Some "real-life" finite element model examples

470 "REAL-LIFE" FINITE ELEMENT EXAMPLES

GEOMETRY OF MIDDLE SURFACE

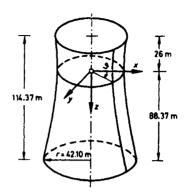
POSITION VECTOR

$$\mathbf{M} = \begin{bmatrix} r \cdot \cos \xi^1 \\ r \cdot \sin \xi^1 \\ \xi^2 \end{bmatrix}$$

WHERE

$$r = 24.85 \sqrt{1 + (\xi^2/64.62)^2}$$

PARAMETER DEFINITION $\xi^1 = 3$, $\xi^2 = z$



MATERIAL DATA

YOUNG'S MODULUS $E = 3 \cdot 10^9 \text{ kp/m}^2$

POISSON'S RATIO $\nu = 0.2$

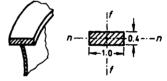
SPECIFIC GRAVITY $7 = 1.0 \text{ kp/m}^3$ (FOR BUCKLING ANALYSIS) DENSITY $9 = 1.0 \text{ kp s}^2/\text{m}^4$ (FOR VIBRATION ANALYSIS)

GEOMETRICAL DATA

SHELL THICKNESS
STIFFENER AT THE TOP
OF THE SHELL

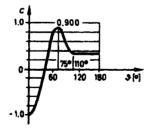
t = 0.14 m $A = 0.4 \text{ m}^2$ $I_{ee} = 3.333 \cdot 10^{\circ}$

 $I_{ff} = 3.333 \cdot 10^{-2} \text{ m}^4$ $I_{nn} = 5.333 \cdot 10^{-3} \text{ m}^4$ $J = 1.597 \cdot 10^{-2} \text{ m}^4$



BOUNDARY CONDITIONS

WINDLOAD



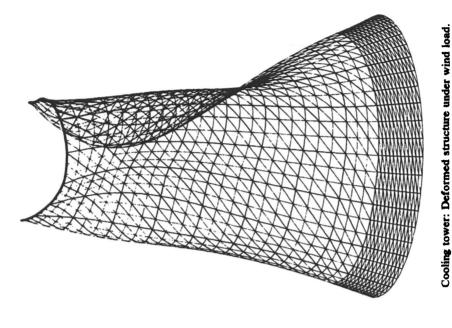
DISTRIBUTION OF WINDLOAD

 $P_{w} = c(\vartheta) \cdot q(z)$

 $q(z) = -100 \text{ kp/m}^2 = \text{constant}$

Cooling tower: Geometry, dimensions and input data. Reproduced by courtesy of Professor J.H. Argyris.

Reproduced by courtesy of Professor J.H. Argyris.



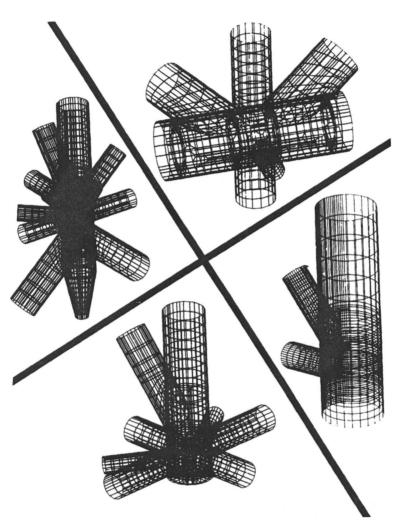
Cooling tower: Triangulation.



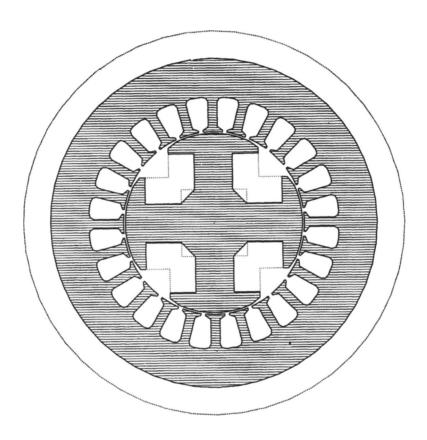
Cooling tower: First vibration mode.

Reproduced by courtesy of Professor J.H. Argyris.

Cooling tower: First buckling mode under dead weight. Reproduced by courtesy of Professor J.H. Argyris.

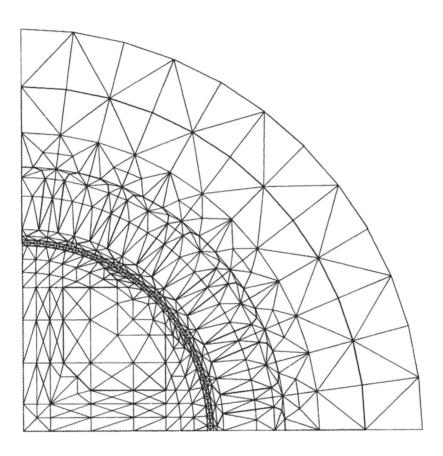


Finite element stress analysis of complex tubular joints. Reproduced by courtesy of Professor C.A. Felippa.



Tetrapolar alternator.

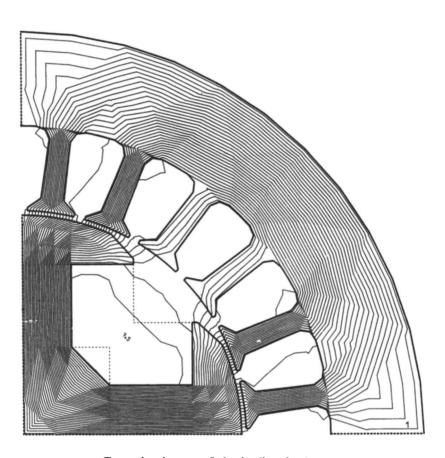
Reproduced by courtesy of Professor R. Glowinski and Mr. A. Marrocco.



Tetrapolar alternator: Example of a triangulation.

Reproduced by courtesy of Professor R. Glowinski and Mr. A. Marrocco.

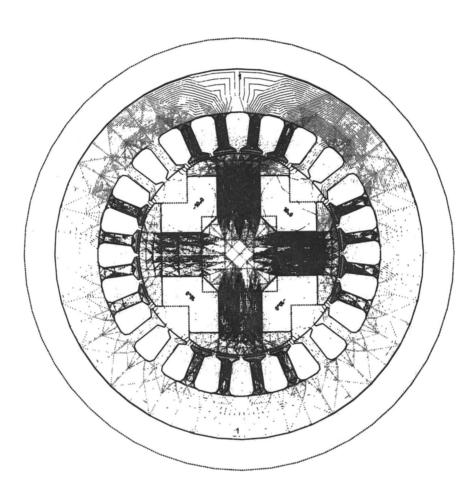
Tetrapolar alternator: Induction lines for J = 2. (J: density of current) Reproduced by courtesy of Professor R. Glowinski and Mr. A. Marrocco.



Tetrapolar alternator: Induction lines for J = 7.5.

(J: density of current)

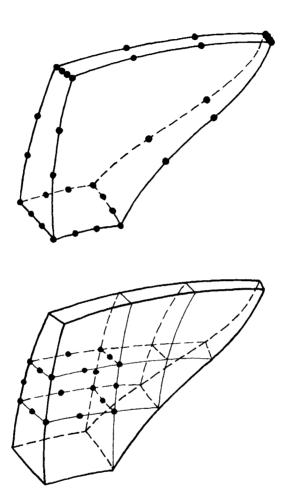
Reproduced by courtesy of Professor R. Glowinski and Mr. A. Marrocco.



Tetrapolar alternator: Induction lines for J = 10.

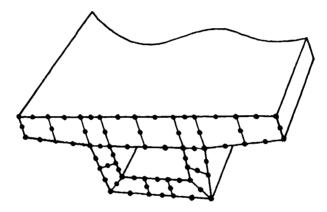
(J: density of current)

Reproduced by courtesy of Professor R. Glowinski and Mr. A. Marrocco.



Arch dam in a rigid valley - Various element subdivisions.

Reproduced from Fig. 9.8 of Professor Zienkiewicz' book: "The Finite Element Method in Engineering Science", McGraw-Hill, London, 1971, by courtesy of Professor O.C. Zienkiewicz, and with permission of the Publisher.



A thick box bridge reduced to a two-dimensional problem with isoparametric, quadratic, elements.

Reproduced from Fig. 13.2 of Professor Zienkiewicz' book: "The Finite Element Method in Engineering Science", McGraw-Hill, London, 1971, by courtesy of Professor O.C. Zienkiewicz, and with permission of the Publisher.

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GLOSSARY OF SYMBOLS

General notation

 $v(\cdot)$, $v(\cdot, \cdot)$, etc. . .: function v of one variable, two variables, etc. . .

 $v(\cdot, b)$: partial mapping $x \to v(x, b)$.

supp $v = \{x \in X; v(x) \neq 0\}^-$: support of a function v.

 $\operatorname{osc}(v; A) = \sup_{x,y \in A} |v(x) - v(y)|.$

 v_A or $v_{|A}$: restriction of a function v to the set A.

 $P(A) = \{P_{|A}; \forall p \in P\}$, where P is any space of functions defined over a domain which contains the set A.

tr v, or simply v: trace of a function v.

 $R(v) = \frac{a(v, v)}{(v, v)}$: Rayleigh quotient.

C(a), C(a, b), etc...: any "constant" which depends solely on a, a and b, etc...

A: interior of a set A.

 ∂A : boundary of a set A.

 \overline{A} or \overline{A} : closure of a set A.

card A: number of elements of a set A.

diam A: diameter of a set A.

CA, or C_XA , or X - A: Complement set of the subset A of the set X. \Rightarrow : implies.

Derivatives and differential calculus

Dv(a), or v'(a): first (Fréchet) derivative of a function v at a point a.

 $D^2v(a)$, or v''(a): second (Fréchet) derivative of a function v at a.

 $D^kv(a)$: k-th (Fréchet) derivative of a function v at a point a.

 $D^k v(a) h^k = D^k v(a) (h_1, h_2, \ldots, h_k)$ if $h_1 = h_2 = \cdots = h_k = h$.

 $\mathcal{R}_k(v;b,a)=v(b)-\left\{v(a)+Dv(a)(b-a)+\cdots+\frac{1}{k!}D^kv(a)(b-a)^k\right\}.$

$$\frac{\partial_i v(A) = Dv(a)e_i,}{\partial_{ijk}v(a) = D^2v(a)(e_i, e_j),}$$

$$\frac{\partial_i v(a) = D^2v(a)(e_i, e_j),}{\partial_{ijk}v(a) = D^3v(a)(e_i, e_j, e_k).}$$
(also used for vector-valued functions)

$$J_F(\hat{x}) = \det(\partial_i F_i(\hat{x})) = \text{Jacobian}$$
 of a mapping $F: \hat{x} \in \mathbb{R}^n \to F(\hat{x}) = (F_i(x))_{i=1}^n \in \mathbb{R}^n$.

$$\operatorname{div} v = \sum_{i=1}^n \partial_i v.$$

 $\nabla v(a) = (\partial_i v)_{i=1}^n$, also denoted $\nabla v(a)$, grad v(a).

$$\Delta v = \sum_{i=1}^{n} \partial_{ii} v, \ \Delta v = (\Delta v_i)_{i=1}^{n}.$$

$$|\alpha| = \sum_{i=1}^{n} \alpha_i$$
, for a multi-index $\alpha = (\alpha_1, \dots, \alpha_n) \in \mathbb{N}^n$.

$$\partial^{\alpha}v(a) = D^{|\alpha|}v(a)(\overline{e_1,\ldots,e_1},\overline{e_2,\ldots,e_2},\ldots,\overline{e_n,\ldots,e_n}).$$

$$v = (v_1, v_2,\ldots,v_n): \text{ unit outer normal vector.}$$

$$\partial_{\nu} = \sum_{i=1}^{n} \nu_{i} \partial_{i}$$
: (outer) normal derivative operator.

 $\tau = (\tau_1, \tau_2)$: unit tangential vector along the boundary of a plane domain.

$$\partial_{\tau}v(a)=Dv(a)\tau=\sum_{i=1}^2\tau_i\partial_iv(a).$$

$$\hat{\sigma}_{\nu\tau}v(a) = D^2v(a)(\nu,\tau) = \sum_{i,j=1}^2 \nu_i \tau_j \hat{\sigma}_{ij}v(a).$$

$$\partial_{\tau \tau} v(a) = D^2 v(a)(\tau, \tau) = \sum_{i=1}^2 \tau_i \tau_i \partial_{ij} v(a).$$

 $(V_I)_{I=1}^{12} = \{\partial^{\alpha} v_{\beta}, |\alpha| \le 1, \beta = 1, 2, \partial^{\alpha} v_3, |\alpha| \le 2\}$ (notation for admissible displacements $v = (v_1, v_2, v_3)$ in shell theory).

Differential geometry

 $(a_{\alpha\beta})$: first fundamental form of a surface.

 $a = \det(a_{\alpha\beta}).$

 (b_{ab}) : second fundamental form of a surface.

 $(c_{\alpha\beta})$: third fundamental form of a surface.

 Γ_{Br}^{α} : Christoffel symbols.

υ_{|β}, υ_{|αβ},...: covariant derivatives along a surface.

 $ds = \sqrt{a} d\xi$: surface element.

 $\frac{1}{D}$: curvature of a plane curve.

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General notation for vector spaces.

 $B(a; r) = \{x \in X; ||x - a|| \le r\}.$

 $\mathcal{L}(X; Y)$: space of continuous linear mappings from X into Y.

 $\mathcal{L}(X) = \mathcal{L}(X;X).$

 $\mathcal{L}_k(X; Y)$: space of continuous k-linear mappings from X^k into Y.

 $\mathcal{L}_2(X_1 \times X_2; Y)$: space of continuous bilinear mappings from $X_1 \times X_2$ into Y.

X': dual of a space X.

 $\|\cdot\|^*$: norm in the space X'.

 $\langle \cdot, \cdot \rangle$: duality pairing between a space and its dual.

 $x + Y = \{x + y; y \in Y\}.$

 $X + Y = \{x + y; x \in X, y \in Y\}.$

 $X \oplus Y = \{x + y; x \in X, y \in Y\} \text{ when } X \cap Y = \{0\}.$

X/Y: quotient space of X by Y.

 $V\{e_{\lambda}, \lambda \in \Lambda\}$: vector space spanned by the vectors $e_{\lambda}, \lambda \in \Lambda$.

I: identity mapping.

: inclusion with continuous injection.

ċ: inclusion with compact injection.

 $\dim X$: dimension of the space X.

 $\ker A = \{x \in X; \ Ax = 0\}.$

Notation for specific vector spaces

$$(u, v) = \int_{\Omega} uv \, dx$$
 (inner product in $L^{2}(\Omega)$).

$$(u, v) = \int_{\Omega} u \cdot v \, dx$$
 (inner product in $(L^2(\Omega))^n$).

 $\mathscr{C}^m(A)$: space of functions m times continuously differentiable on a subset A of \mathbb{R}^n .

$$\mathscr{C}^{\infty}(A) = \bigcap_{m=0}^{\infty} \mathscr{C}^{m}(A).$$

$$\mathscr{C}^{m,\alpha}(A) = \{ v \in C^m(\bar{\Omega}); \forall \beta, |\beta| = m, \exists \Gamma_{\beta}, \forall x, y \in A, \\ |\partial^{\beta}v(x) - \partial^{\beta}v(y)| \leq \Gamma_{\alpha}||x - y||^{\alpha} \}.$$

$$\begin{aligned} &|\partial^{\beta}v(x) - \partial^{\beta}v(y)| \leq \Gamma_{\beta}||x - y||^{\alpha}\}.\\ &\|v\|_{\mathscr{C}^{m,\alpha}(A)} = \|v\|_{m,\infty,A} + \max_{|\beta| = m} \sup_{\substack{(x,y \in A \\ x \neq y}} \frac{|\partial^{\beta}v(x) - \partial^{\beta}v(y)|}{\|x - y\|^{\alpha}}.\end{aligned}$$

$$\mathcal{D}(\Omega) = \{ v \in \mathscr{C}^{\infty}(\Omega); \text{ supp } v \text{ is a compact subset of } \Omega \}.$$

 $\mathfrak{D}'(\Omega)$: space of distributions over Ω .

 $H^{m}(\Omega) = \{v \in L^{2}(\Omega); \forall \alpha, |\alpha| \leq m, \ \partial^{\alpha}v \in L^{2}(\Omega)\}.$

 $H_0^m(\Omega) = \text{closure of } \mathcal{D}(\Omega) \text{ in } H^m(\Omega).$

$$\|v\|_{m,\Omega} = \left(\sum_{|\alpha|=m} \int_{\Omega} |\partial^{\alpha}v|^{2} dx\right)^{1/2}.$$

$$\|v\|_{m,\Omega} = \left(\sum_{i=1}^{m} \int_{\Omega} |\partial^{\alpha}v|^{2} dx\right)^{1/2}.$$

$$\|v\|_{m,\Omega} = \left(\sum_{i=1}^{n} \|v_{i}\|_{m,\Omega}^{2}\right)^{1/2} \text{ (for functions } v = (v_{i})_{i=1}^{n} \text{ in } (H^{m}(\Omega))^{n}).$$

$$\|v\|_{m,\Omega} = \left(\sum_{i=1}^{n} \|v_{i}\|_{m,\Omega}^{2}\right)^{1/2} \text{ (for functions } v = (v_{i})_{i=1}^{n} \text{ in } (H^{m}(\Omega))^{n}).$$

$$\|v\|_{m,\Omega} = \left(\sum_{i=1}^{n} \|v_{i}\|_{m,\Omega}^{2}\right)^{1/2} \text{ (for functions } v = (v_{i})_{i=1}^{n} \text{ in } (H^{m}(\Omega))^{n}).$$

$$\|v\|_{m,\Omega} = \left(\sum_{i=1}^{n} \|v_{i}\|_{m,\Omega}^{2}\right)^{1/2} \text{ (for functions } v = (v_{i})_{i=1}^{n} \text{ in } (H^{m}(\Omega))^{n}).$$

$$\|v\|_{m,\Omega} = \left(\sum_{i=1}^{n} \int_{\Omega} |\partial^{\alpha}v|^{p} dx\right)^{1/p}, \quad 1 \leq p < \infty.$$

$$\|v\|_{m,\rho,\Omega} = \left(\sum_{i=1}^{n} \int_{\Omega} |\partial^{\alpha}v|^{p} dx\right)^{1/p}, \quad 1 \leq p < \infty.$$

$$\|v\|_{m,\rho,\Omega} = \max_{|\alpha|=m} \left\{\text{ess. sup } |\partial^{\alpha}v(x)|\right\}.$$

$$v = \left\{w \in W^{k+1/p}(\Omega); \quad (w - v) \in P_{k}(\Omega)\right\}, \quad \text{notation in the } \text{quotient space } with the proof of the$$

 $H^{1/2}(\Gamma) = \{r \in L^2(\Gamma); \exists v \in H^1(\Omega); \text{ tr } v = r \text{ on } \Gamma\}.$

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 $||r||_{H^{1/2}(\Gamma)} = \inf\{||v||_{1,\Omega}; v \in H^1(\Omega), \text{ tr } v = r \text{ on } \Gamma\}.$

 $H^{-1/2}(\Gamma)$: dual space of $H^{1/2}(\Gamma)$.

 $\|\cdot\|_{H^{-1/2}(\Gamma)}$: norm of $H^{-1/2}(\Gamma)$.

 $\langle \cdot, \cdot \rangle_{\Gamma}$: duality pairing between the spaces $H^{-1/2}(\Gamma)$ and $H^{1/2}(\Gamma)$.

 $W_0^1(\mathbb{R}^3)$ = completion of $\mathcal{D}(\mathbb{R}^3)$ with respect to the norm $|\cdot|_{1,\mathbb{R}^3}$.

 $H(\operatorname{div};\Omega) = \{q \in (L^2(\Omega))^n; \operatorname{div} q \in L^2(\Omega)\}.$

 $\|q\|_{H(\operatorname{div};\Omega)} = (|q|_{0,\Omega}^2 + |\operatorname{div} q|_{0,\Omega}^2)^{1/2}.$

Elasticity

 λ, μ : Lamé's coefficient of a material.

$$E = \frac{\mu(3\lambda + 2\mu)}{\lambda + \mu}$$
: Young's modulus.

$$\sigma = \frac{\lambda}{2(\lambda + \mu)}$$
: Poisson's coefficient.

 $\epsilon_{ii}(v) = \frac{1}{2}(\partial_i v_i + \partial_i v_j)$: components of the (linearized) strain tensor.

 σ_{ii} : components of the stress tensor.

e: thickness of a plate, or a shell.

A: area of a cross section of an arch.

I: moment of inertia of a cross-section of an arch.

 $(\gamma_{\alpha\beta})$: strain tensor (of the middle surface of a shell).

 $(\bar{\rho}_{\alpha\beta})$: change of curvature tensor (of the middle surface of a shell).

Some spaces of polynomials

 P_k : space of all polynomials in x_1, \ldots, x_n of degree $\leq k$.

$$P'_{3} = \{ p \in P_{3}; \quad \phi_{ijk}(p) = 0, \quad 1 \le i < j < k \le n+1 \}, \quad \text{with} \quad \phi_{ijk}(p) = 0 \}$$

$$12p(a_{ijk}) + 2\sum_{l=l,j,k} p(a_l) - 3\sum_{\substack{l,m=l,j,k\\l\neq m}} p(a_{llm}) \text{ (cf. the } n\text{-simplex of type (3'))}.$$

$$P_{3}'' = \{ p \in P_{3}; \quad \psi_{ijk}(p) = 0, \quad 1 \le i < j < k \le n+1 \}, \quad \text{with} \quad \psi_{ijk}(p) = 0 \}$$

$$6p(a_{ijk}) - 2\sum_{l=i,j,k} p(a_l) - \sum_{l=i,j,k} Dp(a_l)(a_l - a_{ijk})$$
 (cf. the Hermite *n*-simplex of type (3')).

$$P_5'(K) = \{ p \in P_5(K); \ \partial_{\nu} p \in P_3(K') \text{ for each side } K' \text{ of } K \}$$

=
$$\{p \in P_5(K); \quad \chi_{ij}(\partial_{\nu}p) = 0, \quad 1 \le i < j \le 3\}, \quad \text{with} \quad \chi_{ij}(v) = 4(v(a_i) + v(a_j)) - 8v(a_{ij}) + Dv(a_i)(a_j - a_{ij}) + Dv(a_j)(a_j - a_{ij}) \text{ (cf. the Bell triangle)}.$$

 Q_k : space of all polynomials in x_1, \ldots, x_n , of degree $\leq k$ with respect to each variable x_i , $1 \leq i \leq n$.

 $Q'_2 = \{ p \in Q_2; 4p(a_9) + \sum_{i=1}^4 p(a_i) - 2 \sum_{i=5}^8 p(a_i) = 0 \}$ (cf. the rectangle of type (2')).

 $Q'_3 = \{p \in Q_3; \psi_i(p) = 0, 1 \le i \le 4\}, \text{ with } \psi_1(p) = 9p(a_{13}) + 4p(a_1) + 2p(a_2) + p(a_3) + 2p(a_4) - 6p(a_5) - 3p(a_6) - 3p(a_{11}) - 6p(a_{12}), \text{ etc.}... \text{ (cf. the rectangle of type (3')).}$

 $T_3(K)$: space of tricubic polynomials (i.e., whose restrictions along any parallel to any side of a triangle K are polynomials of degree ≤ 3 in one variable).

Notation special to Rⁿ

 e_i , $1 \le i \le n$: canonical basis of \mathbb{R}^n , also denoted e^i , for n = 3.

$$||v|| = \left(\sum_{i=1}^{n} |v_i^2|\right)^{1/2}$$
: Euclidean norm of the vector $v = (v_i)_{i=1}^n$.

 $||B|| = \sup_{v \in \mathbb{R}^n} \frac{||Bv||}{||v||}$: norm of the matrix B, induced by the Euclidean vector norm.

 $a \cdot b$: Euclidean scalar product in \mathbb{R}^n of the vectors a and b.

 $a \times b$: vector product of the vectors a and b.

det B: determinant of a square matrix B.

$$meas(A) = dx$$
-measure of a set $A \subset \mathbb{R}^n \Big(= \int_A dx \Big)$.

 $d\gamma$ = superficial measure along a Lipschitz-continuous boundary of an open subset of \mathbb{R}^n .

 $\lambda_j = \lambda_j(x)$: barycentric coordinates of a point $x \in \mathbb{R}^n$, $1 \le j \le n+1$.

$$a_{ij} = \frac{a_i + a_j}{2}, \ i < j.$$

$$a_{iij}=\frac{2a_i+a_j}{3},\ i\neq j.$$

$$a_{ijk} = \frac{a_i + a_j + a_k}{2}, i \neq j, j \neq k, k \neq i.$$

$$L_k(K) = \left\{ x = \sum_{j=1}^{n+1} \lambda_j a_j; \sum_{j=1}^{n+1} \lambda_j = 1, \lambda_j \in \left\{ 0, \frac{1}{k}, \dots, \frac{k-1}{k}, 1 \right\}, 1 \le j \le n+1 \right\}.$$

$$\hat{M}_{k} = \left\{ x = \left(\frac{i_{1}}{k}, \frac{i_{2}}{k}, \dots, \frac{i_{n}}{k} \right) \in \mathbb{R}^{n}; i_{j} \in \{0, 1, \dots, k\}, 1 \leq j \leq n \right\}.$$

$$M_k(K) = F_K(M_k), F_K: x \to F_K(x) = B_K x + b_K, B_K:$$
 diagonal matrix.

Finite Elements (most common notation)

 (K, P, Σ) or (K, P_K, Σ_K) : finite element.

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P = P_K: space of functions p, or p_K: K \to \mathbb{R}.
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 $\Sigma = \Sigma_K$: set of degrees of freedom of a finite element.

 $\varphi_i = \varphi_{i,K}$, $1 \le i \le N$: degrees of freedom of a finite element.

 $p_i = p_{i,K}$, $1 \le i \le N$: basis functions of a finite element.

 \mathcal{N}_K : set of nodes of a finite element.

 $s = s_K$: maximal order of directional derivatives found in the set Σ .

 $\Pi v = \Pi_K v = P$ -, or P_K -, interpolant of a function v.

 $\operatorname{dom} \Pi = \mathscr{C}^{s}(K).$

 $h_K = \operatorname{diam}(K)$.

 $\rho_K = \sup\{\text{diam}(S); S \text{ is a ball contained in } K\}.$

 $\hat{x} \in \hat{K} \rightarrow x = F(\hat{x}) \in K$: bijection between points of \hat{K} and $K = F(\hat{K})$ (F: bijection).

 $\hat{v}: \hat{K} \to \mathbb{R} \to v = \hat{v} \cdot F^{-1}: K \to \mathbb{R}$: bijection between functions defined over \hat{K} and $K = F(\hat{K})$ (F: bijection).

 $F \in (\hat{P})^n \Leftrightarrow F_i \in \hat{P}, \ 1 \le i \le n$, with \hat{P} : space of functions $\hat{p}: \hat{K} \subset \mathbb{R}^n \to \mathbb{R}$.

 $\vec{K} = \vec{F}(\hat{K})$, where $\vec{F} \in (P_1(\hat{K}))^n$ and $\vec{F}(\hat{a}_i) = a_i$, $1 \le i \le n+1$, for isopara $h_K = \operatorname{diam}(\tilde{K}),$ cial elements

 ρ_K = diameter of the sphere inscribed in \tilde{K} .

 $\int_{\mathbb{R}} \varphi(x) dx \sim \sum_{l=1}^{L} \omega_{l} \varphi(b_{l})$: quadrature formula with weights ω_{l} and nodes b_{l} .

$$\hat{E}(\hat{\varphi}) = \int_{\hat{K}} \hat{\varphi}(\hat{x}) \, d\hat{x} - \sum_{l=1}^{L} \hat{\omega}_{l} \hat{\varphi}(\hat{b}_{l}); \text{ quadrature error functional on } \hat{K}.$$

 $E_K(\varphi) = \int_{\mathbb{R}^N} \varphi(x) \, \mathrm{d}x - \sum_{i=1}^L \omega_{i,K} \varphi(b_{i,K})$: quadrature error functional on $K = \sum_{i=1}^L \omega_{i,K} \varphi(b_{i,K})$ $F_K(\hat{K})$, with $\omega_{l,K} = \hat{\omega}_l J_{F_K}(\hat{b}_l)$, $b_{l,K} = F_K(\hat{b}_l)$.

Finite element spaces (most common notation)

 \mathcal{F}_h : triangulation of a set $\bar{\Omega}$.

 X_k : finite element space without boundary conditions.

 $X_{0h} = \{v_h \in X_h; \ v_h = 0 \text{ on } \Gamma\}.$

 $X_{00h} = \{v_h \in X_h; v_h = \partial_\nu v_h = 0 \text{ on } \Gamma\}.$

 V_h : finite element space with boundary conditions.

 Σ_h = set of degrees of freedom of a finite element space X_h .

 φ_h or φ_{kh} , $1 \le k \le M$: degrees of freedom of a finite element space X_h .

 $(w_k)_{k=1}^M$: basis in a finite element space X_h or V_h .

 \mathcal{N}_h : set of nodes of a finite element space X_h .

 $\Pi_h v: X_h$ -interpolant of a function v.

 $\operatorname{dom} \Pi_h = \mathscr{C}^s(\bar{\Omega}), \ s = \max_{K \in \mathcal{F}_h} s_K.$

Various sets of hypotheses concerning the finite element method

(FEM 1): Existence of a triangulation.

(FEM 2): The spaces P_K , $K \in \mathcal{F}_h$, contain polynomials or "nearly polynomials".

(FEM 3): There exists a basis in the finite element space V_h whose functions have "small" support.

$$(\mathcal{T}_h 1) \colon \bar{\Omega} = \bigcup_{K \in \mathcal{T}_h} K.$$

 $(\mathcal{T}_h 2)$: $\forall K \in \mathcal{T}_h, \ \mathring{K} \neq \emptyset$.

 $(\mathcal{F}_h 3)$: $K_1 \neq K_2 \Rightarrow \mathring{K}_1 \cap \mathring{K}_2 = \emptyset$.

 $(\mathcal{F}_h 4)$: For all $K \in \mathcal{F}_h$, the boundary ∂K is Lipschitz-continuous.

 $(\mathcal{F}_h 5)$: Condition on adjacent finite elements.

(H1): Regularity of a family of triangulations.

(H2): All finite elements (K, P_K, Σ_K) , $K \in \bigcup_h \mathcal{F}_h$, are affine-equivalent to a single reference finite element.

(H3): All finite elements (K, P_K, Σ_K) , $K \in \bigcup_h \mathcal{F}_h$, are of class \mathscr{C}^0 .

(H4): The family of triangulations satisfies an inverse assumption.

(H1*): The family (K, P_K, Σ_K) , $K \in \bigcup \mathcal{F}_h$, is almost affine.

(H2*): All finite elements (K, P_K, Σ_K) , $K \in \bigcup_{h=1}^{L} \mathcal{F}_h$, are of class \mathscr{C}^1 .

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Note: An asterisk in the left margin indicates a specific finite element.

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