All of these are standard results and definitions. The class of sub-Gaussian random variables is quite large, so it would be a good choice for our random step sizes.

Definition 1. A random variable $X \in \mathbb{R}$ is called sub-Gaussian of parameter σ^2 if $\mathbb{E}X = 0$ and its moment generating function satisfies

$$\mathbb{E}\exp(sX) \leq \exp\left(\frac{\sigma^2 s^2}{2}\right), \quad \forall s \in \mathbb{R}.$$

These random variables have light tails, i.e., their densities (if they exists) are decaying rapidly at infinity. As a consequence, all random variables taking values in a bounded set are sub-Gaussian.

Lemma 1. *Let X be sub-Gaussian of parameter σ^2. Then for any $t > 0$ it holds*

$$\Pr(X > t) \leq \exp\left(-\frac{t^2}{2\sigma^2}\right), \quad \Pr(X < -t) \leq \exp\left(-\frac{t^2}{2\sigma^2}\right).$$

The absolute moments of these random variables are bounded with functions of the parameter σ^2.

Lemma 2. *Let X be a random variable such that*

$$\Pr(|X| > t) \leq 2\exp\left(-\frac{t^2}{2\sigma^2}\right).$$

Then for any positive integer $k \geq 1$,

$$\mathbb{E}|X|^k \leq (2\sigma^2)^{k/2} k\Gamma(k/2).$$

In our case, we could choose the step sizes H such that the random variable $Z = H - h$ is sub-Gaussian of parameter $\sigma^2 = h^{2p}$. This excludes the log-normal distribution (heavy-tailed).