System Dynamics

We will limit the stretcher motion to one degree of freedom; rotation about the cable axis.

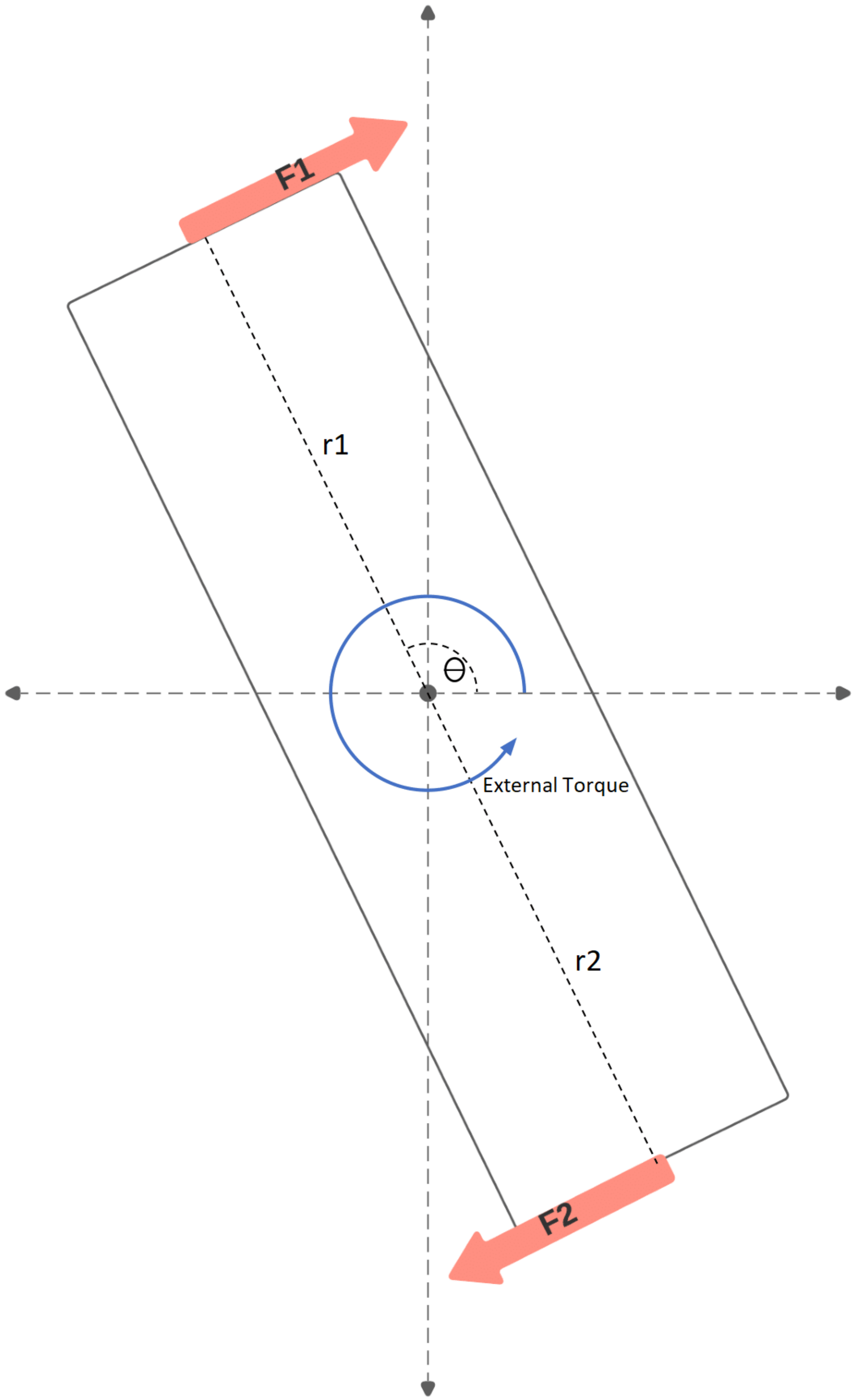


Figure 1: Stretcher free body diagram

To derive the system dynamics, we will make the following assumptions:

1. Our four actuators can be reduced down to two forces, since actuators pointing in opposite directions in a given module will never be activated at the same time
2. The torque from the cable, kθ, and torque from wind resistance, cω are negligible
3. Stretcher is balanced about center, so $r = r1 = r2$
4. Actuation is symmetric, so $\|F_{act}\| = \|F1\| = \|F2\|$

Angular acceleration of stretcher about center:

$$\sum \tau = I\alpha$$

$$\tau_{ext} - 2rF_{act} = I\alpha$$

$$\alpha = \frac{\tau_{ext} - 2rF_{act}}{I}$$

The forces and torque on the system are time varying, so

Angular acceleration: $\alpha(t) = \frac{\tau_{ext}(t) - 2rF_{act}(t)}{I}$

Angular Velocity: $\omega(t) = \omega_{initial} + \int_0^t \alpha(\tau) d\tau$

Absolute angle: $\theta(t) = \theta_{initial} + t\omega_{initial} + \int_0^t \int_0^t \alpha(\tau) d\tau \, d\tau$

While this explains how a stretcher with some arbitrary rotational inertia reacts to any actuator force and external torque, we must further explore how to model these parameters.

Actuator Thrust Model

Typically, a motors RPM is not directly proportional to the input PWM signal. Additionally, RPM to thrust curve for the propellers is also not linear. For example, we can look at other examples of measured motors and propellers.

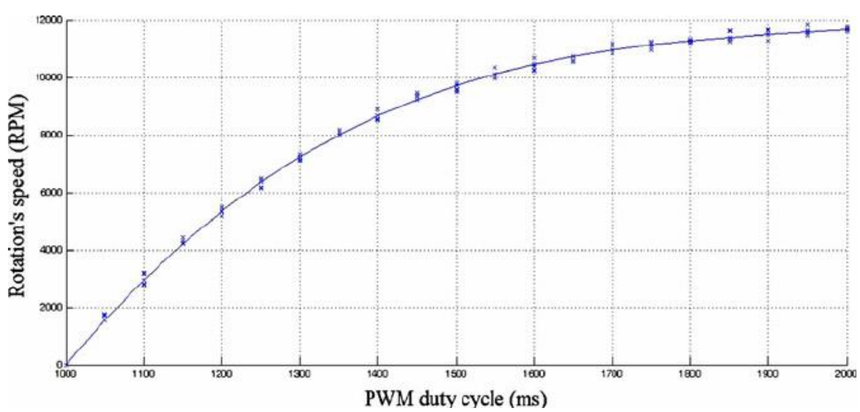


Figure 2: Measured rotational speed of the motor in various PWMs [1]

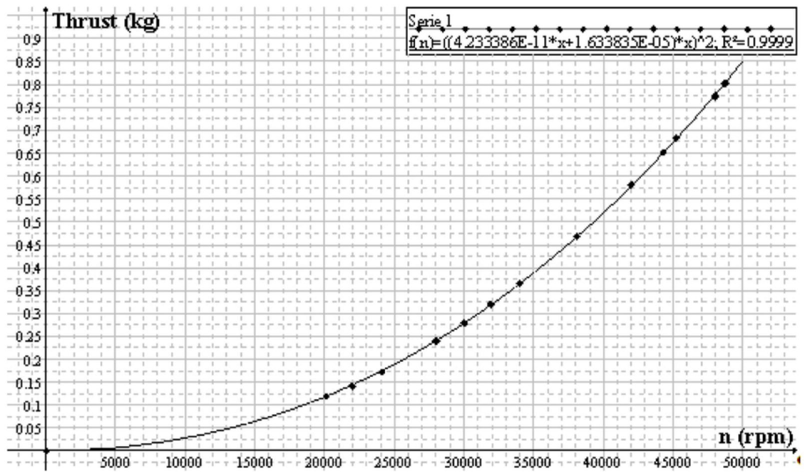
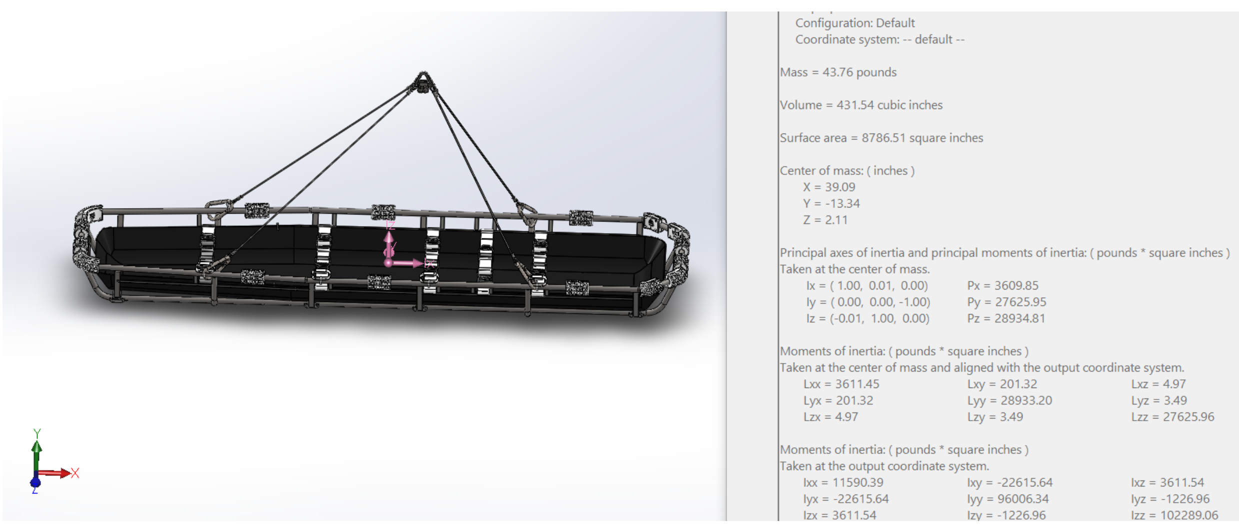


Figure 3: Propeller Thrust (kg) vs. rotational speed (rpm) [2]

These examples are for different motors and propellers than the ones we are using. We cannot find data on our specific actuators, so we will go forward assuming the ESCs and/or actuators have been programmed to output thrust that is directly proportional to the inputs.

<u>Rotational Inertia</u>

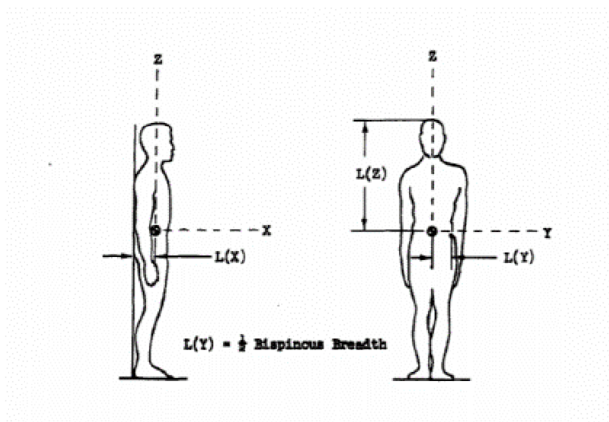
In determining the rotational inertia of the stretcher system, we used the 3D model provided by Lifesavings Systems Corp. After applying materials to all the necessary components, we were able to get information on the center of mass and the axial moments of inertia. Below is an image showing the center of mass of the stretcher and inertial data.



The coord system in pink (created by mass properties feature) is the local coordinate systeme of the stretcher, which is used to define the directions for the moments of inertia. X is towards the head of the stretcher, Y is toward side, and Z is towards the sling and the sky.

This data is for the stretcher-sling system that connects to the rope. If necessary, we can also obtain inertial data for only the stretcher system.

We used a paper on anthropometric data and inertia from the Department of Defense to begin modeling a body inside of a stretcher. By taking data concerning the “standing” position and modifying the axes to fit a body laying down, we can approximate the added inertia of a patient in a stretcher. Below is a summary of inertial data for an upright individual, with standard deviations given for a sample size of 66 participants.



Sample Size 66

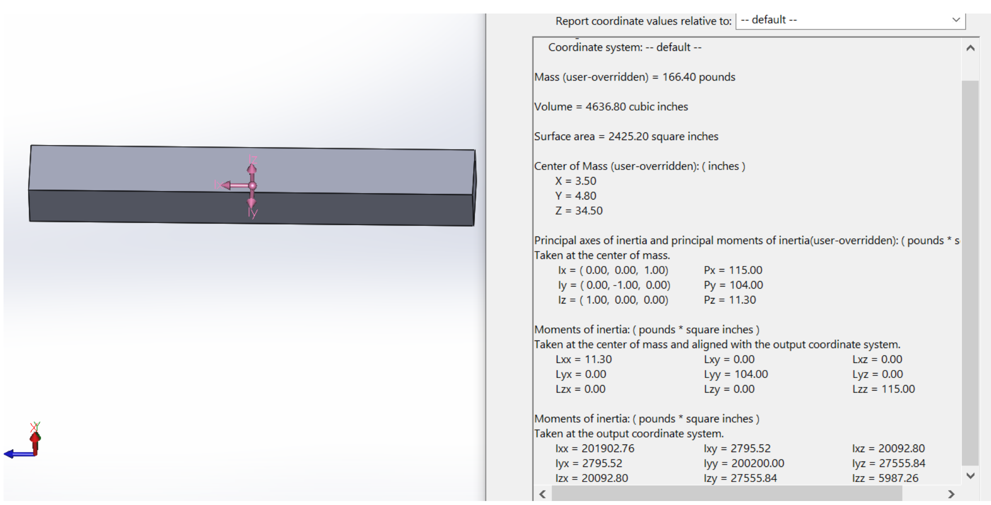
Mean Age 33.2 yrs. S.D. Age 7.2 yrs.

Mean Weight 166.4 lbs. S.D. Weight 19.8 lbs.

Mean Stature 69.4 in. S.D. Stature 2.9 in.

		Axis	Center of Gravity (in.)		Moment of Inertia (lb.in.sec.^{2})	
			Mean	S.D.	Mean	S.D.
1.	Standing	x	3.5	0.20	115.0	19.3
		y	4.8	0.39	103.0	17.9
		z	31.0	1.45	11.3	2.2

In Solidworks, we modeled the body as a prism with overridden mass and inertia properties. Since the body will rotate about its center of gravity and is strapped into the stretcher, this is adequate to model in our simulations. Below is a model based on the mean mass and inertia values.



We plan on modeling several more bodies that represent 1 and 2 standard deviations away from the mean DoD data, as well as extraneous cases where the load is near capacity or the center of gravity does not correspond to that of the stretcher system.

External Torque (Disturbance) Model

While the stretcher is being hoisted, it is subject to downwash from the helicopter, as well as wind from any direction, however, we are not coding a fluid dynamics simulation in python. Our goal is to model wind/downwash as an external torque, with reasonable magnitude and direction over time. We can approximate the bounds of this torque by estimating the maximum force exerted by wind on the stretchers side.

To calculate the force exerted by wind, we will use the following equation,

$$F_{wind} = m_{air} \times V_{air}^2 \times \sin(\theta);$$

$$m_{air} = A_{surface} \times D_{air}$$

which describes the force exerted on a small object for a given wind speed and angle relative to the surface. The maximum wind speed a helicopter can safely operate in is about 90km/hr [3]. Additionally, wind from downwash can travel very quickly as seen from the plot below.

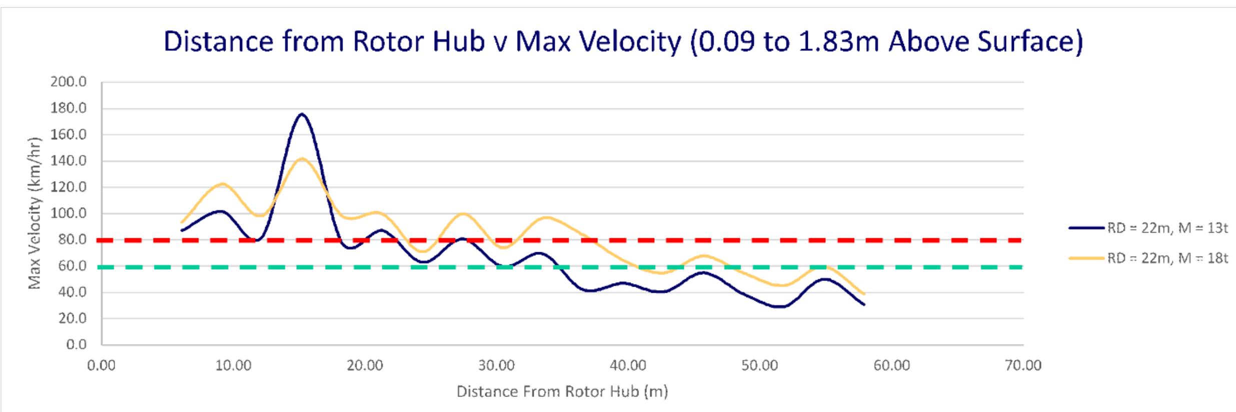


Figure 4: Distance from rotor hub versus maximum downwash velocity [4]

The wind speed of downwash can range from 40km/hr to 180km/hr depending on distance to rotors. However, it is directed down at the stretcher, and thus the angle at which the air hits the sides of the stretcher is likely low.

Using this information, we can calculate the total force wind or downwash could exert on the stretcher:

Force of wind at 90km/hr: $F_{wind} = 1.229 \times 0.38 \times 25^2 = 291N$

Force of downwash at 180km/hr and 10°: $F_{downwash} = 1.229 \times 0.38 \times 50^2 \times \sin(10°) = 203N$

These numbers look very large, but since this wind is exerted equally across the surface area, there is no torque exerted on the stretcher. Torque is only exerted when there is a difference in pressure along the surface area.

Let us assume the worst-case scenario, in which wind only acts on one side of the stretcher. Then the force exerted by the wind would be 145N spread out across one side. For simplicity, lets us assume that equates to a single force acting at the halfway point between the center of the stretcher and the end. This produces a torque of 78 Nm. Likewise, the maximum torque we would see from downwash would be around 54 Nm. While these numbers are larger than the maximum torque output of our actuators (47 Nm), our solution is not designed to handle extreme scenarios like this. However, we will test all scenarios within this range even if our actuators saturate.

Our goal is model the external torque over time (disturbance), and design a controller that can reject this disturbance. Since it is very hard to predict what the actual disturbances look like in the real world, we will conduct many tests with many disturbance models between these bounds.

<u>Mathematical Formulation</u>

The goal of our system is to drive the angular velocity of the stretcher to zero. Thus, the only output we really care about is the angular velocity measured by the IMU.

state: $x = \begin{bmatrix} \omega \\ \alpha \end{bmatrix}$

inputs: $u = [F_{act}]$

disturbance: $w = [\tau_{ext}]$

outputs: o = $[\omega_{\text{measured}}]$

State Equation:

$$x_t = Ax_{t-1} + Bu_{t-1} + Cw_{t-1}$$

$$\begin{bmatrix} \omega_t \\ \alpha_t \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \omega_{t-1} \\ \alpha_{t-1} \end{bmatrix} - \begin{bmatrix} \Delta t \\ 1 \end{bmatrix} \frac{2r}{I} F_{act} + \begin{bmatrix} \Delta t \\ 1 \end{bmatrix} \frac{1}{I} \tau_{ext}$$

Output Equation:

$$o_t = Dx_{t-1} + N$$

$$[\omega_{\text{measured}}] = [1 \quad 0] \begin{bmatrix} \omega_t \\ \alpha_t \end{bmatrix} + Noise$$

References:

1. Mohammadi, Mostafa & Mohammad Shahri, Alireza. (2013). Adaptive Nonlinear Stabilization Control for a Quadrotor UAV: Theory, Simulation and Experimentation. Journal of Intelligent and Robotic Systems. 72. 10.1007/s10846-013-9813-y.

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4. JJ Ryan Consulting
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5. Department of Defense Anthropometric Data

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