Safety Measures Derivation:

Here are the definitions that will be used in this section:

Parameter	Definition	Units
t	Time	Seconds
r	Distance of patient's head to center of rotation	Meters
$\theta(t)$	Angle of stretcher with respect to original position, function of time	Radians
$\omega(t) = \frac{d\theta(t)}{dt}$	Angular Velocity of stretcher, function of time	Radians/Second
$\alpha(t) = \frac{d\omega(t)}{dt}$	Angular Acceleration of stretcher, function of time	Radians/(Seconds^2)
$x(t)$	Position vector of patient's head with respect to standard (non – rotating) reference frame.	Meters
$v(t) = \frac{dx(t)}{dt}$	Velocity vector of patient's head with respect to standard (non-rotating) reference frame.	Meters/Second
$a(t) = \frac{dv(t)}{dt}$	Acceleration vector of patient's head with respect to standard (non-rotating) reference frame.	Meters/(Second^2)
$J(t) = \frac{da(t)}{dt}$	Jerk vector of patient's head with respect to standard (non-rotating) reference frame.	Meters/(Second^3)
$u_R(t)$	Radial Direction vector in current alignment with stretcher	N/A
$u_T(t)$	Tangential direction vector in alignment with current tangential direction of the stretcher	N/A
$v_T(t) =< u_T(t), v(t) >$	Tangential linear velocity of the patient's head	Meters/Second
$a_T(t) =< u_T(t), a(t) >$	Tangential linear acceleration of the patient's head	Meters/(Second^2)
$a_R(t) =< u_R(t), a(t) >$	Radial linear acceleration of the patient's head	Meters/(Second^2)

The position of the patient's head in the stretcher can be modeled by the following vector x(t)

$$x(t) = r * \begin{pmatrix} \cos(\theta(t)) \\ \sin(\theta(t)) \end{pmatrix}$$

Taking the derivative of x(t) with respect to time, we obtain v(t), linear velocity vector

$$v(t) = r * \begin{pmatrix} -\omega(t)\sin(\theta(t)) \\ \omega(t)\cos(\theta(t)) \end{pmatrix}$$

Taking the derivative of v(t) with respect to time, we obtain a(t), acceleration velocity vector

$$a(t) = r * \begin{pmatrix} -\alpha(t)\sin(\theta(t)) - \omega(t)^2\cos(\theta(t)) \\ \alpha(t)\cos(\theta(t)) - \omega(t)^2\sin(\theta(t)) \end{pmatrix}$$

Taking the derivative of a(t) with respect to time, we obtain J(t), the jerk vector

$$J(t) = r * \begin{pmatrix} -\alpha'(t)\sin\big(\theta(t)\big) - \alpha(t)\omega(t)\cos\big(\theta(t)\big) + \omega(t)^3\sin\big(\theta(t)\big) - 2\alpha(t)\omega(t)\cos\big(\theta(t)\big) \\ \alpha'(t)\cos\big(\theta(t)\big) - \alpha(t)\omega(t)\sin\big(\theta(t)\big) - \omega(t)^3\cos\big(\theta(t)\big) - 2\alpha(t)\omega(t)\sin\,(\theta(t)) \end{pmatrix}$$

$$= r * \begin{pmatrix} \big(\omega(t)^3 - \alpha'(t)\big)\sin\big(\theta(t)\big) + \big(-3\alpha(t)\omega(t)\big)\cos\big(\theta(t)\big) \\ -\big(\omega(t)^3 - \alpha'(t)\big)\cos\big(\theta(t)\big) + \big(-3\alpha(t)\omega(t)\big)\sin\big(\theta(t)\big) \end{pmatrix}$$

Define the unit radial and tangential vector as the following

$$u_R(t) = \begin{pmatrix} \cos\big(\theta(t)\big) \\ \sin\big(\theta(t)\big) \end{pmatrix}, u_T(t) = \begin{pmatrix} -\sin\,(\theta(t)) \\ \cos\,(\theta(t)) \end{pmatrix}$$

Using these vectors to evaluate tangential velocity:

$$v_T(t) =< u_T(t), v(t) > = r * \omega(t)$$

Using these vectors to evaluate absolute tangential acceleration:

$$|a_T(t)| = |< u_T(t), a(t) >| = r * |\alpha(t)|$$

Using these vectors to evaluate absolute radial acceleration:

$$|a_R(t)| = |< u_R(t), a(t) >| = r * \omega(t)^2$$

Taking the magnitude of the Jerk vector:

$$|J(t)| = \sqrt{\left(\omega(t)^3 - \frac{d\alpha(t)}{dt}\right)^2 + \big(3\alpha(t)\omega(t)\big)^2}$$

From research, humans can take maximum acceleration of 5gs in direction of spine and 10gs in direction perpendicular to spine. People in the stretcher might be injured, so divide each acceleration value by 10 to obtain bounds with safety measures:

$$Note: 1g = 9.81\left(\frac{m}{s^2}\right), take\ r = 1.1\ m$$

$$|a_R(t)| \leq 0.5g$$

$$1.1 * \omega(t)^2 \leq 0.5 * (9.81)$$

$$|\omega(t)| \leq (2.11)\left(\frac{rad}{s}\right)$$

$$|a_T(t)| \leq 1g$$

$$1.1 * |\alpha(t)| \leq 1 * (9.81)$$

$$|\alpha(t)| \leq 8.91\left(\frac{rad}{s^2}\right)$$