The Real Cost of Inflation under Nominally Rigid Wages

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1 Introduction

The purpose of this article is to measure analytically the effects of inflation on real wages. The perspective is behaviorist in that real consumption is assumed constant and no utility function is assumed.

Nominal wages are also constant, and prices of consumer goods are increasing and influenced by the central bank's inflation target. Savings, on the other hand, are the resulting measure after real consumption has been met. These accrue interest, which is also fixed and settled monthly.

The analysis is carried out in real terms and aims to provide an unbiased measure of the costs of inflation. The reference period is the short-to-medium term, as no adjustment in the nominal wage level and/or interest rate is assumed. This is reasonable in case of normal and moderate inflation, while partially meaningful in case of extreme inflation. In the latter case, however, the risk-free interest adjusts to the level of inflation very slowly, especially if unexpected. So we can be confident that even in this case the model does not lose meaning.

2 Assumptions

The model is based on discrete time intervals, each equal to one month. At time $t = 0$ the nominal wage w is accepted, and it remains constant over time. The household budget is composed as follows:

$$w + R_t = C_t + S_t \tag{1}$$

On the left side of (1) we have income, composed of the nominal wage and the return on investment, and on the right side we have expenditure, composed of consumption and savings.

The return on investment is equal to:

$$R_t = S_{t-1} \cdot (1 + r_0) \tag{2}$$

Where r_0 denotes the market interest on deposit accounts, settled monthly. Also:

$$S_0 = 0 \tag{3}$$

This follows from the fact that wages are granted at time 0 and bestowed initially at $t = 1$.

Consumption can be expressed as follows:

$$C_t = c \cdot P_t \tag{4}$$

With:

$$c \, \epsilon \, (0, 1)$$

We denote by P_t the price level of commonly purchased goods and by c the marginal propensity to consume. (4) indicates that the economic agent maintains the same level of real consumption, regardless of the price level and at the expense of savings.

Price trends are influenced by the central bank, which decides on a target annual inflation level of $(1 + z_y)$. Monthly inflation is then equal to:

$$(1 + z) = (1 + z_y)^{1/12} \tag{5}$$

If we set $P_{t=0} = P_0$, the prices at time t are equal to:

$$P_t = P_0 \cdot (1 + z)^t \tag{6}$$

Normalizing $P_0 = 1$, we also have that $w = 1$ [1]. Substituting the equations above into (1) we get:

$$1 + S_{t-1} \cdot (1 + r_0) = c \cdot (1 + z)^t + S_t \tag{7}$$

[1]The economic agent accepted the wage w at time $t = 0$ with the perspective that at time $t = 1 : \; w = c \cdot P_0 + (1 - c) \cdot P_0$. Normalizing P_0 and being $c \, \epsilon \, (0, 1)$ we have $w = 1$

x	value
r_0	0.0008
$z_{y,1}$	0.0200
$z_{y,2}$	0.0500
$z_{y,3}$	0.1000
c	0.9000

Table 1: Numerical values for the parameters

(7) describes the development of consumption and savings as a function of inflation, given the nominal wage.

3 Savings Trend

In the model, consumption trends are constant and independent. Savings, on the other hand, is a residual category, nominally compressing or expanding as prices rise.

Let us set the numerical values expressed in Table 1 and run a numerical simulation. As can be seen, three levels of inflation are assumed, corresponding to normal, moderate and extreme scenarios, respectively. The propensity to consume is assumed to be 90% of real wages; this is quite reasonable for the middle class [2]

Monthly interest on deposit accounts is 1% per year net and is fixed for each level of inflation. This assumption is reasonable in the short to medium term, where the risk-free interest does not vary with the level of inflation, and for unexpected moderate and extreme levels of inflation.

Finally, the time interval is 36 months, which corresponds to the average length of an employment contract. In fact, it is assumed that after the 3 years, the economic agent agrees on a new wage level or changes jobs.

As seen in Figure 1, in each of the 3 scenarios, savings increase until they reach a maximum point, and then decrease. Over 36 months, the only scenario that hits zero savings is the one with extreme inflation. We can therefore draw the following conclusion:

[2]For the low-income classes it is more appropriate to think of a propensity to consume close to 100%, while for the high-income classes less than or equal to 80%.

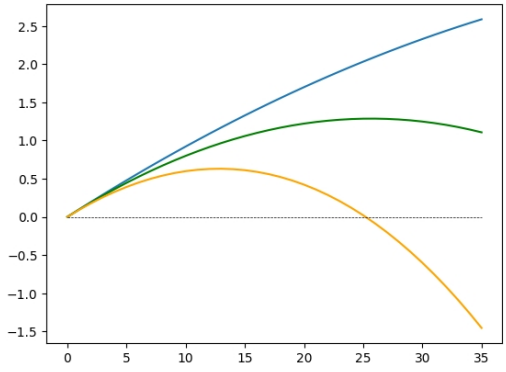

Figure 1: Total saving with respect to time

P1: *Given a fixed interest on risk-free investments and a fixed nominal wage, an exceptional level of inflation reduces real consumption within a 36-month period.*

In the other two scenarios, savings also reach a zero value, but at such a distant time that the economic agent is assumed to have adjusted her nominal wage to the new financial conditions.

4 Total Cost of Inflation

Let us now consider consumption alone and denote the total cost of inflation at time t as follows:

$$IC(t) = c \sum_{i=1}^{t} (P_i - P_0) \tag{8}$$

The argument of the summation in (10) indicates the monthly price increase over the reference period. Substituting (6) and normalizing P_0, we obtain:

$$IC(t) = c \sum_{i=1}^{t} [(1+z)^i - 1] \tag{9}$$

(9) is a geometric series, which can be expressed as follows:

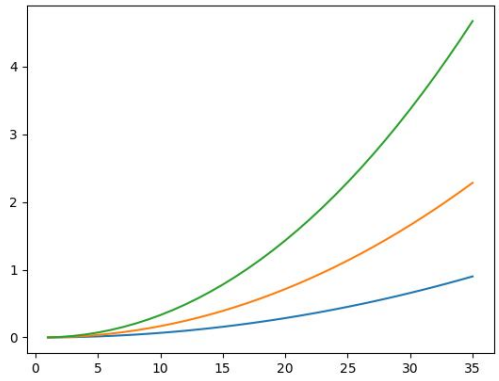

Figure 2: Total costo of inflation with respect to time

$$IC(t) = c\left[\frac{[(1+z)^t - 1](1+z)}{z} - t\right] \tag{10}$$

Let us then calculate the total cost of inflation for each of the three previous scenarios and represent it graphically as in Figure 2. As can be seen, in each scenario, the total cost increases exponentially, but at different rates.

Since wages are normalized to 1, we have that the total cost is expressed in wage units. This is particularly useful, as it allows us to express the cost of inflation in terms of hours worked. Suppose, then, that the nominal wage w corresponds to 160 hours of work and that the employee can work overtime, paid according to w. The hourly wage is then:

$$w_h = \frac{w}{160} \tag{11}$$

We then indicate the cost of inflation in terms of working hours as follows:

$$IC(t)_h = \frac{IC(t)}{w_h} \tag{12}$$

(12), unlike (10), expresses the cost in real term, since it is independent of the monetary unit. After the first year, we have that the cost of inflation in terms of labor hours, approximated to the unit, is respectively:

$$IC(t)_{h,1} = 19h$$

$$IC(t)_{h,2} = 46h$$

$$IC(t)_{h,3} = 92h$$

We can then draw the following conclusion:

> **P2**: *Given a nominally constant wage, in order to maintain the same level of real wages the employee must work 19 hours of overtime within the first year in the case of normal inflation and 78 in the case of extreme inflation.*

The situation obviously gets worse as time passes, and at the end of three years, the overtime required to maintain the same real wage is respectively:

$$IC(t)_{h,1} = 161h$$

$$IC(t)_{h,2} = 410h$$

$$IC(t)_{h,3} = 841h$$

In the case of extreme inflation, we have that the employee must work more than 5 months of overtime in the three years to maintain the same level of real wage.

Finally, we calculate the percntual change in real wages as follows:

$$\Delta_{\%} w_r = \frac{1}{(1+z)^t} - 1 \tag{13}$$

We have that after 36 months the real wage decreased for each assumed scenario respectively by:

$$\Delta_{\%} w_{r,1} = 5.8\%$$

$$\Delta_{\%} w_{r,2} = 14\%$$

$$\Delta_{\%} w_{r,3} = 25\%$$

The cost of inflation described above does not take into account savings, which are able to generate income and counteract inflation. Furthermore, it assumes that prices rise deterministically, rather than randomly.

5 Conclusion

The cost in labor hours turned out to be a particularly unbiased picture of the real costs of inflation. As it turns out, in the case of extreme inflation, this translates into nearly 5 months of overtime needed over the three years to maintain the same level of real wages.

The conclusion we can draw is that at the subjective level inflation is particularly burdensome in extreme cases. Moreover, for the low-income classes, given their high propensity to consume, inflation has costly consequences even in moderate and normal cases.

Further analysis can be carried out from here, especially macroeconomic analysis, aimed at analyzing, for example, labor market rigidity or resignation rates. From a socio-political perspective, it may be interesting to assess how the inflationary burden is distributed among different income classes. This can be useful in explaining voting decisions among poor classes (or regions).

Last point to note is the absence of random variables in the model. This issue turned out to be quite thorny given the nonlinearity of the price variable transformations. However, I undertake to unravel this point in my subsequent articles.