Let us consider a random variable $X \sim \log \mathcal{N}(\mu, \sigma^2)$. We want to find an expression for $\mathbb{E}|X - \mathbb{E}X|^3$. By definition

$$\mathbb{E}|X - \mathbb{E}X|^3 = \int_0^{+\infty} \left| x - \exp\left(\mu + \frac{\sigma^2}{2}\right) \right|^3 \frac{1}{x\sqrt{2\pi}\sigma} \exp\left(-\frac{(\log x - \mu)^2}{2\sigma^2}\right) \mathrm{d}x.$$

Operating the change of variable $y = (\log x - \mu)/(\sqrt{2}\sigma)$ we obtain

$$\mathbb{E}|X - \mathbb{E}X|^3 = e^{3\mu} \int_{-\infty}^{+\infty} \left| e^{\sqrt{2}\sigma y} - e^{\sigma^2/2} \right|^3 \frac{1}{\sqrt{\pi}} e^{-y^2} \, \mathrm{d}y.$$

We now split the integral and expand the cube, thus obtaining

$$\begin{aligned}
\mathbb{E}|X - \mathbb{E}X|^3 = e^{3\mu} &\int_{-\infty}^{\sigma/(2\sqrt{2})} \left(e^{3\sigma^2/2} - e^{3\sqrt{2}\sigma y} - 3e^{\sigma^2 + \sqrt{2}\sigma y} + 3e^{2\sqrt{2}y\sigma^2/2} \right) \frac{1}{\sqrt{\pi}} e^{-y^2} \, \mathrm{d}y \\
- e^{3\mu} &\int_{\sigma/(2\sqrt{2})}^{\infty} \left(e^{3\sigma^2/2} - e^{3\sqrt{2}\sigma y} - 3e^{\sigma^2 + \sqrt{2}\sigma y} + 3e^{2\sqrt{2}y\sigma^2/2} \right) \frac{1}{\sqrt{\pi}} e^{-y^2} \, \mathrm{d}y.
\end{aligned}$$

Considering each term singularly, we obtain the final expression

$$\mathbb{E}|X - \mathbb{E}X|^3 = \exp\left(3\mu + 3\frac{\sigma^2}{2}\right) \left(4 \operatorname{erf}\left(\frac{\sigma}{2\sqrt{2}}\right) - 3e^{\sigma^2} \operatorname{erf}\left(\frac{3\sigma}{2\sqrt{2}}\right) + e^{3\sigma^2} \operatorname{erf}\left(\frac{5\sigma}{2\sqrt{2}}\right) \right), \tag{1}$$

where

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} \, \mathrm{d}t.$$

Let us consider the random variable $Z = H - h$, $0 < h < 1$, where $H \sim \log \mathcal{N}(\mu, \sigma^2)$ and

$$\mu = \log h - \log \sqrt{1 + h^{2p-2}}, \quad \sigma^2 = \log(1 + h^{2p-2}),$$

with $p > 1$. Then, $\mathbb{E}Z = 0$ and the third absolute moment of Z can be expressed as in (1). Let us consider the two factors in (1) separately. For the first, we have

$$\exp\left(\mu + 3\frac{\sigma^2}{2}\right) = h^3 - \left(1 + 3h^{2p-2}\right)^{3/2} + \left(1 + 3h^{2p-2}\right)^{3/2} = h^3. \tag{2}$$

Now, let us consider the second term. We can replace the exponentials of σ^2 and obtain

$$\begin{aligned}
4 \operatorname{erf}&\left(\frac{\sigma}{2\sqrt{2}}\right) - 3e^{\sigma^2} \operatorname{erf}\left(\frac{3\sigma}{2\sqrt{2}}\right) + e^{3\sigma^2} \operatorname{erf}\left(\frac{5\sigma}{2\sqrt{2}}\right) \\
&= 4 \operatorname{erf}\left(\frac{\sigma}{2\sqrt{2}}\right) - 3\left(1 + h^{2p-2}\right) \operatorname{erf}\left(\frac{3\sigma}{2\sqrt{2}}\right) + \left(1 + h^{2p-2}\right)^3 \operatorname{erf}\left(\frac{5\sigma}{2\sqrt{2}}\right) \\
&\leq 4 \operatorname{erf}\left(\frac{\sigma}{2\sqrt{2}}\right) - 3 \operatorname{erf}\left(\frac{3\sigma}{2\sqrt{2}}\right) + \operatorname{erf}\left(\frac{5\sigma}{2\sqrt{2}}\right) \\
&\quad + Ch^{2p-2} \left(\operatorname{erf}\left(\frac{3\sigma}{2\sqrt{2}}\right) + \operatorname{erf}\left(\frac{5\sigma}{2\sqrt{2}}\right) \right).
\end{aligned}$$

where C is a positive constant. Then, since $\operatorname{erf}(x) \leq 2x/\sqrt{\pi}$, we have for the second term

$$\begin{aligned}
Ch^{2p-2} \left(\operatorname{erf}\left(\frac{3\sigma}{2\sqrt{2}}\right) + \operatorname{erf}\left(\frac{5\sigma}{2\sqrt{2}}\right) \right) &\leq Ch^{2p-2} \sqrt{\log(1 + h^{2p-2})} \\
&\leq Ch^{2p-2} h^{p-1} = Ch^{3p-3},
\end{aligned} \tag{3}$$

where the second inequality holds for $0 < h < 1$ and $p > 1$.
Let us now consider the first term, i.e.,

$$\begin{aligned} F(\sigma) &:= 4\,\mathrm{erf}\left(\frac{\sigma}{2\sqrt{2}}\right) - 3\,\mathrm{erf}\left(\frac{3\sigma}{2\sqrt{2}}\right) + \mathrm{erf}\left(\frac{5\sigma}{2\sqrt{2}}\right) \\ &= \frac{2}{\sqrt{\pi}}\left(\int_0^{\sigma/(2\sqrt{2})} e^{-t^2}\mathrm{d}t - 3\int_0^{3\sigma/(2\sqrt{2})} e^{-t^2}\mathrm{d}t + \int_0^{5\sigma/(2\sqrt{2})} e^{-t^2}\mathrm{d}t\right). \end{aligned}$$

With a change of variable in the second and third integral, and considering the Taylor expansion of the exponential, we have

$$\begin{aligned} F(\sigma) &= \frac{2}{\sqrt{\pi}}\int_0^{\sigma/(2\sqrt{2})}\left(4 - 9 + 5 + \sum_{i=1}^{\infty}\frac{(-t^2)^i - 9(-9t^2)^i + 5(-25t^2)^i}{i!}\right)\mathrm{d}t \\ &= \frac{2}{\sqrt{\pi}}\int_0^{\sigma/(2\sqrt{2})}\sum_{i=1}^{\infty}\frac{(-t^2)^i - 9(-9t^2)^i + 5(-25t^2)^i}{i!}\mathrm{d}t. \end{aligned}$$

Since $h < 1 \implies \sigma < 1$, we have for $C > 0$ independent of h

$$F(\sigma) \le C\int_0^{\sigma/(2\sqrt{2})} t^2\,\mathrm{d}t = C\left(\log(1 + h^{2p-2})\right)^{3/2} \le Ch^{3p-3}. \tag{4}$$

Combining (1), (2), (3) and (4), we get

$$\mathbb{E}|Z|^3 \le Ch^{3p}.$$