Portfolio Management
Computational Finance Project

Group 2
Bechi Carlo, Bencini Margherita, Ciotti Alessandro, Delera Giacomo, Riondato Giovanni

Abstract
This study examines portfolio allocation strategies for the S&P500 index, which contains 11 sector indices and 5 factor indices. The analysis focuses on creating efficient portfolios using 2023 financial data through different techniques like the typical efficient frontier, constrained optimization, Black-Litterman model, and Principal Component Analysis. Portfolios are compared to an equally weighted benchmark using performance metrics such as risk, return, and diversification (See Table 2 for more details).

The in-sample analysis reveals the best allocations under various constraints, while the out-of-sample evaluation over 2024 analyzes the stability of these approaches. The results emphasize the trade-offs between risk and return and the impact of constraints on portfolio diversification. This study shows how portfolio theory is actually applied under dynamic market conditions.

1 Standard Efficient Frontier

The first phase of this research requires to identify the standard efficient frontier, a critical notion in portfolio optimization. The efficient frontier refers to portfolios that achieve either the highest feasible return or the lowest risk for a given level of return. This assumes full investment in hazardous assets and no short-selling.
In order to find the efficient frontier, the following steps were taken:

1. **Data Preparation:** Logarithmic returns for 2023 were estimated using price data from 16 measures. The returns were used to compute the mean return vector and covariance matrix, which are crucial for portfolio optimization.
2. **Portfolio Constraints:** The analysis assumes standard constraints:
   - Asset weights (w_i) are non-negative ($w_i \geq 0$).
   - The total allocation sums to 100% ($\sum w_i = 1$).
3. **Efficient Frontier Calculation:** A MATLAB `Portfolio` object was created to model the problem. Using the `estimateFrontier` function, 100 points along the frontier were calculated, each representing a unique portfolio allocation.

1.1 Portfolios analysis and results

Two important portfolios are then studied:

1. **Portfolio A (Minimum Variance Portfolio):**
2. **Portfolio B (Maximum Sharpe Ratio Portfolio):**

Figure 1 shows the efficient frontier on the risk (standard deviation) - return plane. `ptfA` and `ptfB` are critical spots on the curve: while the first portfolio has the lowest risk, making it a good-choice for risk-adverse investors, the second one effectively balances risk and reward, resulting in better returns per unit of risk than other portfolios.

These findings emphasize the inherent risk-return trade-off in portfolio optimization. The efficient frontier allows investors to select portfolios based on their risk tolerance and expected returns, providing a wide range of possibilities.

2 Efficient Frontier with Multiple Constraints

The purpose of this stage is to compute the efficient frontier with additional constraints to analyze how they impact the risk-return trade-off and portfolio diversification. The limitations considered are listed below:

- The total weight of factor indices (the five factor indices) must be greater than 15%.
- The sum of the weights in cyclical and defensive sectors must lie between 40% and 70%.

These constraints aim to provide diversification across sectors and factor indexes while maintaining exposure to major economic drivers.

2.1 Portfolio Calculation

The following steps were followed:

1. The covariance matrix of the returns is used to estimate the risk.
2. The expected returns of the assets were assumed to remain constant.
3. The portfolio optimization problem was solved using the `fmincon` optimization function in MATLAB, which determines the best asset weights to minimize risk while meeting restrictions.

2.2 Results

The efficient frontier with the additional constraints is shown in Figure 2. The key portfolios identified are:

- **Portfolio C (Minimum Variance Portfolio):** This portfolio minimizes risk and satisfies boundaries, resulting in the least volatile choice within the optimization space.
- **Portfolio D (Maximum Sharpe Ratio Portfolio):** maximizes the Sharpe Ratio by balancing risk and return, resulting in the best return per unit of risk within restrictions.

The efficient frontier's position and shape differ from the regular frontier from Section 1, due to extra non linear constraints. Sector and factor exposure constraints limit portfolio allocation options, thereby lowering maximum returns for a given risk level. Limitations can increase diversification by preventing excessive focus in specific businesses or elements.

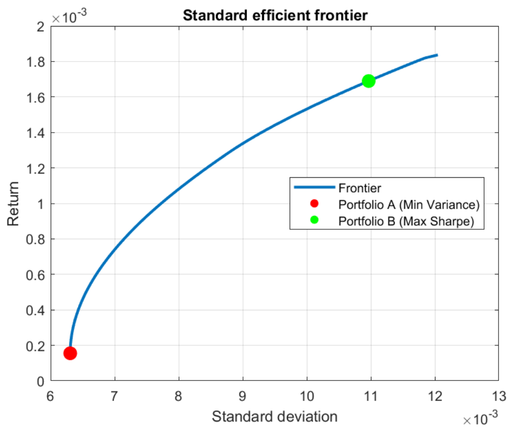


Figure 1: Standard Efficient Frontier.

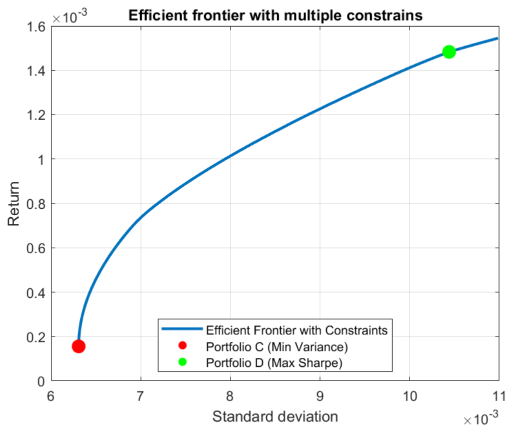


Figure 2: Multiple Constraints Frontier.

3 Robust Portfolios with Resampling

In this part, the efficient frontiers are recalculated with a resampling approach to address portfolio optimization's sensitivity to estimation mistakes in the mean return and covariance matrix. The resampling technique improves portfolio stability and reliability by incorporating variability in input parameters.

The following steps were performed:

1. **Simulation of Returns:**
   - $N = 100$ simulations of asset returns were run using a multivariate normal distribution with the historical mean and log-returns covariance matrix as parameters.
   - A new covariance matrix is computed using simulated returns at each step.
2. **Recalculation of Efficient Frontiers:**
   - For each simulation, a new efficient frontier was computed using the `Portfolio` object.
   - This process was repeated N times to generate N simulated frontiers.
3. **Aggregation of Results:**
   - The average risk-return points from all simulations have been used to create an efficient frontier.
   - Portfolios corresponding to the Minimum Variance (Portfolio E) and Maximum Sharpe Ratio (Portfolio F) from this robust frontier were calculated.
4. **Multiple Constraints:**
   - The same resampling technique was used as mentioned in Section 2, resulting in robust portfolios for Minimum Variance (Portfolio G) and Maximum Sharpe Ratio (Portfolio H).

3.1 Results

Figure 3 and Figure 4 show robust efficient frontiers. The simulated frontiers are plotted to show the variability introduced by the resampling process. The robust frontier is underlined, with significant portfolios highlighted:

- **Portfolio E and F (Standard Constraints):** provide more reliable allocations than their non-robust counterparts.
- **Portfolio G and H (Multiple Constraints):** include diversification constraints, balancing robustness, and restricted allocations.

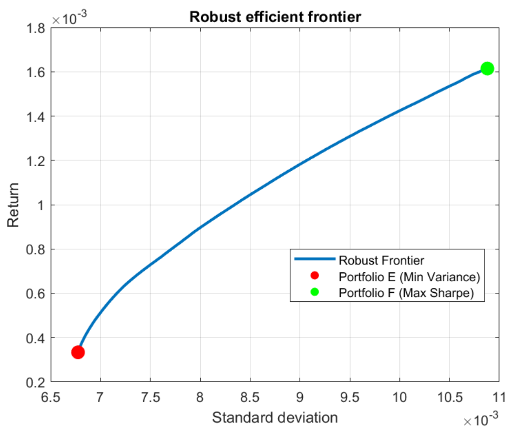


Figure 3: Robust Frontier with Linear Constraints.

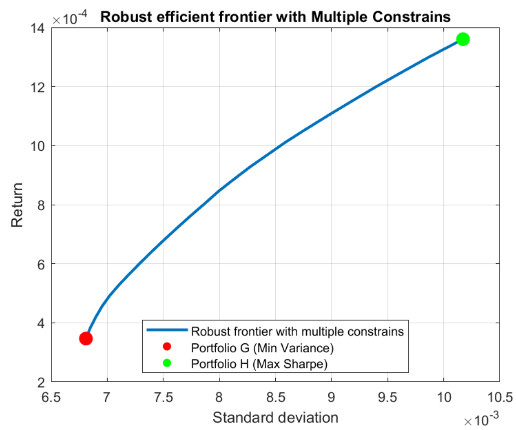


Figure 4: Robust Frontier with Non Linear Constraints.

4 Portfolio Optimization with the Black-Litterman Model

The Black-Litterman model is a robust approach to portfolio optimization that combines:

- **Market Equilibrium Returns:** Computed using market capitalization weights and the covariance matrix of asset returns.
- **Investor Views:** The model incorporates explicit views on the predicted performance of specific assets or sectors, together with corresponding confidence levels.

For this analysis, the following steps were taken:

1. **Define Investor Views:** The Black-Litterman framework includes two perspectives:
   - **View 1:** Cyclical sectors will outperform defensive sectors by 2% annually.
   - **View 2:** The Value factor will outperform the Growth factor by 1% annually.

   These viewpoints represent moderate economic growth and high inflation scenarios, respectively.

2. **Compute Black-Litterman Parameters:**
   - The prior equilibrium returns (μ_{market}) were computed using the market capitalization weights and the covariance matrix.

- The adjusted returns (μ_{BL}) were derived by combining equilibrium returns and views, weighted by confidence levels.

3. **Optimize Portfolios:** Recalculate the efficient frontier based on Black-Litterman expected returns (μ_{BL}) and covariance matrix. Two main portfolios were identified:
   - **Portfolio I:** Minimum Variance Portfolio.
   - **Portfolio L:** Maximum Sharpe Ratio Portfolio.

4.1 Results

Figure 5 depicts the efficient frontier for the Black-Litterman model along with Portfolio I and Portfolio L.
These results highlight the flexibility of this model, that allows investors to incorporate their market perspective into portfolios while preserving a strong risk-return trade-off.

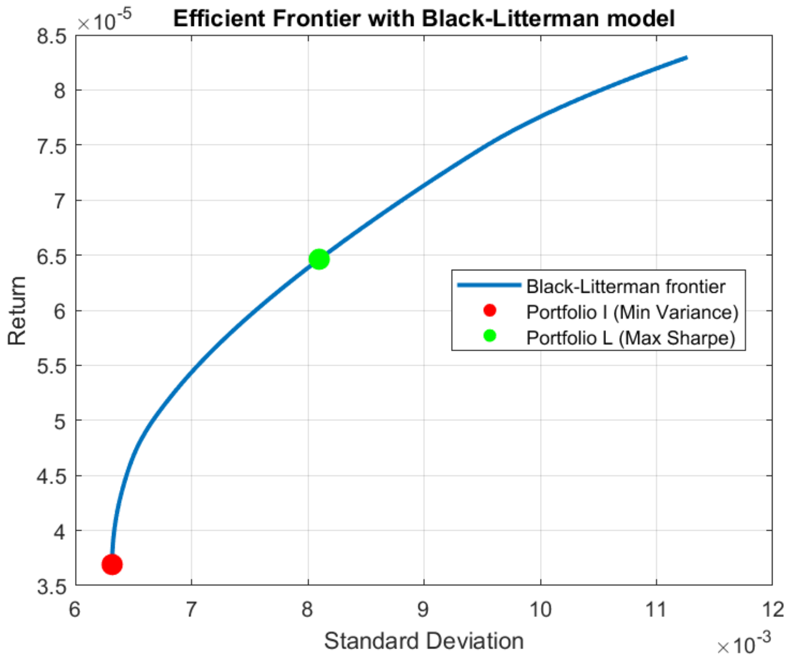


Figure 5: Efficient Frontier with Black-Litterman Model

5 Maximum Diversified Portfolio and Maximum Entropy Portfolio

This section examines two portfolio optimization strategies that try to diversify from different perspectives:

1. **Maximum Diversified Portfolio (Portfolio M):** Maximizes the diversification ratio, defined as:

$$DR(w) = \frac{\sum_{i=1}^{N} w_i \sigma_i}{\sqrt{w' \Sigma w}}$$

where σ_i represents the standard deviation of each asset and Σ is the covariance matrix of returns. A higher diversification ratio indicates better risk distribution across the assets.

2. **Maximum Entropy Portfolio (Portfolio N):** Maximizes entropy in risk contributions:

$$\theta_i = \frac{|\sum_{j=1}^{N} w_i w_j \sigma_i \sigma_j \rho_{ij}|}{\sum_{h=1}^{N} |\sum_{j=1}^{N} w_i w_j \sigma_i \sigma_j \rho_{ij}|}, \quad H_w^{\text{risk}} = -\sum_{i=1}^{N} \theta_i \ln(\theta_i)$$

Both portfolios are optimized under the following constraints:

- **Standard Constraints**
- **Factor Weights:** Weights for factor indices must satisfy $0.05 \leq w_i \leq 0.1$.
- **Deviation from Benchmark:** The sum of the absolute deviations from a benchmark portfolio (market capitalization-weighted) is limited to 50%:

$$\sum |w - w_{\text{benchmark}}| \leq 0.5$$

5.1 Optimization Process and Results

Both the portfolios are obtained maximizing the respective metric using the `fmincon` command. Table 1 shows portfolios' metrics and compared them with the benchmark.

Metrics	Portfolio M	Portfolio N
Diversification Ratio	1.3536	-
Entropy	-	2.7106
Deviation from Benchmark	0.4985	0.4997

Table 1: Comparison of Metrics for Portfolio M and Portfolio N.

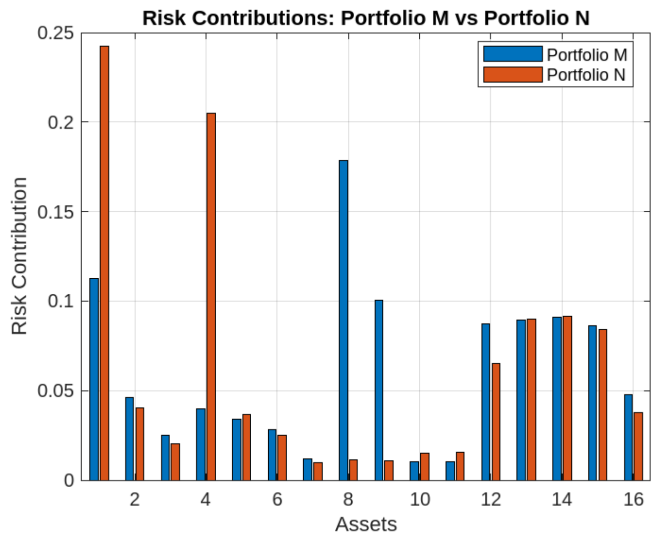


Figure 6: Risk Contributions for Portfolio M and Portfolio N.

Figure 6 illustrates the risk contributions for Portfolio M and Portfolio N, showing how risk is distributed across the assets.

6 Principal Component Analysis (PCA)

PCA is a technique used to reduce the dimensionality of a dataset while retaining as much information as possible. In the context of portfolio optimization, PCA allows for:

- Identifying the principal factors that explain the majority of variance in asset returns.
- Reducing the number of inputs required to calculate expected risk and return, simplifying the model.

The main steps followed are:

1. **Standardization of Returns:** Log-returns are standardized by subtracting the mean and dividing by the standard deviation to remove the effect of different scales among assets.
2. **Calculation of Principal Components:** Using MATLAB's `pca` function, the principal components are identified, each representing a linear combination of assets that maximizes explained variance.
3. **Selection of Principal Factors:** The cumulative sum of explained variances is calculated to determine the minimum number of components needed to explain at least 85% of the total variance.
4. **Reformulation of Covariance:** The covariance matrix of assets is reconstructed using only the selected principal factors, reducing the model's complexity.
5. **Portfolio Optimization:** Using the reconstructed covariance matrix, a portfolio is optimized to maximize expected return at an acceptable level of risk.

6.1 Results

The main results are:

- **Number of Components:** The minimum number of principal components needed to explain at least 85% of the total variance was selected.
- **Reconstructed Efficient Frontier:** The efficient frontier based on the reconstructed covariance matrix was calculated and compared to the original one.
- **PCA Portfolio (Portfolio P):** The optimal portfolio calculated using PCA demonstrates that the approach effectively reduces dimensionality without significantly compromising efficiency. (See Table 2)

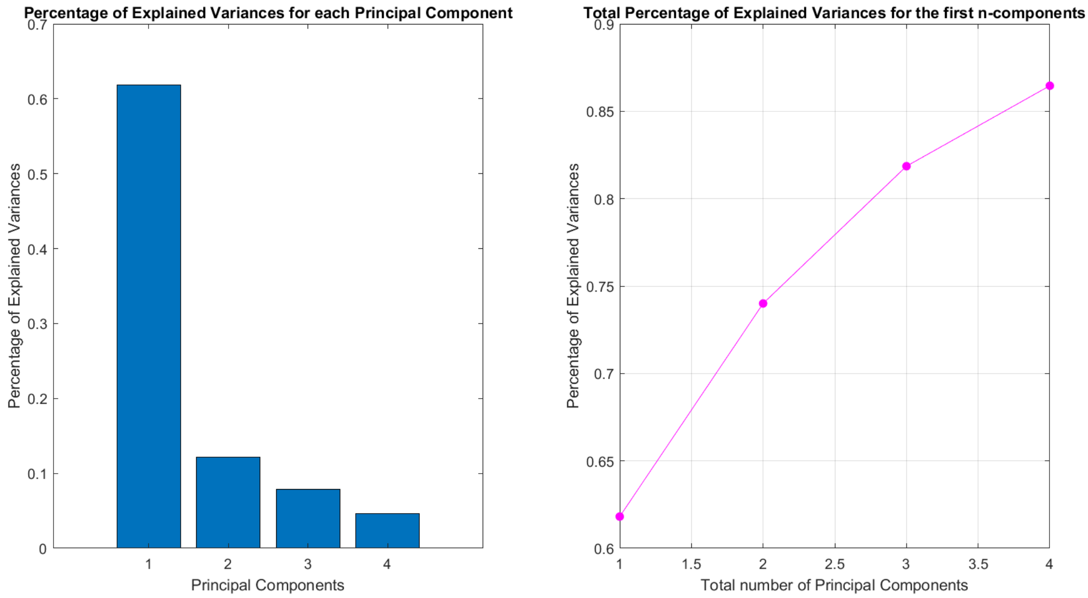


Figure 7: Principal component analysis

7 VaR-modified Sharpe Ratio

In this section, it is determined the portfolio that maximizes, under standard constraints, the Modified Sharpe Ratio (VaR-based Sharpe Ratio). This measure uses the portfolio's Value at Risk (VaR) instead of the standard deviation.

The VaR (Value at Risk) is a statistical measure which indicates the maximum potential loss that a portfolio can face over a given time period at a certain confidence level. It is sometimes preferred over standard deviation as a risk metric because it provides clearer monetary insights about potential losses the portfolio might incur.

The VaR is calculated using the historical method, in which the portfolio's loss distribution is estimated starting from historical returns of the portfolio.

The main steps are:

1. **Calculation of the portfolio's historical returns:** Portfolio weights, multiplied by the historical returns of the assets, are summed together in order to determine portfolio's returns.
2. **Set the confidence level:** The loss distribution is estimated through the historical returns of the portfolio. Then, the confidence level is set.
3. **VaR Calculation:** The VaR is computed simply as the quantile of the loss distribution corresponding to the previously set confidence interval.
4. **Maximum VaR-modified Sharpe Ratio Portfolio:** Among all the portfolios, the one with the highest Modified Sharpe Ratio (VaR-based Sharpe Ratio) is found using the `fmincon` function.

7.1 Results

Two portfolios are computed: one for a confidence level of 0.95 and the other for a confidence level of 0.99. Comparing the two portfolios, the second one appears more diversified hence the 0.99 confidence level was chosen to be the threshold for following computations.
During the optimization process, it was observed that the minimization outcome was highly dependent on the choice of the initial condition, therefore a "MonteCarlo" approach was implemented to ensure robustness. Different initial conditions were used to find the minimum and the one which maximizes the VaR-modified Sharpe Ratio was selected to be `ptfQ`.

8 In-Sample performances

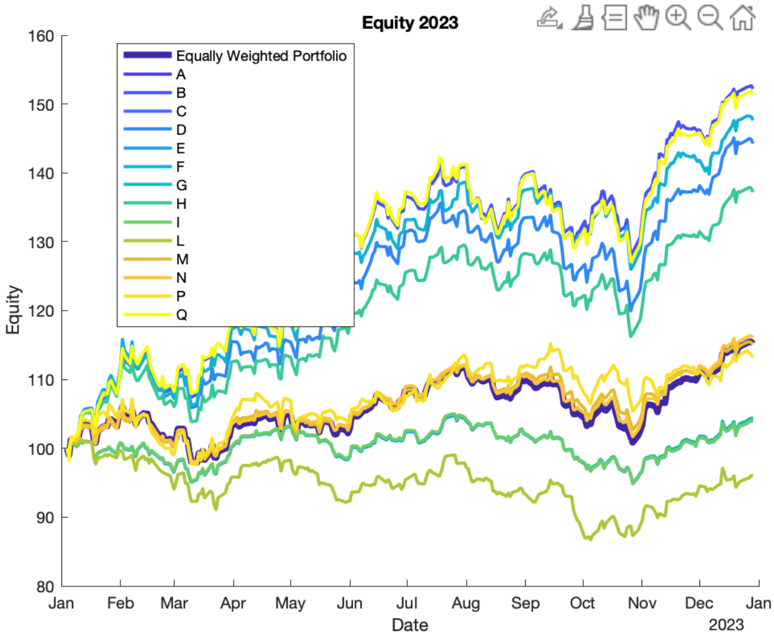
This report has explored various advanced techniques in portfolio optimization, each addressing different aspects of the risk-return trade-off. By employing standard models, incorporating constraints, and integrating robust and factor-based methods, the analysis provides a comprehensive view of portfolio construction strategies under diverse scenarios. For each portfolio, several different metric are computed. The results are displayed in Table 2.

Portfolio Name	Annual Return	Annual Volatility	Annual Sharpe Ratio	MaxDD	Daily Alpha	Beta	Herfindahl Index	Max Weight
Equally weighted	0.14358	0.12152	0.93471	0.09947	-0.00021601	0.96092	0.0625	0.0625
ptfA	0.038665	0.10012	0.086549	0.097391	-0.00040572	0.68009	0.25884	0.37914
ptfB	0.42071	0.1741	2.2441	0.094234	0.00066476	1.2424	0.60562	0.75906
ptfC	0.038699	0.10013	0.086881	0.097402	-0.00040571	0.68025	0.2585	0.37922
ptfD	0.36926	0.16572	2.0472	0.11688	0.00046172	1.238	0.38413	0.44979
ptfE	0.04285	0.10022	0.12822	0.094956	-0.00039135	0.68304	0.24132	0.37237
ptfF	0.39021	0.16974	2.1222	0.092628	0.00054141	1.2434	0.34349	0.48668
ptfG	0.042973	0.10026	0.12939	0.094978	-0.00039241	0.68493	0.23882	0.38409
ptfH	0.33598	0.16173	1.8919	0.1111	0.00033988	1.2237	0.21906	0.34646
ptfI	0.038919	0.10012	0.089086	0.097345	-0.00040505	0.68051	0.25823	0.37845
ptfL	0.11721	0.1461	0.5969	0.12703	-0.00040812	1.0654	0.2216	0.3313
ptfM	0.13982	0.11966	0.91779	0.085714	-0.00020364	0.9276	0.089695	0.14678
ptfN	0.15104	0.11853	1.0212	0.094581	-0.00017219	0.94407	0.070392	0.099996
ptfP	0.11923	0.14048	0.63522	0.089972	-0.00029922	0.94323	0.33626	0.36857
ptfQ	0.41327	0.17647	2.1719	0.10575	0.00062125	1.2589	0.64991	0.80035

Table 2: Portfolios' performances in 2023

Table 2 shows that portfolios which maximize Sharpe Ratio usually generate higher returns. The main drawback of these allocations is an high annual volatility and a high concentration of a few asset classes, leading to a low diversified portfolio.
Along with traditional performance metrics, also more statistical measures are provided. Alpha, in this study daily measured, indicates the extra-profit coming from the portfolio allocation. Generally it represents the possibility to "beat the market". Beta, instead, is an index of correlation between the portfolio and the market. Except for just a few allocations, none of them over-perform the market, showing how difficult it is to create a portfolio which do so. Beta coefficient is positive for all the portfolios, with `PtfQ` being the most responsive to market movements, while `PtfA` demonstrates the least sensitivity.

To better observe it, it is possible to look at the equity curves of the various portfolios.



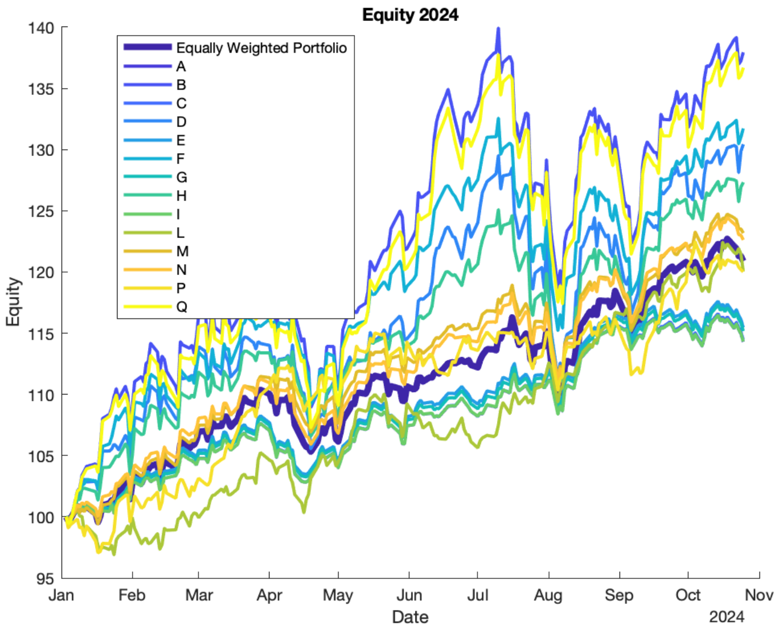
9 Out-of-sample performances

Portfolio Name	Annual Return	Annual Volatility	Annual Sharpe Ratio	MaxDD	Daily Alpha	Beta
Equally weighted	0.17776	0.10304	1.434	0.057582	3.1116e-05	0.79073
ptfA	0.12642	0.08118	1.1877	0.044715	0.00010995	0.47886
ptfB	0.31152	0.21054	1.3371	0.15222	-0.00013296	1.564
ptfC	0.12652	0.081196	1.1887	0.044722	0.00011004	0.47923
ptfD	0.24964	0.18998	1.1561	0.14302	-0.00031984	1.4561
ptfE	0.1363	0.082221	1.2929	0.044348	0.00012512	0.51005
ptfF	0.26446	0.17748	1.3211	0.1316	-0.00014199	1.3554
ptfG	0.13671	0.081801	1.3045	0.043389	0.00013149	0.50589
ptfH	0.21726	0.16238	1.1532	0.11757	-0.00027559	1.2646
ptfI	0.12659	0.081184	1.1898	0.044696	0.00010991	0.47969
ptfL	0.17947	0.10341	1.4455	0.046749	0.00050735	0.34591
ptfM	0.1995	0.10756	1.5758	0.065555	8.9621e-05	0.83546
ptfN	0.19258	0.1065	1.5266	0.06243	4.8589e-05	0.84251
ptfP	0.18214	0.11772	1.2924	0.042507	0.00023739	0.61487
ptfQ	0.29581	0.20238	1.3134	0.14353	-0.00015773	1.515

Table 3: Portfolios' performances in 2024

As seen in the in-sample evaluation, portfolios with the maximum Sharpe Ratio tend to achieve higher returns compared to other allocations. However, this comes with a downside: poor diversification, which leads to greater oscillations in volatility.
An interesting point about these portfolios is that the annual return for 2024 is lower than in 2023. This happens because the portfolio weights are not adjusted over time. By keeping the same allocation regardless of market changes, the portfolio becomes less responsive to new market conditions. This is typical of financial instruments like ETFs (Exchange Traded Funds), which passively track the market. In contrast, actively managed funds try to adjust their exposure to maximize returns. The drawback, however, is these ones come with higher

costs compared to ETFs.
Confirmation of what has been said is once again given by the equity curves.



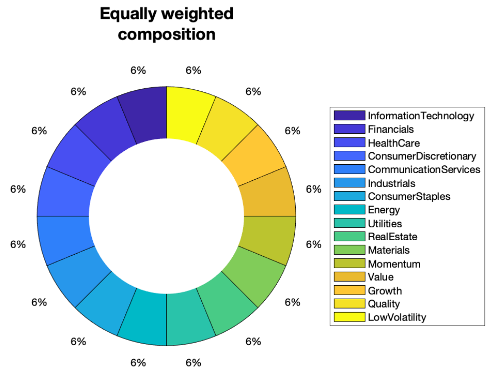
10 `MATLAB` Performances

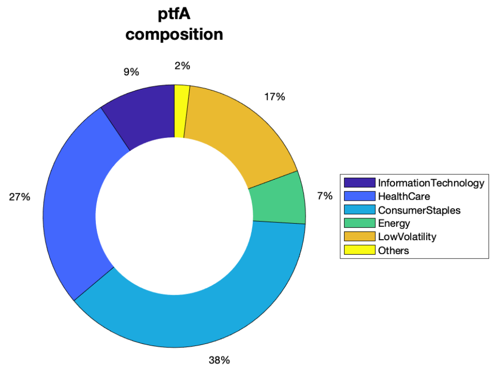
Each item was calculated using two different methods: the first method utilized the **Portfolio object** implemented in MATLAB, while the second method did not. Although both methods produced consistent results, there was a notable difference in performance, with the code using the Portfolio object being slower.

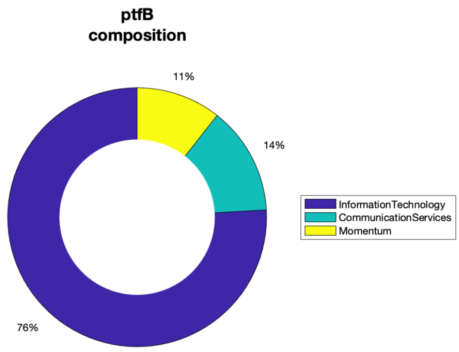
However, the results presented in this report are based on the method using the Portfolio object, as it generates a smoother and more visually comprehensible efficient frontier, improving overall readability.

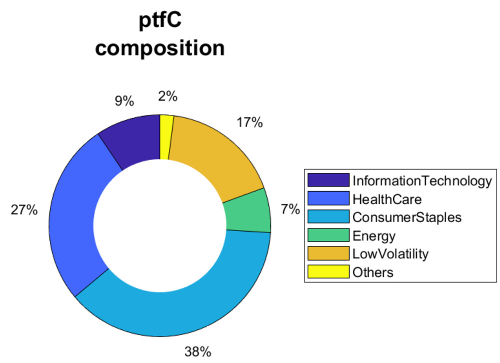
11 Portfolios' composition

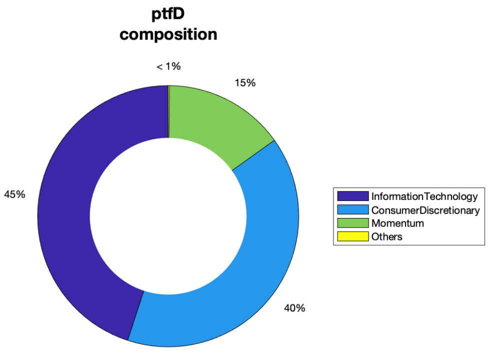
For the sake of completeness, the following images illustrate the various allocations, starting from the equally weighted portfolio to `ptfQ`.

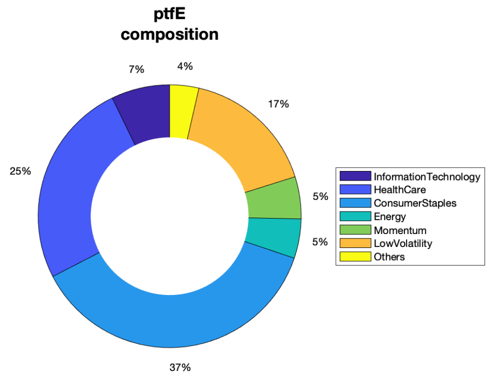


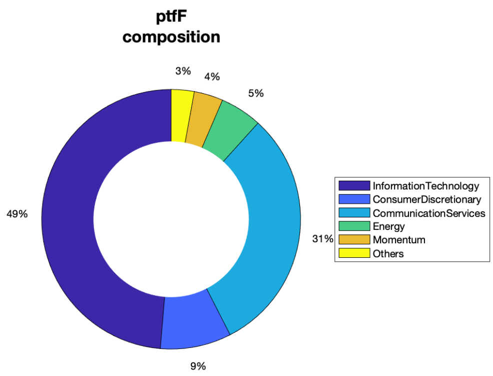








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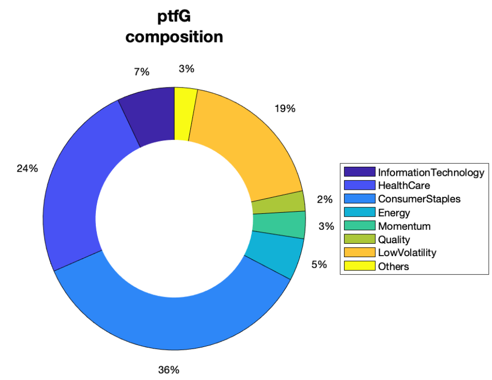
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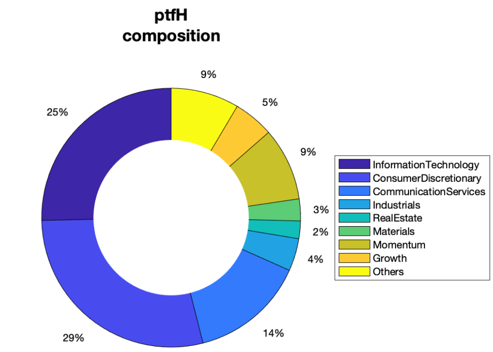
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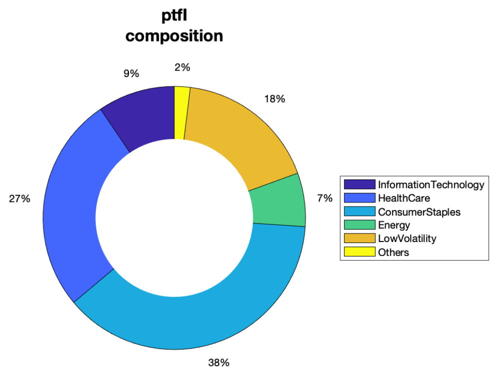
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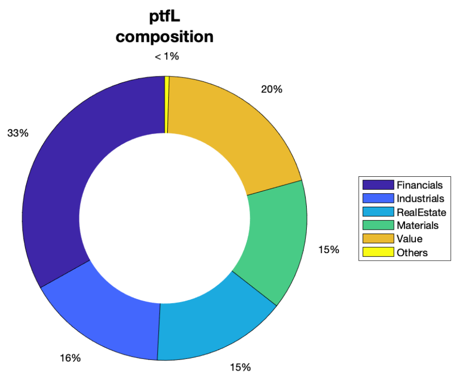
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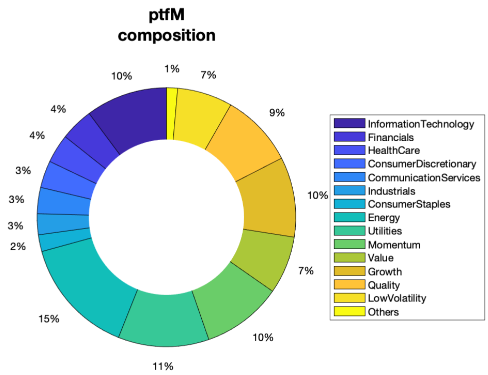
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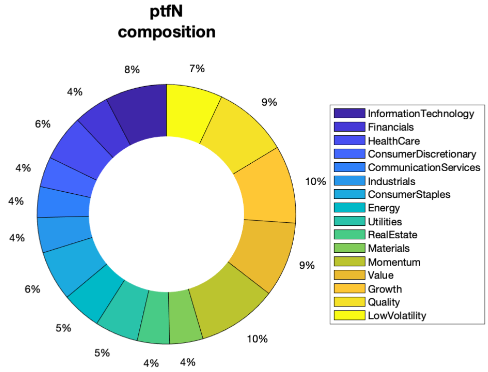
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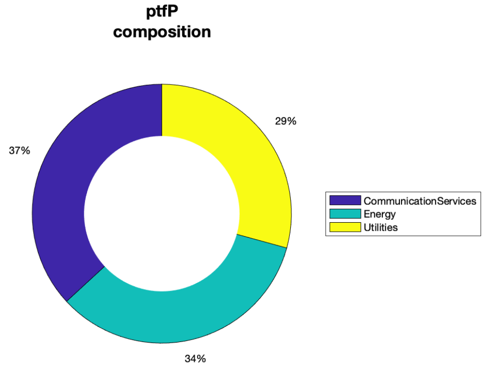
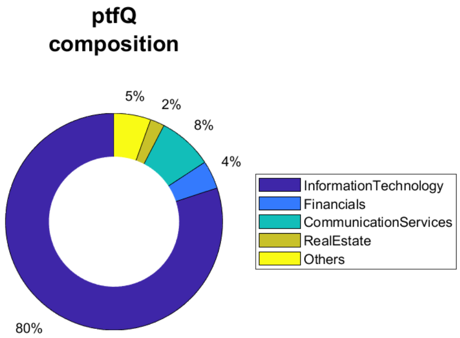
RealEstate
Materials
Momentum
Value
Growth
Quality
LowVolatility


ptfP
composition
37%
29%
34%
CommunicationServices
Energy
Utilities


ptfQ
composition
5%
2%
8%
4%
80%
InformationTechnology
Financials
CommunicationServices
RealEstate
Others