

# Policy Learning with Unobserved Heterogeneity

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# General Problem

- ▶ Map resources into welfare through policies (e.g. transfers to foster development).
  - **Pr1:** Evaluate a given policy's effects (e.g. effects on profits).
  - **Pr2:** Decide who to treat in the full population (e.g. Indian entrepreneurs).
- 
- ▶ **Decide who to treat:**
  - Easy if the policy's effects are constant.
  - Hard if heterogeneous.

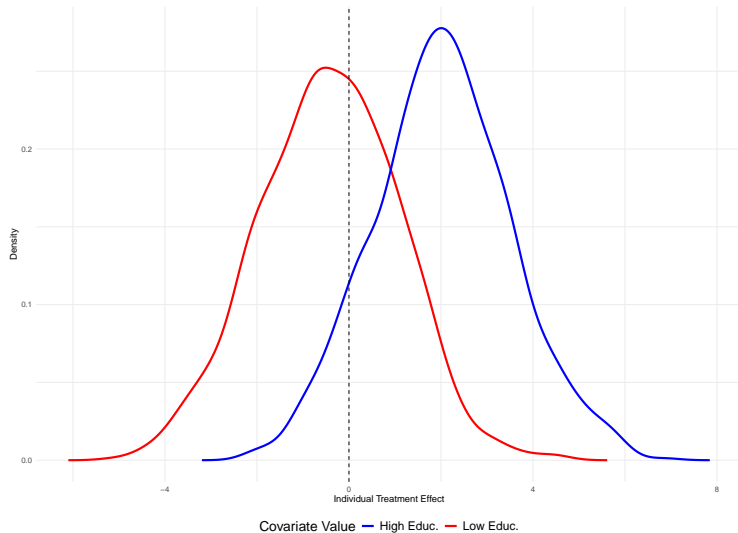
# Problem Description

- ▶ Policies' effects vary between individuals.
  - The same policy can help some and harm others.  
e.g. Alt, Lassen, and Marshall (2016), Hussam, Rigol, and Roth (2022), Biroli et al. (2025), Brynjolfsson, Li, and Raymond (2025).
- ▶ Policymakers test policies before implementing them at scale.
  - Treat only those expected to benefit.
- ▶ Econometric approach: Policy Learning.
  - Use RCTs to learn assignment rules that *perform well* in the population.  
e.g. Kitagawa and Tetenov (2018), Mbakop and Tabord-Meehan (2021), Athey and Wager (2021).
- ▶ State of the art: all relevant dimensions are observed.
  - **When** and **how** to account for **unobserved heterogeneity** in policy learning?

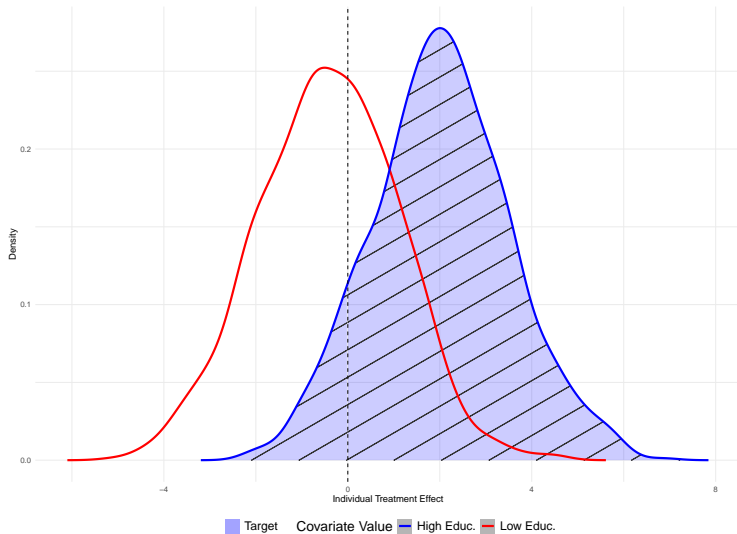
## Stylized Example - Setting Hussam, Rigol, and Roth, 2022 (AER)

- ▶ Binary Policy:  $D_i \in \{0, 1\}$  (e.g. [cash transfer to micro-entrepreneurs](#)).
  - ▶ Random Sample  $\mathcal{S}$  from a population of interest  $\mathcal{P}$  (e.g. [Indian entrepreneurs](#)).
  - ▶ RCT to evaluate the effects on  $Y_i$  (e.g. [profits](#)).
  - ▶ For simplicity,  $X_i \in \{h, l\}$ :  $\tau(l) < 0 < \tau(h)$  (e.g. [high/low education](#)).
  - Who to treat in  $\mathcal{P}$ ? Covariate-Based Policy Rule:  $G_x = \mathbb{1}(X_i = h)$ .
- 
- ▶ Assume now observed, and **unobserved** heterogeneity:  $\tau(X_i, A_i)$ .
  - ▶ For simplicity,  $A_i \in \mathbb{R}$  (e.g. [business skills](#)).
  - ▶ Denote with  $\tau(h), \tau(l)$  the avg. effect for  $X = h, l$ :  $\tau(l) < 0 < \tau(h)$ .
  - Is  $G_x$  still *optimal*?

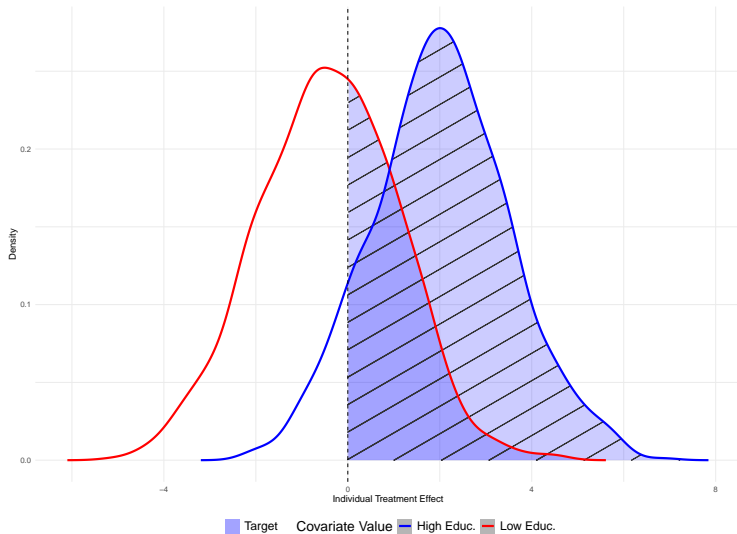
## Stylized Example - Individual Treatment Effects



## Stylized Example - Covariate-Based Policy Rule



## Stylized Example - Oracle Policy Rule



## Research Question

- ▶ **Problem 1:** we only observe  $S$ , and we do not know counterfactuals.  
→ **Solution:** Empirical Welfare Maximization (Kitagawa and Tetenov, 2018).
- ▶ **Problem 2:** we don't observe the realizations of  $A_i$ :  $\alpha_i$  (e.g. business skills).  
→ **Potential solution:** use *estimates* or *proxies*:  $\hat{\alpha}_i$  (e.g. fixed effects, factors, principal components; satellite data, survey questions, ...).

*Can **estimated** latent characteristics improve policy recommendations when treatment effects vary along their **true** values?*



# This Paper in One Slide

Can **estimated** latent characteristics improve policy recommendations when treatment effects vary along their **true** values?

- ▶ **It depends:** trade-off importance of  $\alpha$  vs. estimation error in  $\hat{\alpha}$ .
- **Regret bounds** on welfare for policy rules that include  $\hat{\alpha}$  or not.
- **Connection** with shrinkage: new **regret bounds** for shrinkage-based rules.
- ▶ **New class of rules:** set  $\hat{\alpha}$ 's **importance** depending on the welfare gain produced by including it in the decision rule.
- Data-driven method to set  $\hat{\alpha}$ 's **importance** via cross-validation.
- Adaptively achieves near-optimal welfare.
- ▶ Empirical application in development economics [Hussam et al., 2022 \(AER\)](#).
- Including proxies **halves** the probability of producing welfare losses.

# Related Literature

## ► **Policy Learning.**

e.g. Manski (2004), Bhattacharya and Dupas (2012), Kitagawa and Tetenov (2018), Kitagawa and Tetenov (2021), Mbakop and Tabord-Meehan (2021), Athey and Wager (2021), Viviano and Bradic (2024), Viviano (2024).

- Unobserved heterogeneity introduces a new approximation-estimation error trade-off.
- Connection with shrinkage (Yamin, 2025; Moon, 2025).
- Data-driven procedure can solve this trade-off.

## ► **Applied Microeconomics** (development, education, political economy, labor).

e.g. Leuven, Oosterbeek, and Klaauw (2010), Alt, Lassen, and Marshall (2016), Hussam, Rigol, and Roth (2022), Bryan, Karlan, and Osman (2024), Biroli et al. (2025), Brynjolfsson, Li, and Raymond (2025).

- How to scale up interventions when treatment effects vary between individuals.
- We can evaluate policy recommendations before recommending them!

# Roadmap

Introduction

Formal Setting

Potential Outcomes

Policy Rules

Welfare

Theoretical Results

Assumptions

Results

Empirical Application

Work in Progress

Conclusion

# Potential Outcomes

Consider:

$$(Y_i(0), X_i, A_i) \sim P_{y(0), x, \alpha}, \quad D_i \sim \mathcal{B}(1, e(Z_i))$$

that takes values  $(y_i(0), x_i, \alpha_i) \in \mathcal{Y} \times \mathcal{X} \times \mathcal{A}$  and  $e(Z_i) = p$ .

**Potential Outcomes:**

$$Y_i(0), \quad Y_i(1) = Y_i(0) + \tau(X_i, A_i)$$

**Observable Data:**

$$(Y_i, X_i, D_i), \quad Y_i = D_i \cdot Y_i(1) + (1 - D_i) \cdot Y_i(0)$$

## Policy Rules and Classes

**Policy Rule:** A mapping from variables ( $\mathcal{Z} = \mathcal{X}$  or  $\mathcal{Z} = \mathcal{X} \times \mathcal{A}$ ) to target set  $\{0, 1\}$ :

$$G_{\mathcal{Z}} : \mathcal{Z} \rightarrow \{0, 1\}$$

**Classes of rules:**

$$\underbrace{\mathcal{G}_{\mathcal{X}} = \{G_{\mathcal{X}} : \mathcal{X} \rightarrow \{0, 1\}\}}_{\text{Covariate-Based (CB)}}, \quad \underbrace{\mathcal{G}_{\mathcal{X}, \alpha} = \{G_{\mathcal{X}, \alpha} : \mathcal{X} \times \mathcal{A} \rightarrow \{0, 1\}\}}_{\alpha\text{-Augmented } (\alpha\text{-CB)}}$$

**Example:** Grant Allocation.

- ▶  $G_{\mathcal{X}}$ : Assign by age and education.
- ▶  $G_{\mathcal{X}, \alpha}$ : Also include business skills  $\alpha$ .

## Feasible $\alpha$ -Augmented Rules

The realized value of  $A_i$ ,  $\alpha_i$  is not observed, but it can be estimated with  $\hat{\alpha}_i$ .

$$\hat{\alpha}_i = \alpha_i + \varepsilon_i, \quad \varepsilon_i \sim \mathcal{N}(0, \sigma_\varepsilon^2)$$

Feasible  $\alpha$ -Augmented rules:

$$\underbrace{\mathcal{G}_{x, \hat{\alpha}} = \{G_{x, \hat{\alpha}} : \mathcal{X} \times \mathcal{A} \rightarrow \{0, 1\}\}}_{\hat{\alpha}\text{-Augmented } (\hat{\alpha}\text{-CB)}}$$

## Example: (Linear) Threshold Rules

Threshold rules:

► CB rules:

$$\mathcal{G}_x = \{G_x = \mathbb{1}(x > t)\}$$

►  $\alpha$ -CB rules:

$$\mathcal{G}_{x,\alpha} = \{G_{x,\alpha} = \mathbb{1}(x + \alpha > t)\}$$

►  $\hat{\alpha}$ -CB rules:

$$\mathcal{G}_{x,\hat{\alpha}} = \{G_{x,\hat{\alpha}} = \mathbb{1}(x + \hat{\alpha} > t)\}$$

## Welfare and Policy Learning

**Welfare** generated by a (general) Policy  $G_z$ :

$$W(G_z) := \frac{1}{n} \sum_{i=1}^n [Y_i(1) \cdot 1(i \in G_z) + Y_i(0) \cdot 1(i \notin G_z)]$$

**Oracle Rule:** Find  $G_z$  that maximizes expected welfare:

$$G_z^* := \arg \max_{G_z \in \mathcal{G}_z} E_{P^n}[W(G_z)]$$

**Challenge:**  $G_z^*$  depends on counterfactuals and solves a population-wide problem.  
Need for a feasible empirical analogue.



# Empirical Welfare Maximization (EWM) - Kitagawa and Tetenov (2018)

**EWM Rule:**

$$\hat{G}_z := \arg \max_{G_z \in \mathcal{G}_z} \{W_n(G_z)\}$$

with

$$W_n(G_z) := \frac{1}{n} \sum_{i=1}^n \left[ \frac{Y_i D_i}{e(Z_i)} \cdot 1(i \in G_z) + \frac{Y_i(1 - D_i)}{1 - e(Z_i)} \cdot 1(i \notin G_z) \right]$$

**Regret:**

$$R(\hat{G}_z) := E_{P^n}[W(G_z^*) - W(\hat{G}_z)]$$

*Measures average welfare loss from using  $\hat{G}_z$  instead of  $G_z^*$ .*

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# Assumption 1 - Kitagawa and Tetenov (2018) (1/2)

## i. Bounded Outcomes

$$|Y_i| \leq M/2$$

*Potential outcomes are uniformly bounded by a constant.*

## ii. Clean Design (Unconfoundedness + SUTVA)

$$D_i \perp (Y_i(0), Y_i(1)) | (X_i, A_i) \quad ; \quad Y_i(D_i, \mathbf{D}_{-i}) = Y_i(D_i)$$

*Treatment assignment is as good as random; no spillovers.*

## Assumption 1 - Kitagawa and Tetenov (2018) (2/2)

### iii. Strict Overlap

$$\Pr(D_i = 1|X_i, A_i) \in [k, 1 - k] \text{ for some } k > 0$$

*All units have a positive chance of receiving treatment.*

### iv. Finite VC-Dimension

$$\text{VC}(\mathcal{G}_z) = v_z < \infty$$

*The policy class has finite complexity.*

## Assumption 2 - Novel

### i. Proxy Representation

$\hat{\alpha}_i$  can be written as  $\hat{\alpha}_i = \alpha_i + \varepsilon_i$ , and  $\varepsilon_i \sim \mathcal{N}(0, \sigma_\varepsilon^2)$ .

### ii. Lipschitz Treatment Effects

$$|\tau(x, \alpha + \gamma) - \tau(x, \alpha)| \leq L \cdot |\gamma|, \quad L \in \mathbb{R}^+$$

*Small changes in  $\alpha$  lead to smooth changes in  $\tau$ .*

# Regret Bound for Covariate-Based Rules

**Theorem 1:** Under Assumptions 1–2, regret of Covariate-Based policy rules satisfies:

$$R(\hat{G}_x) := E_{P^n}[W(G_{x,\alpha}^*) - W(\hat{G}_x)] \leq C_1 \frac{M}{k} \sqrt{\frac{v_x}{n}} + \sigma_{\tau|x}$$

where  $C_1$  is a universal constant and  $\sigma_{\tau|x} = E_X \left[ \sqrt{\text{Var}(\tau(X_i, A_i) | X_i)} \right]$

► **Two terms:**

1.  $C_1 \frac{M}{k} \sqrt{\frac{v_x}{n}}$ : regret due to empirical analogue.
2.  $\sigma_{\tau|x}$ : regret due to ignoring  $\alpha$ .

## Regret Bound for $\hat{\alpha}$ -Augmented Rules

**Theorem 2:** Under Assumptions 1–2, regret of  $\hat{\alpha}$ -Augmented policy rules satisfies:

$$R(\hat{G}_{x,\hat{\alpha}}) := E_{P^n}[W(G_{x,\alpha}^*) - W(\hat{G}_{x,\hat{\alpha}})] \leq 2C_1 \frac{M}{k} \sqrt{\frac{v_{x,\hat{\alpha}}}{n}} + c_1 \sigma_\varepsilon$$

where  $C_1$  is a universal constant and  $c_1 := 4L\sqrt{2/\pi}$ .

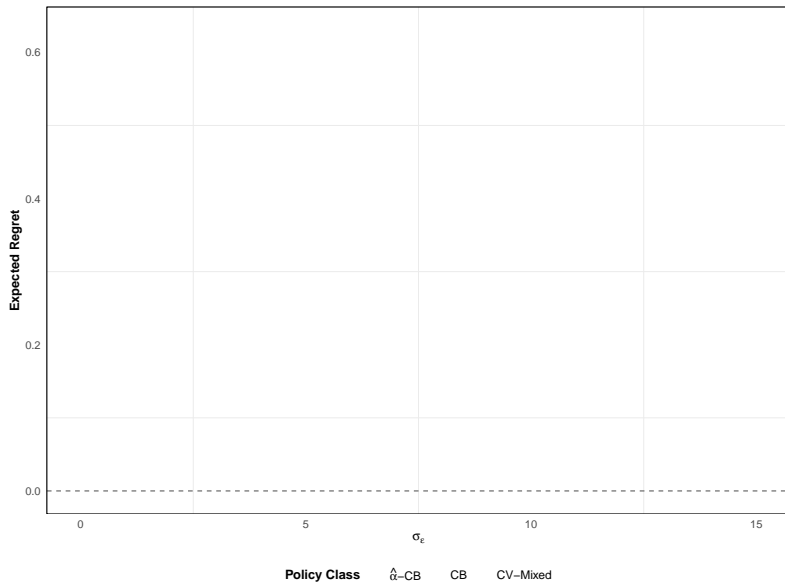
► **Two terms:**

1.  $2C_1 \frac{M}{k} \sqrt{\frac{v_{x,\hat{\alpha}}}{n}}$ : regret due to empirical analogue.
2.  $c_1 \sigma_\varepsilon$ : regret due to noisy estimates of  $\alpha$ .

► **Key insight:** trade-off between estimation (due to noise in  $\hat{\alpha}$ ) and approximation (due to importance of  $\alpha$ ) error.

# Simulations Results - Regret

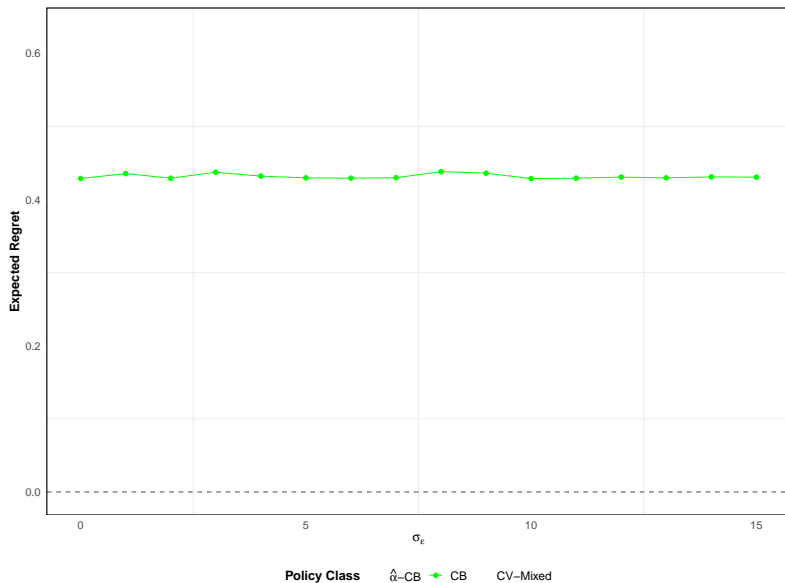
Simulations DGP



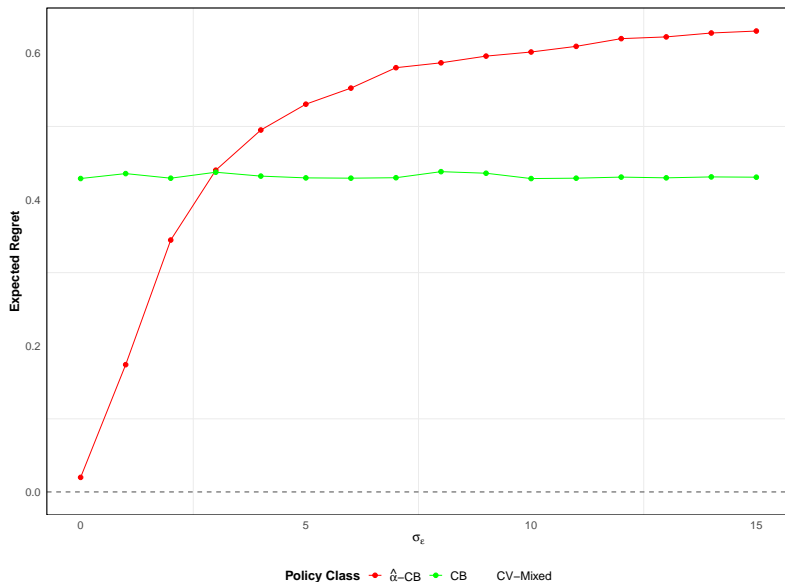


# Simulations Results - Regret CB Rules

Simulations DGP



# Simulations Results - Regret $\hat{\alpha}$ -CB Rules

[Simulations DGP](#)[Back to Empirics](#)

# Motivation for Shrinkage Steps

Formal motivation:

$$\begin{aligned} R(\hat{G}_{x,\hat{\alpha}}) &= E_{P^n}[W(G_z^*) - W(\hat{G}_{x,\hat{\alpha}})] = \\ &= E_{P^n}[W(G_z^*) - W(\hat{G}_{x,\alpha})] + E_{P^n}[W(\hat{G}_{x,\alpha}) - W(\hat{G}_{x,\hat{\alpha}})] = \\ &= \underbrace{E_{P^n}[W(G_{x,\alpha}^*) - W(\hat{G}_{x,\alpha})]}_A + \underbrace{E_{P^n}[W(\hat{G}_{x,\alpha}) - W(\hat{G}_{x,\hat{\alpha}})]}_B \end{aligned}$$

And:

$$B \leq KT18 + 4L \cdot \sqrt{2/\pi} \cdot \sqrt{E_{P^n} \left[ \frac{1}{n} \sum_{i \in S^n} (A_i - \hat{\alpha}_i)^2 \right]}$$

# Empirical Bayes - $\hat{\alpha}^{EB}$ -Augmented Rules

## Assumption 3

The unobserved factor  $A_i$  is normally distributed:  $A_i \sim \mathcal{N}(\mu_\alpha, \sigma_\alpha^2)$ .

Define:

$$\hat{\alpha}_i^{EB} = \omega \hat{\alpha}_i + (1 - \omega) \bar{\alpha}$$

where  $\omega = \frac{\sigma_\alpha^2}{\sigma_\epsilon^2 + \sigma_\alpha^2}$ .

The class of  $\hat{\alpha}^{EB}$ -Augmented Rules is then defined as:

$$\mathcal{G}_{x, \hat{\alpha}^{EB}} = \{G_{x, \hat{\alpha}^{EB}} : \mathcal{X} \times \mathcal{A}^B \rightarrow \{0, 1\}\}, \quad \mathcal{A}^B = \{\hat{\alpha}^{EB} | \mathcal{S}_n\}$$

## Regret Bound on $\hat{\alpha}^{EB}$ -Augmented Rules

**Theorem 2:** Under Assumptions 1–2, regret of  $\hat{\alpha}$ -Augmented policy rules satisfies:

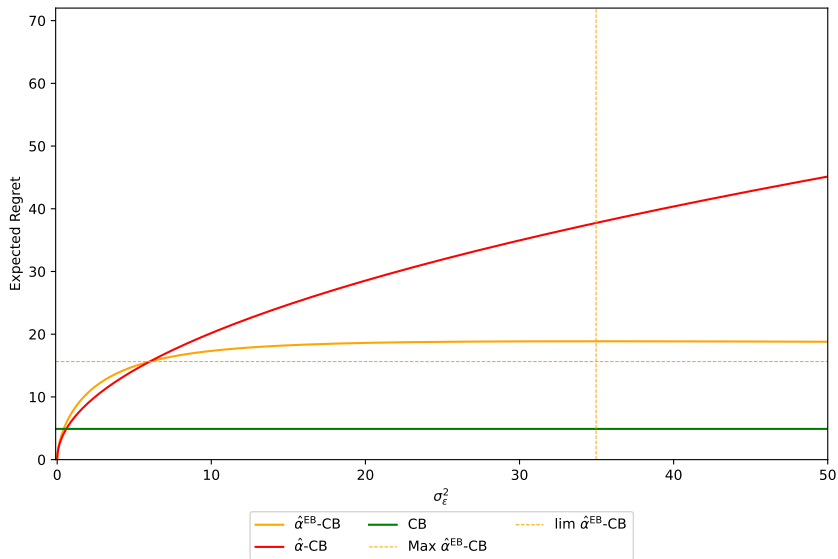
$$R(\hat{G}_{x,\hat{\alpha}}) := E_{P^n}[W(G_{x,\alpha}^*) - W(\hat{G}_{x,\hat{\alpha}})] \leq 2C_1 \frac{M}{k} \sqrt{\frac{v_{x,\hat{\alpha}}}{n}} + c_1 \sigma_\varepsilon$$

**Proposition 1:** Under Assumptions 1-3, regret of  $\hat{\alpha}^{EB}$ -Augmented policy rules satisfies:

$$\begin{aligned} R(\hat{G}_{x,\hat{\alpha}^{EB}}) &:= E_{P^n}[W(G_{x,\alpha}^*) - W(\hat{G}_{x,\hat{\alpha}^{EB}})] \leq \\ &\leq 2C_1 \frac{M}{k} \sqrt{\frac{v_{x,\hat{\alpha}^{EB}}}{n}} + c_1 \left[ \frac{1 + \sqrt{n}}{\sqrt{n}} (\omega \sigma_\varepsilon + (1 - \omega) \sigma_\alpha) + \frac{\sigma_\varepsilon}{\sqrt{n}} \right] \end{aligned}$$

where  $C_1$  is a universal constant and  $c_1 := 4L\sqrt{2/\pi}$ .

# Asymptotic Comparison - Analytical Bounds



# Data-Driven Shrinkage based on Welfare

Mixed Rules:

$$\mathcal{G}(\lambda) = \{G(\lambda) : \mathcal{X} \times \tilde{\mathcal{A}} \rightarrow \{0, 1\}\}, \quad \tilde{\mathcal{A}} := \Lambda \times \mathcal{A} := \{\lambda \hat{\alpha} \in \mathbb{R} : \lambda \in \Lambda, \hat{\alpha} \in \mathcal{A}\}$$

For any class  $\mathcal{G}$ ,  $\mathcal{G}(0) = \mathcal{G}_x$ , and  $\mathcal{G}(1) = \mathcal{G}_{x, \hat{\alpha}}$ .

How to select  $\lambda$ ?

- ▶ Split  $\mathcal{S}$  of size  $N$  into a training set  $\mathcal{S}_{\text{train}}$ , and a validation set  $\mathcal{S}_{\text{val}}$ .
- ▶ Estimate  $\hat{G}(\lambda)$  on  $\mathcal{S}_{\text{train}}$  and select  $\hat{\lambda}$  on  $\mathcal{S}_{\text{val}}$ :

$$\hat{G}(\hat{\lambda}) := \operatorname{argmax}_{\lambda \in \Lambda, i \in \mathcal{S}_{\text{val}}} \{W_m(\hat{G}(\lambda))\}$$

## Cross-Validated Mixed Rules - Example Algorithm

**Mixed** (threshold) **Rules**:

$$G(\lambda) = \mathbb{1}(x + \lambda \cdot \hat{\alpha} > t(\lambda)), \quad \lambda \in \Lambda \subset [0, 1]$$

→  $\lambda = 0$ : Covariate-Based rule  $G_x$

→  $\lambda = 1$ :  $\hat{\alpha}$ -Augmented rule  $G_{x, \hat{\alpha}}$

**Idea:** Learn the optimal weight to attach to  $\hat{\alpha}$  via cross-validation using the welfare generated out-of-sample.



## Optimality of CV-Mixed Rules

**Proposition 3:** Under Assumption 1, with probability at least  $1 - \delta$ ,

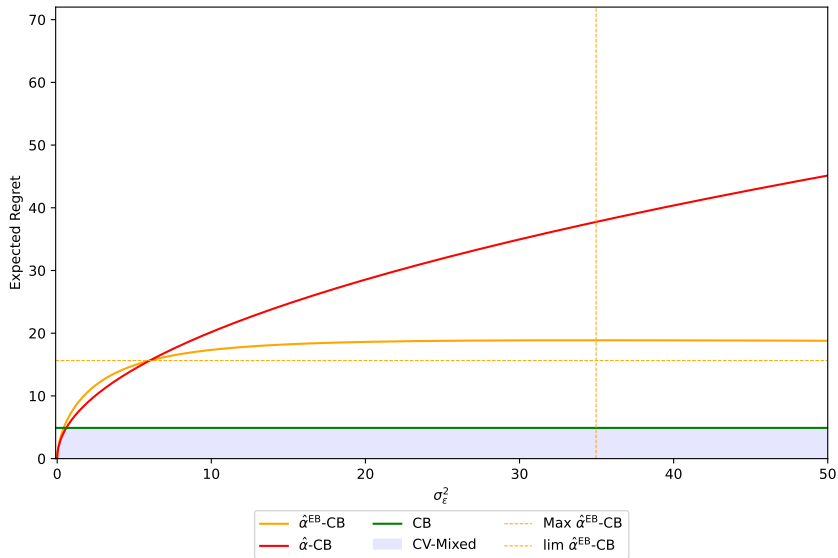
$$E_{P^n}[W(\hat{G}(\hat{\lambda}))] \geq \max\{E_{P^n}[W(\hat{G}(0))], E_{P^n}[W(\hat{G}(1))]\} - 2\gamma_m,$$

where:

$$\gamma_m := M \cdot \sqrt{\frac{1}{2m} \log \left( \frac{2r}{\delta} \right)}$$

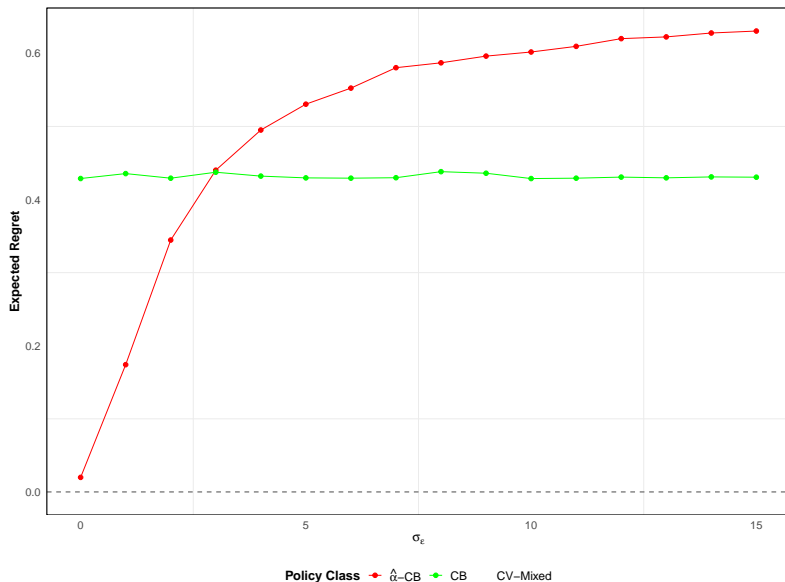
- **Key Insight:** with high probability, CV-Mixed rules outperform the best between Covariate-Based and  $\hat{\alpha}$ -Augmented rules.

# Asymptotic Comparison - Analytical Bounds

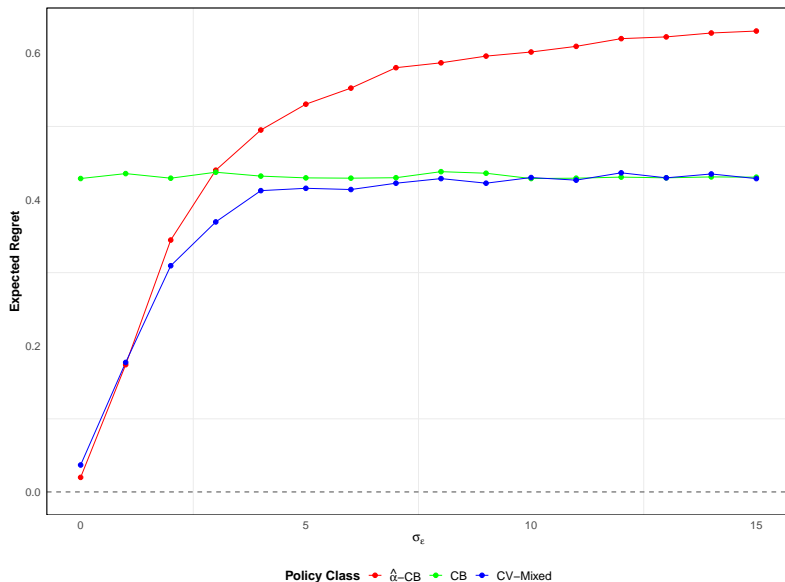


# Simulation Results - Regret CV-Mixed Rules

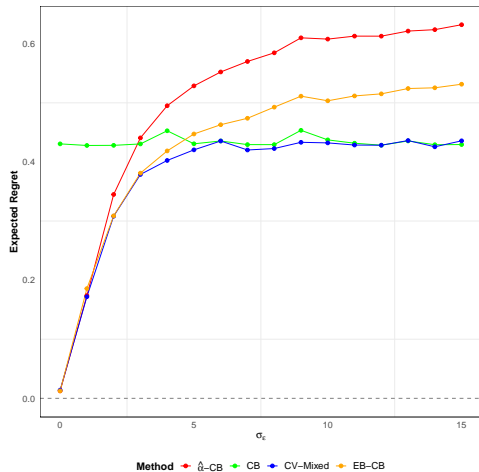
Simulations DGP



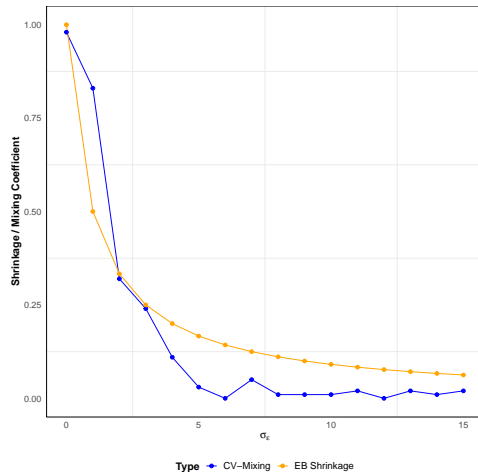
# Simulation Results - Regret CV-Mixed Rules

[Is  \$\hat{\lambda}\$  Interpretable?](#)[Different  \$\sigma\_{\tau|x}\$](#) [Back to Empirics](#)

# Simulation Results - $\hat{\lambda}$ and $\omega$ [Back](#)



Panel A. Regret



Panel B. Shrinking Parameters

# This Presentation

Introduction

Formal Setting

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## Empirical Application - Setting

*Targeting High Ability Entrepreneurs Using Community Information: Mechanism Design in the Field* (Hussam, Rigol, and Roth, 2022 (AER)):

- ▶ RCT with 1500 Indian microentrepreneurs.
- ▶ Treatment: cash for business development.
- ▶ Outcome: profits.
- ▶ Heterogeneity dimension: community ratings as a proxy for business skills.

Main point of the paper: **demonstrate that community knowledge can help target high-growth microentrepreneurs.**

## Policy Learning Exercise

Does including community ratings as a targeting variable *actually* increase welfare?

- ▶ Status Quo: don't scale up.
- ▶ Random rule  $G_{\text{rand}}$ : scale up randomly.
- ▶ CB threshold rules  $G_x$ : age, education.
- ▶  $\hat{\alpha}$ -CB threshold rules  $G_{x,\hat{\alpha}}$ : covariates + community rating.
- ▶ CV-Mixed threshold rules  $G(\lambda)$ : selects the weight  $\lambda$  of community ratings via cross-validation.

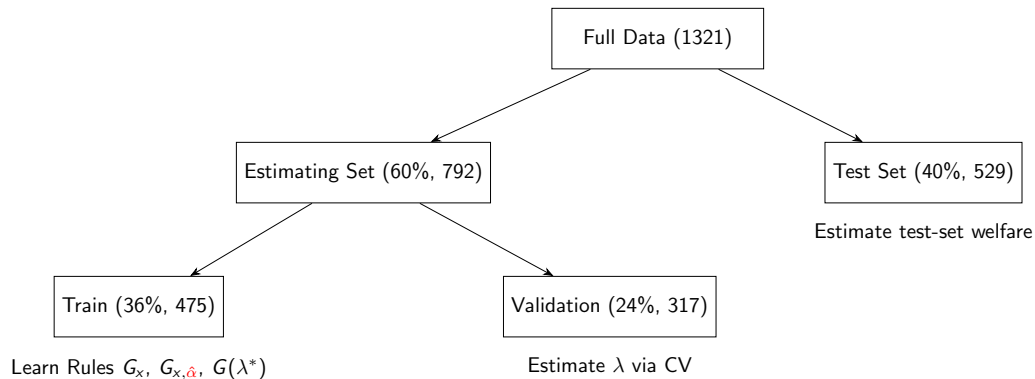


# Ranking Rules

Formal Algorithm

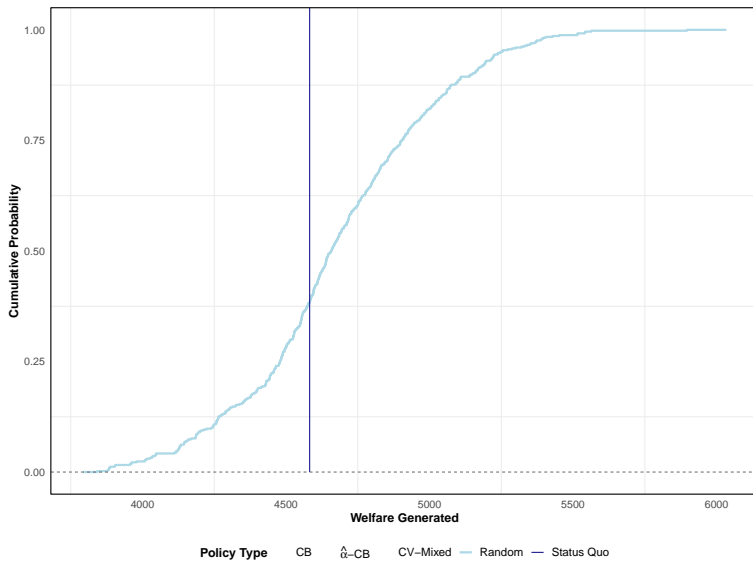
Welfare

Randomly split the data into an estimating and testing sample:

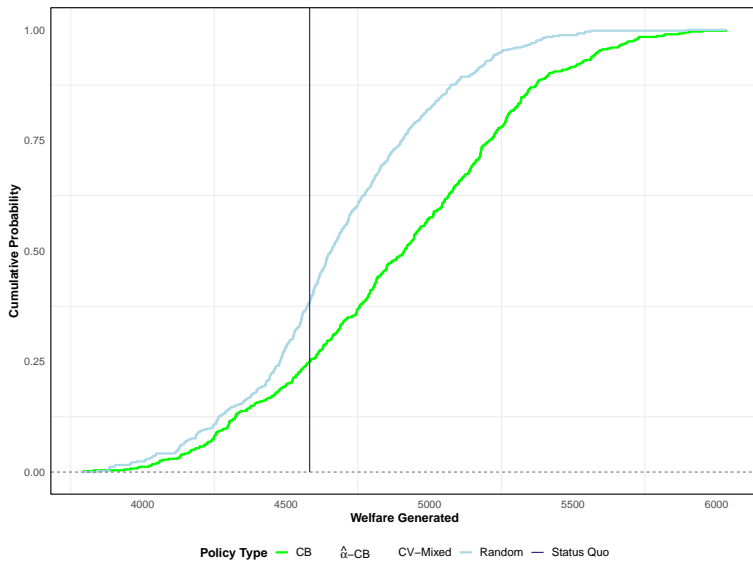


Repeat the random split  $B = 500$  times.

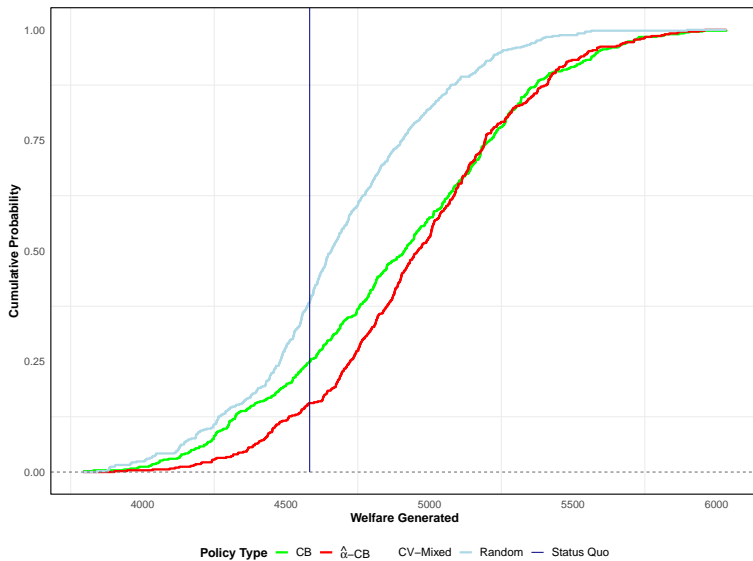
# Distribution of Welfare - Random Rules



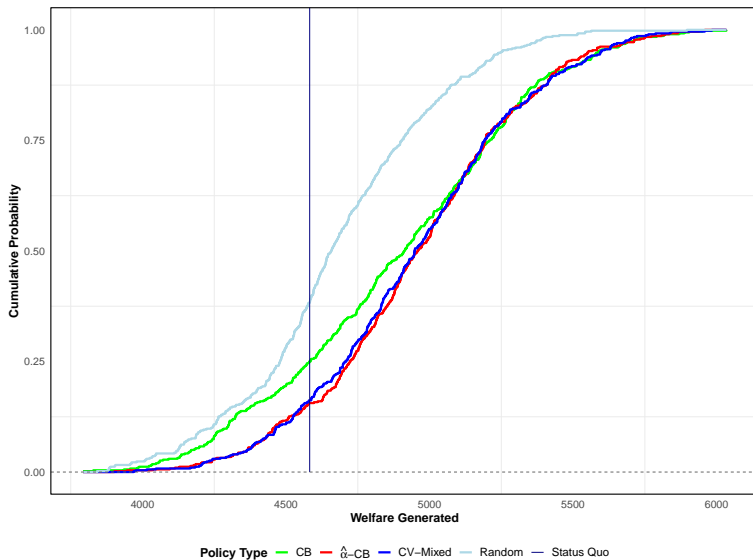
# Distribution of Welfare - CB Rules



# Distribution of Welfare - $\hat{\alpha}$ -CB Rules



# Distribution of Welfare - CV-Mixed Rules

[Noise Increase](#)[Table Summary](#)

# Ethical Policy Learning

- ▶ Allocation of a scholarship.
- ▶ The policymaker can access genetic data as a proxy for individuals' innate ability.
- ▶ High-ability (type  $H$ ) students benefit more from the policy.

Would that be ethical to assign the scholarship based on the genetic information?

I define as ethical the set of allocations that do not enlarge pre-existing differences in outcomes between groups defined through the proxy of the unobserved characteristic:

$$\max_{G \in \mathcal{G}} E_{P^n}[W(G)] \quad \text{s.to:} \quad \mathcal{G} = \{G : E_{P^n}[W(G)|i \in H] - E_{P^n}[W(G)|i \in L] \leq Q_a\}$$

- ▶ **Main insight:** Including estimated latent variables introduces an approximation-estimation error trade-off.
  - Improves policy recommendations if  $\alpha$ 's importance  $>$   $\hat{\alpha}$ 's estimation error.
- ▶ **CV-Mixed rules:** adaptively set the importance of  $\hat{\alpha}$  via cross-validation.
  - Theoretically and empirically shown to achieve **near-optimal welfare**.
- ▶ **Empirical application** in development economics.
  - Intuitive procedure to rank policy recommendations.

*Thanks for your attention!*

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## Data-Generating Process:

- ▶ Covariates:  $X_i \in \mathbb{R}^1$ .
- ▶ Unobserved characteristic:  $\alpha_i \sim N(0, \sigma_\alpha^2)$ .
- ▶ Unobserved characteristic's estimate:  $\hat{\alpha}_i = \alpha_i + N(0, \sigma_\varepsilon^2)$ .
- ▶ Potential outcomes:

$$Y_i(0) = g(X_i) + A_i + \varepsilon_i, \quad Y_i(1) = Y_i(0) + \tau(X_i, A_i)$$

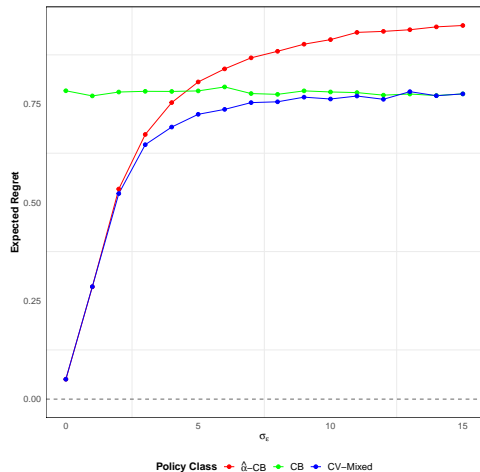
- ▶ Treatment assignment:  $D_i \sim \text{Bernoulli}(0.5)$ .

## Treatment Effect:

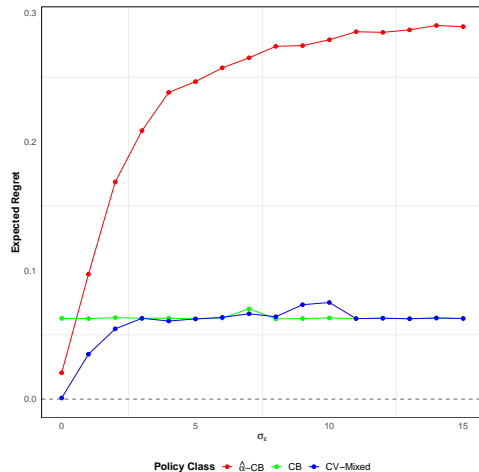
$$\tau(x, \alpha) = x + \gamma \cdot \alpha$$

Linear in  $(x, \alpha)$  with varying  $\gamma$  to control unobserved heterogeneity.

# Simulation Results - Different $\sigma_{\tau|x}$

[Back to CV-Mixed](#)[Simulations DGP](#)

Panel A. High  $\sigma_{\tau|x}$



Panel B. Low  $\sigma_{\tau|x}$

$$\begin{aligned}
 E_{P^n}[W(\hat{G}_{X,\alpha}) - W(\hat{G}_{X,\hat{\alpha}})] &\leq KT18 + 4 \cdot E_{P^n} \left[ \frac{1}{n} \sum_{i=1}^n (\tau(X_i, A_i) - \tau(X_i, \hat{\alpha})) \right] \leq \\
 &\leq KT18 + 4 \cdot E_{P^n} \left[ \frac{1}{n} \sum_{i \in \mathcal{S}^n} |\tau(X_i, A_i) - \tau(X_i, \hat{\alpha})| \right] \\
 &\leq KT18 + 4L \cdot E_{P^n} \left[ \frac{1}{n} \sum_{i \in \mathcal{S}^n} |A_i - \hat{\alpha}_i| \right] = \\
 &= KT18 + 4L \cdot \sqrt{2/\pi} \cdot \sigma_\epsilon = KT18 + 4L \cdot \sqrt{2/\pi} \cdot \sqrt{E_{P^n} \left[ \frac{1}{n} \sum_{i \in \mathcal{S}^n} (A_i - \hat{\alpha}_i)^2 \right]}
 \end{aligned}$$

---

### Algorithm Cross-Validated Mixed Rules

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**Require:** Data  $(X_i, \hat{\alpha}_i, Y_i, D_i)$  for  $i = 1, \dots, n + m$ , grid  $\Lambda = \{\lambda_1, \dots, \lambda_r\} \subset [0, 1]$ .

- 1: Randomly split data into training set  $\mathcal{S}_{\text{train}}$  of size  $n$  and validation set  $\mathcal{S}_{\text{val}}$  of size  $m$ .
  - 2: **for** each  $\lambda \in \Lambda$  **do**:
  - 3:     Estimate thresholds  $t(\lambda)$  on  $\mathcal{S}_{\text{train}}$ .
  - 4:     Define  $G(\hat{t}(\lambda))$ .
  - 5:     Estimate empirical welfare  $W_m(G(\hat{t}(\lambda)))$  on  $\mathcal{S}_{\text{val}}$ .
  - 6: **end for**.
  - 7: Estimate  $\hat{\lambda} = \arg \max_{\lambda \in \Lambda} W_m(G(\hat{t}(\lambda)))$ .
  - 8: **return** Final policy rule  $\hat{G}(\hat{\lambda})$ .
-

# Can we compute the distribution of the performance?

[Back](#)

Consider different realizations of the sample splitting:

---

## Algorithm Welfare Evaluation

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- 1: **for**  $b = 1$  to  $B = 500$  **do**
  - 2:     Set random seed to  $b$ .
  - 3:     Random split:  $\mathcal{S} = \mathcal{S}_{\text{est}}^b \cup \mathcal{S}_{\text{est}}^b$  and  $\mathcal{S}_{\text{est}}^b = \mathcal{S}_{\text{train}}^b \cup \mathcal{S}_{\text{val}}^b$ ,  $\mathcal{S}_{\text{train}}^b \cap \mathcal{S}_{\text{val}}^b \cap \mathcal{S}_{\text{test}}^b = \emptyset$
  - 4:     **Estimate Rules:**
  - 5:     Estimate  $G_x$  and  $G_{x,\hat{\alpha}}$  using  $\mathcal{S}_{\text{train}}^b$ .
  - 6:     Estimate  $G(\lambda^*)$  using  $\mathcal{S}_{\text{train}}^b$  and  $\mathcal{S}_{\text{val}}^b$ .
  - 7:     **Evaluate Rules:**
  - 8:     Estimate  $\hat{W}_{\text{test}}^b(\hat{G}_{\text{rand}})$ ,  $\hat{W}_{\text{test}}^b(\hat{G}_x)$ ,  $\hat{W}_{\text{test}}^b(\hat{G}_{x,\hat{\alpha}})$ , and  $\hat{W}_{\text{test}}^b(\hat{G}(\lambda^*))$ .
  - 9: **end for**
-

Rules are defined as:  $\hat{G}_z := \arg \max_{G_z \in \mathcal{G}_z} \{W_n(G_z)\}$ , where:

$$W_n(G_z) := \frac{1}{475} \sum_{i \in \mathcal{S}} \left[ \frac{Y_i D_i}{0.3} \cdot 1(i \in G_z) + \frac{Y_i(1 - D_i)}{0.7} \cdot 1(i \notin G_z) \right]$$

and:

- ▶  $Y_i$  is profits 30 days after the intervention.
- ▶  $D_i$  takes value one if  $i$  received the grant.

## What if Community Rankings Were More Noisy?

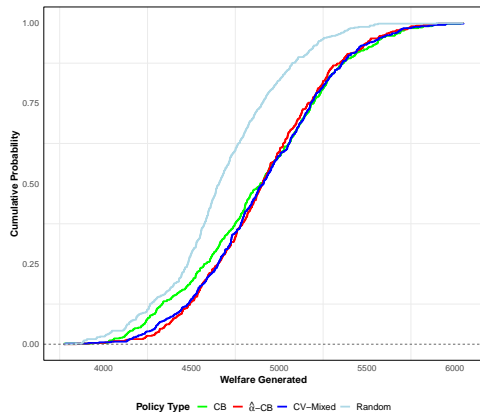
[Back to Welfare](#)

Are these findings robust to an increase in  $\sigma_\varepsilon^2$ ? Add random noise  $\zeta_i \sim \mathcal{N}(0, \sigma_\zeta^2)$  to the original variable:

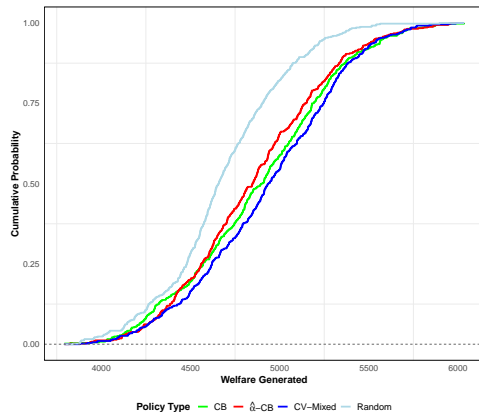
$$\tilde{\alpha}_i = \hat{\alpha} + \zeta_i$$

And apply the same algorithm to compute welfare gains.

# Noise Increase - Welfare Gains

[Simulations 1](#)[Simulations 2](#)[Back to Graph](#)

Panel A.  $\sigma_{\zeta}^2 = 1$



Panel B.  $\sigma_{\zeta}^2 = 5$



# Welfare Gains - Summary Table [Back to Graph](#)

How can we summarize welfare gains?

| Policy Rule        | Harm Rate | Rand.       | CB           | $\hat{\alpha}$ -CB | CV-Mixed     |
|--------------------|-----------|-------------|--------------|--------------------|--------------|
| Status Quo         | -         | +98\$ (+2%) | +310\$ (+7%) | +380\$ (+8%)       | +375\$ (+8%) |
| Rand.              | 0.38      | -           | +212\$ (+5%) | +282\$ (+6%)       | +277\$ (+6%) |
| CB                 | 0.25      | -           | -            | +69\$ (+1%)        | +64\$ (+1%)  |
| $\hat{\alpha}$ -CB | 0.16      | -           | -            | -                  | -5\$ (-0%)   |
| CV-Mixed           | 0.16      | -           | -            | -                  | -            |
| Status Quo         | 4,582\$   | -           | -            | -                  | -            |

**Notes:** Each cell reports the difference in mean welfare between the policy class in the column and the one in the row. Positive values indicate that the column policy performs better. Harm Rate denotes the probability that the policy yields lower welfare than the status quo.

## Future Directions (1/2)

Three different policy learning problems:

- ▶ Find a treatment rule that generalizes well. **Today!**
- ▶ Find an optimal subset of a given sample. **Work in Progress...**
  - New matching estimator to estimate ITEs and find the subgroup that maximizes *Synthetic Welfare*.
- ▶ Treat/not decision on a single unit. **Work in Progress...**
  - New probabilistic bounds on ITEs.

Policy recommendations meet policy learning.

- ▶ Take most cited papers with an RCT published on a top-5.
- ▶ Formalize their policy recommendations.
- ▶ Evaluate their performance.
- ▶ Compare them with what policy learning would suggest.