Notation and symbols

If A and B are sets, then $x \in B$ means that "x is an element of B," whereas $A \subset B$ means that "A is contained in B." For $x \notin B$ and $A \not\subset B$ substitute "is not" for "is" in these statements. Furthermore, $A \cup B = \{x: x \in A \textit{ or } x \in B\}$, $A \cap B = \{x: x \in A \textit{ and } x \in B\}$, $A \backslash B = \{x: x \in A \textit{ and } x \notin B\}$, and $A \times B = \{(x,y): x \in A \textit{ and } y \in B\}$, respectively, define the **union, intersection, difference,** and **Cartesian product** of two sets A and B. Symbol $\emptyset$ denotes the **empty set,** and

$$1_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}$$

is the **characteristic** (or **indicator**) **function** for set A.

When $a < b$ the **closed interval** $[a,b] = \{x: a \leqslant x \leqslant b\}$, whereas the **open interval** $(a,b) = \{x: a < x < b\}$. The half-open intervals $[a,b)$ and $(a,b]$ are similarly defined. The **real line** is denoted by R, and the positive half-line by R^+. If A is a set and $\alpha \in R$, then $\alpha A = \{y: x \in A \text{ and } y = \alpha x\}$.

The notation $f: A \to B$ means that "f is a function whose **domain** is A and whose **range** is in B," or "f maps A into B." Given two functions $f: A \to B$ and $g: B \to C$, then $g \circ f$ denotes the **composition** of g with f and $g \circ f: A \to C$. If f maps R (or a subset of R) into R, and b is a positive number, then

$$g(x) = f(x) \qquad (\text{mod } b)$$

means that $g(x) = f(x) - nb$, where n is the largest integer less than or equal to $f(x)/b$. $\|f\|_{L^p}$ and $\langle f, g\rangle$, respectively, denote the L^p **norm** of the function f, and the **scalar product** of the functions f and g. $\bigvee_a^b f$ is used for the **variation** of the function f over the interval $[a,b]$.

The following is a list of the most commonly used symbols and their meaning:

a.e.	almost everywhere
$\mathscr{A}$	σ-algebra
$\mathscr{B}$	Borel σ-algebra
$d(g, \mathscr{F})$	L^1 distance between function g and $\mathscr{F}$

d_+f/dx	right lower derivative
$D, D(X, \mathcal{A}, \mu)$	set of densities
$D^2(\xi)$	variance of random variable ξ
$\mathfrak{D}(A)$	domain of an infinitesimal operator A
$E(\xi)$	mathematical expectation of a random variable ξ
$E(V \mid f)$	expected value of V with respect to f
$\mathrm{E}i(x)$	exponential integral
$\{\eta_t\}$	continuous time stochastic process
f	an element of L^p, often a density
f_*	stationary density
$\mathcal{F}$	L^p set of functions, σ-algebra in probability space
$\{\mathcal{F}_t\}$	family of σ-algebras
$g_{\sigma b}(x)$	Gaussian density with variance $\sigma^2/2b$
g_{ij}^{ϕ}	Riemannian metric
$H_n(x)$	Hermite polynomial
$H(f)$	entropy of a density f
$H(f \mid g)$	conditional entropy of f with respect to g
I	identity operator
$K(x, y)$	stochastic kernel
$L^p, L^p(X, \mathcal{A}, \mu)$	L^p space
$L^{p'}$	space adjoint to L^p
$\mu(A)$	measure of a set A
$\mu_f(A)$	measure of a set A with respect to a density f
μ_w	Wiener measure
$\{N_t\}_{t \geqslant 0}$	counting process
ω	an element of Ω; angular frequency
Ω	space of elementary events
$(\Omega, \mathcal{F}, \mathrm{prob})$	probability space
P	Markov or Frobenius–Perron operator
$P_\varepsilon, \overline{P}$	Markov operator
$\{\hat{P}_t\}_{t \geqslant 0}$	continuous semigroup generated by the linear Boltzmann equation
prob	probability measure
Prob	probability measure on a product space
R_λ	resolvent operator
S	transformation
$S^{-1}(A)$	counterimage of a set A under a transformation S
S_m	Chebyshev polynomial
S^1	unit circle
$\{S_t\}_{t \in R}, \{S_t\}_{t \geqslant 0}$	dynamical or semidynamical system

$\sigma(\xi)$	standard deviation of a random variable ξ
T	transformation
T^d	d-dimensional torus
$\{T_t\}_{t\geq 0}$	semigroup corresponding to an infinitesimal operator A
U	Koopman operator
V	Liapunov function, potential function
$\{w(t)\}_{t\geq 0}$	Wiener process
$(X, \mathcal{A}, \mu)$	measure space
ξ, ξ_i	random variables
$\{\xi_n\}, \{\xi_t\}$	discrete or continuous time stochastic process