Quant II

Lab 2: Regression

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Today's plan

- DAGs
- Regression
- Heterogeneity and effective samples
- Running regressions in practice

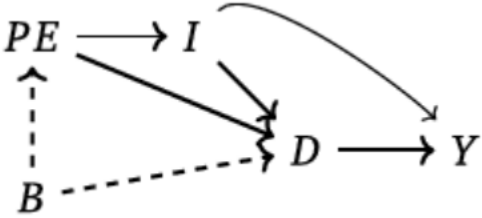
Conditioning in DAGs

Conditioning on some variable w in a DAG is equivalent to do the following steps:

- If w is a collider, link all pairs of parents of w by drawing an undirected edge between them
- For any ancestor of w, if this ancestor is itself a collider, link all pairs of parents of this ancestor with undirected edges to connote induced dependencies
- Erase w from the graph and all the edges connected with w

DAGs

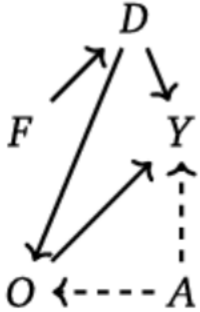
An example from the Mixtape



DAG simulations

- Let's see these properties at work, with some simulations
- Examples and code from The Mixtape, (pp. 108-113)

Case 1: the effect of gender discrimination on women income



DAG simulations

```r
library(tidyverse)
library(stargazer)

# Set seed
set.seed(123)

# Simulate our data
tb <- tibble(
  female = ifelse(runif(10000) >= 0.5, 1, 0),
  ability = rnorm(10000),
  discrimination = female,
  occupation = 1 + 2*ability + 0*female - 2*discrimination + rnorm(10000),
  wage = 1-1*discrimination + 1*occupation + 2*ability + rnorm(10000)
)

# Estimate regressions
lm_1 <- lm(wage ~ female, tb)
lm_2 <- lm(wage ~ female + occupation, tb)
lm_3 <- lm(wage ~ female + occupation + ability, tb)
```

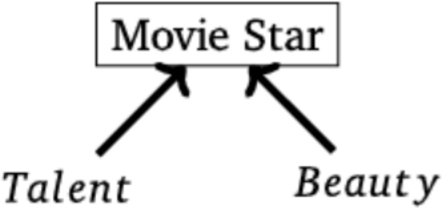
DAG simulations

```
# Compare
stargazer(lm_1, lm_2, lm_3, type = "text",
          column.labels = c("Biased unconditional",
                            "Biased",
                            "Unbiased Conditional"))
```

```
##
## ===========================================================================================================
##                                                  Dependent variable:
##                        ------------------------------------------------------------------------------------
##                                                         wage
##                           Biased unconditional           Biased               Unbiased Conditional
##                                   (1)                      (2)                        (3)
## -----------------------------------------------------------------------------------------------------------
## female                         -3.066***                0.587***                  -1.050***
##                                 (0.085)                  (0.030)                    (0.028)
##
## occupation                                              1.796***                   0.987***
##                                                          (0.006)                    (0.010)
##
## ability                                                                            2.033***
##                                                                                     (0.022)
##
## Constant                        2.023***                0.222***                   1.025***
##                                 (0.060)                  (0.020)                    (0.017)
##
## -----------------------------------------------------------------------------------------------------------
## Observations                     10,000                   10,000                     10,000
## R2                                0.114                    0.912                      0.952
## Adjusted R2                       0.114                    0.912                      0.952
## Residual Std. Error         4.265 (df = 9998)         1.347 (df = 9997)          0.994 (df = 9996)
## F Statistic          1,292.306*** (df = 1; 9998) 51,551.530*** (df = 2; 9997) 65,927.470*** (df = 3; 9996)
## ===========================================================================================================
## Note:                                                                         *p<0.1; **p<0.05; ***p<0.01
```

DAG simulations

Case 2: Talent and beauty

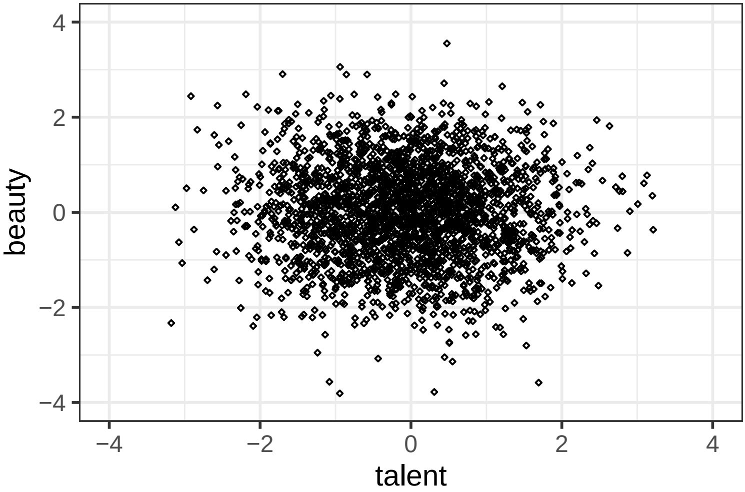


DAG simulations

```r
library(tidyverse)

# Set seed
set.seed(3444)

# Simulate data
star_is_born <- tibble(
  beauty = rnorm(2500),
  talent = rnorm(2500),
  score = beauty + talent,
  c85 = quantile(score, .85),
  star = ifelse(score >= c85, 1, 0)
)
```

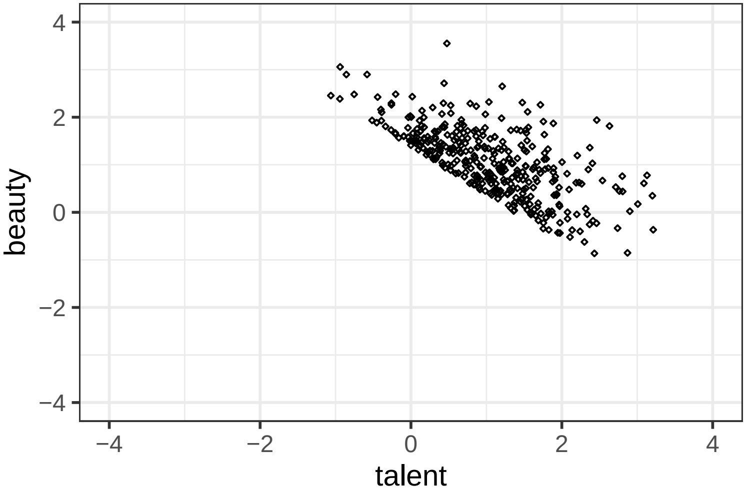
DAG simulations

```
star_is_born %>%
  lm(beauty ~ talent, .) %>%
  ggplot(aes(x = talent, y = beauty)) +
  geom_point(size = 0.5, shape = 23) + xlim(-4, 4) + ylim(-4, 4) +
  theme_bw()
```



DAG simulations

```
star_is_born %>%
  filter(star == 1) %>% lm(beauty ~ talent, .) %>%
  ggplot(aes(x = talent, y = beauty)) +
  geom_point(size = 0.5, shape = 23) + xlim(-4, 4) + ylim(-4, 4) +
  theme_bw()
```



Conclusion

- Don't control for/condition on colliders
- Endogenous sample selection is a form of collider bias!
  - See discussion in Knox et al (2020) on admin data

Additional resources:

- Elwert and Winship (2014), Endogenous selection bias: the problem of conditioning on a collider variable
- Knox, Lucas, and Cho (2022), Testing causal theories with learned proxies
- Schneider (2020), Collider bias in economic history research

Frisch-Waugh-Lovell theorem: a refresher

- We have a linear model with K covariates. In matrix form: $y = X'\beta + \epsilon$
- FWL gives a formula for the OLS estimate of the k^{th} coefficient.

$$\hat{\beta}_k = (X_k' M_{[X_{-k}]} X_k)^{-1} X_k' M_{[X_{-k}]} y$$

- In other words we can do the following:
  - Regress the individual variable X_k on all the other covariates and take the residuals
  - Regress the outcome variable y on all the covariates, except X_k, and take the residuals
  - Regress the residuals of y on the residuals for X
- Note that to get $\hat{\beta}_k$ it is enough to regress the non-residualized y on residualized X_k (because the matrix M is idempotent), but the SE won't be right
- Useful because typically we are interested in just one regressor (e.g. a treatment indicator), so we can reduce the dimensionality of the model

FWL in practice

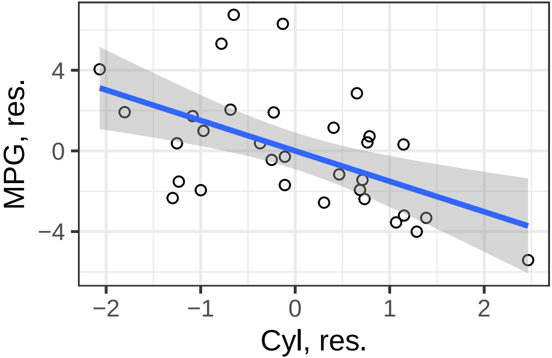
```r
# Import a dataset
data("mtcars")

# Multivariate regression
fit <- lm(mpg ~ cyl + drat + wt, mtcars)

# FWL
resy <- lm(mpg ~ drat + wt, mtcars) %>% residuals()
resx <- lm(cyl ~ drat + wt, mtcars) %>% residuals()
fit2 <- lm(resy ~ resx)

# Compare results
out <- c(coefficients(fit)["cyl"], coefficients(fit2)["resx"])
names(out) <- c("Multivariate", "Univariate Residualized")
out
```

```
##            Multivariate Univariate Residualized
##               -1.509577               -1.509577
```

FWL in practice

With residual-residual plots we can show the relationship between two variables while "controlling" for the others

```
as.data.frame(cbind(resy, resx)) %>% rename(mpg_res = resy, cyl_res = resx) %>%
  ggplot(aes(x=cyl_res, y=mpg_res)) + geom_point(size=1.5, colour="black", shape=21) +
  geom_smooth(method="lm") +
  labs(x = "Cyl, res.", y = "MPG, res.") + theme_bw()
```



Heterogeneous treatment effects

With heterogeneous treatment effects, OLS estimators have some features to be aware of

- Two groups in the population, A and B. Represented by covariate $X_i \in \{A, B\}$. We draw a random sample. We want to estimate the ATT.
- There is treatment effect heterogeneity by group: $ATT_A \neq ATT_B$. CIA holds: $(Y_{0i}, Y_{1i}) \perp D_i | X_i$
- CIA intuitively means we can do the following:
  1. Fix a value of X_i
  2. Estimate the effect of D on Y for units with that value of X_i. This is a causal estimate
  3. Repeat for all values of X_i
  4. Aggregate all these causal estimates: a weighted average

Heterogeneous treatment effects

Call α_A the share of units in group A among the treated in the population, and α_B the share of units in group B. Then,

$$
\begin{aligned}
&ATT = \alpha_A * ATT_A + \alpha_B * ATT_B \\
&\alpha_A + \alpha_B = 1
\end{aligned}
$$

We can rewrite as:

$$
\begin{aligned}
&ATT = \color{red}{\alpha_A} * ATT_A + \color{red}{\alpha_B} * ATT_B \\
&ATT = \color{red}{P(X_i = A|D_i = 1)}\{E[Y_{1i} - Y_{0i}|X_i = A, D_i = 1]\} + \\
&\color{red}{P(X_i = B|D_i = 1)}\{E[Y_{1i} - Y_{0i}|X_i = B, D_i = 1]\}
\end{aligned}
$$

The terms in red are the weights we use in the weighted average: the share of units with the same ATT.

By CIA, the terms in black are identified. So we can estimate ATT with sample analogues.

Heterogeneous treatment effects

An unbiased estimator is

$$A\hat{T}T = \frac{\sum_{x \in \{A,B\}} \hat{\delta}_X \hat{P}(D_i = 1|X_i = x)\hat{P}(X_i = x)}{\sum_{x \in \{A,B\}} \hat{P}(D_i = 1|X_i = x)\hat{P}(X_i = x)}$$

OLS regression also estimates a weighted average of individual-unit treatment effects (proof in lecture slides and MHE)

$$A\hat{T}T_R = \frac{\sum_{x \in \{A,B\}} \hat{\delta}_X \hat{P}(D_i = 1|X_i = x)(1 - \hat{P}(D_i = 1|X_i = x))\hat{P}(X_i = x)}{\sum_{x \in \{A,B\}} \hat{P}(D_i = 1|X_i = x)(1 - \hat{P}(D_i = 1|X_i = x))\hat{P}(X_i = x)}$$

This is not the weighted average we have started with: instead of weighting more the group that represents more units, it weights more the group where the treatment status has higher variance.

How distributed the treatment needs to be in a group in order for it to have the highest weight?

Effective sample

- Aronow and Samii (2016) show that under some assumptions about the functional form of the treatment assignment the following result holds:

$$\hat{\beta} \xrightarrow{p} \frac{E[w_i \tau_i]}{E[w_i]}, \text{ where } w_i = (D_i - E[D_i|X_i])^2$$

so that

$$E[w_i|X_i] = E[D_i - E[D_i|X_i]|X_i]^2 = Var[D_i|X_i]$$

- This result implies that regression re-weights units in ways that are not detectable at first sight
- Units in groups/Covariates strata where the treatment has a higher conditional variance receive more weight
- Equivalent to run the regression on an *effective* sample different from the one we think we are working with
- To characterize the effective sample we can estimate the w_is

Effective sample

Let's rewrite the last expression.

$$E[w_i|X_i] = E[(D_i - E[D_i|X_i]|X_i)^2] = Var[D_i|X_i]$$

- If we assume linearity of the treatment assignment in X_i, the weight is equal to the square of the residual of regressing the treatment indicator on X_i
- Intuition: higher conditional variance of treatment $\implies$ treatment has more residual variance not explained by the covariates $\implies$ higher error term
- The regression exploits as much as possible this identifying variation
- We can estimate the regression weights by the following procedure:
  1. Run the regression $D_i = X_i\gamma + e_i$
  2. Take residual $\hat{\epsilon}_i = D_i - X_i\hat{\gamma}$ and square it

Effective sample: example

- Let's study the effective sample in an actual paper
- Egan and Mullin (2012) look at how people form their attitudes based on personal experiences
- They use local weather variation to estimate the effect of experiencing weather changes on beliefs about global warming
- To understand the effective sample, we need to ask where weather is most variable (conditional on covariates)

Effective sample: example

```r
# Import the data
library(haven)
d <- read_dta("gwdataset.dta")

# Import state IDs
zips <- read_dta("zipcodetostate.dta")
zips <- zips %>% select(c(statenum, statefromzipfile)) %>% unique()
zips <- zips %>% filter(!(statenum == 8 & statefromzipfile == "NY"))

# Import population data
pops <- read.csv("population_ests_2013.csv")

# Format
pops$state <- tolower(pops$NAME)
d$getwarmord <- as.double(d$getwarmord)
```

Effective sample: example

```r
# Estimate primary model of interest:
d$doi <- factor(d$doi)
d$statenum <- factor(d$statenum)
d$wbnid_num <- factor(d$wbnid_num)
Y <- "getwarmord"
D <- "ddt_week"
X <- names(d)[c(15,17,42:72)]
reg_formula <- paste0(Y, "~", D, "+", paste0(X, collapse = "+"))
reg_out <- lm(as.formula(reg_formula), d)

# Or
out <- lm(getwarmord~ddt_week+educ_hsless+educ_coll+educ_postgrad+
          educ_dk+party_rep+party_leanrep+party_leandem+
          party_dem+male+raceeth_black+raceeth_hisp+
          raceeth_notwbh+raceeth_dkref+age_1824+age_2534+
          age_3544+age_5564+age_65plus+age_dk+ideo_vcons+
          ideo_conservative+ideo_liberal+ideo_vlib+ideo_dk+
          attend_1+attend_2+attend_3+attend_5+attend_6+
          attend_9+as.factor(doi)+as.factor(statenum)+
          as.factor(wbnid_num),d)
```

Base Model

```
summary(reg_out)$coefficients[1:10,]
```

```
##                  Estimate  Std. Error   t value   Pr(>|t|)
## (Intercept)   1.945740062 0.771478843 2.5220913 0.01169077
## ddt_week      0.004857915 0.002475887 1.9620908 0.04979656
## wbnid_num3103 0.843451519 0.922666490 0.9141456 0.36067588
## wbnid_num3154 1.575071541 0.973391215 1.6181280 0.10568587
## wbnid_num3159 1.903629413 1.021302199 1.8639237 0.06237963
## wbnid_num3804 1.406498119 0.794035963 1.7713280 0.07655528
## wbnid_num3810 1.330878449 0.806312016 1.6505750 0.09887602
## wbnid_num3811 1.082204367 0.798796489 1.3547936 0.17553267
## wbnid_num3812 1.219327925 0.803974284 1.5166255 0.12941222
## wbnid_num3813 0.986084952 0.829563706 1.1886790 0.23461152
```

Estimate the weights

```r
# Regress treatment indicator on the vector of covariates
D_formula <- paste0(D, "~", paste0(X, collapse = "+"))
outD <- lm(as.formula(D_formula),d)

# Extract the residuals and take their square
eD2 <- residuals(outD)^2
```

Effective sample statistics

- We can use these estimated weights to study the effective sample

```
# Take some relevant variables
compare_samples<- d[, c("wave", "ddt_week", "ddt_twoweeks",
  "ddt_threeweeks", "party_rep", "attend_1", "ideo_conservative",
  "age_1824", "educ_hsless")]

# Compute statistics with and without weights
compare_samples <- t(apply(compare_samples,2,function(x)
  c(mean(x),sd(x),weighted.mean(x,eD2),
    sqrt(weighted.mean((x-weighted.mean(x,eD2))^2,eD2)))))
colnames(compare_samples) <- c("Nominal Mean", "Nominal SD",
      "Effective Mean", "Effective SD")
```

Effective Sample Statistics

```
compare_samples
```

```
##                   Nominal Mean Nominal SD Effective Mean Effective SD
## wave                3.09693726  1.4252527     3.20788200    1.5609143
## ddt_week            3.83548593  5.9047249     5.11579140   10.8980228
## ddt_twoweeks        3.85505617  5.4572382     5.00137435    9.2262827
## ddt_threeweeks      3.96719696  4.7689594     5.10859485    8.4348180
## party_rep           0.29527208  0.4561989     0.28978321    0.4536617
## attend_1            0.11433244  0.3182383     0.12343459    0.3289354
## ideo_conservative   0.31132917  0.4630715     0.29325249    0.4552532
## age_1824            0.07195956  0.2584402     0.06881146    0.2531333
## educ_hsless         0.34151056  0.4742516     0.31219962    0.4633908
```

Effective sample maps

- We can depict the samples visually
- What places in the US are “over-represented” in the effective samples?

Effective sample maps

```
# Construct the "effective sample weights" for each state
wts_by_state <- tapply(eD2, d$statenum, sum)
wts_by_state <- wts_by_state/sum(wts_by_state)*100
wts_by_state <- data.frame(eff = wts_by_state,
                           statenum = as.numeric(names(wts_by_state)))

# Merge to the state name variable
data_for_map <- merge(wts_by_state, zips, by="statenum")

# Construct the "nominal sample weights" for each state
wts_by_state <- tapply(rep(1,6726),d$statenum,sum)
wts_by_state <- wts_by_state/sum(wts_by_state)*100
wts_by_state <- data.frame(nom = wts_by_state,
                           statenum = as.numeric(names(wts_by_state)))

# Add to the other data
data_for_map <- merge(data_for_map, wts_by_state, by="statenum")
```

Effective sample maps

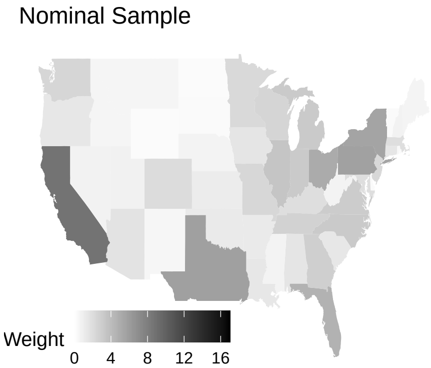
```r
# Get correct state names
require(maps,quietly=TRUE)
data(state.fips)

# Add them to the dataset
data_for_map <- left_join(data_for_map, state.fips,
                          by = c("statefromzipfile" = "abb"))

# More data prep
data_for_map$state <- sapply(as.character(data_for_map$polyname),
                             function(x)strsplit(x,":")[[1]][1])
data_for_map <- data_for_map %>% group_by(statefromzipfile) %>%
  summarise_all(first) %>% ungroup() %>% select(-polyname)

# Diff between nominal and effective weights
data_for_map$diff <- data_for_map$eff - data_for_map$nom

# Merge with population data
data_for_map <- left_join(data_for_map, pops, by="state")

# Actual "weight" of each state in the US
data_for_map$pop_pct <- data_for_map$POPESTIMATE2013/sum(
  data_for_map$POPESTIMATE2013)*100

# Different representativity of the two samples
data_for_map <- mutate(data_for_map,
                       pop_diff_eff = eff - pop_pct,
                       pop_diff_nom = nom - pop_pct)
data_for_map <- mutate(data_for_map,
                       pop_diff = pop_diff_eff - pop_diff_nom)

require(ggplot2,quietly=TRUE)
state_map <- map_data("state")
```

More setup

```r
# Plot the weights in each sample
plot_eff <- ggplot(data_for_map, aes(map_id = state)) +
  geom_map(aes(fill=eff), map = state_map) +
  expand_limits(x= state_map$long, y = state_map$lat) +
  scale_fill_continuous("% Weight", limits=c(0,17), low="white", high="black") +
  labs(title = "Effective Sample") +
  theme(legend.position=c(.2,.1),legend.direction = "horizontal",
        axis.line = element_blank(), axis.text = element_blank(),
        axis.ticks = element_blank(), axis.title = element_blank(),
        panel.background = element_blank(),
        plot.background = element_blank(),
        panel.border = element_blank(),
        panel.grid = element_blank())


plot_nom <- ggplot(data_for_map, aes(map_id = state)) +
  geom_map(aes(fill=nom), map = state_map) +
  expand_limits(x=state_map$long, y=state_map$lat) +
  scale_fill_continuous("% Weight", limits=c(0,17), low="white", high="black") +
  labs(title="Nominal Sample") +
  theme(legend.position=c(.2,.1),legend.direction = "horizontal",
        axis.line = element_blank(), axis.text = element_blank(),
        axis.ticks = element_blank(), axis.title = element_blank(),
        panel.background = element_blank(),
        plot.background = element_blank(),
        panel.border = element_blank(), panel.grid = element_blank())
```

Maps

```r
require(gridExtra,quietly=TRUE)
grid.arrange(plot_nom,plot_eff,ncol=2)
```

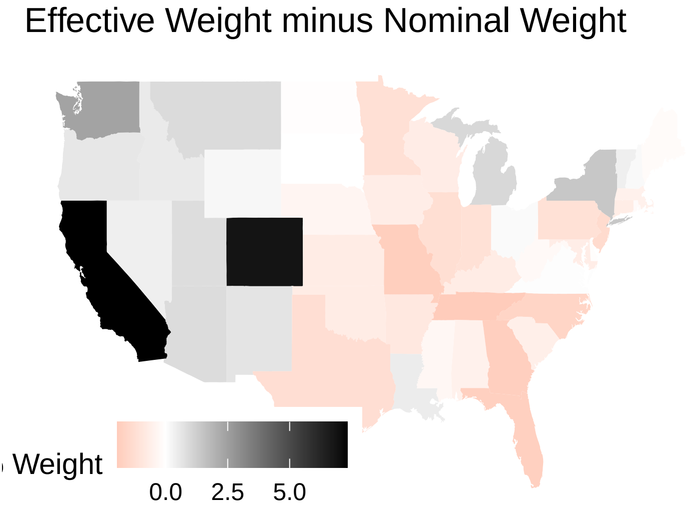


Setup comparison plot

```
plot_diff <- ggplot(data_for_map,aes(map_id=state)) +
  geom_map(aes(fill=diff), map = state_map) +
  expand_limits(x = state_map$long, y = state_map$lat) +
  scale_fill_gradient2("% Weight", low = "red", mid = "white", high = "blac
  labs(title = "Effective Weight minus Nominal Weight") +
  theme(legend.position=c(.2,.1),legend.direction = "horizontal",
        axis.line = element_blank(), axis.text = element_blank(),
        axis.ticks = element_blank(), axis.title = element_blank(),
        panel.background = element_blank(),
        plot.background = element_blank(),
        panel.border = element_blank(), panel.grid = element_blank())
```

Difference in weights

```
plot_diff
```



Linear regression in practice

- In R the command `lm()` implements the OLS estimation
- Now a variety of packages handle robust SEs and FEs much better (more on this in the coming weeks)
  - `estimatr::lm_robust()`. Good for design of randomized experiments
  - `fixest::feols()`. Good for panel data with many FEs
  - `lfe::felm()`. Once popular for panel data, now on the way to be abandoned
- Stata: `reg[ress]`. Better default than `lm()` but also superseded in contemporary empirical research
  - `xtreg`, `areg`, `reghdfe` all used in empirical research due to the handling of FEs, the latter seems to be taking over
- Packages to export output in tables: `stargazer`, `modelsummary`
- In Stata: `outreg2`, `estout`