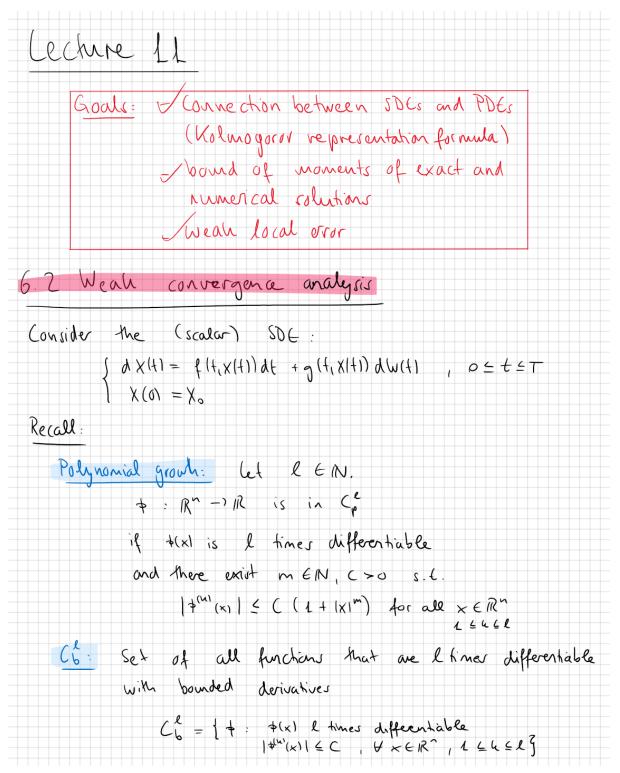


NISDE: Screenshots of Lecture 11





```
A numerical nethod (Xh)_{n\geq n} is said to have weak order of convergence r, if there exists a constant C s.t.
                1 E (+(Xn)]- E(+(X(+n))] < C 4"
       for any to = nh & COITS and all 7 & Cp
                                                         ( h Small enough)
A Connection between SDEs and PDEs
 Consider a (one-dim.) SDE for s = T
    (S) \begin{cases} \chi(s) = \chi(s,\chi(s)) & \text{if } s = \chi \\ \chi(s) = \chi & \text{if } s = \chi \end{cases}
    where x EIR
 Assume that f and g satisfy the assumptions on existence k uniqueness of a solution and denote X^{4/*}(s) the solution of this SDE.
  Next define the differential operator for a smooth
  Quection 4 (€, x) : C=17 > R ->112
         1 u(x,x) = f(x,x) 0xu(x,x) + 4 g2(x,x) 0xx u(x,x)
Further consider the parabolic PDE for SET
   (P) \begin{cases} \partial_{\xi} u(\xi,x) + \Delta u(\xi,x) = 0, & 0 \leq \xi \leq s \\ u(\xi,x) = \varphi(x) \end{cases}
Bachward holmogorov
```



```
Classical result.
     If fig: Co.T] x R -> 12 are continuous with
     f(\(\), g(\(\), \): IR →1R in (\(\) uniformly in \(\) \(\) \(\) \(\)
     and 4 € ( , m = 2
     Then there exists an unique solution u(t,x) of (P)
     with u(((x), ) ; byu((x), ) or u(x) continuous with polynomial
    growth len
  (without proof)
Than L: (Voluogorov representation formula)
    Assume that the assumptions of the classical
    result hold with m > 2.
    Then the solution u(E,x) of (P) has the
    representation
           4(4,x)= E[4(X,x))), 0585
Proof: Applying the It's formula gields
  u(s,x^{\xi\alpha}(s)) = u(\xi,X^{\xi\alpha}(\xi))
          + ) { ( > 2 mlv, x 1, x (-1) + f (r, x 1, x (-1) dx mlv, x 1, x (-1) + 2 g2 (r, x 1, x m) dx mlv, x 1, x m) dr
          Sina u solver the PDE IPI, we have
  u(s, X * (s)) - u(t, x) = ) = (r, X * (r)) > u(r, X * (r)) dw(r)
   Applying the expectation and using the proporties of Ito integrals
          E \left( \frac{\chi(s_1 | \chi^{s_1 x}(s_1))}{\varphi(\chi^{s_1 x}(s_1))} = E \left( \frac{\chi(s_1 x)}{\varphi(s_1 x)} \right) = \chi(s_1 x)
```









We make the usual assumption on find g to have existence and uniqueness.

Further we assume sufficient smoothwass to apply successively the Ito formula.

Defin: We say that a numerical nethod [Xn]==0 has bean local order r, if for any function $(p \in C_p^{2r+2})$ there exist constants $C_1 \times S$. t. $(p \in C_p^{2r+2}) = C(1 + C(1 \times 1 \times 1)) = C(1 \times 1 \times 1 \times 1)$ Rem.: If a numerical nethod has weak local order r, then we have $(p \in C_p^{2r+2}) = C(1 + C(1 \times 1 \times 1)) = C(1 \times 1 \times 1 \times 1) = C(1 \times 1$