

## NOTES ON QUANTITATIVE FINANCIAL ANALYSIS

	AAA	AA	A	BBB	BB	B	CCC/C	D
AAA	91.386	7.947	0.508	0.093	0.062	0.001	0.001	0.001
AA	0.603	90.65	7.936	0.603	0.062	0.114	0.021	0.010
A	0.052	1.991	91.427	5.858	0.44	0.157	0.031	0.042
BBB	0.021	0.171	4.112	89.508	4.561	0.812	0.182	0.288
BB	0.033	0.044	0.276	5.799	83.508	8.114	0.992	1.235
B	0.001	0.057	0.215	0.351	6.249	82.27	4.766	6.091
CCC/C	0.001	0.001	0.322	0.472	1.426	12.56	54.139	31.079

**Table VIII.34** Average Transition rates from 1981. Source: “Credit Risk Modeling” by Löffler & Posch

We can continue in this way. Though we have eight bins, we only need to compute seven thresholds.

	AAA	AA	A	BBB	BB	B	CCC/C	D
A	0.052	1.991	91.427	5.858	0.440	0.157	0.031	0.042
Bin	( $+\infty$ ,3.28]	(3.28,2.04]	(2.04,-1.51]	(-1.51,-2.47]	(-2.47,-2.83]	(-2.83,-3.18]	(-3.18,-3.34]	(-3.34, $-\infty$ )

**Table VIII.35** Binning procedures for transition from A grade

The getThresholdsMatrix(TransitionDataset) function allows to compute all the thresholds for the bins of the distribution for all the rating grades. The input argument is the Transition matrix. The output of the function has been reported in the below Figure.

The usual representation of the grades is applied: from top to down and from right to left, the ratings become worse and worse.

-1.36498	-2.4751	-2.95173	-3.21598	-4.01281	-4.10748	-4.26489
2.5098	-1.35656	-2.40438	-2.86729	-2.97814	-3.42271	-3.71902
3.26881	2.04454	-1.5119	-2.47296	-2.83379	-3.18252	-3.33927
3.54008	2.89266	1.71656	-1.56809	-2.23162	-2.59715	-2.76114
3.41407	3.17084	2.69495	1.54222	-1.26236	-2.00897	-2.24606
4.26489	3.24854	2.77856	2.49827	1.48532	-1.23417	-1.54718
4.26489	4.10748	2.72245	2.41074	2.00992	1.04583	-0.493612

**Figure VIII.32** Upper thresholds of the bins

As stated above, this representation is more convenient to perform simulations on the transition rates. Let us

imagine that the normal density is shifted to the left, i.e. it assumes a negative mean rather than zero. The probability of a transition is the probability of ending up in the associated bin. This probability is equal to the area enclosed by the boundaries of the bin and the density function. Therefore, a shift to the left would increase the probabilities of downgrades as well as the probability of default.

Importantly, we still have fully specified transition probabilities, albeit ones that are different from those we used for the threshold determination. Likewise, we could reduce the probabilities of downgrade and default by shifting the distribution to the right.

Transition probabilities that result after the shift can be computed calling the `getAdjustedTransitionMatrix` function which takes as input the transition matrix and the magnitude of the shift that is called “**credit index**”. A negative number of the credit index means that the distribution function is shifted to the left, thus increasing the probabilities of downgrade and default. The probability that a normal variable with mean  $m$  and standard deviation 1 ends up to the left of a threshold is given by  $\Phi(\text{threshold} - m)$ . In order to achieve the probability of ending up in a bin, we use this formula to obtain the probability of ending up below the upper threshold of the bin, and then subtract the probability of ending up below the lower threshold of the bin. We could compute the latter with the normal distribution, but we can also sum over the cells in the same row that are located to the right of the bin we are in.

For the AAA bins (first column of the tables), we exploit the fact that transition probabilities sum up to 1. Using a credit index equal to -0.25, we obtain the adjusted transitions reported in the Figure below. This is a useful tool in order to perform what-if analysis on the transition rates or adjusting the historical transitions if an unlikely event has occurred, which can cause a discontinuity in the economic scenario, like for example the covid-19 pandemic.

0	1	2	3	4	5	6	7	8
1	86.757	11.9393	0.958837	0.194039	0.142459	0.0026726	0.00275444	0.00297367
2	0.289185	86.2867	11.8636	1.11737	0.124692	0.242948	0.0493931	0.0261184
3	0.0216742	1.06629	88.5627	9.03836	0.822322	0.320499	0.0677865	0.100326
4	0.00752982	0.0761815	2.378	88.1646	6.99758	1.43021	0.344164	0.60171
5	0.0124119	0.0188024	0.130291	3.49338	80.7768	11.6392	1.63275	2.29636
6	0.000316747	0.0230741	0.0994692	0.17669	3.83469	79.6141	6.52315	9.72847
7	0.000316747	0.000341097	0.147059	0.242124	0.80148	8.5604	49.8717	40.3766

**Figure VIII.33** Transitions rates using a credit index equal to -0.25

## FURTHER READINGS

De Laurentis G., Maino R., Molteni L. – “Developing, Validating and using Internal Ratings: methodologies and case studies” – Wiley (2010).