Random time step probabilistic methods for uncertainty quantification in chaotic and geometric numerical integration

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Review for *SINUM*

I have major concerns about the aims and relevance of this work. I can best express my concerns in a sequence of related bullet points.

1. Given a deterministic ODE and a deterministic numerical method, it is not clear to me what "uncertainty" means, or why the idea needs to be introduced.

2. It is not clear to me why we need a probability measure over the numerical solution. Further, it is not clear what a successful measure would look like.

3. The abstract and certain parts of the main text give the impression that the work applies to general ODE solving. Other parts of the text imply that the approach is designed exclusively for chaotic systems and geometric integration. I will assume the latter.

4. Definion 3.1, Theorem 3.5, etc. have $k = 1, 2, \ldots, N$. I assume $Nh = T$ here, for some fixed T; i.e., convergence over finite time. (As in the statement of Theorem 4.5.)

5. The analysis in Section 4 is based on classical global Lipschitz and Gronwall inequality arguments. Without the benefit of any further explanation in the text, I can only conclude that the new approach can do just as well as a standard method, on average, over a finite time interval with an increasingly stringent choice of (random) timestep sequence. I do not see any benefits.

6. The limitation to global Lipschitz f and finite time interval (and the lack of backward error analysis) make it difficult for me to see the relevance to chaotic systems and to geometric integration. In the setting of section 4, my understanding is that the underlying fixed timestep method will have a global error expansion, and the probability measure over the numerical solution will tell us something about the leading term. But why is this any more useful than a traditional error estimate?

7. (I note that Lemma 4.5 imposes a global Lipschitz assumption on f, but this is not mentioned in Theorem 4.5, which uses the lemma.)

8. The numerical tests emphasize "capturing chaotic behavior" but this phrase is not explained, the idea is not quantified, and I do not see how the previous (finite time) analysis is relevant.

Overall, it is not clear to me what fundamental drawbacks the manuscript is addressing, nor is it clear how these drawbacks have been overcome.

[For info I note that random time stepping has been suggested in other contexts, e.g., *Exponential Timestepping with Boundary Test for Stochastic Differential Equations*, Kalvis M. Jansons and G. D. Lythe, SIAM J. Sci. Comput., 24(5), 2003, 1809–1822 and *Multidimensional Exponential Timestepping with Boundary Test*, Kalvis M. Jansons and G. D. Lythe, SIAM J. Sci. Comput., 27(3), 2005, 793–808.]