$p_i = 0.115\%, p_s = 3.962\%, \omega_i = 21.950\%, \omega_j = 27.590\%$ with an objective function equal to 185.79.
Recalling the four steps for measuring a portfolio credit risk reported at the beginning of this section, we are now able to set the Monte Carlo engine for our credit portfolio. In particular, the default correlations are modelled by linking defaults to a continuous variable: the asset value A. Borrower i defaults if its asset value falls below a certain threshold d_1 chosen to match the specified PD_1 in accordance with:

$$\text{Default}_i \Leftrightarrow y_i = 1 \Leftrightarrow A_i \leq d_i \quad \textit{(Eq. VIII.143)}$$

$$\text{No Default}_i \Leftrightarrow y_i = 0 \Leftrightarrow A_i > d_i \quad \textit{(Eq. VIII.144)}$$

If the asset values are assumed to be standard normally distributed, we would set $d_i = \Phi^{-1}(PD_i)$, where Φ denotes the cumulative standard normal distribution function
Correlation in asset values can be modeled through factor models. We start with a simple one containing only one systematic factor Z:

$$A_i = \omega_i Z + \sqrt{1 - \omega_i^2}\epsilon_i,\ \text{cov}(\epsilon_i, \epsilon_j) = 0,\ i \neq j,\ \text{cov}(Z, \epsilon_i) = 0,\ \forall_i \quad \textit{(Eq. VIII.145)}$$

$$Z \sim N(0,1),\ \epsilon_i N(0,1), \forall_i \quad \textit{(Eq. VIII.146)}$$

In other words, we assume that systematic (Z) and idiosyncratic (ϵ) shocks are independent. In the asset value approach, the standard way of obtaining the portfolio distribution (i.e., the fourth step in the logical flow reported at the beginning of this chapter) is to run a Monte Carlo simulation, which typically has the following structure:

1. Randomly draw asset values for each obligor in the portfolio.
2. For each obligor, check whether it defaulted; if yes, determine the individual loss $LGD_i \times EAD_i$.
3. Aggregate the individual losses into a portfolio loss.
4. Repeat steps 1-3 sufficiently to reach a distribution of credit portfolio losses.

In the proposed example, we have already computed the PD_i, LGD_i, EAD_i and asset correlations using a model discussed previously. We use a one-factor model with normally distributed asset values so the correlations are fully specified once we have specified the factor sensitivities ω_i. The data set used has been retrieved from the "Credit Risk modeling" by Löffler and Posch and it is imported in the IDE through a csv file which consists of five fields: the portfolio Identification number (ID), the Probability of Default (PD), the Loss Given Default (LGD), the Exposure at Default (EAD) and the asset correlation (W).
Starting from this information, the logical steps (1-4) have been implemented in the Python function MonteCarloSim. This function takes four input arguments: the data set, the number of simulations, the confidence at which the risk measures are computed and a Boolean variable which allows to plot the histogram of the portfolio losses. The output structure is a tuple with three elements:
- a $(1 \times NSim)$ array with the simulated credit portfolio losses.
- the Value at Risk at a given confidence (common choices are 95% or 99%). Mathematically, VaR_α is the $(1 - \alpha)$ quantile of the loss distribution.

- the Expected Shortfall at a given confidence (common choices are 95% or 99%). Mathematically, the expected shortfall at $\alpha\%$ level is the expected return on the portfolio in the worst $\alpha\%$ of cases.

Graphics is a Boolean variable and if it is true, the function also returns the empirical distribution of the simulated losses on the analysed credit portfolio in the graphical device of the Python IDE.

Running the function with 100.000 simulations, 95% of confidence on the Credit Portfolio Data set, we obtain a VaR = 300 and ES = 381.365. The empirical distribution of losses is shown in the Figure below.

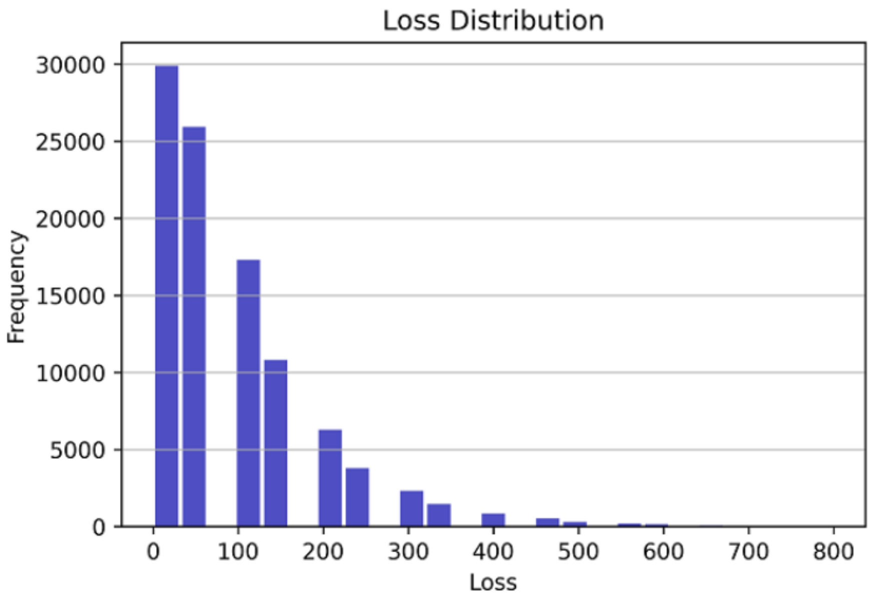

Figure VIII.34 Statistical distribution of the simulated losses on the credit portfolio ($NSim = 100{,}000$)

FURTHER READINGS

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