In the $K\pi$ mixing analysis, we computed confidence contours in 2 dimensions, $n = 2$. This was done by computing $-2\Delta \log \mathcal{L}$, that is, by taking twice the difference in log likelihood between the fit with mixing and another fit where $x'^2 = y' = 0$, but allowing all the other parameters which were allowed to float in the standard mixing fit to float here as well. It was shown using toy Monte Carlo, that $-2\Delta \log \mathcal{L}$ was distributed like χ^2 for $n = 2$ degrees of freedom. We would like to construct regions in parameter space which encompasss the true parameter value with some probability, called the *coverage probability*, $1 - \alpha$

The probability α for exceeding χ^2 with n degrees of freedom is given by

$$\alpha(\chi^2; n) = \left[2^{n/2}\Gamma\left(\frac{n}{2}\right)\right]^{-1} \int_{\chi^2}^{\infty} t^{\frac{n}{2}-1} e^{-t/2} dt \tag{1}$$

$n = 1$: Using $\Gamma(1/2) = \sqrt{\pi}$, Eq. 1 becomes:

$$\alpha(\chi^2; 1) = \frac{1}{\sqrt{2\pi}} \int_{\chi^2}^{\infty} \frac{1}{\sqrt{t}} e^{-t/2} dt \tag{2}$$

Using the variable substitution $t/2 = y^2$, Eq. 2 becomes

$$\alpha(\chi^2; 1) = \frac{2}{\sqrt{\pi}} \int_{\sqrt{\frac{\chi^2}{2}}}^{\infty} e^{-t} dt = 1 - \mathrm{erf}\left[\sqrt{\chi^2/2}\right] \tag{3}$$

n=2: Eq. 1 becomes:

$$\alpha(\chi^2; 2) = \frac{1}{2} \int_{\chi^2}^{\infty} e^{-t/2} dt = e^{-\chi^2/2} \tag{4}$$

For higher values of n, the following recursion relation can be used:

$$\alpha(\chi^2; n+2) = \alpha(\chi^2; n) + \frac{\left[\chi^2/2\right]^{n/2} e^{-\chi^2/2}}{\Gamma\left(\frac{n}{2} + 1\right)} \tag{5}$$

The difference $-2\Delta \log \mathcal{L}$ due to statistical errors was determined to be 23.9 units. This was reduced by a factor 1.3 to 18.4 to account for systematic uncertainties. This change in log likelihood corresponds to a p-value of 0.00001 for $n = 2$. To convert this to “standard deviations” we use the following expression

$$1 - \alpha = \frac{1}{\sqrt{2\pi}} \int_{-S}^{+S} e^{-y^2/2} dy = \mathrm{erf}\left[S/\sqrt{2}\right] \tag{6}$$

Thus,

$$S = \sqrt{2}\mathrm{erf}^{-1}(1-\alpha) \tag{7}$$

In my opinion, use of this formula to express standard deviations in the multivariate case is not internally consistent since this formula gives the confidence interval for a measurement of a single quantity with Gaussian errors. In the multivariate case, one usually quotes the coverage probability $1-\alpha$ for joint estimation of n parameters. This is simply the probability that the given region contains the true values of the parameters.

For particular values of the coverage probability $1-\alpha$, Table **??** lists the change in log likelihood $-2\Delta\log\mathcal{L}$ for $n=1$ and $n=2$. For $n=1$, the change in log likelihood as a function of the coverage probability $1-\alpha$ is given by

$$-2\Delta\log\mathcal{L} = 2\left[\mathrm{erf}^{-1}(1-\alpha)\right]^2 \tag{8}$$

For $n=2$, the corresponding change in log likelihood is

$$-2\Delta\log\mathcal{L} = 2\log(1-\alpha). \tag{9}$$

$1-\alpha$	$n=1$	$n=2$
0.6827	1.00	2.30
0.9	2.71	4.61
0.95	3.84	5.99
0.9545	4.00	6.18
0.99	6.63	9.21
0.9973	9.00	11.83
0.999	10.83	13.82
0.9999	14.14	18.42

Table 1: The change in log likelihood $-2\Delta\log\mathcal{L}$ corresponding to values of the coverage probability $1-\alpha$, for joint estimates of n parameters, in the large data sample limit.