Solutions to Homework 7

1

Introduction Expanding the product

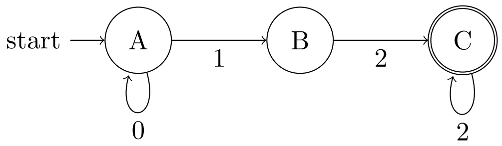
$$\dots M^{[i]} M^{[i+1]} M^{[i+2]} M^{[i+3]} \dots = \dots \begin{pmatrix} \mathbb{1} & X_i + X_{i+1} + X_{i+2} + X_{i+3} \\ 0 & \mathbb{1} \end{pmatrix} \dots$$

so, in order to close the product

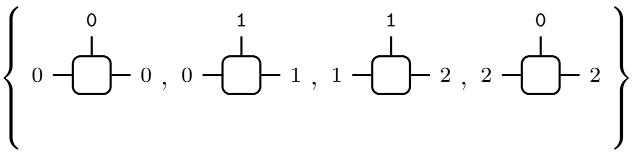
$$M^{[1]} = \begin{pmatrix} \mathbb{1} & X_1 \end{pmatrix}, \quad M^{[1]} = \begin{pmatrix} X_N \\ \mathbb{1} \end{pmatrix}.$$

Notice that in general, open boundary conditions imply that $M^{[1]i,j}_{a,b} = M^{i,j}_{1,b}$ and $M^{[N]i,j}_{a,b} = M^{i,j}_{a,\chi}$.

Nearest-Neighbor Interaction Compared to the previous example, we need to introduce an additional internal state



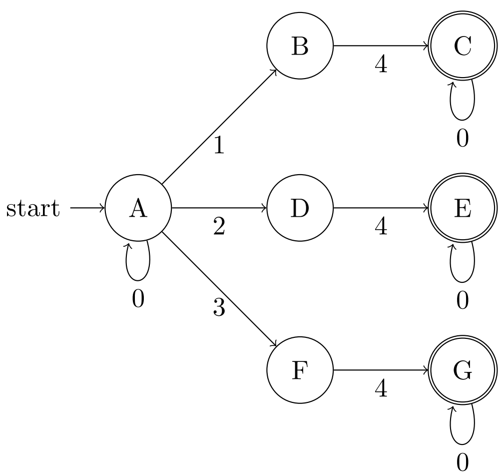
We need to add an additional rule to transition from an 1 to the next 1.



so we obtain the following MPO

$$M = \begin{pmatrix} \mathbb{1} & X & 0 \\ 0 & 0 & X \\ 0 & 0 & \mathbb{1} \end{pmatrix},$$

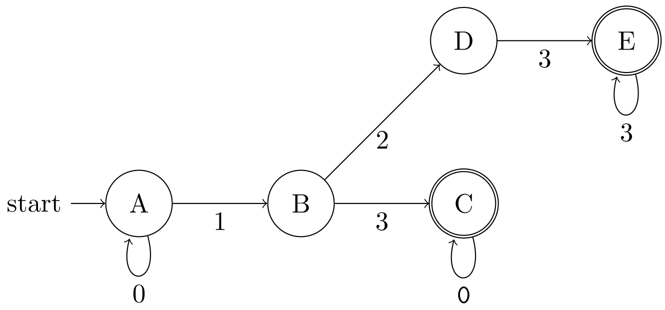
Heisenberg Model Using the labels $\mathbb{1} \sim 0$, $\sigma^x \sim 1$, $\sigma^x \sim 2$, $\sigma^z \sim 3$, we obtain the transitions map



which corresponds to

$$M = \begin{pmatrix} \mathbb{1} & \sigma^x & \sigma^y & \sigma^z & 0 \\ 0 & 0 & 0 & 0 & \sigma^z \\ 0 & 0 & 0 & 0 & \sigma^y \\ 0 & 0 & 0 & 0 & \sigma^x \\ 0 & 0 & 0 & 0 & \mathbb{1} \end{pmatrix}.$$

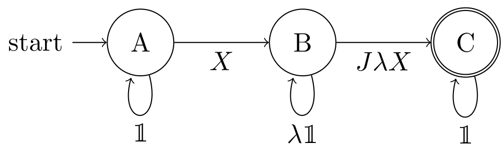
Next Nearest-Neighbor Interaction In the following map we associate the weights J_1 with node C and J_2 with node E.



The MPO is then

$$M = \begin{pmatrix} \mathbb{1} & Z & 0 & 0 \\ 0 & 0 & \mathbb{1} & J_1 Z \\ 0 & 0 & 0 & J_2 Z \\ 0 & 0 & 0 & \mathbb{1} \end{pmatrix}.$$

Long-Range Interactions By now you should be able to see that, in the finite state automaton picture, we obtain the picture below.



The interpretation is the following: once we hit a first X, we can propagate for any length r, each time hitting a λ factor, until we end up in another Z. Once we end up in the second X, we do nothing. The Hamiltonian is then the sum

$$H = \sum_i \sum_{r>0} J\lambda^r X_i X_{i+r}.$$

Defining $\lambda = e^{-1/\xi}$, we obtain the desired result.

2

No solution is presented for this exercise.