Quant II

Lab 3: DAG review, bootstrap, clustering

Giacomo Lemoli

February 9, 2023

Today's plan

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- DAG Review
- Bootstrap

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- Bootstrap
- Clustering

Housekeeping

- Homework 1 due February 14 before class
- Email to me, cc Cyrus
- Send code and output (.Rmd or pdf + file)
- Math can be scanned and attached (or from tablet), no need to compile in LaTeX

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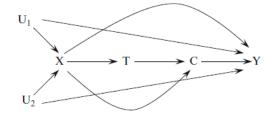
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 - Exclusion restrictions: where assumptions are
- Acyclic: cannot come back to starting point

Important concepts:

- \bullet A \rightarrow B: B is children of A, A is parent of B
- A \rightarrow C \leftarrow B: C is a *collider*
- Causal path (relative to T): arrows go from T towards Y
- Non-causal path (relative to T): the others
- Backdoor path: non-causal path between T and Y
- Colliders block non-causal paths
- Conditioning on colliders open new non-causal paths



Conditioning on some variable w in a DAG is equivalent to do the following steps:

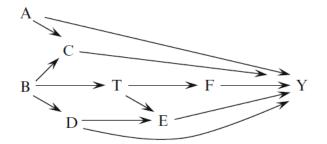
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- For any ancestor of w, if this ancestor is itself a collider, link all pairs
 of parents of this ancestor with undirected edges to connote induced
 dependencies
- Erase w from the graph and all the edges connected with w



Collider bias

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- Parametric, Semi-parametric, Non-parametric
- Non-parametric: based on re-sampling
- Random re-sampling from the sample approximates random sampling from the population
- The sampling distribution of statistics from re-samples approximates the true sampling distribution

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Non-parametric bootstrap

- From our sample of size N, draw a random sample of size N with replacement
- On this sample, compute the estimate
- 3 Repeat many times (e.g. 1000)
- Obtain a distribution of estimates from the resamples (a bootstrapped sampling distribution)
- For inference, compute moments of the bootstrapped distribution:
 e.g. sd, quantiles

Non-parametric bootstrap

Advantages:

- No distributional assumptions
- Inference on functions of estimators

```
set.seed(123)
# Population
pop <- rnorm(10000)

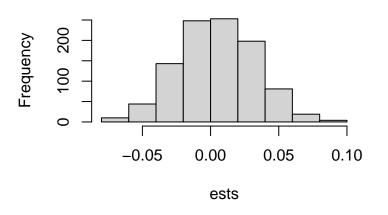
# Sample
S <- sample(pop, 500)

# Estimate Mean(x) - Median(x)
delta <- mean(S)-median(S)

# Bootstrap inference
nboot <- 1000
ests <- rep(NA, nboot)
for (i in 1:nboot){
   ids <- sample(1:length(S), length(S), replace=T)
   s <- S[ids]
   ests[i] <- mean(s)-median(s)
}</pre>
```

```
# Bootstrapped distribution
hist(ests, main="Bootstrapped sampling distribution")
```

Bootstrapped sampling distribution



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-0.03465047 0.07370951

```
# SE

sd(ests)

## [1] 0.02840985

# CI

delta + quantile(ests, c(0.025, 0.975))

## 2.5% 97.5%
```

Non-parametric bootstrap

We can also use the boot function

```
set.seed(123)
library(boot)
# Function for the bootstrap estimate
bootfn <- function(data, id){
    s <- data[id]
    out <- mean(s)-median(s)
    out
}
bs <- boot(S, statistic = bootfn, R=1000)</pre>
```

[1] 0.03066007

```
# Output:
## ORDINARY NONPARAMETRIC BOOTSTRAP
## Call:
## boot(data = S, statistic = bootfn, R = 1000)
## Bootstrap Statistics :
         original bias std. error
## t1* 0.01479699 -0.01066407 0.03066007
# Bias:
mean(bs[["t"]])-delta
## [1] -0.01066407
# Std. error
sd(bs[["t"]])
```

- Code manually
- lm + sandwich

qsec

```
library(sandwich); library(lmtest)
set.seed(000)
data("mtcars")
fit <- lm(mpg ~ qsec, mtcars)
coeftest(fit, vcov=vcovBS(fit, cluster=NULL, R=1000))
## t test of coefficients:
              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -5.11404 8.47717 -0.6033 0.550862
```

1.41212 0.48884 2.8887 0.007116 ** ## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1

• lm + car

```
## Call:
## Coll:
## Coll:
## Coll:
## Coll:
## boot::boot(data = dd, statistic = boot.f, R = R, .fn = f, parallel = p_type,
## ncpus = ncores, cl = cl2)
##
## Bootstrap Statistics:
## original bias std. error
## tt* -5.114038 -1.07603420 9.1139840
## t2* 1.412125 0.05996816 0.5247496
## You can replace coef with more complex functions
```

In Stata:

```
** Regression with bootstrap SEs set seed 000 reg mpg qsec, vce(bootstrap, reps(1000))
```

Output:

Linear regression

```
      Number of obs
      =
      32

      Replications
      =
      1,000

      Wald chi2(1)
      =
      9.01

      Prob > chi2
      =
      0.0027

      R-squared
      =
      0.1753

      Adj R-squared
      =
      0.1478

      Root MSE
      =
      5.5637
```

mpg	Observed Coef.	Bootstrap Std. Err.	Z	P> z		-based Interval]
qsec	1.412125	.4705516	3.00	0.003	.4898608	2.334389
_cons	-5.11404	8.153858	-0.63	0.531	-21.09531	10.86723

Clustering

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 - For instance, we randomize a treatment at the housing block level, assignment is correlated within cluster
- In both cases, relative to simple random process, units are contributing less information
- Extreme case: perfect correlation
- If we treat units as independent, we overstate the information in the data, and underestimate the variance of the estimator

Guide to empirical practice

Source: MacKinnon, Nielsen, and Webb (JE, 2023)

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 - $\bullet\,$ E.g. individual-level data, geographic distances at place level
- Tests for cluster level (Ibragimov and Mueller): estimate model for each cluster and compare variation in estimates with the one under finer clusters. Only works if treatment varies within cluster

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 - Influential cluster (analogous to single observations)
- Recommended to plot distribution of estimates from exclusion of each cluster sequentially

Main problems:

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 - Example: Delaware contains 50+% of US incorporations
- The opposite (a few small clusters) is generally not problematic
- Only a few clusters have treated observations

Summary guide

- List all possible clustering dimensions and levels for the data, make informed decision regarding clustering structure
- For each possible level, report the n. of clusters and the distribution of cluster sizes
- For key regression specifications, report info on leverage, partial leverage and influence, especially the effective number of clusters for coefficients of interest => summclust in Stata
 - Is leverage collinear with cluster size?
- In addition to the CV1 estimator (the most common), use also the jack-knife based CV3 (\implies summclust) and WCR (\implies boottest)
- If treatment at cluster level and few treated/control clusters, check with other methods (e.g. randomization inference)

If you have many clusters of similar size and good treatment variation across clusters you are fine

Summclust

G*(0) is 16.18657

summclust mw, yvar(hours2) xvar(black female) fevar(educ age year state) cluster(state) rho(0.5) jack

					clustered by		
Regressio	on O	utput					
s.e.		Coeff	Sd. Err.	t-stat	P value	CI-lower	CI-upper
CV3 -0.15389		.153891 .153891 .153891	0.067127			-0.279051 -0.288720 -0.288720	-0.019061
Cluster V				rage	Partial L.	bata no d	
Statisti	Lo	Ng	Deve	Luge	rurciur mi	Deca no g	
ni	in	258.0	1.01	5250	0.000837	-0.177645	_
mi	in	258.0 2495.0	1.01	5250 6757	0.000837	-0.177645 -0.156898	
mi media	in q1	258.0 2495.0 7082.0	1.01 1.13 1.41	5250 6757 3468	0.000837 0.003932 0.009442	-0.177645 -0.156898 -0.153682	
mi media mea	in q1 an	258.0 2495.0 7082.0 9663.3	1.01 1.13 1.41 1.54	5250 6757 3468 9020	0.000837 0.003932 0.009442 0.019608	-0.177645 -0.156898 -0.153682 -0.153896	_
mi media mea	in g1 an an	258.0 2495.0 7082.0	1.01 1.13 1.41 1.54	5250 6757 3468 9020	0.000837 0.003932 0.009442 0.019608	-0.177645 -0.156898 -0.153682 -0.153896 -0.150470	

Summclust

Table 3
Summary statistics for cluster heterogeneity.

Clustering	G	$G^*_{\beta}(0)$	\bar{N}_g	min.	1st quart.	Median	3rd quart.	max.
Hours data: N	= 492,827							
State-year	765	79.4	644	6	176	480	860	3,052
State	51	16.2	9,663	258	2,495	7,082	13,481	35,995
Year	15	6.6	32,855	28,262	28,839	30,733	40,224	40,394
Region	9	7.5	54,759	27,849	37,396	50,489	65,389	96,337
Employment a	nd student da	ata: $N = 1,531$,360					
State-year	765	66.0	2,002	42	524	1,413	2,426	10,794
State	51	13.1	30,027	927	7,363	22,845	37,020	144,914
Year	15	6.5	102,091	92,701	95,589	102,319	108,858	110,528
Region	9	7.0	170,151	74,172	104,703	181,767	208,099	291,955

Notes: The values of $G_{\beta}^{\epsilon}(0)$ are calculated using 28 regressors after the state dummies have been partialed out. The β subscript emphasizes the fact that they correspond to the coefficient β in (40), Because there are state fixed effects, values of $G_{\beta}^{\epsilon}(1)$ are not reported; see MacKinnon et al. (2022b).

Cluster-robust standard errors in regression

-0 15389068 0 06231373 -2 46961125 0 01352633

```
library(sandwich)
# Work with MNW example dataset
d <- read.csv("min_wage_teen_hours2.csv")

# lm + sandwich
fit <- lm(hours2 ~ mw + black + female + factor(educ) + factor(age) + factor(year) + factor(statefip), d)

coeftest(fit, vcov=vcovCL(fit, cluster = ~ statefip))["mw",]

## Estimate Std. Error t value Pr(>|t|)
```

```
# lfe
library(lfe)
fit <- felm(hours2 ~ mw + black + female | educ + age + year + statefip | 0 | statefip, d)
summary(fit)
##
## Call:
     felm(formula = hours2 ~ mw + black + female | educ + age + year + statefip | 0 | statefip, data = d)
## Residuals:
             10 Median
      Min
                            30
## -35.152 -6.934 -1.124 5.994 47.645
##
## Coefficients:
         Estimate Cluster s.e. t value Pr(>|t|)
       -0.15389
                      0.06231 -2.470
## mw
                                         0.017 *
## black 0 98832 0 11714 8 437 3 54e-11 ***
## female -2.61304 0.08368 -31.227 < 2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 9.887 on 492748 degrees of freedom
## Multiple R-squared(full model): 0.1927 Adjusted R-squared: 0.1926
## Multiple R-squared(proj model): 0.01753 Adjusted R-squared: 0.01738
## F-statistic(full model, *iid*): 1508 on 78 and 492748 DF, p-value: < 2.2e-16
## F-statistic(proj model): 357.1 on 3 and 50 DF, p-value: < 2.2e-16
## *** Standard errors may be too high due to more than 2 groups and exactDOF=FALSE
```

0.01753

Within R2

```
# fixest
library(fixest)
fit <- feols(hours2 ~ mw + black + female | educ + age + year + statefip, cluster=~statefip, data=d)
etable(fit, keep = c("mw"))
                              fit
## Dependent Var.:
                          hours2
       -0.1539* (0.0623)
## Fixed-Effects: -----
## educ
                              Yes
## age
                              Yes
                              Yes
## year
                              Yes
## statefip
## S.E.: Clustered
                    by: statefip
                        492,827
## Observations
                         0.19270
## R2
```

Cluster-robust standard errors in regression

In Stata:

vce(cluster clustervar)

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- Re-estimate the model for all parameters and find $\hat{\beta}_h$

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- Compute *t*-statistic *t_b* from the new estimates
- Repeat many times
- ullet Compute bootstrap p-values by counting the share of simulated t_b to the left/right of the observed one t

- In Stata: boottest
- Computes bootstrap p-value and confidence interval, but not the standard error

```
. boottest mw, cluster(styear) noci reps(999) seed(123)

Overriding estimator's cluster/robust settings with cluster(styear)

Wild bootstrap-t, null imposed, 999 replications, Wald test, clustering by styear, bootstrap clustering by styear, Rademacher weights:

mw

t(764) = -3.3823

Proby|t| = 0.0020
```

In R:

• Option cluster in sandwich::vcovBS()

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- The package fwildclusterboot is a translation of Stata's boottest (same options)
 - Function boottest works with objects of class lm, felm, fixest
- Another option is the package multiwaycov and the function cluster.boot which can be used for post-estimation SE calculation (e.g. in coeftest or stargazer)

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