

Contents

Preface	<i>page</i> ix
Chapter 1. Introduction	1
1.1 A simple system generating a density of states	1
1.2 The evolution of densities: an intuitive point of view	4
1.3 Trajectories versus densities	10
Chapter 2. The toolbox	13
2.1 Measures and measure spaces	13
2.2 Lebesgue integration	15
2.3 Convergence of sequences of functions	27
Chapter 3. Markov and Frobenius–Perron operators	32
3.1 Markov operators	32
3.2 The Frobenius–Perron operator	36
3.3 The Koopman operator	42
Chapter 4. Studying chaos with densities	45
4.1 Invariant measures and measure-preserving transformations	45
4.2 Ergodic transformations	52
4.3 Mixing and exactness	59
4.4 Using the Frobenius–Perron and Koopman operators for classifying transformations	63
4.5 Kolmogorov automorphisms	73
Chapter 5. The asymptotic properties of densities	77
5.1 Weak and strong precompactness	77
5.2 Properties of the averages $A_n f$	80
5.3 Asymptotic periodicity of $\{P^n f\}$	86
5.4 The existence of stationary densities	90
5.5 Ergodicity, mixing, and exactness	92

5.6	Asymptotic stability of $\{P^n\}$	94
5.7	Markov operators defined by a stochastic kernel	101
5.8	Conditions for the existence of lower-bound functions	111
Chapter 6. The behavior of transformations on intervals and manifolds		114
6.1	Functions of bounded variation	114
6.2	Piecewise monotonic mappings	119
6.3	Piecewise convex transformations with a strong repeller	128
6.4	Asymptotically periodic transformations	131
6.5	Change of variables	140
6.6	Transformations on the real line	147
6.7	Manifolds	151
6.8	Expanding mappings on manifolds	158
Chapter 7. Continuous time systems: an introduction		163
7.1	Two examples of continuous time systems	163
7.2	Dynamical and semidynamical systems	165
7.3	Invariance, ergodicity, mixing, and exactness in semidynamical systems	169
7.4	Semigroups of the Frobenius–Perron and Koopman operators	173
7.5	Infinitesimal operators	179
7.6	Infinitesimal operators for semigroups generated by systems of ordinary differential equations	183
7.7	Applications of the semigroups of the Frobenius–Perron and Koopman operators	188
7.8	The Hille–Yosida theorem and its consequences	200
7.9	Further applications of the Hille–Yosida theorem	206
7.10	The relation between the Frobenius–Perron and Koopman operators	215
Chapter 8. Discrete time processes embedded in continuous time systems		219
8.1	The relation between discrete and continuous time processes	219
8.2	Probability theory and Poisson processes	220
8.3	Discrete time systems governed by Poisson processes	226
8.4	The linear Boltzmann equation: an intuitive point of view	229
8.5	Elementary properties of the solutions of the linear Boltzmann equation	232
8.6	Further properties of the linear Boltzmann equation	236

8.7	Effect of properties of the Markov operator on solutions of the linear Boltzmann equation	238
8.8	Linear Boltzmann equation with a stochastic kernel	241
8.9	The linear Tjon–Wu equation	244
Chapter 9.	Entropy	248
9.1	Basic definitions	248
9.2	Entropy of $P^n f$ when P is a Markov operator	254
9.3	Entropy $H(P^n f)$ when P is a Frobenius–Perron operator	257
9.4	Behavior of $P^n f$ from $H(P^n f)$	260
Chapter 10.	Stochastic perturbation of discrete time systems	266
10.1	Independent random variables	266
10.2	Mathematical expectation and variance	269
10.3	Stochastic convergence	273
10.4	Discrete time systems with randomly applied stochastic perturbations	277
10.5	Discrete time systems with constantly applied stochastic perturbations	282
10.6	Small continuous stochastic perturbations of discrete time systems	289
Chapter 11.	Stochastic perturbation of continuous time systems	293
11.1	One-dimensional Wiener processes (Brownian motion)	293
11.2	d -Dimensional Wiener processes (Brownian motion)	302
11.3	The stochastic Itô integral: development	304
11.4	The stochastic Itô integral: special cases	309
11.5	Stochastic differential equations	313
11.6	The Fokker–Planck (Kolmogorov forward) equation	317
11.7	Properties of the solutions of the Fokker–Planck equation	322
11.8	Semigroups of Markov operators generated by parabolic equations	326
11.9	Asymptotic stability of solutions of the Fokker–Planck equation	329
11.10	An extension of the Liapunov function method	336
	References	345
	Notation and symbols	350
	Index	353

