\bullet Monte Carlo estimation of the likelihood (section 3.2), given M number of trajectories of the probabilistic method. Then

$$\int \mathcal{L}\left(d, U^{h,\sigma}(\theta, \xi)\right) d\xi = \frac{1}{M} \sum_{i=1}^{M} \mathcal{L}\left(d, U_{i}^{h,\sigma}(\theta, \xi_{i})\right),$$

where the index i indicates the draws from the solution $U^{h,\sigma}$. Is it necessary to perform a Monte Carlo estimation of the likelihood at each MCMC iteration?