POLITECNICO
MILANO 1863

Assignment 2 RM - Group 11

Financial Engineering - A.A. 2023-2024

Group Components:

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Q1: Baseline Case

Present Value

Warning: In the develop of the code, we haven't fixed an initial seed. Thus the results can change on some decimals level. The main requirements of the section were to compute the Forward Values at 1y of the Investment Grade rate, in order to do that we introduced a new function called FV_risky_bond. The method implemented takes consideration of a single factor structure only, hence $\rho = 0$.

The Forward Values can be computed as sum of the semiannual coupons discounted by the defaultable discount factor. Moreover it's necessary to take into account the recovery rate Π if the default happens in the future time:

- **Case "Possible Future Default after 1y"**:

$$FV_{1y} = \sum_{i=0}^{N} \bar{c}_i \cdot \bar{B}_{1y}(t_{1y}, t_i) + 100 \cdot \pi \cdot \sum_{i=0}^{N-1} B_{1y}(t_{1y}, t_{i+1}) \cdot (P(t_{1y}, t_i) - P(t_{1y}, t_{i+1}))$$

- **Case "Certain Default before 1y"**:

$$FV_{1y} = 100 \cdot \pi$$

The results obtained can be seen in the following table:

	IG	HY	Default
FV	100.51	98.43	40

Table 1: Forward Values from the IG state

The **Expected Forward Value (EFV)** can be instead computed through the formula $EFV_{IG} = \sum_i FV_i * Q_{IG,i}$ and the result obtained was $EFV = 99.76$.

In order to simulate numerically the migration and default probability from the transition table, we applied a Montecarlo simulation using the Gordy Model. The Gordy Model took care of the correlation between different issuers on the available bonds, meanwhile the Montecarlo simulation refined the computation of the final probability. The probabilities computed at the end of the process were **identical to the ones given by the text** of the assignment.

VaR 99.9% - Only Default

The VaR can be obtained from the previous Montecarlo simulation estimating the quantile at "$1 - 99.9\%$", considering only the default losses. The result obtained was $VaR_{99.9\%} = 1.49$.

VaR 99.9% - Default and Migration

The VaR can be obtained from the previous Montecarlo simulation estimating the quantile at "$1 - 99.9\%$", considering both default losses and migration changes. The result obtained was $VaR_{99.9\%} = 1.25$.

Q2: AVR Correlation Significance

The Q2 required to change the **AVRs correlation** in the computation of the VaR, considering 200 issuers and taking into account defaults and migrations. The results are represented in the table below.

	$\rho = \sqrt{0.12}$	$\rho = \sqrt{0.2135}$	$\rho = \sqrt{0.24}$
VaR	4.21	6.93	7.80

Table 2: Forward Values from the IG state

Q3: Concentration Significance

The Q3 required to change the **Concentration Risk** in the framework, thus we used only 20 bonds issuers instead of the 200 issuers of previous questions. The results obtained can be seen in the following table:

	$\rho = \sqrt{0.12}$	$\rho = \sqrt{0.2135}$	$\rho = \sqrt{0.24}$
VaR	8.84	9.68	9.99

Table 3: VaR Values changing concentration

Discussion

Q: Inclusion of migration risk at very high confidence level has no material impact on VaR measurement if the portfolio is well diversified ?

A: False. In a well diversified portfolio, $\rho \to 0$, the difference between the VaR Default and Migration and Only Default is low but we cannot consider it with no material effect. The low effect generated adjusting the VaR to a high notional will correspond to a quite huge change.

Q: Portfolio VaR is very sensitive to AVR correlations ?

A: True. As we have discussed in Q2, changing the correlation coefficient will have a high impact on the VaR value. This can be seen in the table provided for Q2.

Q: Inclusion of migration risk causes the increase of VaR under any correlation assumptions ?

A: True, for high number of issuers. Changing the AVR correlation the VaR Default and Migration will always be greater than the one computed in case of Only Default. In the following table, we have reported the differences between the values for 200 issuers and 10M simulations:

	$\rho = \sqrt{0.12}$	$\rho = \sqrt{0.2135}$	$\rho = \sqrt{0.24}$
VaR - Only Default	3.88	6.27	7.17
VaR - Default and Migration	4.21	6.93	7.80
Difference	0.33	0.66	0.63

Table 4: VaR Only Default - Default and Migration

It's necessary to notice that if reduce the number of issuers to a lower value, in case of $\rho \to 0$ we will have a decrease of VaR with respect to the case of Only Default.

Q: A Credit Portfolio Model is not sensitive to concentration risk if it is based on a single systematic factor ?

A: False. A Credit Portfolio Model based on a single systematic factor, $\rho = 0$, is sensitive to concentration risk as shown in the following table:

	Number of Issuers = 20	Number of Issuers = 200
VaR - Only Default	5.98	1.49
VaR - Default and Migration	5.82	1.25

Table 5: VaR - Changing issuers