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ECE 141 Final Design Project

Current Control Loop Design

$P_{elec}(s)$

Power Amplifier: $\frac{V_a(s)}{V_c(s)} = 5$

Back EMF: $V_b(s) = \frac{K_f^2 s}{m_1 s^2 + b_1 s + k_1} I_c(s)$

Actuator:

$$-V_a + V_b + L\frac{dI_c}{dt} + (R_c + R_s)I_c = 0$$

$$-V_a(s) + \frac{K_f^2 s}{m_1 s^2 + b_1 s + k_1} I_c(s) + LsI_c(s) + (R_c + R_s)I_c(s) = 0$$

$$\frac{I_c(s)}{V_a(s)} = \frac{1}{\frac{K_f^2 s}{m_1 s^2 + b_1 s + k_1} + Ls + (R_c + R_s)}$$

Sensing Amplifier: $\frac{V_s(s)}{I_c(s)} = 5R_s$

$$P_{elec}(s) = \frac{V_s(s)}{V_c(s)} = \frac{V_a(s)}{V_c(s)} \times \frac{I_c(s)}{V_a(s)} \times \frac{V_s(s)}{I_c(s)}$$

Finally,

$$P_{elec}(s) = \frac{25R_s}{\frac{K_f^2 s}{m_1 s^2 + b_1 s + k_1} + Ls + (R_c + R_s)}$$

Assuming $V_b = 0$,

$$-V_a(s) + LsI_c(s) + (R_c + R_s)I_c(s) = 0$$

$$\frac{I_c(s)}{V_a(s)} = \frac{1}{Ls + (R_c + R_s)}$$

Then,

$$P_{elec}(s) = \frac{25R_s}{Ls + (R_c + R_s)}$$

At high frequencies, the term $\frac{K_f^2 s}{m_1 s^2 + b_1 s + k_1}$ becomes very small,

Thus, at high frequencies,

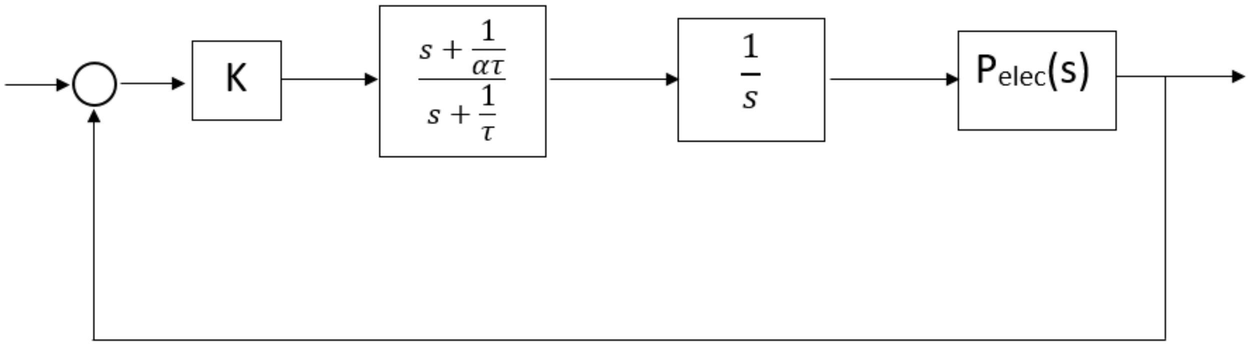
$$P_{elec}(s) \approx \frac{25R_s}{0 + Ls + (R_c + R_s)} = \frac{25R_s}{Ls + (R_c + R_s)}$$

C_{elec}(s)

$$\frac{V_c(s)}{V_s(s)} = -\frac{R_3 C_1 s + 1}{R_2 s(R_3 C_1 C_2 s + C_1 + C_2)}$$

$$-\frac{V_c(s)}{V_s(s)} = \frac{1}{C_2 R_2} \times \frac{s + \frac{1}{R_3 C_1}}{s + \frac{C_1 + C_2}{R_3 C_1 C_2}} \times \frac{1}{s}$$

We can see that this transfer function takes the form of a gain, lead compensator, and integrator. The block diagram is shown below.



partially compensated plant, $P(s) = \frac{1}{s} \times P_{elec}(s)$

At the desired crossover frequency, w_g = 6x10^5, The phase is $\angle P(jw_g) = -178°$

Designing the lead compensator to add $\phi_m = 60° + 10° - (180° - 178°) = 68°$

to the phase of P(s) at w_g will result in $\angle P(jw_g) = -110°$, which gives us a PM of 70°

$$\alpha = \frac{1 + \sin(\phi_m)}{1 - \sin(\phi_m)} = \frac{1 + \sin(68°)}{1 - \sin(68°)} = 26.4$$

$$\tau = \frac{1}{w_g\sqrt{\alpha}} = 3.24 \times 10^{-7}$$

This lead compensator adds the desired phase at w_g, but now we need to move the crossover frequency to w_g by adding gain K_{lead}.

Since $\left|C_{lead}(jw_g) \times P(jw_g)\right| = -157.4dB$,We must add 157.4dB of gain at w_g

This is accomplished by $K_{lead} = 10^{\frac{157.4}{20}} = 10^{7.87}$

Equating the theoretical gain and lead compensator, we have:

$$K_{lead} = \frac{1}{C_2 R_2}$$

$$\frac{s + \frac{1}{\alpha\tau}}{s + \frac{1}{\tau}} = \frac{s + \frac{1}{R_3 C_1}}{s + \frac{C_1 + C_2}{R_3 C_1 C_2}}$$

From the DC gain condition, R_2 = 5000ohms

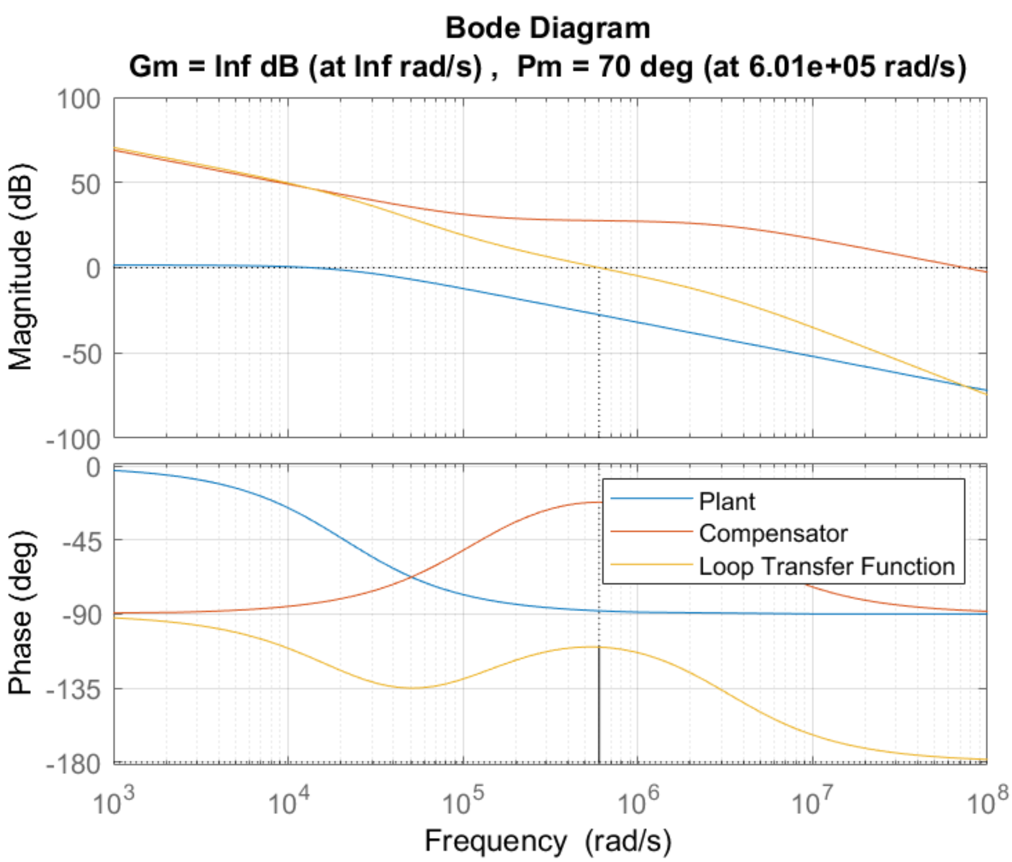
Using the values of α, τ, K_{lead} to get C_1, C_2, R_3 yields:

$$C_1 = 6.85 \times 10^{-11} F$$

$$C_2 = 2.69 \times 10^{-12} F$$

$$R_3 = 125 K\Omega$$

As we can see on the bode plot, the open loop transfer function has a PM of 70deg at w = 6x10^5



FTS Plant System Identification

Using the spring mass damper model, $G_p(s) = \frac{\frac{1}{m_1}}{s^2+\frac{b_1}{m_1}s+\frac{k_1}{m_1}}$

This can be modeled as the general second order system, $G_p(s) = k\frac{w_n^2}{s^2+2\zeta w_n+w_n^2}$

By looking at the frequency response data and ignoring the delay for now

$$10 = \frac{1}{2\zeta\sqrt{1-\zeta^2}} \Longrightarrow \zeta = 0.05 \quad \text{and} \quad w_r = 10^3 = w_n\sqrt{1-2\zeta^2} \Longrightarrow w_n = 1002.5$$

and $k = 10^{-4}$

Matching up these values with the physical model yields:

$$m_1 = 0.01, \qquad b_1 = 1, \qquad k_1 = 10{,}000$$

This gives us the model without delay.

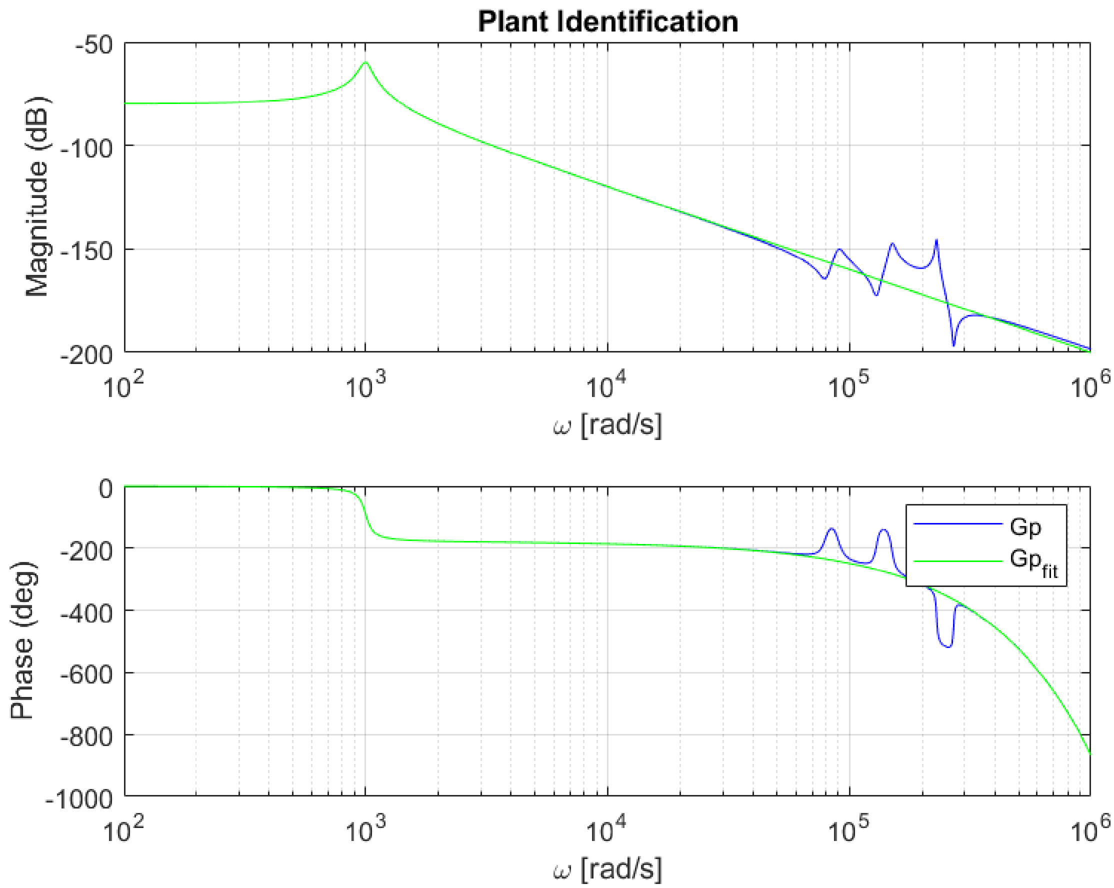
To estimate the delay, trial and error is used, eventually resulting in a delay of

$$e^{-1.2\times10^{-5}s}$$

Thus, the final FTS plant model is as follows:

$$G_p(s) = e^{-1.2\times10^{-5}s} \times \frac{100}{s^2+100s+10^6}$$

Plotting the estimated model with the actual data yields decent results



As we can see, the low frequency response is modeled accurately. Likewise, the delay is modeled well. The only issue is the deviations in the magnitude and phase that could not be modeled with a second order system. However, the simplicity of the modeled plant makes it easy to work with, and is a worthwhile trade off.

Position Control Loop Design

The first step to shaping the loop is to plot the uncompensated loop transfer function.

In other words, set $C_{mech}(s) = -1$.

After checking the frequency response, the uncompensated crossover frequency is at w = 10^4, and the phase is slightly below -180deg

To add some positive phase margin, I designed a lead compensator to give us a phase margin of about 30deg.

$$C_{lead}(s) = \frac{s + \frac{1}{\alpha\tau}}{s + \frac{1}{\tau}};\ \alpha = 10.315, \tau = 3.15 \times 10^{-5}$$

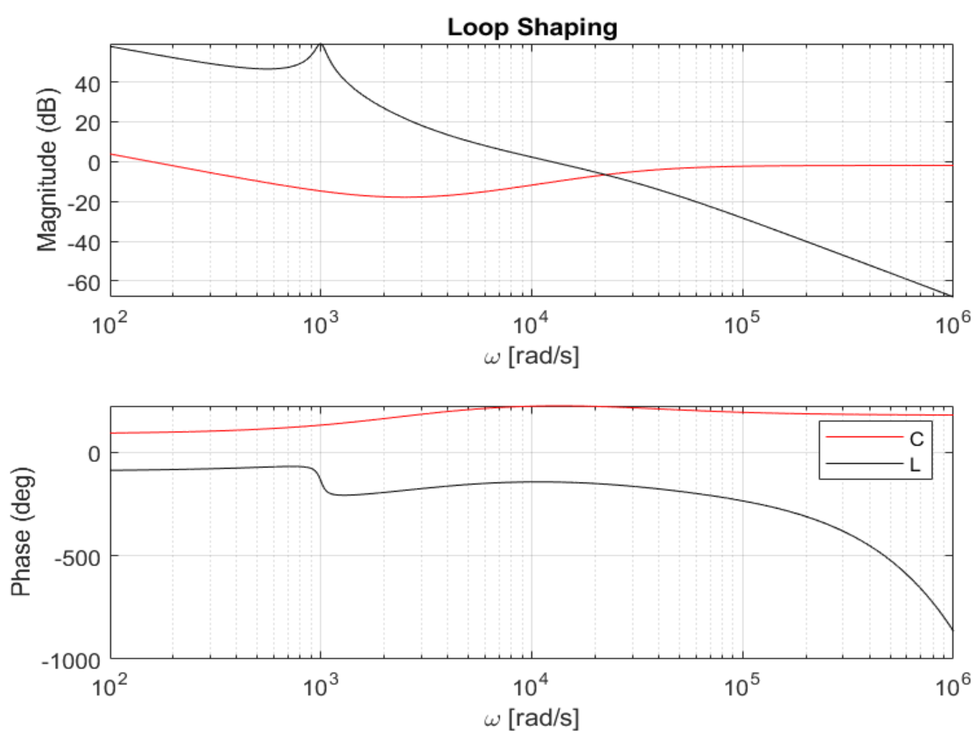
Next, I designed a PI controller to increase the low frequency gain.

$$C_{PI}(s) = 1 + \frac{K_I}{s}$$

After experimenting with K_I for some time, I decided to leave it at about one fifth the crossover frequency $K_I = 0.2 \times 10^4$

Finally, I used a constant gain to multiply the whole compensator for further tweaking. After some experimentation, I decided that the best gain was about 0.8

$$C_{mech}(s) = -1 \times (0.8) \times C_{lead}(s) \times C_{PI}(s)$$



With this compensator design

RMS SS step tracking error (nm): 0.0182

RMS SS sinusoidal tracking error (um): 0.9923

RMS SS noise error (nm): 2.7575

These specs are acceptable because I wanted to prioritize the steady state step tracking error, and then try to minimize the sinusoidal tracking error. The trade off was deciding where I wanted the highest gain. To track steady step input, high gain at low frequency is optimal. However, to track the SS sinusoid, high gain at it's resonant frequency is optimal. The other consideration is rejecting noise disturbance, which is achieved by making the high frequency gain as low as possible.

Percent Overshoot: 45%

This PO could have been reduced by increasing the phase margin. However, this would lower the low frequency gain, so I decided to leave it at 45%. While this is not great, I decided it is an acceptable trade off.

The remaining plots can be found by running my MATLAB code, as it would be redundant to put them here