconomic variables will respond differently to output gap shocks, i.e. $\beta_{GAP}^{High-uncertainty} \neq \beta_{GAP}^{Low-uncertainty}$. In general, assuming we controlled for any other major variation across groups of countries, the remaining difference in β should come only from the variation in the different degrees of uncertainty.

The other covariates are designed to capture two other incentives that drives policy makers' decision on fiscal policy. The first one is debt stabilization: the higher the debt, or the cost of debt, the less fiscal policy will be expansionary. The most straightforward way to capture this motive would be to include debt-to-GDP ratio in the list of regressors. However, debt-to GDP is a stock variable that tend to move at a much lower frequency than flow variables that move at the economic cycle frequency, hence it may not be informative. For this reason, we use first difference of debt-to-GDP ratio. This variable also controls for the fiscal capacity specific of each country: lower increase in debt-to-GDP means that the country has higher ability to borrow from capital markets. Furthermore, to avoid potential endogeneity between change in debt and primary balance, we lag the former. In the Appendix, we check for robustness of our results using other possible proxies for debt.

The second motive we want to capture is the political incentive to raise spending before upcoming elections in order to increase support for the incumbent government or legislature. We then use the lead of this variable in our list of regressors: we assume that politicians start to expand the fiscal budget for electoral reasons 1 year in advance to gather consensus. We collect data from the Varieties of Democracy Research Project ("v-dem.net/") to construct a dummy variable for each country, taking value of 1 if either a presidential or a legislative election took place in a given year.

Initial conditions of the government fiscal capacity play an important role in the magnitude of the response to a shock. Thus, we estimate the fiscal rule adding an interaction term between output gap and debt increase in the previous period, as specified in Equation (10):

$$\begin{aligned} CAPB_{i,t|t+S} =& \beta_0 + \beta_{GAP} GAP_{i,t|t+S} + \beta_{DEBT} \Delta DEBT_{i,t-1|t+S} + \beta_{Elect} Elect_{i,t+1} \\ & + \beta_{1,GAP} GAP_{i,t|t+S} \times \Delta DEBT_{i,t-1|t+S} + \epsilon_{i,t} \end{aligned} \tag{10}$$

Policy makers may have an incentive to react differently to positive and negative shocks to the economy. In particular, one would expect to see a strong fiscal reaction in bad times and a weak consolidation in good times. To check for the presence of asymmetries in the fiscal rule, we augment Equation (9) with a dummy variable that take value 1 if the current output gap is negative and 0 otherwise; we then interact this variable with output gap itself, as shown in Equation (11):

$$\begin{aligned} CAPB_{i,t|t+S} =& \beta_0 + \beta_{GAP} GAP_{i,t|t+S} + \beta_{DEBT} \Delta DEBT_{i,t-1|t+S} + \beta_{Elect} Elect_{i,t+1} \\ & + \beta_{1,crisis} \mathbb{I}[GAP_t < 0] + \beta_{2,crisis} \mathbb{I}[GAP_t < 0] \times GAP_{i,t|t+S} + \epsilon_{i,t} \end{aligned} \tag{11}$$

Given the persistent nature of shocks, we lag all the aforementioned regressors.

2.3.2 Instrumental variables

However, Equations (9), (10) and (11) suffer from endogeneity issues, arising mainly from the fact that output gap is affected by the fiscal stance in the first place. For this reason, we need to find a suitable instrument for the output gap. A large body of literature (see, e.g. Galí et al. (2003) and Alesina, Campante, and Tabellini (2008)) uses the output gap of an external economy considered to be a source of output gap shocks that is not affected by fiscal policy in the countries of interest. In particular, they use the output gap of the US to instrument for the output gap of every country in the Eurozone, arguing that European countries' fiscal policy does not affect output gap in the US, but shocks to US output gap is a meaningful source of shocks to European output gaps.

Building on this, we use a weighted average of output gaps from other countries in the same regional grouping of the WEO[3]. The key assumption is that shocks to output gaps of neighboring countries are an external source of shocks for the output gap of the country itself, but fiscal policy in any given country is not enough to impact this weighted

[3] The IMF has five regional area departments: African (AFR), Asia Pacific (APD) European (EUR), Middle East and Central Asia (MCD), and Western Hemisphere (WHD).

average:

$$\bar{GAP}^{IV}_{g,t|t+S} = \frac{\sum_{i \in g} GDP_{i,t|t+S} GAP_{i,t|t+S}}{\sum_{i \in g} GDP_{i,t|t+S}} \tag{12}$$

where $g \in \{AFR, APD, EUR, MCD, WHD\}$ is the geographical department of WEO. This methodology echoes the instrument introduced by Jaimovich and Panizza (2007), who used the share of exports from i to j as weights to construct an instrument specific to country i.

2.3.3 Results

Table 4 shows the results for the estimated fiscal reaction functions using fixed effect models. Random effects model regressions are implemented for robustness checks in the Appendix. Under symmetric fiscal reaction function (Columns (1) and (2)), fiscal policy is counter-cyclical for AE when variables are measured in real-time, confirming the results from Cimadomo (2012), with CAPB-to-GDP ratio increasing by 0.612 percentage points for a 1 percentage point increase in output gap. We find evidence of counter-cyclicality in EM as well, although the magnitude is lower. CAPB-to-GDP increases on impact by 0.517 percentage points in response to a 1 percentage point increase in output gap. However, EM seem to react more to lags in output gap, with an increase of 1.3 percentage points in CAPB-to-GDP one year after output gap increased by 1 percentage point. In contrast, AE react only by 0.5 percentage points.

We do not find evidence that initial fiscal capacity affects AE significantly, either in the symmetric specification of Column (1) nor in interacting with output gap itself in Column (3). On the other hand, there is weak evidence that debt dynamics affects both the reaction function in EM. In the symmetric specification (Column (2)), an increased debt-to-GDP ratio in the previous year leads to a fiscal consolidation, increasing CAPB-to-GDP by 0.5 percentage points. Furthermore, from Column (4) of Table 4 we see that countries that experienced a higher debt increase in the past period reduce the sensitivity of CAPB to output gap by 0.014 percentage points. This result is robust to the asymmetric specification. Lastly, there is no evidence that fiscal reaction functions are asymmetric depending on the sign of the shock.

In conclusion, the magnitude of the coefficient associated with output gap varies substantially across income groups, as AE are more responsive to an output gap shocks on impact, and EM being more constrained by fiscal capacity. If our model is correct in controlling for other factors, the lower sensitivity may be a consequence of higher uncertainty: as output gap measurement is much more uncertain, policy makers may want to be more cautious in their response to shocks.

Table 4: **Real-time fiscal reaction functions, Fixed Effects models.**

	Dependent variable:					
	Cyclically adjusted primary balance-to-GDP$_t$					
	AE (9)	EM (9)	AE (10)	EM (10)	AE (11)	EM (11)
	(1)	(2)	(3)	(4)	(5)	(6)
$CAPB_{t-1}$	0.243	−0.532	0.327	−0.299	0.375	−0.343
	(0.190)	(0.385)	(0.572)	(0.267)	(0.384)	(0.252)
GAP_t	0.612***	0.517***	0.639***	0.533***	1.096	0.671
	(0.067)	(0.154)	(0.217)	(0.109)	(4.278)	(0.923)
GAP_{t-1}	0.505***	1.310**	0.558***	0.806*	0.752***	0.964***
	(0.118)	(0.587)	(0.153)	(0.428)	(0.180)	(0.293)
$\Delta DEBT_{t-1}$	0.046	0.511**	0.112	0.222	0.144	0.279*
	(0.051)	(0.231)	(0.299)	(0.197)	(0.189)	(0.148)
Election Year$_{t+1}$	−0.267	0.691	−0.266	−0.367	−0.253	−0.136
	(0.470)	(1.059)	(0.476)	(0.855)	(0.551)	(0.792)
$\mathbb{I}[GAP_t < 0]$					2.520	1.322
					(2.533)	(2.099)
$GAP_t \times \Delta DEBT_{t-1}$			0.010	−0.014*	0.014	−0.014*
			(0.047)	(0.008)	(0.027)	(0.008)
$\mathbb{I}[GAP_t < 0] \times GAP_t$					−0.125	−0.055
					(4.263)	(0.872)
Observations	100	109	100	109	100	109
R^2	0.714	0.265	0.713	0.459	0.665	0.411
Adjusted R^2	0.535	−0.221	0.527	0.087	0.429	−0.025
F Statistic	163.360***	28.292***	158.857***	57.436***	126.351***	49.212***

*p<0.1; **p<0.05; ***p<0.01

3 Model

We adapt the standard closed economy New Keynesian model from Woodford (2003) and Galí (2015), with some modifications to accomodate for active fiscal policy. In particular, we adapt the framework proposed in Vitek (2023).

3.1 Households

A fraction ϕ^C of households is financially constrained: they cannot save or borrow, so they have to consume all their income in the current period. Hence they can only choose the amount of hours worked, N_t^C, and consume the totality of their income:

$$\max_{C_t^C, N_t^C} \frac{(C_t^C)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \frac{(N_t^C)^{1+\varphi}}{1+\varphi} \tag{13}$$

subject to

$$P_t C_t^C = (1-\tau)(W_t N_t^C + \Theta_t) \tag{14}$$

Θ_t represents the profits of firms rebated lump sum to the households. Both labour income and profits are subject to proportional taxation with tax rate τ.

The labor supply of this type of agents has the same form as the unconstrained household's:

$$(C_t^C)^{\sigma}(N_t^C)^{\varphi} = (1-\tau)\frac{W_t}{P_t} \tag{15}$$

The remaining $1-\phi^C$ fraction of households are instead unconstrained, and maximizes the lifetime expected utility by choosing consumption, amount of nominal bonds to hold and hours worked.

The problem of the unconstrained household is:

$$\max_{(C_t^U, B_t^n, N_t^U)_{t\geq 0}} \mathbb{E}_0 \sum_{t\geq 0} \beta^t \left[\frac{(C_t^U)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \frac{(N_t^U)^{1+\varphi}}{1+\varphi} \right] \tag{16}$$

subject to

$$B_t^n = (1+i_t)[B_{t-1}^n + (1-\tau)(W_t N_t^U + \Theta_t) - P_t C_t^U] \tag{17}$$

B_t^n represents the demand for nominal government bonds.

The solution to the unconstrained household's problem yields the standard Euler equation and the labor supply equation:

$$\beta \mathbb{E}_t \left[\left(\frac{C_{t+1}^U}{C_t^U} \right)^{-\frac{1}{\sigma}} \frac{(1+i_t)}{\Pi_{t+1}} \right] = 1 \tag{18}$$

$$(C_t^U)^{\frac{1}{\sigma}} (N_t^U)^{\varphi} = (1-\tau) \frac{W_t}{P_t} \tag{19}$$

3.2 Firms

There are two types of firms: a continuum of intermediate good producers which use labor as input to produce each their own variety of intermediate good, and a final good producer which aggregates the intermediate goods.

3.2.1 Final Good Producer

The final good producer is perfectly competitive, featuring a CES production function with elasticity ε. The producer chooses the quantity of each intermediate goods to minimize the cost:

$$\min_{(Y_t(i))_i} \int P_t(i) Y_t(i) di \tag{20}$$

subject to

$$\left(\int Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} = Y_t \tag{21}$$

The solution to the minimization problem yields the individual demand for input i:

$$Y_t^d(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t \tag{22}$$

3.2.2 Intermediate Goods Producers

The producer of each variety i is a monopolist facing an isoelastic demand $Y_t^d(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t$ and production function: $Y_t(i) = A_t N_t(i)$.

Firms are subject to staggered price setting *á la* Calvo: at any time t they can adjust their price with probability $1-\vartheta$. If they can adjust, they will chose a reset price $P_t^*(i)$

that maximizes the future stream of real profits, i.e. it solves:

$$\max_{P_t^*(i)} \sum_{s\geq 0} \vartheta^s \Lambda_{t,t+s} \left[\frac{P_t^*(i) Y_{t+s}(i)}{P_{t+s}} - \frac{W_{t+s} Y_{t+s}(i)/A_{t+s}}{P_{t+s}} \right] \tag{23}$$

subject to

$$Y_{t+s}(i) = \left(\frac{P_t^*(i)}{P_{t+s}} \right)^{-\varepsilon} Y_{t+s} \tag{24}$$

$\Lambda_{t,t+s}$ is the stochastic discount factor of the unconstrained household, that is the ratio between marginal utility of consumption at time $t+s$ and t. This is because the ownership of the firms is held by the households, and the unconstrained type is the only one concerned with dynamics.

3.3 Government

The fiscal authority chooses the amount of government spending G_t, and the amount of real public debt B_t to issue. The government budget constraint is:

$$P_{t+1} B_t = (1 + i_t)(P_t B_{t-1} + P_t G_t - \tau P_t Y_t) \tag{25}$$

The monetary authority, on the other hand, sets the nominal interest rate according to a Taylor rule:

$$(1 + i_t) = \beta^{-1} (\Pi_t)^{\phi_\pi} \left(\frac{Y_t}{Y_t^p} \right)^{\phi_y} \tag{26}$$

with $\Pi_t = \frac{P_t}{P_{t-1}}$ being the gross inflation rate and Y_t^p being potential output.

3.4 Equilibrium

Goods market clearing condition implies:

$$C_t^C + C_t^U + G_t = Y_t \tag{27}$$

This condition can be rewritten as:

$$(1 - \phi^C) C_t + G_t = (1 - (1 - \tau)\phi^C) Y_t \tag{28}$$

with $C_t = C_t^C + C_t^U$.

Labor market clearing implies:

$$N_t = N_t^U + N_t^C \tag{29}$$

where $N_t = \frac{Y_t}{A_t}$. The equilibrium condition for the labour market can be rewritten as:

$$C_t^{\frac{1}{\sigma}} N_t^{\varphi} = (1-\tau)\frac{W_t}{P_t}[(1-\phi^C)^{-\frac{1}{\sigma\varphi}} + (\phi^C)^{-\frac{1}{\sigma\varphi}}]^{\varphi} \tag{30}$$

Lastly, we have the bond market clearing condition:

$$B_t^n = P_{t+1}B_t \tag{31}$$

3.5 Log-linear economy

The Euler equation (18) is log-linearized as:

$$c_t = \mathbb{E}_t c_{t+1} - \sigma(i_t - \mathbb{E}_t \pi_{t+1}) \tag{32}$$

Equation (28) for goods market clearing implies:

$$(1-\phi^C)(1-\gamma)c_t = (1-(1-\tau)\phi^C)y_t - \gamma g_t \tag{33}$$

where γ is government spending ratio in steady state. Combining the two equations we thus have a closed form of the Dynamic IS curve that accounts for fiscal policy:

$$\hat{y}_t = \mathbb{E}_t\hat{y}_{t+1} + \frac{\gamma}{1-(1-\tau)\phi^C}(\hat{g}_t - \mathbb{E}_t\hat{g}_{t+1}) - \sigma\frac{(1-\phi^C)(1-\gamma)}{1-(1-\tau)\phi^C}(i_t - \mathbb{E}_t\pi_{t+1} - r_t^n) \tag{34}$$

where $r_t = i_t - \mathbb{E}_t\pi_{t+1}$ is the real interest rate and r_t^n is its natural level. From now on we will refer to $\mu^g = \frac{\gamma}{1-(1-\tau)\phi^C}$ as the fiscal multiplier, and $\chi = \sigma\frac{(1-\phi^C)(1-\gamma)}{1-(1-\tau)\phi^C}$.

Inflation dynamics can be derived from the solution of Problem (23). This yields the standard New Keynesian Phillips curve augmented to account for fiscal transfers:

$$\pi_t = \kappa\hat{y}_t + \beta\mathbb{E}_t\pi_{t+1} \tag{35}$$

The log-linearized Taylor rule (26) is:

$$i_t = r_t^n + \phi_y \hat{y}_t + \phi_\pi \pi_t \tag{36}$$

Government debt dynamics derives from equation (25), the fiscal authority budget constraint:

$$\beta \hat{b}_t = \hat{b}_{t-1} + \delta\gamma\hat{g}_t - \delta\tau\hat{y}_t + \beta(i_t - \mathbb{E}_t\pi_{t+1} - r_t^n) \tag{37}$$

where $\delta = \frac{Y}{B}$ is the inverse of debt-to-GDP ratio in steady state and τ is the income tax rate.

3.6 Benchmark Optimal Fiscal Policy

We will now discuss the policy maker's optimal choice of government spending in a benchmark case with rational expectations and perfect foresight. We do not formally derive the social welfare function as in Woodford (2003) and Benigno and Woodford (2003). Instead, we assume that the government aims at stabilizing the output gap at 0 taking into account the effect of excess spending on future debt $\hat{b}_t$, which should not deviate from a target level $\bar{b}$. The external shocks that the government has to counter in order to stabilize output gap and debt have not been explicitly modeled so far: we will take a reduced form approach and assume that the dynamic IS curve can be hit by shocks:

$$\hat{y}_t = \mathbb{E}_t\hat{y}_{t+1} + \mu^g(\hat{g}_t - \mathbb{E}_t\hat{g}_{t+1}) - \chi(i_t - \mathbb{E}_t\pi_{t+1} - r_t^n) + x_t \tag{38}$$

$$x_t = \rho x_{t-1} + \epsilon_t \qquad \epsilon \sim N(0,1) \tag{39}$$

The policy maker's objective function is thus a quadratic loss with relative weight of debt λ:

$$\max -\frac{1}{2}\mathbb{E}_0 \sum_{t\geq 0} \beta^t [\hat{y}_t^2 + \lambda(\hat{b}_t - \bar{b})^2] \tag{40}$$

subject to

$$
\begin{aligned}
\hat{y}_t &= \mathbb{E}_t\hat{y}_{t+1} + \mu^g(\hat{g}_t - \mathbb{E}_t\hat{g}_{t+1}) - \chi(i_t - \mathbb{E}_t\pi_{t+1} - r_t^n) + x_t && IS\\
\beta\hat{b}_t &= \hat{b}_{t-1} + \delta\gamma\hat{g}_t - \delta\tau\hat{y}_t + \beta(i_t - \mathbb{E}_t\pi_{t+1} - r_t^n) && BC\\
\pi_t &= \kappa\hat{y}_t - \kappa^g\hat{g}_t + \beta\mathbb{E}_t\pi_{t+1} && PC\\
i_t &= r_t^n + \phi_y\hat{y}_t + \phi_\pi\pi_t && MP\\
x_t &= \rho x_{t-1} + \epsilon_t
\end{aligned}
$$

This can be assumed without loss of generality, as Problem (40) closely resembles the problem of the social planner in Vines and Stehn (2007) and Leith and Wren-Lewis (2013).

We adopt the Lagrangean solution method as in Currie and Levine (1993). In particular, we recast the problem as follows:

$$
\max -\frac{1}{2}\mathbb{E}_0\sum_{t\geq 0}\beta^t\mathbf{x}_t'R\mathbf{x}_t \tag{41}
$$

subject to

$$
A\mathbf{x}_t = B\mathbb{E}_t\mathbf{x}_{t+1} + C\mathbf{x}_{t-1} + D\mathbf{u}_t + \boldsymbol{\epsilon}_t
$$

where $\mathbf{x_t}$ is a vector containing all the endogenous variables of the model and $\mathbf{u}_t$ is the vector of controls (in our case $\mathbf{u}_t = g_t$). In Lagrangean form, the problem can be rewritten as:

$$
\begin{aligned}
&\max\mathbb{E}_0\sum_{t\geq 0}\beta^t\mathbf{L}_t && (42)\\
&\mathbf{L}_t = -\frac{1}{2}\mathbf{x}_t'R\mathbf{x}_t + \boldsymbol{\mu}_t'(B\mathbb{E}_t\mathbf{x}_{t+1} + C\mathbf{x}_{t-1} + D\mathbf{u}_t + \boldsymbol{\epsilon}_t - A\mathbf{x}_t) && (43)
\end{aligned}
$$

where $\boldsymbol{\mu}_t$ is a vector of multipliers associated with each constraint. The first order conditions imply:

$$
\begin{aligned}
\mathbf{0} &= -R\mathbf{x}_t + \beta^{-1}\boldsymbol{\mu}_{t-1}'B + \beta\mathbb{E}_t\boldsymbol{\mu}_{t+1}'C - \boldsymbol{\mu}_t'A && (\mathbf{x}_t) && (44)\\
\mathbf{0} &= \boldsymbol{\mu}_t'D && (\mathbf{u}_t) && (45)
\end{aligned}
$$

The system of equations consisting of the first order conditions, together with the laws of motion represented by the constraints, form a dynamic linear system that pins down the dynamics of all the endogenous variables, the multipliers and the controls.

The dynamics of our economy is represented by the 4 constraints of the government problem together with the following system of first order conditions

$$
\begin{aligned}
0 =& -\hat{y}_t - \mu_t^{IS} + \beta^{-1}\mu_{t-1}^{IS} - \delta\tau\mu_t^{BC} + \kappa\mu_t^{PC} + \phi_y\mu_t^{MP} && (\hat{y}_t)\\
0 =& -\lambda(b_t - \bar{b}) - \beta\mu_t^{BC} + \beta\mathbb{E}_t\mu_{t+1}^{BC} && (b_t)\\
0 =& \mu^g\mu_t^{IS} - \beta^{-1}\mu^g\mu_{t-1}^{IS} + \delta\gamma\mu_t^{BC} && (g_t)\\
0 =& \chi\beta^{-1}\mu_{t-1}^{IS} - \mu_{t-1}^{BC} - \mu_t^{PC} + \mu_{t-1}^{PC} + \phi_\pi\mu_t^{MP} && (\pi_t)\\
0 =& -\chi\mu_t^{IS} + \beta\mu_t^{BC} - \mu_t^{MP} && (i_t)
\end{aligned}
$$

where we labeled each multiplier with the name of the corresponding constraint.

3.7 Optimal Fiscal Policy under Uncertainty

To capture the idea of output gap uncertainty, we will add a slight modification to the robustness literature pioneered by Hansen and Sargent (2008). In particular, we assume that the government perceives the Dynamic IS equation as misspecified when evaluated in real-time. As outlied in Section 2, output gap estimates are unstable due to uncertainty surrounding the data generating process (i.e. the probabilistic model adopted by the statistician). Thus, model misspecification is well suited to treat the policy maker's attitude towards uncertainty. Furthermore, to capture the idea of imperfect policy implementation, the debt accumulation equation in real-time is perceived to be misspecified as well. This is due to the fact that, as the government plans a certain level of fiscal spending for the current period, the realized g_t may differ from that level, thus implying a different accumulation of debt than what originally planned.

We will use the notation $\hat{y}_{t|t}$ to denote the real-time estimate of output gap and, analogously, all the other variables. Misspecification in real-time is assumed to take the following form:

$$\hat{y}_{t|t} = \mathbb{E}_t\hat{y}_{t+1|t} + \mu^g(\hat{g}_{t|t} - \mathbb{E}_t\hat{g}_{t+1|t}) - \chi(i_{t|t} - \mathbb{E}_t\pi_{t+1|t} - r_{t|t}^n) + x_t + \sigma_w w_t \tag{46}$$

$$\beta\hat{b}_{t|t} = \hat{b}_{t-1|t} + \delta\gamma\hat{g}_{t|t} - \delta\tau\hat{y}_{t|t} + \beta(i_{t|t} - \mathbb{E}_t\pi_{t+1|t} - r_{t|t}^n) + \sigma_v v_t \tag{47}$$

with $(w_t, v_t)_{t\geq 0}$ being random errors.

Then, an uncertainty averse (robust) decision maker in the sense of Hansen and Sargent (2008) solves for the optimal fiscal policy rule considering a worst-case scenario among a set of possible disturbances. The set of feasible disturbances is such that the relative entropy between the disturbances and the reference model with no misspecification is bounded:

$$\frac{1}{2}\sum_{t\geq 0}\beta^t(w_t^2+v_t^2)\leq\frac{\eta}{1-\beta} \tag{48}$$

We can now restate the problem for the robust government in real-time:

$$\max \min_{(w_t,v_t)_{t\geq 0}} -\frac{1}{2}\sum_{t\geq 0}\beta^t[\hat{y}_{t|t}^2+\lambda(\hat{b}_{t|t}-\bar{b})^2] \tag{49}$$

subject to

$$\begin{aligned}
\hat{y}_{t|t} &= \mathbb{E}_t\hat{y}_{t+1|t}+\mu^g(\hat{g}_{t|t}-\mathbb{E}_t\hat{g}_{t+1|t})-\chi(i_{t|t}-\mathbb{E}_t\pi_{t+1|t}-r^n_{t|t})+x_t+\sigma_w w_t\\
\beta\hat{b}_{t|t} &= \hat{b}_{t-1|t}+\delta\gamma\hat{g}_{t|t}-\delta\tau\hat{y}_{t|t}+\beta(i_{t|t}-\mathbb{E}_t\pi_{t+1|t}-r^n_{t|t})+\sigma_v v_t\\
\pi_{t|t} &= \kappa\hat{y}_{t|t}-\kappa^g\hat{g}_{t|t}+\beta\mathbb{E}_t\pi_{t+1|t}\\
i_{t|t} &= r^n_{t|t}+\phi_y\hat{y}_{t|t}+\phi_\pi\pi_{t|t}\\
x_t &= \rho x_{t-1}+\epsilon_t\\
\frac{1}{2}&\sum_{t\geq 0}\beta^t(w_t^2+v_t^2)\leq\frac{\eta}{1-\beta}
\end{aligned}$$

In particular, if we write down the Lagrangian for the last constraint:

$$\max \min_{(w_t,v_t)_{t\geq 0}} -\frac{1}{2}\sum_{t\geq 0}\beta^t\left\{\hat{y}_{t|t}^2+\lambda(\hat{b}_{t|t}-\bar{b})^2+\theta^{-1}\left[\eta-w_t^2-v_t^2\right]\right\} \tag{50}$$

subject to

$$\begin{aligned}
\hat{y}_{t|t} &= \mathbb{E}_t\hat{y}_{t+1|t}+\mu^g(\hat{g}_{t|t}-\mathbb{E}_t\hat{g}_{t+1|t})-\chi(i_{t|t}-\mathbb{E}_t\pi_{t+1|t}-r^n_{t|t})+x_t+\sigma_w w_t\\
\beta\hat{b}_{t|t} &= \hat{b}_{t-1|t}+\delta\gamma\hat{g}_{t|t}-\delta\tau\hat{y}_{t|t}+\beta(i_{t|t}-\mathbb{E}_t\pi_{t+1|t}-r^n_{t|t})+\sigma_v v_t\\
\pi_{t|t} &= \kappa\hat{y}_{t|t}-\kappa^g\hat{g}_{t|t}+\beta\mathbb{E}_t\pi_{t+1|t}\\
i_{t|t} &= r^n_{t|t}+\phi_y\hat{y}_{t|t}+\phi_\pi\pi_{t|t}\\
x_t &= \rho x_{t-1}+\epsilon_t
\end{aligned}$$

As stated by Hansen and Sargent (2008), the inverse of the multiplier θ^{-1} represents the sensitivity of the policy maker to the measurement errors of output gap and debt respectively. The higher the parameter, the more pessimistic the worst-case scenario will be and hence the decision maker will be more cautious.

The nature of the problem does not change, and the first order conditions with re-

Table 5: **Calibrated parameters of the model.**

Variable	Parameter	Value	Source
Discount factor (annual)	β	0.8235	Average long term real rate
Target/SS Debt-to-GDP (inverse δ)	$\bar{b}$	0.4892	Average debt-to-GDP ratio
SS tax-to-GDP	τ	0.28	Average revenues-to-GDP ratio
SS government spending-to-GDP	γ	0.285	$\tau+$ average CAPB-to-GDP ratio
CRRA	σ	1	Standard
Relative weight on debt	λ	0.51	Calibrated (details in the text)
Persistence of output gap shock	ρ	0.5	Bhattacharya and Patnaik (2013)
Fraction of constrained HH	ϕ^C	0.786	Bhattacharya and Patnaik (2013)
Slope of the Phillips curve	κ	0.2	Standard
SD of the error in the IS equation	σ_w	1.558	S.D. of output gap FFE
SD of the error in the debt equation	σ_v	1.352	S.D. of CAPB FFE
Taylor rule output gap coefficient	ϕ_y	0.7	Standard
Taylor rule inflation coefficient	ϕ_π	1.3	Standard
Uncertainty aversion	θ	0.3226	Calibrated (details in the text)

spect to the endogenous variables of the system and the controls are the same as in the benchmark problem. However, we have to add 2 additional conditions, coming from the minimization problem:

$$\theta^{-1}w_t + \mu_t^{IS} = 0 \qquad (w_t) \qquad (51)$$

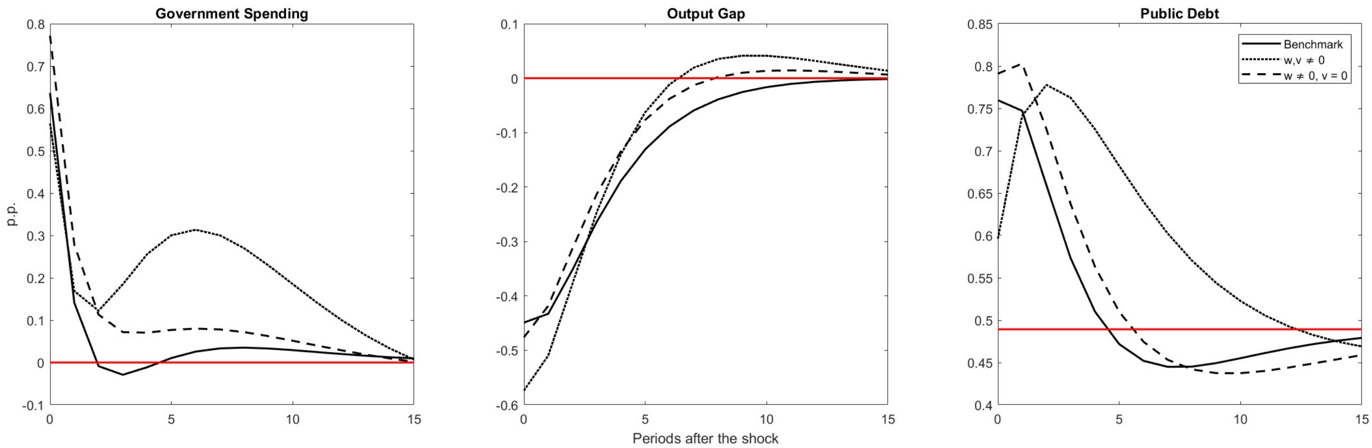
$$\theta^{-1}v_t + \mu_t^{BC} = 0 \qquad (v_t) \qquad (52)$$

This will not change the tradeoff between debt and output gap stabilization that the policy maker is facing. It will however change the quantitative response to an exogenous shock, generating a different path of policy and endogenous variables.

3.8 Calibration and Simulation

We use the real-time data set from the first section to calibrate all the parameters deriving from steady state objects, focusing on EM only. In particular, for the discount factor β, we compute the average of the gross long term real interest rate across the EM sample we have available over the selected time period, and we compute the inverse. As for public debt-to-GDP ratio $\bar{b}$ and tax rate τ we simply compute the average of debt-to-GDP ratio and revenues-to-GDP ratio. We then compute the share of government spending γ by adding the average CAPB-to-GDP ratio to the calibrated value of τ. The share of financially constrained households ϕ^C is taken from Bhattacharya and Patnaik (2013), who computed the share of Indian households with no access to banking before the financial sector liberalization reform in 1991. We also compute the standard deviations of final forecast errors for output gap and CAPB to calibrate σ_w and σ_v respectively. Lastly,

Figure 4: **Response to a 1% shock to output gap: benchmark case vs different degrees of uncertainty aversion**



λ, the relative weight the policy maker attaches to debt in the maximization problem, is set to generate a response of fiscal spending to an output gap shock that is consistent with the coefficients estimated in the fiscal rule in Section 2. In particular, we assumed that the main difference in the fiscal response between EM and AE stems from uncertainty. Hence, we set λ so that the coefficient on the shock ϵ_t of the policy function of g_t in the benchmark case with no uncertainty matches the coefficient β_{GAP} estimated for AE in Equation (9). Taylor rule coefficients ϕ_y and ϕ_π and the slope of the Phillips curve κ are standard in New Keynesian literature. Although these parameters are usually calibrated for the US economy, they also ensure the stability of the system. We summarize our calibration in Table 5.

We will now study how the government optimally responds to a 1% negative shock to output gap under different degrees of uncertainty aversion. We are first interested in describing the behavior of the economy when there is no uncertainty, i.e. what we call the benchmark case. Second, we analyse the situation in which the policy maker is *equally* concerned about output gap and fiscal policy implementation uncertainty. The multiplier θ which captures risk aversion is set to 0.3226, as stated in Table 5. This figure is obtained by matching the coefficient on the shock ϵ_t of the policy function of g_t to β_{GAP} from Equation (9) estimated for EM. Finally, we remove fiscal implementation uncertainty, and see how the economy react when the policy maker is only concerned about output gap uncertainty. Since the literature agrees that this is a crucial problem for EM, as stated in Section 1, it is natural to focus on what the optimal response would be in this case.

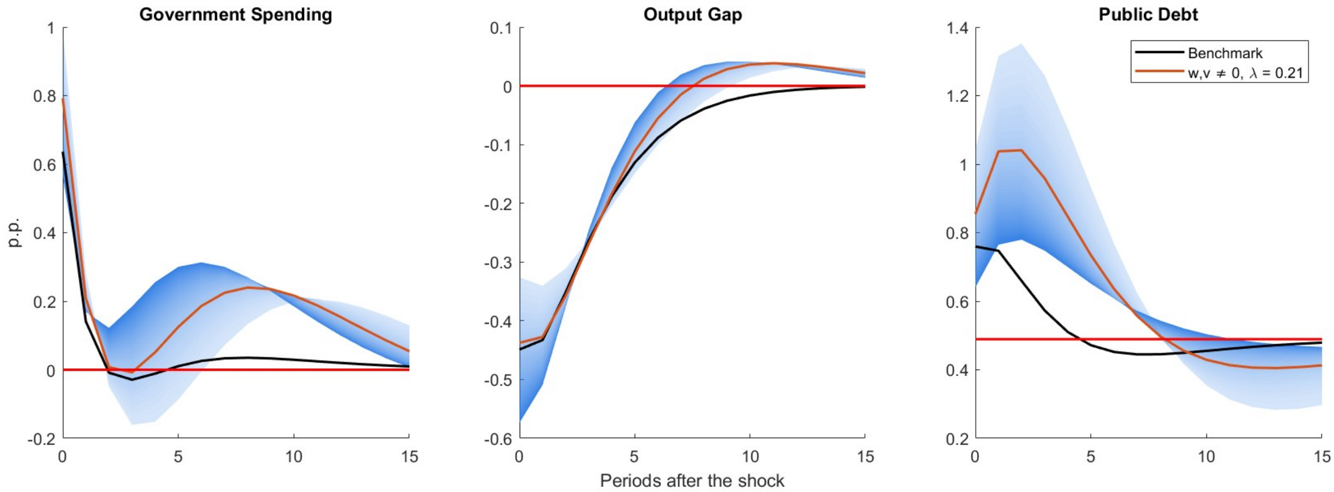
Figure 4 plots the impulse responses of debt, output gap and the implied dynamics of government spending to a 1% shock to output gap under different degrees of uncertainty aversion. The solid line represents the benchmark case without uncertainty; the dotted line shows the case when the policy maker is equally concerned about output gap and fiscal implementation uncertainty (hence both disturbances w and v are non zero); the dashed line shows the case when the policy maker is only concerned about output gap uncertainty (hence the disturbances on the law of motion of debt are constant at 0).

In the benchmark case, government spending increases by 0.63% on impact and sharply falls in subsequent periods to dampen the effect of Ricardian equivalence that would significantly reduce consumption of the financially unconstrained households. As a result, debt increases to 0.75%, and output gap decreases by 0.44%. When the policy maker is concerned about both sources of uncertainty, the fiscal reaction on impact is, as expected, less counter-cyclical, increasing by 0.56%. It is however more persistent, displaying a hump-shaped dynamics. A less counter-cyclical fiscal response implies that output gap falls by 0.57% on impact, more than the benchmark case. It does however recover faster than the benchmark, due to the dynamics of fiscal spending. The dynamics of the fiscal policy response is mirrored by public debt, which surges on impact to 0.6%. As the policy maker grows concerned only about output gap uncertainty, the fiscal reaction grows stronger, with government spending increasing by 0.77% and being more persistent than the benchmark scenario. This means that output gap drops by 0.48% on impact, more than the benchmark case due to the effect of Ricardian equivalence, but it reverts back to 0 faster due to sustained and increased fiscal spending. As a result, public debt jumps to 0.79%, higher than the benchmark scenario, but it reverts faster thanks to the increased growth that buoys tax revenues.

These findings can be rationalized as follows. If the policy maker is only concerned about output gap uncertainty, the worst-case scenario entails a much larger deviation of output gap from the optimal value of 0 compared to the benchmark case, thus triggering a stronger response of the government and a larger public debt accumulation. This will make output gap converge faster to the target, even though it may be costlier on impact. On the other hand, if the government is also concerned about fiscal policy implementation uncertainty, the worst-case scenario implies that, once the policy maker starts increasing government spending after the negative output gap shock, the public debt greatly deviates from its target $\bar{b}$; thus, the government reaction becomes more timid in order to control debt dynamics. The resulting effect is an under-reaction to the output gap shock compared to the benchmark, consistent with observations in Section 2.

The weaker counter-cyclicality of government spending to output gap shocks in the presence of both sources of uncertainty induces welfare losses from the point of view of an uncertainty neutral policy maker. We therefore modify the core objective function by lowering the relative weight λ, so that the policy maker is more concerned about output gap stabilization. Figure 5 shows the range of impulse responses of government spending, output gap and public debt as λ varies between 0.01 and 0.51. It also highlights the case $\lambda = 0.21$, which is the value that minimizes the distance between the response of output gap and the benchmark response. This shows that the policy maker can replicate the benchmark case of no uncertainty by lowering its relative preference for debt stabilization, hence triggering a higher and more persistent fiscal response compared to what would otherwise be the case. It is important to note that this does not necessarily means that public debt will be negatively impacted: a faster recovery of output implies that tax revenues will also increase faster, implying that public debt stabilizes in the long run.

Figure 5: **Responses to a 1% shock to output gap for $\lambda \in [0.01, 0.51]$. Darker blue implies higher λ**



4 Conclusions

This paper contends that policy makers try to stabilize output, while also attempting to minimize debt accumulation, using output gap estimates as an indicator of the cyclical position of the economy. However, uncertainty surrounding these estimates owing to measurement errors, model specifications, and data revisions ex post could give rise to Type I and II errors, leading to unnecessary debt accumulation or tipping an economy into recession, respectively. Using data from past WEO vintages, we found significant dispersion in real-time estimates for both output gap and fiscal stance. Furthermore, output gap dispersion is significantly higher for EM than it is for AE. Despite this, we found that fiscal policy is counter-cyclical for EM when estimated in real-time, although the responsiveness to output gap shocks is moderate. We rationalize these two facts by building a model of optimal fiscal policy within a New Keynesian general equilibrium framework. The model is then calibrated to quantitatively match an average EM country.

When there is uncertainty around output gap and fiscal policy implementation, the model produces less counter-cyclical fiscal responses to output gap shocks compared to the benchmark under certainty. This entails that the presence of underlying uncertainty and uncertainty aversion produce a loss in efficiency in terms of output gap stabilization. On the other hand, the response is more counter-cyclical when aversion towards output gap uncertainty is the only one taken into account. In particular, this means that the policy maker tends to overshoot the fiscal response when compared to the benchmark. This suggests that, as output gap uncertainty is a major issue in policymaking in EM, fiscal response should increase its counter-cyclical stance for these countries. We showed that this can be achieved by significantly lowering the relative weight that policy makers assign to public debt targeting in favor of output gap stabilization in their objective function. Notably, this does not necessarily imply an explosion of public debt, which stabilizes in the long run thanks, to a faster recovery of the economy.

The findings in this paper contribute to the debate on the scope and pace of fiscal spending to raise or stabilize output amid rising public debt accumulation, especially in EMs with constrained fiscal space. Overall, the results suggest the need for continued caution in relying on real-time output gap estimate in EM for policy formulation and implementation, especially where fiscal policy is the main instrument against asymmetric shocks to the economy.

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Appendix

Fiscal Reaction Function Robustness Checks

Table 6 shows the results of the estimation of Equations (9), (10) and (11) using random effects models. Columns (1) through (4) show strong counter-cyclical behavior of fiscal policy on impact for both AE and EM. However, countrary to the fixed effect models estimated in Section 2.3, we find that fiscal policy is much more persistent, with a very high autoregressive coefficient. Furthermore, we also find that EM are prone to consolidating their balance once they are hit by an output gap shock: their cyclically adjusted primary balance moves in the opposite direction of the lagged output gap. Lastly, random effect models show that EM governments have strong political incentive, as they implement expansionary fiscal policy in the year leading up to an election.

The Hausman test statistic rejects the use of random effect models for Equations (9) and (11) although it does not for Equation (10) as shown in Table 7. However, we think it is important to capture country-specific differences even in the case that would deliver a statistically inefficient result. Hence we prefer fixed effects models reported in Table 4.

An alternative way to Equation (10) to account for the impact of initial conditions on fiscal reactions is given by Golinelli and Momigliano (2006), who estimate the Equation (53)

$$\Delta CAPB_{i,t|t+S} = \alpha + \rho CAPB_{i,t-1|t+S} + \beta GAP_{i,t-1|t+S} + \mathbf{x}'_{i,t|t+S}\gamma + \epsilon_{i,t} \tag{53}$$

where we collected the proxy for the fiscal space in the vector $\mathbf{x}$.

Table 8 shows the results of the estimation. Fiscal policy appears to be more history-dependent for EM, as the coefficient on lagged CAPB-to-GDP is twice the one for AE in both specifications we try.

Table 6: **Real-time fiscal reaction functions, Random Effects models.**

	Dependent variable:					
	Cyclically adjusted primary balance-to-GDP$_t$					
	AE	EM	AE	EM	AE	EM
	(1)	(2)	(3)	(4)	(5)	(6)
$CAPB_{t-1}$	1.029***	1.136***	0.889***	0.968***	1.206***	0.820***
	(0.073)	(0.123)	(0.138)	(0.174)	(0.283)	(0.200)
GAP_t	0.611***	0.568***	0.408**	0.562***	−12.264	−0.612
	(0.077)	(0.165)	(0.162)	(0.166)	(10.858)	(1.028)
GAP_{t-1}	0.194	−0.450**	0.018	−0.517***	0.515	−0.333**
	(0.123)	(0.190)	(0.192)	(0.196)	(0.499)	(0.142)
$\Delta DEBT_{t-1}$	0.124**	−0.046	−0.084	−0.171	0.265	−0.054
	(0.049)	(0.067)	(0.178)	(0.118)	(0.305)	(0.140)
Election Year$_{t+1}$	−0.542	−1.850***	−0.736	−2.195***	−0.474	−1.849***
	(0.442)	(0.668)	(0.517)	(0.703)	(0.948)	(0.673)
$\mathbb{I}[GAP_t < 0]$					−4.651	0.385
					(5.478)	(2.482)
$GAP_t \times \Delta DEBT_{t-1}$			−0.042	−0.014	0.029	0.002
			(0.035)	(0.011)	(0.058)	(0.018)
$\mathbb{I}[GAP_t < 0] \times GAP_t$					13.420	1.252
					(11.254)	(1.003)
Constant	0.159	0.847	−0.572	0.904	7.760	1.097
	(0.352)	(0.632)	(0.641)	(0.647)	(6.948)	(1.935)
Observations	100	109	100	109	100	109
R^2	0.750	0.555	0.701	0.512	0.395	0.599
Adjusted R^2	0.737	0.533	0.682	0.483	0.342	0.567
F Statistic	296.966***	139.328***	242.831***	113.736***	70.466***	170.504***

*p<0.1; **p<0.05; ***p<0.01

Table 7: **Hausman test: Fixed Effect vs Random Effect models for Equations (9), (10) and (11).**

	AE (9)	EM (9)	AE (10)	EM (10)	AE (11)	EM (11)
Statistic	19.728	40.366	7.475	1.024	38.628	35.423
p-value	0.001	0.000	0.486	0.961	0.000	0.000

Table 8: **Real-time fiscal reaction functions according to Equation (53).**

	Dependent variable:			
	Δ Cyclically adjusted primary balance-to-GDP			
	AE	EM	AE	EM
	(1)	(2)	(3)	(4)
$CAPB_{t-1}$	-0.481^{**}	-0.928^{***}	-0.498^{*}	-1.148^{**}
	(0.200)	(0.141)	(0.269)	(0.449)
GAP_t	0.547^{***}	0.572^{***}	0.289	0.414^{*}
	(0.137)	(0.219)	(0.190)	(0.250)
GAP_{t-1}	0.176^{***}	0.110		
	(0.051)	(0.069)		
$DEBT_{t-1}$			-3.485^{*}	-0.120
			(2.012)	(0.329)
Long Term real rate$_{t-1}$	0.077	-0.369	-0.185	0.057
	(0.474)	(0.513)	(0.680)	(0.602)
Observations	135	144	124	63
R^2	0.221	0.269	0.116	0.379
Adjusted R^2	-0.076	-0.035	-0.221	0.104
F Statistic	28.323^{***}	59.467^{***}	8.529^{*}	28.677^{***}

$^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Table 9 shows the results of the estimation of fiscal reaction function when fiscal capacity is measured by debt-to-GDP in levels. The numbers are consistent with what found in Section 2.3.3.

Table 9: **Real-time fiscal reaction functions, Debt in Levels.**

	Dependent variable:					
	Cyclically adjusted primary balance-to-GDP$_t$					
	AE	EM	AE	EM	AE	EM
	(1)	(2)	(3)	(4)	(5)	(6)
$CAPB_{t-1}$	0.299**	0.191	0.320**	0.108	0.460	0.096
	(0.141)	(0.119)	(0.143)	(0.109)	(0.640)	(0.124)
GAP_t	0.618***	0.518***	0.519***	1.258***	−2.328	1.636
	(0.056)	(0.094)	(0.101)	(0.239)	(19.290)	(2.059)
GAP_{t-1}	0.420***	0.237	0.406***	0.219	0.630	0.299*
	(0.097)	(0.192)	(0.098)	(0.166)	(0.388)	(0.156)
$DEBT_{t-1}$	0.028	0.009	0.032	−0.028	0.118	−0.023
	(0.038)	(0.060)	(0.039)	(0.051)	(0.333)	(0.064)
Election Year$_{t+1}$	−0.216	−0.669	−0.160	−0.634*	−0.457	−0.504
	(0.334)	(0.428)	(0.340)	(0.374)	(1.559)	(0.573)
$\mathbb{I}[GAP_t < 0]$					0.222	1.476
					(10.058)	(2.819)
$GAP_t \times DEBT_{t-1}$			0.001	−0.015***	−0.0004	−0.018***
			(0.001)	(0.005)	(0.004)	(0.006)
$\mathbb{I}[GAP_t < 0] \times GAP_t$					3.234	−0.130
					(19.867)	(1.807)
Observations	135	144	135	144	135	144
R^2	0.740	0.292	0.739	0.417	0.663	0.374
Adjusted R^2	0.637	−0.012	0.632	0.158	0.514	0.077
F Statistic	289.325***	54.046***	285.071***	81.167***	206.203***	72.384***

*p<0.1; **p<0.05; ***p<0.01

Model Solution

Household Problem

The Lagrangean for the unconstrained household is:

$$\max_{\{(C^U)_t, B_t^n, (N^U)_t\}_t} \mathbb{E}_0 \sum_{t\geq 0} \beta^t \{ \frac{(C_t^U)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \frac{(N_t^U)^{1+\varphi}}{1+\varphi} \tag{54}$$

$$+ \Lambda_t^U[-P_t C_t^U - B_t^n + (1+i_{t-1})B_{t-1}^n + (1-\tau)(W_t N_t^U + \Theta_t)]\} \tag{55}$$

Then, the FOC are

$$\begin{aligned}
&(C_t^U)^{-\frac{1}{\sigma}} - P_t \Lambda_t^U = 0 && (C_t^U)\\
&-\Lambda_t^U + \beta(1+i_t)\Lambda_{t+1}^U = 0 && (B_t^n)\\
&-(N_t^U)^{\varphi} + \Lambda_t^U(1-\tau)W_t = 0 && (N_t^U)
\end{aligned}$$

Combining the three equations yields the Euler equation and the labour supply. As for the constrained household, his Lagrangean is:

$$\max_{\{C_t^C, N_t^C\}_t} \frac{(C^C)_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \frac{(N^C)_t^{1+\varphi}}{1+\varphi} + \Lambda_t^C[-P_t C_t^C + (1-\tau)(W_t N_t^C + \Theta_t)] \tag{56}$$

The FOC

$$\begin{aligned}
&(C_t^C)^{-\frac{1}{\sigma}} - P_t \Lambda_t^C = 0 && (C_t^C)\\
&-(N_t^C)^{\varphi} + \Lambda_t^C(1-\tau)W_t = 0 && (N_t^C)
\end{aligned}$$

yielding the labour supply curve.

Final Good Producer

The Lagrangean of the final good producer is:

$$\min_{\{Y_t(i)\}_i} \int P_t(i) Y_t(i) di + \Lambda_t^f \left[Y_t - \left(\int Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \right] \tag{57}$$

Hence, taking the FOC:

$$
\begin{aligned}
P_t(i) - \Lambda_t^f Y_t(i)^{-\frac{1}{\varepsilon}} Y_t^{\frac{1}{\varepsilon}} &= 0 \qquad \forall i \\
\frac{P_t(i)}{P_t(j)} &= \left(\frac{Y_t(i)}{Y_t(j)}\right)^{-\frac{1}{\varepsilon}} \\
Y_t(i) &= \left(\frac{P_t(i)}{P_t(j)}\right)^{-\varepsilon} Y_t(j) \\
Y_t(i) P_t(j)^{-\varepsilon} &= P_t(i)^{-\varepsilon} Y_t(j) \\
Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} P_t(j)^{-(\varepsilon-1)} &= P_t(i)^{-(\varepsilon-1)} Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}}
\end{aligned}
$$

Integrating both sides in dj yields:

$$
\begin{aligned}
Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \int P_t(j)^{-(\varepsilon-1)} dj &= P_t(i)^{-(\varepsilon-1)} Y_t^{\frac{\varepsilon-1}{\varepsilon}} \\
Y_t(i) &= \left(\frac{P_t(i)}{(\int P_t(j)^{1-\varepsilon} dj)^{\frac{1}{1-\varepsilon}}}\right)^{-\varepsilon} Y_t
\end{aligned}
$$

Since the final good producer is perfectly competitive, she makes 0 profits. Hence, the no-profit condition pins down the price index P_t:

$$
P_t Y_t = \int P_t(i) Y_t(i) di = \int P_t(i) \left(\frac{P_t(i)}{(\int P_t(j)^{1-\varepsilon} dj)^{\frac{1}{1-\varepsilon}}}\right)^{-\varepsilon} di Y_t
$$

$$
P_t = \frac{\int P_t(i)^{1-\varepsilon} di}{(\int P_t(j)^{1-\varepsilon} dj)^{\frac{\varepsilon}{1-\varepsilon}}} = \left(\int P_t(i)^{1-\varepsilon} di\right)^{1-\frac{\varepsilon}{1-\varepsilon}} = \left(\int P_t(i)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}} \tag{58}
$$

The individual demand for input i is thus:

$$
Y_t^d(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t \tag{59}
$$

Intermediate Goods Producers

$$
\max_{\{P_t^*(i)\}} \sum_{s \geq 0} \vartheta^s \Lambda_{t,t+s} \left[\frac{P_t^*(i) Y_{t+s}(i)}{P_{t+s}} - \frac{TC_{t+s}(i)}{P_{t+s}}\right] \tag{60}
$$

subject to

$$
Y_{t+s}(i) = \left(\frac{P_t^*(i)}{P_{t+s}}\right)^{-\varepsilon} Y_{t+s} \tag{61}
$$

where $TC_{t+s}(i) = \frac{W_{t+s} Y_{t+s}(i)}{A_{t+s}}$, and hence $MC_{t+s}(i) = MC_{t+s} = \frac{W_{t+s}}{A_{t+s}}$. The solution is then given by:

$$\sum_{s\geq 0}\vartheta^s\Lambda_{t,t+s}\left[\frac{Y_{t+s}(i)}{P_{t+s}}+\frac{P_t^*(i)}{P_{t+s}}\frac{\partial Y_{t+s}(i)}{\partial P_t^*(i)}-\frac{MC_{t+s}}{P_{t+s}}\frac{\partial Y_{t+s}(i)}{\partial P_t^*(i)}\right]=0$$
$$\frac{\partial Y_{t+s}(i)}{\partial P_t^*(i)}=-\varepsilon\frac{P_t^*(i)^{-\varepsilon-1}}{P_{t+s}^{-\varepsilon}}Y_{t+s}$$

$$\sum_{s\geq 0}\vartheta^s\Lambda_{t,t+s}\left[\left(\frac{P_t^*(i)}{P_{t+s}}\right)^{-\varepsilon}\frac{Y_{t+s}(i)}{P_{t+s}}-\varepsilon\frac{P_t^*(i)^{-\varepsilon}}{P_{t+s}^{-\varepsilon}}\frac{Y_{t+s}}{P_{t+s}}+MC_{t+s}\varepsilon\frac{P_t^*(i)^{-\varepsilon-1}}{P_{t+s}^{-\varepsilon}}\frac{Y_{t+s}}{P_{t+s}}\right]=0$$
$$\sum_{s\geq 0}\vartheta^s\Lambda_{t,t+s}\left[(1-\varepsilon)P_{t+s}^{\varepsilon-1}Y_{t+s}+\varepsilon MC_{t+s}P_t^*(i)^{-1}P_{t+s}^{\varepsilon-1}Y_{t+s}\right]=0$$

$$P_t^*(i)=\frac{\varepsilon}{\varepsilon-1}\frac{\sum_{s\geq 0}\vartheta^s\Lambda_{t,t+s}MC_{t+s}P_{t+s}^{\varepsilon-1}Y_{t+s}}{\sum_{s\geq 0}\vartheta^s\Lambda_{t,t+s}P_{t+s}^{\varepsilon-1}Y_{t+s}}=P_t^* \tag{62}$$

Let $X_t^{aux,1}$ and $X_t^{aux,2}$:

$$X_t^{aux,1}=MC_tP_t^{\varepsilon-1}Y_t+\vartheta\Lambda_{t,t+1}X_{t+1}^{aux,1} \tag{63}$$
$$X_t^{aux,2}=P_t^{\varepsilon-1}Y_t+\vartheta\Lambda_{t,t+1}X_{t+1}^{aux,2} \tag{64}$$

Then the optimal reset price becomes:

$$P_t^*=\frac{\varepsilon}{\varepsilon-1}\frac{X_t^{aux,1}}{X_t^{aux,2}} \tag{65}$$

Hence, aggregating all prices into the price index:

$$P_t=\left(\int P_t(i)^{1-\varepsilon}di\right)^{\frac{1}{1-\varepsilon}}$$
$$=\left((1-\theta)P_t^{*1-\varepsilon}+\theta P_{t-1}^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}$$

Hence we can write aggregate inflation as

$$\Pi_t=\left((1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\varepsilon}+\theta\right)^{\frac{1}{1-\varepsilon}}$$
$$=\left((1-\theta)\left(\Pi_t^*\Pi_t\right)^{1-\varepsilon}+\theta\right)^{\frac{1}{1-\varepsilon}}\qquad \Pi_t^*=\frac{P_t^*}{P_t}$$

Let $\tilde{X}_t^{aux,1}=\frac{X_t^{aux,1}}{P_t^{\varepsilon}}$ and $\tilde{X}_t^{aux,2}=\frac{X_t^{aux,2}}{P_t^{\varepsilon-1}}$. Let $MC_t^r=\frac{MC_t}{P_t}$. Then

$$1=(1-\theta)\left(\Pi_t^*\right)^{1-\varepsilon}+\theta\Pi_t^{\varepsilon-1} \tag{66}$$

$$\Pi_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{\tilde{X}_t^{aux,1}}{\tilde{X}_t^{aux,2}} \tag{67}$$

$$\tilde{X}_t^{aux,1} = MC_t^r Y_t + \theta \Lambda_{t,t+1} \Pi_{t+1}^{\varepsilon} \tilde{X}_{t+1}^{aux,1} \tag{68}$$

$$\tilde{X}_t^{aux,2} = Y_t + \theta \Lambda_{t,t+1} \Pi_{t+1}^{\varepsilon-1} \tilde{X}_{t+1}^{aux,2} \tag{69}$$

Equilibrium

The goods market equilibrium condition reads:

$$C_t^C + C_t^U + G_t = Y_t$$

Since $C_t^U = (1 - \phi^C) C_t$ and

$$\begin{aligned} P_t C_t^C &= (1-\tau)(W_t N_t^C + \phi^C \Theta_t) \\ &= (1-\tau)\phi^C (W_t N_t + P_t Y_t - W_t N_t) \\ &= (1-\tau) P_t Y_t \end{aligned}$$

we conclude that:

$$\phi^C (1-\tau) Y_t + (1-\phi^C) C_t + G_t = Y_t$$

$$(1-\phi^C) C_t + G_t = (1 - (1-\tau)\phi^C) Y_t \tag{70}$$

The labor market equilibrium

Let $N_t = \int N_t(i) di$ be the total demand for labor of intermediate goods firms. Then, labor market clearing implies:

$$\begin{aligned} N_t &= N_t^U + N_t^C \\ &= \left((1-\tau) \frac{W_t}{P_t} (C_t^U)^{-\frac{1}{\sigma}} \right)^{\frac{1}{\varphi}} + \left((1-\tau) \frac{W_t}{P_t} (C_t^C)^{-\frac{1}{\sigma}} \right)^{\frac{1}{\varphi}} \\ &= \left((1-\tau) \frac{W_t}{P_t} C_t^{-\frac{1}{\sigma}} \right)^{\frac{1}{\varphi}} [(1-\phi^C)^{-\frac{1}{\sigma\varphi}} + (\phi^C)^{-\frac{1}{\sigma\varphi}}] \end{aligned}$$

$$C_t^{\frac{1}{\sigma}} N_t^{\varphi} = (1-\tau) \frac{W_t}{P_t} [(1-\phi^C)^{-\frac{1}{\sigma\varphi}} + (\phi^C)^{-\frac{1}{\sigma\varphi}}]^{\varphi} \tag{71}$$

Log-linearization

The economy is log-linearized around the non-inflationary steady state, i.e. $\Pi_{ss} = 1$ and $Y_{ss} = Y_{ss}^p$.

Log-linearizing equations (66) to (69) we obtain

$$
\begin{aligned}
0 &= (1-\vartheta)(1-\varepsilon)(\Pi^*)^{-\varepsilon}\pi_t^* + \vartheta(\varepsilon-1)(\Pi)^{\varepsilon}\pi_t \implies \pi_t^* = \frac{\vartheta}{1-\vartheta}\pi_t \\
\pi_t^* &= \tilde{x}_t^{aux,1} - \tilde{x}_t^{aux,2} \\
\tilde{x}_t^{aux,1} &= \frac{MC^rY}{X^{aux,1}}\hat{mc}_t + \frac{MC^rY}{X^{aux,1}}y_t + \vartheta\beta\tilde{x}_{t+1}^{aux,1} + \vartheta\beta\lambda_{t,t+1} + \varepsilon\vartheta\beta\mathbb{E}_t\pi_{t+1} \\
&= (1-\vartheta\beta)\hat{mc}_t + (1-\vartheta\beta)y_t + \vartheta\beta\tilde{x}_{t+1}^{aux,1} + \vartheta\beta\lambda_{t,t+1} + \varepsilon\vartheta\beta\mathbb{E}_t\pi_{t+1} \\
\tilde{x}_t^{aux,2} &= \frac{Y}{X^{aux,2}}y_t + \vartheta\beta\tilde{x}_t^{aux,2} + \vartheta\beta\lambda_{t,t+1} + (\varepsilon-1)\vartheta\beta\mathbb{E}_t\pi_{t+1} \\
&= (1-\vartheta\beta)y_t + \vartheta\beta\tilde{x}_t^{aux,2} + \vartheta\beta\lambda_{t+1} + (\varepsilon-1)\vartheta\beta\mathbb{E}_t\pi_{t+1}
\end{aligned}
$$

Combining all of the equations above we obtain:

$$
\begin{aligned}
\frac{\vartheta}{1-\vartheta}\pi_t =&(1-\vartheta\beta)\hat{mc}_t + (1-\vartheta\beta)y_t + \vartheta\beta\tilde{x}_{t+1}^{aux,1} + \vartheta\beta\lambda_{t,t+1} + \varepsilon\vartheta\beta\mathbb{E}_t\pi_{t+1} - \\
&(1-\vartheta\beta)y_t - \vartheta\beta\tilde{x}_t^{aux,2} - \vartheta\beta\lambda_{t,t+1} - (\varepsilon-1)\vartheta\beta\mathbb{E}_t\pi_{t+1} \\
=&(1-\vartheta\beta)\hat{mc}_t + \vartheta\beta(x_{t+1}^{aux,1} - x_{t+1}^{aux,2}) + \vartheta\beta\mathbb{E}_t\pi_{t+1} \\
=&(1-\vartheta\beta)\hat{mc}_t + \vartheta\beta\pi_{t+1}^* + \vartheta\beta\mathbb{E}_t\pi_{t+1} \\
=&(1-\vartheta\beta)\hat{mc}_t + \frac{\vartheta\beta}{1-\vartheta}\pi_{t+1} \\
\pi_t =&\frac{(1-\vartheta)(1-\vartheta\beta)}{\vartheta}\hat{mc}_t + \beta\pi_{t+1}
\end{aligned}
$$

The relevant log-linearized equations of the supply side are therefore:

$$
\pi_t = \psi\hat{mc}_t + \beta\mathbb{E}_t\pi_{t+1} \tag{72}
$$

$$
\hat{mc}_t = w_t - a_t \tag{73}
$$

The log linearized equilibrium conditions are:

$$
(1-\phi^C)(1-\gamma)c_t + \gamma g_t = (1-(1-\tau)\phi^C)y_t \tag{74}
$$

$$
\sigma^{-1}c_t + \varphi n_t = w_t \tag{75}
$$

$$
y_t = a_t + n_t \tag{76}
$$

Hence, combining Equation (73) with Equations (74), we obtain a modified Phillips curve:

$$
\begin{aligned}
\hat{mc}_t &= \sigma^{-1}c_t + \varphi n_t - a_t \qquad (77)\\
&= \left(\sigma^{-1}+\varphi\right)\frac{1-(1-\tau)\phi^C}{(1-\phi^C)(1-\gamma)}y_t - \frac{\gamma}{(1-\phi^C)(1-\gamma)}g_t - (1+\varphi)a_t \qquad (78)
\end{aligned}
$$

and hence

$$
\pi_t = \kappa\hat{y}_t + \beta\mathbb{E}_t\pi_{t+1} \tag{79}
$$

Real debt accumulation dynamics follows:

$$P_{t+1}B_t = (1+i_t)(P_tB_{t-1} + P_tG_t - \tau P_tY_t) \tag{80}$$

$$B_t = \frac{1+i_t}{\Pi_{t+1}}(B_{t-1} + G_t - \tau Y_t) \tag{81}$$

Log-linearization around non-inflationary steady state yields

$$\begin{aligned} Bb_t &= (1+i)[Bb_{t-1} + B(i_t - \pi_{t+1}) + Tg_t - \tau y_t] + \\ \beta b_t &= b_{t-1} + \frac{T}{B}\hat{g}_t - \frac{Y}{B}\tau y_t + \beta(i_t - \mathbb{E}_t\pi_{t+1}) \end{aligned}$$

with $1+i = \beta^{-1}$. Since in ss we have $T = \gamma Y$, the equation above can be more compactly expressed as

$$\beta b_t = b_{t-1} + \frac{Y}{B}(\gamma g_t - \tau y_t) + \beta(i_t - \mathbb{E}_t\pi_{t+1}) \tag{82}$$

In deviation from natural:

$$\beta\hat{b}_t = \hat{b}_{t-1} + \delta(\gamma g_t - \tau y_t) + \beta(i_t - \mathbb{E}_t\pi_{t+1} - r_t^n) \tag{83}$$

Benchmark problem

FOC of the Lagrangean

$$\begin{aligned} 0 =& -\hat{y}_t - \mu_t^{IS} + \beta^{-1}\mu_{t-1}^{IS} - \delta\tau\mu_t^{BC} + \kappa\mu_t^{PC} + \phi_y\mu_t^{MP} && (\hat{y}_t) \\ 0 =& -\lambda(b_t - \bar{b}) - \beta\mu_t^{BC} + \beta\mathbb{E}_t\mu_{t+1}^{BC} && (b_t) \\ 0 =& \mu^g\mu_t^{IS} - \beta^{-1}\mu^g\mu_{t-1}^{IS} + \delta\gamma\mu_t^{BC} && (g_t) \\ 0 =& \chi\beta^{-1}\mu_{t-1}^{IS} - \mu_{t-1}^{BC} - \mu_t^{PC} + \mu_{t-1}^{PC} + \phi_\pi\mu_t^{MP} && (\pi_t) \\ 0 =& -\chi\mu_t^{IS} + \beta\mu_t^{BC} - \mu_t^{MP} && (i_t) \end{aligned}$$

Conditions (i_t) and (g_t) imply $\mu_t^{MP} = -\chi\mu_t^{IS} + \beta\mu_t^{BC}$ and $\delta\gamma\mu_t^{BC} = -\mu^g(\mu_t^{IS} - \beta^{-1}\mu_{t-1}^{IS})$. Thus, plugging this in the other conditions

$$\begin{aligned} \hat{y}_t =& (\delta\gamma/\mu^g - \delta\tau + \phi_y\beta)\mu_t^{BC} + \kappa\mu_t^{PC} - \phi_y\chi\mu_t^{IS} && (\hat{y}_t) \\ \lambda(b_t - \bar{b}) =& -\beta\mu_t^{BC} + \beta\mathbb{E}_t\mu_{t+1}^{BC} && (b_t) \\ \Delta\mu_t^{PC} =& \chi(\beta^{-1}\mu_{t-1}^{IS} - \phi_\pi\mu_t^{IS}) - (\mu_{t-1}^{BC} - \phi_\pi\beta\mu_t^{BC}) && (\pi_t) \end{aligned}$$

Combining the first and the third:

$$\hat{y}_t = \frac{\beta^{-1}\delta\chi(\gamma - \mu^g\tau)}{\mu^g}\mu_t^{IS} \tag{84}$$

so

$$b_t = \bar{b} - \frac{\beta\mu^g}{(\gamma - \mu^g\tau)\lambda\delta\tau}\hat{y}_t + \frac{\beta\mu^g}{(\gamma - \mu^g\tau)\lambda\delta\tau}\mathbb{E}_t\hat{y}_{t+1} \tag{85}$$