# The Stochastic Effect of the Expansionary Business Cycle on the Real Structure of the Balance Sheet

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## 1 Introduction

The purpose of this article is to analytically assess the effects that the business cycle has on the corporate balance sheet. In particular, an attempt will be made to answer the following question:

> *How does the balance sheet assets interfere with the business cycle and corporate capital?*

The discussion is carried out using microeconomic and corporate finance models. The balance sheet is then evaluated not by the book value, but by real one. This procedure became necessary because the former does not allow deviations in the short run from historical cost.

As mentioned, the time horizon is that of the short and medium run. In particular, it is assumed that firms cannot reorganize production following an exogenous macroeconomic shock. This greatly simplifies the model and is in line with macroeconomic models, where it is accepted that in the short run even financial shocks can have real effects.

## 2 Assumptions

Assume a firm with a balance sheet valued at his real value. The balance sheet is then composed of the sum of $n$ assets divided not by accounting category, but by risk and time to claim settlement. In contrast, the company's debts to third parties are fixed. The Equity is thus equal to:

$$K(\mathbf{r}, \mathbf{t}) = A(\mathbf{r}, \mathbf{t}) - L^* \tag{1}$$

With:

$$\begin{aligned} \mathbf{r} &= < r_1, ..., r_n > \\ \mathbf{t} &= < t_1, ..., t_n > \end{aligned} \tag{2}$$

Where $K(.)$ denotes precisely the Equity, or capital stock, $A(.)$ the sum of the real value of the assets, and $L^*$ the debts owed to third parties; $\mathbf{r}$ represents the vector of the returns of all the debtor firms, and $\mathbf{t}$ the different bond maturity times. Assuming that for each company the bonds have the same maturity time, each receivable from company $i$ is described by the tuple $(r_i, t_i)$.

The value of each asset is given by the face value of the debt, appropriately discounted by risk and time to maturity. In formulas:

$$A_i = \left(\frac{1}{1 + r_i}\right)^{t_i} \cdot x_i P_i \tag{3}$$

In (3), the element in parentheses is the discount factor and $x_i P_i$ the face value of the loan; the latter is expressed in terms of the market price of the debtor company's shares, denoted by $P_i$.

Substituting (3) into (1) we get:

$$K(\mathbf{r}, \mathbf{t}) = \left[\sum_{i=1}^{n} \left(\frac{1}{1 + r_i}\right)^{t_i} \cdot x_i P_i\right] - L^* \tag{4}$$

Assume further that for each firm $i$:

$$r_i = r_{i,u} + r_{i,s} \tag{5}$$

(5) indicates that each firm's return is given by the sum of unsystemic (firm specific) and systemic (market specific) return. The latter, in line with the SML model [1], can be expressed as follows:

[1] Security Market Line

$$r_{i,s} = r_0 + \beta_i \cdot (r_m - r_0) \tag{6}$$

With:

$$\beta_i = \frac{\sigma(i,m)}{\sigma_m} \tag{7}$$

Where $r_0$ denotes the interest rate of government bonds, $r_m$ and $\sigma_m$ the return and volatility of the market portfolio, and $\sigma(i,m)$ the covariance between the return of asset $i$ and the market portfolio.

(7) tells us that the systemic return of each asset depends on the sensitivity of that asset with the market portfolio, which can be seen as a proxy for the performance of the economy [2].

We also assume that the financial market is competitive and that there are no arbitrage opportunities. This results in the equality of the real value of each asset:

$$\left(\frac{1}{1+r_{i,s}}\right)^{t_i} \cdot P_i = \left(\frac{1}{1+r_{j,s}}\right)^{t_j} \cdot P_j \tag{8}$$

As (8) shows, only systemic risk is taken into account, as firm-specific risk can be neutralized by portfolio diversification; the market therefore does not remunerate the investor for taking on this risk.

We substitute (6) into (8) to get:

$$[1 + r_0 + \beta_i \cdot (r_m - r_0)]^{-t_i} \cdot P_i = [1 + r_0 + \beta_i \cdot (r_m - r_0)]^{-t_j} \cdot P_j \tag{9}$$

To make the discussion clearer, we simplify (19) to the continuous case:

$$e^{-(r_0+\beta_i\cdot(r_m-r_0))\cdot t_i} \cdot P_i = e^{-(r_0+\beta_j\cdot(r_m-r_0))\cdot t_j} \cdot P_j \tag{10}$$

Applying the natural logarithm and moving the terms, we get:

$$\beta_j = -\frac{r_0 \cdot (t_j - t_i) - ln(P_j) + ln(P_i)}{(r_m - r_0) \cdot t_j} + \frac{\beta_i \cdot t_i}{t_j} \tag{11}$$

[2]This proposition is not totally correct as it is necessary to make a distinction between the real and monetary economy. The market portfolio gives us information about liquidity and the propensity to save, which translate into the real economy only through a nonlinear, lagged mechanism on

As (11) shows, given market risk and $n$ different assets, it is possible to express the systemic risk of one of them through the risk of the other $n-1$ assets.

To simplify the discussion, assume $n = 2$ and $\mathbf{t} = (1, 2)$. (11) becomes:

$$\beta_2 = \frac{\beta_1}{2} - \frac{r_0 - ln(P_2) + ln(P_1)}{(r_m - r_0) \cdot 2} \tag{12}$$

Substituting to (4) and making some adjustments [3], we get:

$$K = e^{-r_0 - \beta_1 \cdot (r_m - r_0)} \cdot (e^{-r_{1,u}} \cdot x_1 P_1 + e^{-[2 \cdot r_{2,u} + ln(P_2) - ln(P_1)]} \cdot x_2 P_2) - L^* \tag{13}$$

(13) expresses the value of capital stock as a function of both financial parameters, such as the returns of firms in the market, and macroeconomic variables, such as the interest on government bonds and the return on the market portfolio.

# 3 Exogenous Systemic Shock in the Very Short Run

Assume now that, given the money supply [4], we enter the expansionary phase of the business cycle. This results in a change in market portfolio and riskiness.

To calculate the total effect on capital stock, we write $r_m$ as a function of $\sigma_m$ in (14) and explicitly rewrite $\beta$ as in (7):

$$K(r_m(\sigma_m)) = e^{-\frac{\sigma(1,m)}{\sigma_m} \cdot (r_m(\sigma_m) - r_0)} \cdot C - L^* \tag{14}$$

With:

$$C = (e^{-r_{1,u}} \cdot x_1 P_1 + e^{-[2 \cdot r_{2,u} + ln(P_2) - ln(P_1)]} \cdot x_2 P_2) \tag{15}$$

In (15), capital is assumed to be a function of market portfolio returns alone. This may be true in a short-term perspective, where firms have not yet reorganized production according to the new market conditions.

[3] (4) has been updated to the continuous case as in (11)

[4] In other words, the parameter $r_0$ remains constant

To compute the total effect on $K(r_m(\sigma_m))$, we compute the derivative of these with respect to $\sigma_m$:

$$\frac{dK}{d\sigma_m} = -\sigma_{1,m} \cdot A_0 \cdot \frac{dr/d\sigma_m \cdot \sigma_m - (r_m - r_0)}{\sigma_m^2} \tag{16}$$

Where in (17) we denote by $A_0$ the value of assets before the macroeconomic shock [5]. We note that the effect on capital stock is the larger, the greater the real value of assets and the covariance between them and the market portfolio.

To evaluate the total effect, we place (16) greater than zero and simplify:

$$\frac{dr_m}{d\sigma_m} < \frac{(r_m - r_0)}{\sigma_m} \tag{17}$$

In (17), the right-hand member, the Sharpe Ratio [6], indicates the performance of the market portfolio before the exogenous shock. Indicating the changes in percentage terms instead, we get:

$$\Delta_{\%} r_m > a \cdot \Delta_{\%} \sigma_m \iff \Delta\sigma < 0 \tag{18}$$

With:

$$a = \left(1 - \frac{r_0}{r_m}\right) \tag{19}$$

In other words, the total effect on the balance sheet is positive as long as the percentage change in market return is less than the appropriately discounted percentage change in risk. Moreover, the greater the difference between $r_m$ and $r_0$, the greater the chance that a macroeconomic shock has a negative effect on the balance sheet. However, it is not possible to say a priori whether the total change is positive or negative, since usually $r_m$ varies more than $\sigma_m$.

Consider the most common case, where the expansionary business cycle increases the propensity to consume and thus aggregate demand. This results in a reduction in aggregate savings; as a result, firms must offer a higher return (or return on capital) for the same risk. The left member of (17) is thus negative, and (17) is always true.

We can therefore draw the following conclusion:

[5] $A_0$ was stostituted to $K + L$, the liability side of the balance sheet

[6] $\frac{(r_m - r_0)}{\sigma_m}$ is also the slope of the capital market line, in the model of the same name

**T1**: *Given a competitive capital market and an absence of arbitrage opportunities, assuming costant non-systemic risk, the expansionary phase of the business cycle increases the real value of the firm capital stock.*

In the next section we will consider the general case where firm-specific risk also varies.

# 4 Medium Term and Adaptation of the Unsistemic Risk

Not fixed firm-specific risk can be seen as a medium term perspective, in which firms reorganize production according to new market conditions. However, real output still does not vary, since it is a function of the technical conditions of production. Consequently, the real output expressed by (5) also remains constant [7]. We then rewrite (14) as a function of also $r_{1,u}$ and $r_{2,u}$ and calculate the total derivative:

$$dK(r_m(\sigma_m), r_{1,u}, r_{2,u}) = \frac{\delta K}{\delta r_m}\frac{\delta r_m}{\delta \sigma_m} \cdot \Delta r_m + \frac{\delta K}{\delta r_{1,u}} \cdot \Delta r_{1,u} + \frac{\delta K}{\delta r_{2,u}} \Delta r_{2,u} \quad (20)$$

Carrying out the calculations, we get [8]:

$$\Delta K = -\sigma_{1,m} \cdot A_0 \cdot \sigma^2(\Delta r_m - (r_m - r_0) \cdot \Delta\sigma) - C(\Delta r_{1,u} - 2\Delta r_{2,u}) \quad (21)$$

By shifting the terms of (14) we find that:

$$C = A_0 \cdot e^{\frac{\sigma(1,m)}{\sigma_m} \cdot (r_m(\sigma_m) - r_0)} \quad (22)$$

We then substitute in (21) and place greater than zero as in (17) to find the conditions under which capital stock grows as a result of the exogenous shock:

$$-\sigma_{1,m} \cdot \sigma^2[\Delta r_m - (r_m - r_0) \cdot \Delta\sigma] > e^{\frac{\sigma(1,m)}{\sigma_m} \cdot r_m - r_0)} \cdot (\Delta r_{1,u} - 2\Delta r_{2,u}) \quad (23)$$

[7] In formulas we have in the medium run $\Delta r_{i,u} = -\Delta r_{i,m}$

[8] $\Delta K$ has been substituted for $dK(r_m(\sigma_m), r_{1,u}, r_{2,u})$

We also consider here the most common case, where an increase in the propensity to consume increases the market portfolio return, reducing riskiness. According to the CAPM model, this results in an increase in firms' non systemic risk at the expense of systemic risk. Thus, we have the left member of (23) negative [9] and the left member with indeterminate sign; if greater than zero, (23) is never true. In formulas:

$$\Delta r_{1,u} > 2\Delta r_{2,u} \implies \Delta K < 0 \tag{24}$$

We can then draw the following conclusion:

> **T2**: *Given a competitive market and assets of only two types, the expansion phase of the business cycle reduces the real value of the capital stock if the firm-specific risk of the firm with short-maturity obligations is greater than the firm with long-maturity obligations.*

The proposition **T2** is obviously dependent on the assumption that $t_1 = 1$ and $t_2 = 2$. Moreover, the reverse is not true, namely, that the change in equity is positive if the (24) does not hold. To measure under what conditions this occurs, we give the following numerical values to the variables in the model:

$$r_0 = 0.025$$
$$r_m = 1$$
$$\sigma = 1$$
$$\sigma_{1,m} = 0,667$$
$$\Delta r_m = 0,02$$
$$\Delta\sigma = -0,001$$

We substitute at (23) and adjust the terms:

$$\Delta r_{1,u} < -0,023 + 2 \cdot \Delta r_{2,u} \tag{25}$$

Now, since by definition $\Delta r_{1,u}$, greater than zero, we have that (25) is false as long as:

[9] $\sigma(1, m)$ and $\sigma^2$ are positive by definition, $(r_m - r_0)$ is trivially positive, $\Delta r_m$ and $\Delta\sigma$ are positive and negative, respectively, by assumption.

$$\Delta r_{2,u} < 0.015 \tag{26}$$

We can therefore draw the following conclusion:

> **T3**: *Given an expansive phase of the business cycle, an inadequate change in the non-systemic return on long-maturity assets reduces the real value of equity, regardless of the return on short-maturity assets.*

The proposition **T3**, although based on a numerical simulation, is important in that it decides the value of the capital asset based on a single asset type.

However, it was assumed that the relationship between $\beta_1$ and $\beta_2$ remains unchanged as described by (12). From a long term perspective, it is important to consider this factor as well in relation to the new macroeconomic conditions. Same for the covariance in (7).

However, the business cycle is a phenomenon confined to the short run; therefore, it is reasonable to leave the above parameters unchanged.

# 5 Conclusion

To the question we posed in the introduction, namely, how much does the business cycle affect capital stock, we can give multiple answers, depending on the time horizon.

In the short run, before macroeconomic effects are transmitted on firms' specific riskiness, the expansionary phase of the cycle improves assets value and thus capital stock. This conclusion is obviously linked to the assumption that the market is competitive and that the exogenous shock is stochastic and totally unexpected.

In the medium term we have a partially different situation. The positive effects of the cycle are infuenced by the firm specific return. In particular, an excessive return of the short-maturity obligated firm relative to the long one reduces the real value of capital. In addition, giving reasonable values to the parameters of the system, the long-maturity bonds alone decide the trend of real capital value, regardless of other credits.

Therefore, we can conclude that within the medium run, the viariable deciding on the change of capital stock is the non-systemic risk of the company with long-maturity bonds.

# Reference

The model developed in this paper is based on microeconomic and corporate financial theory. Eexcellent reference texts are:

*Walter Nicholson, Christopher M. Snyder*, Microeconomic Theory: Basic Principles and Extensions; 12th Edition, 2016, Cengage Learning.

*Jonathan Berk, Peter DeMarzo*, Corporate Finance, 5th Edition, 2019, Pearson.