I apologise for the incompleteness of some comments in my first review, e.g. comment no. 86. As before, I do not include a numerical rating. The reasons are unchanged.

The revisions indicate that the authors have put in significant effort to address the feedback in the first review. In particular, the proof of Theorem 6 is much clearer, thanks to Lemma 6, Lemma 7, and Lemma 8. Minor revisions are needed to improve the readability of some of the statements and to make it clearer why some statements are true.

Currently, the equation count in the paper is 175, which is rather high. Please consider removing equation numbers, especially if the equations are not referenced later in the paper. I think the reader will be grateful for being able to quickly skip equations that are not numbered, especially when they want to understand a step in a proof and the justification was given many pages earlier. On the other hand, if a mathematical statement is repeatedly used after its first appearance, I strongly recommend putting the statement in the equation environment and labelling the statement for easy reference.

Explanatory remarks, equation numbers and references are extremely useful in making it easy for readers who prioritise rigour and therefore want to check that each assertion in a mathematical paper is properly justified. In my first report, I recommended that the authors reference equations and statements in their proofs. While the authors have added explanations and equation references in some proofs (and I am grateful for these), I think there is still room for improvement. I have made some concrete suggestions in the 'detailed comments' section below.

I thank the authors for their work on the revision and commend them for a thorough mathematical analysis of the method they have proposed.

Section 1: Modifications of Section 7
Numbers correspond to the items in this section of the authors' response letter.

1. OK.
3. OK.
5. OK. In the authors' response, all instances of `(75)' in the response should be replaced with `(73)', I believe.
6. OK.

7. OK.
8. OK.
9. OK.
10. OK.
11. OK.
12. OK.

Section 2: Reviewer #1
Numbers correspond to the items in this section of the authors' response letter. Thank you for the service of specifying in brackets the places to which my comments refer.

1. OK.
2. OK. I would recommend that a sentence be inserted into the text that mentions the boundedness of the attractor of the FitzHugh-Nagumo system and explains how this justifies treating the map x\mapsto x^\top x as a function with uniformly bounded derivatives, for readers like myself who do not know (at the time of reading) about this property and might be confused (as I was when I read the first version).
3. OK.
4. OK.
5. OK.
6. OK. Thank you for modifying Assumption 4 and for adding the useful remark, and for highlighting the changes to Definition 1, Lemma 1, and Theorem 1.
7. "We thank the referee for pointing this imprecision out." You're welcome. Thank you for your changes. I agree with the proof of Theorem 1 in the revision.
8. OK. I observed one typographical error in Assumption 4; see comment #2 in the "Detailed comments" section below.
9. OK.
10. OK.
11. OK.
12. OK.
13. OK.
14. OK.
15. OK.
16. OK.
17. OK.
18. OK. Thank you for the explanation.
19. OK.

20. OK.

Detailed comments on the first revision of the paper
Page numbers refer to the page numbers of the revision itself. As a service to the authors, the comments in the square brackets provide additional information for identifying the relevant parts of the text.

1. Page 7, lines 42-43 [proof of Lemma 1]: the period at the end of the second equation in (19) should be replaced by a comma, since (19) is followed by 'which implies'.

2. Page 8, line 14 [Assumption 4]: "... but not on Φ or h." Here \Phi should be \psi.

3. Page 8, line 18-19 [Remark 4]: "This fact will be indeed exploited ..." I would remove the 'indeed', as it places an unnatural emphasis on 'exploited' in the sentence.

4. Page 9, line 7-8 [Remark 5]: "SDE whichs depends" -> "SDE which depends"

5. Page 10, lines 19-20 [First comment after the statement of Lemma 4]: "The proof of Lemma 4 follows from the definition of the flow and the Gronwall inequality". The Gronwall inequality is not used here; it is the global Lipschitz continuity of the right-hand side / driving vector field f that is used for (37).

6. Page 10, lines 20-21 [Second comment after the statement of Lemma 4]: "... and the proof of Lemma 2 follows from the discrete Gronwall inequality". This comment should be moved so that it appears immediately after Lemma 2.

7. Page 11, lines 39-40 [text after equation (49)]: "where M is the number of number of realisations". There is an unnecessary 'number of' in the phrase that should be removed.

8. Page 12, Lines 2-3 [text before equation (54)]: "can be trivially bounded exploiting" ->"can be trivially bounded by exploiting"

9. Page 12, Lines 3-4 [text before equation (54)]: "the independence of the samples by:" I would remove the 'by'.

10. Page 12, lines 5-8 [equation and inequalities in (54)]: I understand that the second inequality follows by the Lipschitz continuity of \Phi. However, the justification for the first inequality is unclear. Please provide a more explicit proof for the first inequality.

11. Page 12, lines 20-26 [equation (56)]: I think that one must be careful here with the cases when p<q\leq 2p and 2p<q. Take the case that 2p<q, for example. In this case we want both terms in (51) to be O(h^{4p}), which means that M^{-1} must be O(h^{2p}). However, the condition that M is O(h^{-2p}) does not imply that M^{-1} is O(h^{2p}).

12. Page 12, lines 29-30 [Remark 10]: "numerical solution would be a Diract delta". It should be `Dirac', not `Diract'.

13. Page 16, lines 13-14 [text after (73)]: "which, in light of (70), satisfy $\vert \eta_j\vert \leq CH_j e^{-\kappa/H_j}$ almost surely." Since the almost sure inequality on \vert \eta_j\vert is important later on - e.g. it is referenced in the proof of (169) - I would write the inequality in an equation environment and refer to the equation whenever it is used.

14. Page 16, lines 40-42 [Lemma 6]: The formulation of the lemma needs improvement for clarity and logical order. Furthermore, since the parameter \bar{h} does not assume any particularly interesting values in the paper, it might be simpler to just replace \bar{h} to 1, so that the reader does not need to keep track of another parameter. Moreover, in Theorem 6 h is assumed to satisfy 0<h\leq 1. For example: "Suppose that Assumption 1, Assumption 3 and Assumption 6 hold true, and suppose that $0<h\leq 1$. Then the random variables η_j satisfy ..."

15. Page 16, lines 52-54 [Lemma 7]: Prior to the statement of Lemma 7, I recommend reminding the reader that the integer parameter N in Lemma 7 is not T/h.

16. Page 17, lines 23-28 [Lemma 8]: The formulation of Lemma 8 needs improvement for clarity and logical order. Although it may seem repetitive to do so, the parameter N should be properly introduced in the statement of the lemma, since Lemma 8 and Lemma 7 are presented as independent assertions.

17. Page 17, lines 40-45 [Theorem 6]: The statement of the theorem should be reformulated for clarity, e.g. "Let $0<h\leq 1$. Suppose that Assumption 1 holds with $p\geq 3/2$, and that Assumption 3, Assumption 5, and Assumption 6 hold."
- The statement of the theorem is unclear as to what is the difference between the numerical solution Y_n and the solution given by the RTS-RK method.

- I recommend breaking up the long sentence into shorter sentences that are easier to understand.
- I recommend using the full "if … then …" construction instead of writing "if … there exist a constant …", because this will make it easier for a reader to distinguish the conclusion from the hypotheses.
- I recommend stating the condition that Y_0=y_0.

18. Page 18, lines 9-10 [text before (86)]: "Hence, replacing the definition of \widetilde{Q} and \widehat{Q}_j, we get" -> "Hence, by applying the definitions (69) and (72) of \widetilde{Q} and \ widehat{Q}_j respectively, we get"

19. Page 18, lines 33-34 [text before (89)]: "Now, considering (163), we obtain". I think (163) should be replaced by (164).

20. Page 19, lines 4-6 [Equation (95)]: it would be helpful to the reader if justifications were given (e.g. by references to previous equations or inequalities) to explain why the second inequality follows from the first.

21. Page 28, line 17-18 [Caption of Figure 9]: "The mean was computed averaging 20 realisations" -> "The mean was computed by averaging 20 realisations"

22. Page 30, lines 16-18 [Conclusion]: "a novel probabilistic integrators for ODEs built on Runge-Kutta numerical integrators based on a random selection of time steps" -> "a novel probabililstic integrator built on Runge-Kutta numerical integrators with random time steps"

23. Page 31, lines 19-20 [Text after (134)]: "where C is positive constant" -> "where C is a positive constant"

24. Page 31, lines 32-33 [First equation in (137)]: In the exponential term inside the parentheses on the right-hand side, the power of e is missing a factor of h.

25. Page 31, lines 34-35 [Second equation in (137)]: The prefactor of h is missing in front of the second parentheses on the right-hand side.

26. Page 31, lines 36-37 [Third equation in (137)]: The exponent of h in the O(h^{2p+1}) term, should be 4p+1, not 2p+1.

27. Page 32, lines 15-16 [Proof of Theorem 7]: For clarity, I would write the proof of Theorem 7 as "The proof follows by replacing $\ mathcal{L}$ with $\widetilde{\mathcal{L}}$ and Lemma 1 with

Lemma 9 in the proof of Theorem \ref{thm:theorem1}."

28. Page 32, line 39 [Proof of Lemma 6]: "the desired inequlity" -> "the desired inequality"

29. Page 32, lines 39-40 [Proof of Lemma 6]: "This is because Assumption 6 implies that $M\geq 1$". Strictly speaking, Assumption 6 alone does not exclude the possibility that $0<M< 1$. However, Assumption 6 and Assumption 1.(ii), together with monotonicity of expectation, imply that M\geq 1.

30. Page 32, line 40 [Proof of Lemma 6]: "because $e^{-r\kappa/x}$ is a growing function of x, so that $e^{-r\kappa/h}\leq e^{-r\kappa/(Mh)}$". It seems to me that this explanation would be more useful if it appeared immediately after (145). It is not needed to explain how (147) follows from (146).

31. Page 32, lines 40-41 [Proof of Lemma 6]: "and because Mh can be bounded by $M\bar{h}$." As mentioned before in comment 14, it might be easier (in terms of keeping track of which parameter does what) to simply set $\bar{h}=1$.

32. Page 33, line 24 [text after (152)]: "Substituting the expressions we obtained in (150)" -> Substituting the expressions (151) and (152) in (150)"

33. Page 33, lines 58-59 [(156)]: Please give a more thorough explanation of why the first inequality in (156) is true.

34. Page 34, lines 4-9 [(157)]: All instances of C_\Delta should have the exponent 2. Moreover, as a service to the reader I would use equation references here. For example, the first inequality follows by (156), while the first equation follows by (74).

35. Page 34, lines 27-28 [text before (161)]: "Now, since $k+l\geq 2q+1$". I would add the explanation that this is because $k\geq q+1$ and $l\geq q$ in the definition of R_2(a) in (79).

36. Page 34, lines 36-38 [(163)]: I would explicitly state that the inequality and equation follow from (156) and (74) respectively.

37. Page 35, lines 20-21 [text after (172)]: "Since $\vert \eta_j\vert\leq CH_i e^{-\kappa /H_i}$" -> Here, the `i' subscripts should be replaced by `j' subscripts.

38. Page 35, lines 58-59 [Reference 6]: This paper has been published. https://epubs.siam.org/doi/abs/10.1137/17M1139357

39. Page 36, lines 31-33 [Reference 16]: This paper has been published.
https://link.springer.com/article/10.1007/s11222-019-09898-6?wt_mc=alerts.TOCjournals&utm_source=toc&utm_medium=email&utm_campaign=toc_11222_29_6

40. Page 36, lines 47-48 [Reference 21]: This paper has been published.
https://link.springer.com/article/10.1007/s11222-019-09902-z?wt_mc=alerts.TOCjournals&utm_source=toc&utm_medium=email&utm_campaign=toc_11222_29_6