Quant II

Lab 8: Mediators and Moderators

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Today's plan

- Mediation (review and practice)
- Moderation and TE Heterogeneity (review and practice)

Mediation

Concepts we have seen:

- Natural Direct Effect: $\zeta_i(t) = Y_i(1, M_i(t)) - Y_i(0, M_i(t))$
- "Natural Mediation Effect": $\delta_i(t) = Y_i(t, M_i(1)) - Y_i(t, M_i(0))$
- Total Effect: $\tau_i = Yi(1, Mi(1)) - Yi(0, Mi(0))$
- $\tau = \delta(t) + \zeta(1 - t)$

δ and ζ can be identified under sequential ignorability.

Sequential ignorability

$$T_i \perp (Yi(t', m), M_i(t))|X_i = x$$
$$M_i(t) \perp Y_i(t', m)|T_i, X_i = x$$

The first part is CIA, satisfied by construction in a randomized experiment.

The second part is "new": it means that there can't be omitted post-treatment confounders or other mediators causally connected to M.

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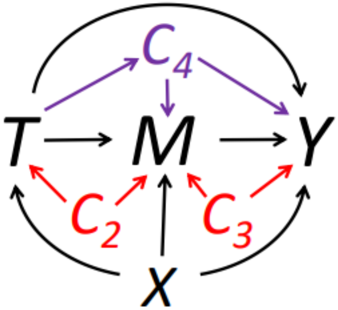
The second part is "new": it means that there can't be omitted post-treatment confounders or other mediators causally connected to M.

It is a strong assumption.

Sequential ignorability



Causal mediation analysis

The R package `mediation` performs causal mediation analysis under sequential ignorability.

Idea: Identify the effect of T on M given X and the effect of M on Y given T and X. Then use them to compute the direct/mediation effects. In the special case of linear models you can multiply the coefficients.

Causal mediation analysis

Working example from the `mediation` package: Brader et al (2008)

- T: Media stories about immigration
- Y: Letter about immigration policy to representative in Congress
- M: Anxiety
- X: Age, education, gender, income

Casual mediation analysis

```r
library(mediation)

data(framing)

set.seed(2014)

# Model for the mediator (T + X)
med.fit <- lm(emo ~ treat + age + educ + gender + income, data = framing)

# Model for the outcome (M + T + X)
out.fit <- glm(cong_mesg ~ emo + treat + age + educ + gender + income, data = framing,
               family = binomial("probit"))

# Compute the mediation effects
med.out <- mediate(med.fit, out.fit, treat = "treat", mediator = "emo",
                   robustSE = TRUE, sims = 100)
```

Causal mediation analysis

```
summary(med.out)
```

```
##
## Causal Mediation Analysis
##
## Quasi-Bayesian Confidence Intervals
##
##                          Estimate 95% CI Lower 95% CI Upper p-value
## ACME (control)             0.0791       0.0351         0.15  <2e-16 ***
## ACME (treated)             0.0804       0.0367         0.16  <2e-16 ***
## ADE (control)              0.0206      -0.0976         0.12    0.70
## ADE (treated)              0.0218      -0.1053         0.12    0.70
## Total Effect               0.1009      -0.0497         0.23    0.14
## Prop. Mediated (control)   0.6946      -6.3109         3.68    0.14
## Prop. Mediated (treated)   0.7118      -5.7936         3.50    0.14
## ACME (average)             0.0798       0.0359         0.15  <2e-16 ***
## ADE (average)              0.0212      -0.1014         0.12    0.70
## Prop. Mediated (average)   0.7032      -6.0523         3.59    0.14
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Sample Size Used: 265
##
##
## Simulations: 100
```

Controlled Direct Effect

Controlled Direct Effect: $\kappa_i(m) = Y_i(1, m) - Y_i(0, m)$

Effect of T on Y when M has the same value for all units.

Relative to NDE and NME is identified also in presence of intermediate confounders.

The natural approach would be to include M as control in the regression. If there are intermediate confounders, this introduces post-treatment bias. The CDE is an estimand that allows to overcome this issue.

Sequential g-estimation

Popularized in polisci by Acharya, Blackwell, and Sen (2016)

Procedure in two stages:

- Regress Y on M, T, pre-treatment and intermediate variables
- Subtract from Y the effect of M: this is the "demediated" Y, $\tilde{Y} = Y - \hat{\beta_M} M$
- Regress $\tilde{Y}$ on T and pre-treatment variables

This can be done by hand, but in this way one would ignore the variability in $\tilde{Y}$, due to the fact that it is estimated, resulting in wrong SEs.

Use the package `DirectEffect` or bootstrap.

Center the mediator at the value you want to "fix" it at.

Sequential g-estimation

Why is it important?

Often questions we are interested in involve comparisons that "hold fixed" things realized after the treatment.

- Do natural shocks impact political development even in absence of town destruction?
- Does ethnic diversity lead to conflict even in absence of government instability?

We may also want to rule out specific causal mechanisms

- Are the effects of slavery/famine just due to subsequent changes in racial/ethnic composition? (Acharya, Blackwell, and Sen (2016); Rozenas and Zhukov (2019) resp.)

An example

Data from Alesina, Giuliano, and Nunn (2013): provided with the `DirectEffects` package.

- Y: share of political positions held by women in 2000
- A_i: relative proportion of ethnic groups that traditionally used the plow within a country
- M_i: log GDP per capita in 2000, mean-centered
- Z_i: post-treatment, pre-mediator intermediate confounders
  - civil conflict, interstate conflict, oil, European descent, communist, polity2..)
- X_i: pre-treatment characterisrics of the country
  - tropical climate, agricultural suitability, large animals, political hierarchies, economic complexity, rugged

An example

```r
library(DirectEffects)

data("ploughs")

## ATE
ate_mod <- lm(women_politics ~ plow + agricultural_suitability + tropical_climate +
                large_animals + political_hierarchies + economic_complexity + rugged,
              data = ploughs)

summary(ate_mod)[[4]]["plow",]
```

```
##   Estimate Std. Error    t value   Pr(>|t|)
## -2.1031536  2.1270350 -0.9887725  0.3244216
```

An example

```r
## Formula for sequential_g
form_main <- women_politics ~ plow + agricultural_suitability + tropical_climate +
  large_animals + political_hierarchies + economic_complexity + rugged | # pre-treatment vars
  years_civil_conflict + years_interstate_conflict  + oil_pc + european_descent +
  communist_dummy + polity2_2000 + serv_va_gdp2000 | # intermediate vars
  centered_ln_inc + centered_ln_incsq # mediating vars

## Sequential g-estimation
direct <- sequential_g(formula = form_main, data = ploughs)
```

An example

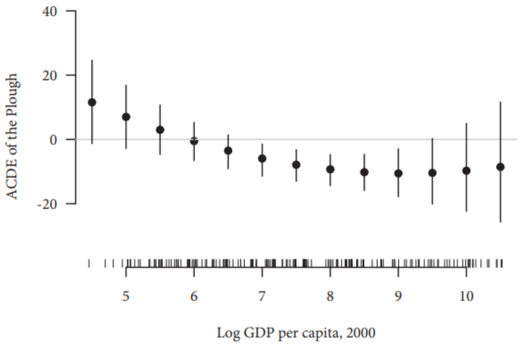
```
summary(direct)
```

```
##
## t test of coefficients:
##
##                          Estimate Std. Err. t value Pr(>|t|)
## (Intercept)              12.18450   3.64442  3.3433 0.001121 **
## plow                     -4.83879   2.34467 -2.0637 0.041312 *
## agricultural_suitability  4.57388   3.10477  1.4732 0.143458
## tropical_climate         -2.18919   2.10505 -1.0400 0.300554
## large_animals            -1.33001   3.40008 -0.3912 0.696401
## political_hierarchies     0.49575   1.09060  0.4546 0.650283
## economic_complexity      -0.10521   0.42973 -0.2448 0.807029
## rugged                   -0.30869   0.47821 -0.6455 0.519888
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

More on DirectEffects

- You can conduct sensitivity analysis using `cdesens` function
- Can center mediator at different values to see how CDE varies at different values of M

FIGURE 8. The ACDE of the Plough as a Function of the Fixed Level of Current-day Income



Note: vertical lines are 95% confidence intervals from 1,000 bootstrapped replications.

Moderation

Essentially characterizing treatment effect heterogeneity.

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Why do we care?

- Knowledge: going beyond the aggregation
- Policy: on what sub-populations the intervention is more effective
- Mechanisms: understanding what units drive the average effect gives insights about what the treatment is doing

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Broadly speaking: regression-based methods vs non-parametric methods

Moderation in regression

Classical approach is to use interaction terms. Let's start from the case of binary treatment D_i and binary moderator Z_i.

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$$y_i = \alpha + \beta D_i + \gamma Z_i + \delta D_i * Z_i + \epsilon_i$$

Assume all terms are identified. What do the coefficients measure?

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In this linear model:

- $E[y_i|D_i = 0, Z_i = 0] = \alpha$
- $E[y_i|D_i = 1, Z_i = 0] = \alpha + \beta$
- $E[y_i|D_i = 0, Z_i = 1] = \alpha + \gamma$
- $E[y_i|D_i = 1, Z_i = 1] = \alpha + \beta + \gamma + \delta$

Moderation in regression

$$y_i = \alpha + \beta D_i + \gamma Z_i + \delta D_i * Z_i + \epsilon_i$$

We have:

- $E[y_i|D_i = 0, Z_i = 0] = \alpha$
- $E[y_i|D_i = 1, Z_i = 0] = \alpha + \beta$
- $E[y_i|D_i = 0, Z_i = 1] = \alpha + \gamma$
- $E[y_i|D_i = 1, Z_i = 1] = \alpha + \beta + \gamma + \delta$

Thus:

- $\beta = E[y_i|D_i = 1, Z_i = 0] - E[y_i|D_i = 0, Z_i = 0]$: effect of D_i when $Z_i = 0$
- $\gamma = E[y_i|D_i = 0, Z_i = 1] - E[y_1|D_i = 0, Z_i = 0]$: effect of Z_i when $D_i = 0$
- $\delta = [E[y_i|D_i = 1, Z_i = 1] - E[y_i|D_i = 0, Z_i = 1]] - [E[y_i|D_i = 1, Z_i = 0] - E[y_i|D_i = 0, Z_i = 0]]$: increase in the effect of D_i when Z_i goes from 0 to 1: a DiD!

Moderation in regression

With continuous D_i and/or Z_i: restate in terms of marginal effects (increase the variable by 1 unit).

Which of the above are of interest? Depends on what we want!

In the standard DiD, what we care about is the interaction term alone, i.e. the ATT estimate under parallel trends.

In this setting, we care about how ATE/ATT varies between strata of Z. We need to be careful about how to interpret each coefficient.

Interpreting moderation in regression

Now let's generalize to continuous Z.

$$y_i = \alpha + \beta D_i + \gamma Z_i + \delta D_i * Z_i + \epsilon_i$$

Recall:

- β: effect of D_i when $Z_i = 0$
- δ: increase in the effect of D_i when Z_i increases by 1
- $\beta + \delta * z$: average effect of D_i when $Z_i = z$

β is an ATE for a specific sub-group without necessarily a substantive value: may not even exist in the data.

If instead we center Z_i, e.g. interact by $Z^* = (Z_i - \bar{Z}_i)$ then β is the ATE at the mean of the moderator (interpretable as population ATE).

Moderation effect vs sub-group effects

δ tells us by how much the ATE *varies* in a sub-group relative to a reference sub-group.

It is *not* the ATE for a subgroup. E.g. ATE(z) is given by $\beta + \delta * z$.

We can compute the effect of D for sub-groups with different values of Z using standard packages:

- `margins` in Stata
- `margins` and `marginaleffects` in R

Issues with linear interaction terms

Researchers use linear interaction terms to study how the treatment effect evolves over the distribution of the moderator.

Hainmueller, Mummolo, and Xu (2019) point out that this practice relies on requirements that can be violated:

- The TE changes linearly in the moderator at any point of its distribution
  - It may be non-linear or non-monotonic
- There is common support between treatment and moderator
  - If not, we rely on extrapolation from the linear model (recall our discussion about conditioning)

If these fail, our estimates can be model dependent.

Working example

Slavery, Reconstruction, and Bureaucratic Capacity in the American South

PAVITHRA SURYANARAYAN *Johns Hopkins University*
STEVEN WHITE *Syracuse University*

Conventional political economy models predict taxation will increase after franchise expansion to low-income voters. Yet, contrary to expectations, in ranked societies—where social status is a cleavage—elites can instead build cross-class coalitions to undertake a strategy of bureaucratic weakening *to limit future redistributive taxation. We study a case where status hierarchies were particularly extreme: the post-Civil War American South. During Reconstruction, under federal oversight, per capita taxation was higher in counties where slavery had been more extensive before the war, as predicted by standard theoretical models. After Reconstruction ended, however, taxes fell and bureaucratic capacity was weaker where slavery had been widespread. Moreover, higher intrawhite economic inequality was associated with lower taxes and weaker capacity after Reconstruction in formerly high-slavery counties. These findings on the interaction between intrawhite economic inequality and pre-War slavery suggest that elites built cross-class coalitions against taxation where whites sought to protect their racial status.*

interflex

Hainmueller, Mummolo, and Xu (2019) propose a more flexible procedure implemented by the `interflex` package (in both R and Stata)

Binning estimator

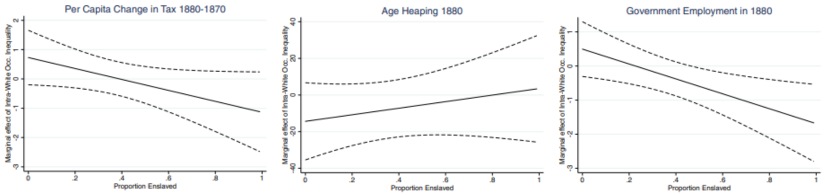
Divide the support of Z into j bins (e.g. terciles), indicated by G_j, and estimate

$$y_{ij} = \sum_{j=1}^{3} \{\alpha_j + \beta_j D_{ij} + \gamma_j (Z_{ij} - Z_j^M) + \delta_j (Z_{ij} - Z_j^M) D_{ij}\} G_j + \psi X_{ij} + \epsilon_{ij}$$

where Z_j^M is the median value of Z inside bin j.Given the specification β_js are the conditional ATEs at the center of each bin.

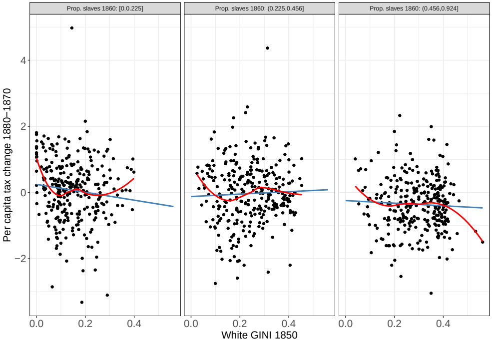
Moderation using the binning estimator

FIGURE 7. Marginal Effect of Intrawhite Inequality on Taxation and Bureaucratic Quality



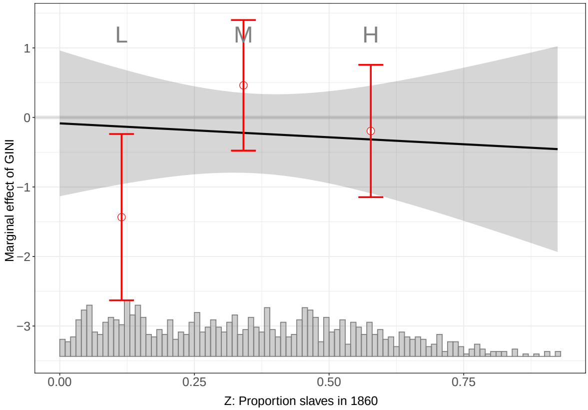
Moderation using the binning estimator

```
library(interflex); library(haven); library(tidyverse)

# Import the data
d <- read_dta("suri_white_preprocessed.dta") %>% as.data.frame()

# Raw data plot
# Note we are not including controls used in the paper, so no causal interpretation
interflex(estimator = "raw", data = d, Y = "tax_diff", D = "county_sei_gini_whitemale_1850",
                X = "pslave1860", ylab = "Marginal effect of GINI",
                xlab = "Z: Proportion slaves in 1860", theme.bw = T, ncols=3,
          Dlabel = "White GINI 1850", Ylabel = "Per capita tax change 1880-1870",
          Xlabel = "Prop. slaves 1860")
```



Moderation using the binning estimator

```
# Heterogeneous TE with interflex (note in this notation Z and X are inverted)
out <- interflex(estimator = "binning", data = d,
                 Y = "tax_diff", D = "county_sei_gini_whitemale_1850",
                 X = "pslave1860", ylab = "Marginal effect of GINI",
                 xlab = "Z: Proportion slaves in 1860", theme.bw = T)

out$figure
```



interflex

Kernel estimator

Allow the treatment effect to vary over the whole distribution of the moderator, estimating the following semiparametric model (again, notation in their paper inverts X and Z)

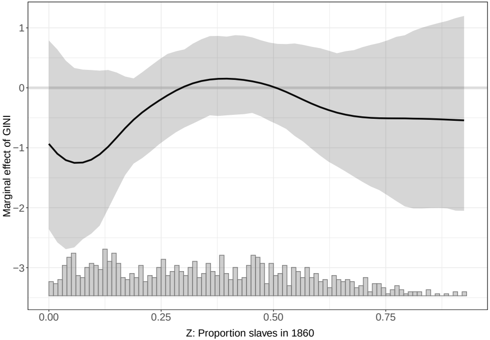
$$y_i = f(Z_i) + g(Z_i)D_i + h(Z_i)X_i + \epsilon_i$$

Moderation with the kernel estimator

```
set.seed(123)
outk <- interflex(estimator = "kernel", data = d,
                  Y = "tax_diff", D = "county_sei_gini_whitemale_1850",
                  X = "pslave1860", ylab = "Marginal effect of GINI",
                  xlab = "Z: Proportion slaves in 1860", theme.bw = T)
```

```
## Cross-validating bandwidth ...
## Parallel computing with 4 cores...
## Optimal bw=0.1222.
## Number of evaluation points:50
## Parallel computing with 4 cores...
##
```

```
outk$figure
```



Diagnostic tools

`interflex` also gives diagnostic tools for model specification.

For example, Wald tests for the hypothesis that the simple linear interaction is correct.

With a slight reparametrization, the null hypothesis is that the coefficients within each bin but one are jointly 0, i.e. constant coefficients.

```
out$tests$p.wald
```

```
## [1] "0.151"
```