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Chapter 1

Introduction

The electromagnetic spectrum extends from low energies to high energies, X-rays and γ-rays which are measured in electron volt (eV), where:

$$1\,\mathrm{eV} = 1.602 \times 10^{-19}\mathrm{C} \cdot 1\,\mathrm{V} = 1.602 \times 10^{-19}\mathrm{J}$$

The photon energy is:

$$E = h\nu = \frac{hc}{\lambda}$$

When we refer to electron energy we intend a kinetic energy that can be associated with a temperature:

$$E = k_B T_e$$

X-rays and γ-rays can easily penetrate atoms due to their wavelength smaller than the Bohr radius – equal to $0.5\,\text{Å}$. In addition, on the basis sof the conversion between energy and mass both the masses of the electron an proton can be expressed in terms of energy.

1.1 Luminosity and flux

The luminosity is the intrinsic energy emitted by a source per unit of time, does not depend on the solid angle therefore neither on the direction and the distance of the observer. Usually is evaluated in a specific wavelength or within an interval:

$$L_{\nu_1-\nu_2} = \int_{\nu_1}^{\nu_2} L_\nu d\nu$$

Applying this definition we can define $L_{0.2\ -\ 2\,\mathrm{keV}}$ as soft X-ray luminosity and $L_{2\ -\ 10\,\mathrm{keV}}$ as hard X-ray luminosity. When observing in different ranges of the spectrum different instruments are mandatory and is possible to combine them in order to obtain the Spectral Energy distribution and define the bolometric luminosity as the total luminosity integrated from radio to γ-rays:

$$L_{\mathrm{bol}} = \int_0^\infty L_\nu d\nu$$

A SED is a low-resolution spectrum made by photometric data obtained in different intervals. On the x-axis there is the frequency and on the y-axis the flux density or the $\log_{10}$ value. The instruments measure the flux, flux and luminosity are related via:

$$F = \frac{L}{4\pi R^2}$$

Instruments deal with the specific intensity, which is the energy emitted by a source in units of area, time, frequency and solid angle:

$$L = \frac{dE_\nu}{dA\cos\theta dt d\nu d\Omega}$$

When the source is resolved the flux is the energy flowing across the unit area from all directions:

$$F_\nu = \int_\Omega I_\nu \cos\theta d\Omega$$

At hih energies the photon intensity is commonly used in place of the specific intensity due to the possibility of photon counting:

$$n = \frac{dn}{dA\cos\theta dt d\nu d\Omega}$$

It is possible to convert from photon flux to energy flux multiplying by $h\nu$.

1.2 Atmospheric transmission

Earth atmosphere is always in turbulent regime which changes locally the refractive index of the air and cause the wavefront to be segmented in small linear undisturbed parts with a size of few centimeters. The net effect is called seeing and to solve this problem modern ground-based telescopes are equipped with a complex instrumentation called adaptive optics system. The alternative consists in space telescopes.
The atmosphere is opaque at some wavelengths, such as the UV, X-rays and par of the infrared. To carry studies in these ranges the use of space telescopes is mandatory. In addition the earth is a powerful infrared source that causes a rapid saturation of the background.

1.3 Telescope resolving power

According to the Huygens-Fresnel principle, a point source emits spherical wavelets whose overlap forms a spherical wavefront. When a collimated beam of light reaches a small aperture, each point of the aperture converts the plane wavefront into spherical wavelets and their overlap and interference produce the diffraction patterns. In case of a circular aperture we obtain an image known as the Airy disk, where the position of the first maximum of intensity is given by:

$$\theta = 1.22\frac{\lambda}{a}$$

In case of two point sources observed through a circular aperture the Airy disks overlap producing a new pattern. The Rayleigh criterion specifies the minimum separation between two light sources that can be resolved into distinct objects. If the first maximum of a source coincides with the first minimum of the other then we can resolve the two sources. If the circular aperture is a telescope then this gives us the theoretical resolving power of the instrument.
The image of a point-like source on the detector is the result of the contribution of aberrations and seeing (for ground-based telescopes) and is called Point Spread Function (PSF).

Chapter 2

X-ray telescopes

2.1 Grazing incidence

X-ray telescopes are different from the optical ones because of the wavelength of the incident photons. The energy of 1 keV corresponds to a radiation of 12.4 Å that can pass through the mirror and/or be absorbed. Therefore a trick must be applied to focus X-ray photons.

According to the Snell's law of refraction the direction of the propagation changes when passing between two different mediums:

$$n_1 \sin \alpha_i = n_2 \sin \alpha_r$$

In particular when $n_1 > n_2$ we have $\alpha_r > \alpha_i$. When $\alpha_r = 90$ we can define the critical angle:

$$\alpha_c = \arcsin \left(\frac{n_2}{n_1} \right)$$

Beyond this angle the light is reflected. Keeping in mind that X-rays telescopes are in space it means that mirrors for X-rays must be coated with materials having $n_2 < 1$.
The X-ray refractive index may be written in the form:

$$n = 1 - \delta - i\beta$$

where the real part (with $\delta \sim 10^{-5} - 10^{-4}$) accounts for the refraction effect and the imaginary part (with $\beta \sim 10^{-6} - 10^{-5}$) is related to the X-ray photoelectric absorption. The δ and β parameters represent the optical constants of the material.
The smallness of δ causes extremely small deviations to the direction of incidence X-rays, therefore the use of lenses for X-rays would have a focal length too long for a single spacecraft. On the other hand the use of thick lenses for X-rays would be ruled out by the too-large absorption coefficients. As a consequence practical X-ray optics have to be just reflective and in a grazing incidence configuration.

In X-ray astronomy is commonly used θ_C which is complementary to critical angle α_c. Now since the imaginary part is much smaller than the real one we can assume that $n \approx 1 - \delta$. Therefore:

$$\sin \alpha_c = \cos \theta_c = n_2 \approx 1 - \delta$$

Because of the small value do δ also θ_c is small so that we can apply the approximation:

$$\cos \theta_c \approx 1 - \frac{\theta_c^2}{2} = 1 - \delta$$
$$\theta_c \approx \sqrt{2\delta} \quad \text{in radians}$$
$$\theta_c \approx 81\sqrt{\delta} \quad \text{in degrees}$$

The parameter δ is given by the following formula:

$$\delta = \frac{\rho A_0 r_e \lambda^2 f_1}{2\pi W_m}$$

where ρ is the density of the material, A_0 is the Avogadro number, r_e is the classical electron radius, f_1 is the first atomic scattering coefficient and W_m is the molar weight of the reflecting material. At high energies f_1 is almost equal to the atomic number Z we can write:

$$\theta_c \propto \frac{\sqrt{\rho}}{E}$$

that is that the critical angle is proportional to the square root of the material density and the atomic number and the atomic number, and inversely proportional to the photon energy. To increase θ_c we have to increase the material density. Indeed, high-Z materials are used as coating layers for X-ray mirrors on top of lighter supporting plates. For a fixed incident angle, only photons below a certain cut-off energy can be reflected. It must be noticed that in addition to photoelectric absorption the impossibility to obtain a perfectly smooth surface must be taken into account. In conclusion only near-total reflection is really possible.

2.2 Bragg reflection

At high energies, typically larger than 1 keV it becomes more difficult to obtain reflection due to very small angles. A possible solution is using multiple layers of alternate high-Z and low-Z materials. The solution is based on Bragg reflection: incoming rays are in phase while outcoming rays are out of phase and produce interference. When the optical path difference $\Delta = 2d\sin\theta$ is an integer multiple m of the wavelength λ we observe constructive interference:

$$m\lambda = 2d\sin\theta$$

Reflection above 10 keV is achieved choosing the layer optical thickness in order to produce constructive interference between each coating pair and summing the reflection amplitude of all the stacks. The Bragg equation must be modified to take into account the effect of refraction in the layers:

$$m\lambda = 2d_M\sin\theta\sqrt{1 - \frac{2\delta}{\sin^2\theta}}$$

where δ is the period-averaged index of refraction and d_M is the period of the multi-layer system.

2.3 Wolter telescope

The idea of using grazing incidence reflection to focus X-rays was proposed in 1960 by Giacconi and Rossi: a truncated parabolic mirror not located in the vertex of the parabola, like in optical telescopes but in the arms. The configuration works well only if the rays are parallel to the optical axis but for different inclinations the telescope would be strongly affected by coma therefore the useful field of view of the optics would be too small to produce any image in the focal plane.
The solution was found by Wolter in 1952 who proposed to use an even number of reflections from confocal conic-like optics. This combination of mirrors must fulfill the so-called Abbe sine condition to avoid coma aberration by ensuring the same optical path for all incident X-ray photons:

$$\frac{h}{\sin\theta} = R$$

where R is a constant radius. For astronomical objects all rays must be considered parallel and so the Abbe condition is satisfied if the incident rays intersect the reflected ray direction in a spherical surface, called principal Abbe surface centered on the focus.
The principal surface is not a sphere but a paraboloid that is well approximated by a sphere near the vertex of the optical system, meaning that the Abbe condition is really verified in the angular region close to the center of the fov.

There are three main configurations:

- **Wolter I** – Hyperbolic mirror with focus F_2 and parabolic mirror with focus F_1
- **Wolter II** – Hyperbolic mirror with fous in F_2 and parabolic mirror with focus in F_2
- **Wolter III** – Elliptical mirror with focus in F_1 and F_2 and parabolic mirror with focus in F_1.

The main difference is the ratio between the focal length and the system length. A Wolter I telescope has a ratio smaller than 1, a Wolter II has a larger focal length and system length and a Wolter III has the shortest focal length.

2.4 Effective area

While in optical telescopes mirrors are orthogonal to the optical axis and therefore fully oriented toward the source to collect the incoming photons, in X-ray telescopes mirrors are cylinders. The consequence of high inclination of the surface mirror with respect to the direction of the rays is the strong reduction of the effective area which implies an obvious loss of efficiency. In case of an optical telescope the effective area is:

$$A_{\text{eff}} = \pi \frac{\phi^2}{4}$$

while for an X-ray telescope is:

$$A_{\text{eff}} = \pi \frac{(\phi_1 - \phi_2)^2}{4}$$

In order to increase the effective area of an X-ray telescope and so increase the efficiency in collecting and focusing photons it is possibile to nest together many confocal shells and the final effective area will be the sum of the effective areas of each mirror. This solution has the additional advantage to increase the energy range of the collected photons since each shell is characterized by a different incident angle. Among the configurations the Wolter I is the favorite since it is possible to make nested mirrors. The effective area of that kind of telescope is given by:

$$A_{\text{eff}}(E) = 8\pi f L \theta^2(E) R^2(E)$$

with f focal length, L length of the parabolic mirror θ the incident angle and $R(E)$ the reflectivity.

2.5 Angular resolution

In X-ray telescopes the angular resolution is given by the half-power diameter (HPD) which is also called half-energy width (HEW). It is more efficient than FWHM because of the shape of X-ray mirrors.
A good angular resolution allows to have a low probability for a background event that can contaminate the signal of the source.

Chapter 3

Alternative instruments

In addition to telescopes there are other instruments used in high energy astrophysics: passive collimators for both X-ray and γ-ray, coded masks for X-rays and Compton telescopes for γ-rays.

3.1 Photon-matter interaction

Photons are detected by being stopped in matter and producing a signal. The interaction between photons and matter occurs:

- photo-electric absorption at energies lower than a few hundreds keV where the absorption requires high-Z materials
- compton scattering at $100\,\mathrm{keV} < E < 30\,\mathrm{MeV}$ where absorption is difficult and both masks and collimators become transparent
- pair production at energies higher than a few MeV (to few GeV)

3.1.1 Photoelectric absorption

It occurs when the photon has enough energy to ionize the atom and generate a free electron. Most of the absorption of X-ray photons occurs in the inner K or L shells. When an electron is removed another electron can move form an outer and high energetic shell to fulfill the hole in a process that release energy (fluorescence).
The probability to have photoelectric absorption is related to the cross section:

$$\sigma_{ph} \propto \frac{Z^n}{E_{ph}^3} \quad n = 4 - 5$$

therefore this process dominates at low energies and its cross section increases rapidly with the atomic number. In addition σ_{ph} is characterized by discontinuities, called absorption edges.

3.1.2 Compton scattering and pair production

X-ray photons can scatter off atomic electrons while they pass through matter and a part of the photon energy is transferred to the recoiling electron. This electron is in one of the outer orbits and its binding energy is significantly less than the energy of the photon. The process is known as compton scattering.
The energy of the scattered photon depends on the angle θ and its original energy:

$$E'_{ph} = \frac{E_{ph}}{1 + \frac{E_{ph}}{m_e c^2}(1 - \cos\theta)}$$

The cross section for compton scattering is described by the Klein-Nishina formula and predicts the angular distribution of photons after scattering. Its probability decreases with increasing Z of the absorber. This interaction is more probable in the middle photon energy range (0.1 - 1 MeV and light materials. The emitted photon can be absorbed locally.

When the energy of the photon is greater than twice the rest mass of the electron and the γ-ray comes into the near vicinity of the nucleus, it decades into a pair of electron and positron. Pair production is the dominant process at

high energies but is not important until a photon energy of 4 MeV is reached. When the positron has expended its kinetic energy in the medium it will annihilate with a free electron and two 0.51 MeV photons are emitted in opposite directions.
If the interaction takes place with an electron then there is the creation of an extra electron called recoil electron constituting what is called a triplet. This process is more probable in low-Z atoms.

Measuring the pair direction and their energy it allows to reconstruct the primary photon energy.

3.2 Mechanical collimator

A passive mechanical collimator is made by a detector surrounded by absorbent walls that act like shields reducing the angular size od the sky that can be observed. It has an high sensitivity but is collects photons from both the source and the background. A passive collimator does not produce an image and it suffers form the contamination by nearby sources.

3.3 Coded mask

If there is nothing in between a detector cannot spatially distinguish the direction of the incoming photons from a source and no image is formed. By using an aperture (mask) it is possible to select the incoming light rays and create an image of the source (pinhole camera).
A coded mask is like a chess board with some parts absorbing radiation and others that let the radiation reach the detector. It is less sensitive than a collimator but it allows to generate images reducing the problem of source confusion. The result onto the detector is not a real image of the fov but a pattern therefore in order to restore the image is necessary to apply a deconvolution process.

3.4 Compton telescope

In the MeV range the compton effect hampers any tentative to shield efficiently the detector. A compton telescope is essentially made by two planes of detectors, one acting as a scatter and the other one as an absorber. This instrument has low sensitivity and poor angular resolution that can be improved using coded masks.

Chapter 4

Charge-Coupled Device

Detection systems are based on semiconductors or solid state materials like silicon, germanium, cadmium-telluride or cadmium-zinc-telluride. Once the ionizing radiation generated an electron-hole pair, its motion within an applied electric field create the detector electrical signal. The discrete energy levels in atoms become energy bands in solids. The separation between the valence band and the conduction band is called band gap and defines a solid as an insulator ($> 5\,\mathrm{eV}$) or a semiconductor ($\geq 1\,\mathrm{eV}$).
Without sufficient energy to cross the band gap, electrons cannot move and the conduction band remains empty. Thanks to relatively small gaps, semiconductors can allow conductivity through excitation processes like an increase of temperature or an absorption of a photon. In both cases the electron moves to the conduction band leaving a hole in the valence band and generating an electron-hole pair. Through the application of an electric field the electron moves through the detector while the hole moves in the opposite direction.

4.1 CCD readout

A CCD pixel is a capacitor equipped with three electrodes or gates aligned along a direction whose function is to move the charges collected in the depletion region. After the CCD is exposed to the light, each pixel has collected a charge packet proportional to the number of incident photons. Then the packet are moved through the pixels off to the chip.

4.2 Quantum efficiency

Each CCD is characterized by a quantum efficiency curve which described the ability to capture photons incident on it. The QE is defined by the ratio of recorded photons to received photons and varies with the wavelength. There is a quantity called the absorption length that depends on the wavelength and is defined as the distance for which 63% of the incident photons will be absorbed.
CCDs are distinguished mainly into thick front-side illuminated and thin back-side illuminated. Thick front side are about $300\,\mu\mathrm{m}$ thick and thus efficient up to the nIR. They are easier to build and therefore cheaper. They are illuminated from the front that is from the side on which gates are placed therefore photons are partially reflected and partly absorbed by the gates and the layers of material they encounter before reaching the depletion region. This causes an overall reduction of the QE.
The thin back-side illuminated have a thickness of $15\,\mu\mathrm{m}$ and are illuminated from the back, therefore the silicon is facing the light. With the addition of an anti-reflection layer there is a strong increase in the QE. The thinning process allows for increased efficiency in the blue but there is a loss in efficiency in the red. To solve the problem in the red and nIR CCDs can be made twice as thick as normal thin CCDs. These CCDs are called deep depleted and are also back-side illuminated. They are able to easily capture large wavelength photons, thus increasing the QE in the red.

4.3 CCDs for X-rays

A visible light photon produces a single electron-hole pair. In order to measure a signal a large number of electrons must be collected and counted therefore they need a large exposure time.
Viceversa an X-ray photon has enough energy to produce many electrons and holes through the process of secondary ionization by the primary photoelectron. For example a single $6\,\mathrm{keV}$ photon generated 1630 electrons, enough to be measured. This means that a long exposure time is not necessary and we can use the detector as a photon counter. No more than 1 photon must be incident on each pixel in any mage frame thus typical exposure times are less than a second. However long total exposure times are mandatory and can be obtained through the co-addition of many

short exposures.

Using the CCD in photon counting mode gives us three information:

- position: we can identify which pixel catches the photon
- energy: we can measure the energy of the incoming photon
- arrival time: we can measure when the photon is detected

counting the number of photons as a function of their energy we obtain the low-resolution spectra and using short exposure times we can measure the variability of the source through its light-curve.

Another important property of X-ray CCDs is the size of the depletion region that is on the order of 30 - 300 µm, larger than in visible light. This is necessary to store the larger number of electrons. In order to improve the efficiency in collecting charges these CCDs have larger pixels because of the secondary ionization and thus secondary production of electrons. In addition in order to improve the efficiency these CCDs have larger pixels with respect to those used for visible light.

However an X-ray photon can deposit energy in more than one pixel because of secondary ionization and thus secondary production of electrons. Usually a multiplet of pixels is analyzed for each event. When the event is limited to a single pixel we have a perfect single situation and the quality of the signal is called grade 0. Depending on the distribution of charges in the surrounding pixels, other grades are defined with an increasing number that corresponds to progressively worse situations.
In the table of events is possible to see that each pixel has an associated grade, an evaluation of the quality of the detection. This evaluation is performed on board of the telescope and in space where the cosmic rays are also present. The instrumentation is able to distinguish between X-ray photons and cosmic rays since the latter cause more complex structures on the detector. In case of extremely bright objects is quite difficult to apply exposure times sufficiently short to detect only one photon. If more photons reach the same pixel there is an unavoidable extra charge called pile-up condition.

In visible light the QE is plotted as a function of the wavelength while at high energies is given as a function of the energy itself. The QE in X-ray is a combination of two processes: the transmission through the dead layers and the absorption in the depletion region.

CCDs for X-rays are sensitive to visible photons and this cause noise and calibration issues. Thus materials that absorb visible light are applied on top of the CCDs. The consequence is a QE curve slightly lower, especially at low energies.
The energy and angular resolution defines the ability of the instrument to discriminate photons of different energies and positions.

Chapter 5

X-ray diffraction gratings

In X-ray astronomy is it possible to obtain a spectrum with a detector because we can distinguish photons by energy. But to have an high resolution spectra we need gratings.
XrDG are placed at the exit aperture of a focusing optics, that is in the focused beam. According to the kind of grating, transmission or reflection we have two possible configurations. Because of the grazing incidence in X-ray optics, spectrographs must be slit-less and then they work effectively only with point-like sources. In particular is the angular size of the source that determines the spectral resolution that is the angular resolution off the instrument is directly affecting the spectral resolution.

Typical **transmission gratings** are made by a period nanostucture consisting of finely spaced parallel gold bars supported on a thin plastic membrane. If i is the angle of incidence and θ is the dispersion angle, the equation of the grating is:

$$d(\sin\theta - \sin i) = m\lambda$$

where d is the period. If Δi is the angular resolution of the telescope that is the intrinsic angular spread of the incident beam, the diffracted beam will have an angular spread $\Delta\theta = \Delta i$ for a given order of wavelength. In turn $\Delta\theta$ is related to $\Delta\lambda$ by the dispersion equation:

$$d(\sin\theta - \sin i) = m\Delta\lambda = \frac{d}{m}\cos\theta\Delta\theta$$

Both i and θ are small, and if $i = 0$ we can compute the resolving power:

$$R = \frac{\lambda}{\Delta\lambda} = \frac{\tan\theta}{\Delta\theta} \approx \frac{\theta}{\Delta\theta}$$

It is clear that to increase R it is necessary to decrease d or m or to have a telescope with an higher angular resolution.

Reflection gratings are made of a thin silicon carbide substrate coated with gold. Because of the grazing incidence, angles are not measured from the normal to the grating but from the surface of the grating. Therefore the equation is:

$$d(\cos\theta - \cos i) = m\lambda$$

For a given angular resolution Δi the diffracted beam has a spread $\Delta\theta$ corresponding to a wavelength distribution:

$$\Delta\lambda = \frac{d}{m}\sin i\Delta i$$

and the resolving power is:

$$R = \frac{\lambda}{\Delta\lambda} = \frac{\cos\theta - \cos i}{\sin i\Delta i}$$

Is possible to increase the resolving power through higher resolution telescope or smaller incidence angle.

Gratings are placed in a converging beam instead of a parallel beam. The consequence is that both the incidence and the dispersion angle are varying across the grating. The way to solve the problem is curving the grating or produce gratings with variable periodicity across its surface. The best solution was found in 19th century and is named Rowland circle: it is a circle of radius R that lies tangent to a concave curved diffraction grating of radius $2R$. In addition the groove density is constant when projected in a plane tangent to the grating.

Chapter 6

Astrostatistics

To measure an observable quantity we need to collect a sample of data that will be characterized by fluctuations. The underlying set of values from which this quantity is sampled is called a distribution, and is related to the probability to obtain a particular value in an experiment. They can be based on a continuous variable or a discrete variable and are generally normalized.

$$f(x) \rightarrow \int_x f(x)dx = 1 \quad \text{continuous}$$
$$f(k) \rightarrow \sum_k f(k) = 1 \quad \text{discrete}$$

A distribution is characterized by moments: 1 = mean, 2 = variance, 3 = skewness, 4 = kurtosis.
Mean and variance are given by:

$$E[x] = \int_x x \cdot f(x)dx \qquad E[k] = \sum_k k \cdot f(k)$$
$$V[x] = \int_x x^2 \cdot f(x)dx - (E[x])^2 \qquad V[k] = \sum_k k^2 f(k) - (E[k])^2$$

The skewness measures the asymmetry and the kurtosis the tailedness of the distribution, that is the degree of flatness.

The most popular distribution is the Gaussian, defined in the real numbers domain by moment 1 and 2, while 3 and 4 are zero.

$$f(x) = \frac{1}{2\pi\sqrt{\sigma}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

the mean is $E[x] = \mu$ and the variance is $V[x] = \sigma^2$.
On the logarithmic scale the gauss function becomes a parabola and the fourier transform of the gauss function is still a gauss function.
The value of σ is related to the FWHM (FWHM $= \sqrt{8 \ln 2}\sigma$), increasing σ the peak decreases due to the normalization.

The Poisson distribution is used for discrete measurement and describes the probability of observing k counts knowing the expected value λ.

$$f(k) = \frac{\lambda^k}{\Gamma(k+1)} e^{-\lambda} = \frac{\lambda^k}{k!} e^{-\lambda}$$

The Poisson distribution is not symmetric, is strongly skewed as λ goes to zero because the range of the function is bounded. At large λ it can be approximated with the gaussian function.
The poisson distribution is useful in determining the probability that a certain number of events occurs over a given time interval. In astrophysics it is the fundamental underlying distribution that is used to describe all of high energy photon counts data. The mean is given by $E[k] = \lambda$ because the distribution is normalized by definition. The variance is still λ. The standard deviation is $\sigma = \sqrt{\lambda}$ and it is the error in counting statistics. It means that in case of a bright source which emits an high number of photons, the statistical noise is a smaller fraction of the source signal.

6.1 Signal to noise

If s is the count rate from the source, b the count rate from the background and t the exposure time, the total number of counts is:

$$N_{tot} = (s+b)t \pm \sqrt{(s+b)t}$$

If we can neglect the uncertainties on the background: $N_{bg} = bt$. Then the total source photons is given by:

$$N_{src} = N_{tot} - N_{bg} = st \pm \sqrt{(s+b)t}$$

and the SNR is:

$$SNR = \frac{N_{src}}{\sqrt{N_{tot}}} = \frac{st}{\sqrt{(s+b)t}}$$

When the SNR is around 2-3, the source is barely detected and we need a longer exposure time or a larger telescope or both. A solid detection is when the SNR is at least 5. In case of SNR=10 it is possible to do science. When SNR=100 we have very good measurements and the analysis can be accurate and detailed. The inverse of SNR corresponds to the relative error of the measurements therefore SNR=100 is a 1% error.

To be more specific, the SNR is a combination of the effective area of the telescope, the bandwidth, the dark current rate and the read out noise of the detector.
As possible sources of the background we mention high energy cosmic rays, particles hitting the satellite and producing X-rays, low energy photons, and soft X-ray galactic foreground.

6.2 Fitting

We have a set of measurements and we want to compare it with an astrophysical model which is a function of several variables called parameters. We define the χ^2:

$$\chi^2 = \sum_i \frac{(D_i - M_i)^2}{\sigma_i^2}$$

which is the appropriate statistics to use when fitting data whose errors are normally distributed.

Since gaussian probability density is a measurement of the likelihood of the data for the specific model, minimizing the χ^2 means that we are maximizing the likelihood (better results).
When a model having m parameters is fitted to a dataset having N bins we can define the degrees of freedom:

$$\nu = N - m$$

A fit is possible only if we have $N > m$.

A typical situation is that we don't have a unique result from the fitting procedure since the result depends on the initial guess parameters, therefore we obtain an ensemble of good fits and we need to analyze the distribution of the χ^2.
A popular criterion is the so-called reduced-χ^2, $\frac{\chi^2}{\nu}$, normalized to the degrees of freedom. When:

$$\frac{\chi^2}{\nu} \gg 1 + \sqrt{\frac{2}{\nu}}$$

the fit is not good. In general we can say that both the χ^2 and the reduced-χ^2 are used to evaluate the goodness of a fit.

High energy data are often characterized by a small number of counts for which a Poisson distribution is more suited therefore we can't use the χ^2 minimization but we have to use the C-statistics.

6.3 p-value

Best-fit parameters and error bars are often insufficient to test an hypothesis. An additional tool is the p-value used to understand whether the hypothesis are correct or no. The p-value is the area under a distribution integrated between a particular value to the end of the domain. It is correlated with the null hypothesis, the default distribution for which we do not observe any significant difference between a hypothesized value of a parameter and its value estimated from a sample drawn from that distribution.

Chapter 7

Radiative processes

The main source of radiation in relativistic astrophysics is given by accelerated particles. The number of particles N as a function of their kinetic energy ($\mathcal{E}$) follows a power-law distribution:

$$dN \propto \mathcal{E}^{-s} d\mathcal{E}$$

where s is called power index. The formula says that is more probable to find low-energy particles than high-energy ones.
A charge is accelerated by an electric or magnetic field an when accelerating it emits radiation. Acceleration must be intended in terms of change of velocity.

7.1 Bremsstrahlung

When a free charge, in general an electron, passes nearby an ion, the electron changes velocity losing the kinetic energy that it is converted into radiation called Bremsstrahlung or free-free radiation.
If the electron moves within an electric field then we have thermal Bremsstrahlung because the electron temperature changes. If the electron moves within a magnetic field then we have cyclotron or synchrotron radiation depending on the energy of the particle (non-relativistic or relativistic).

Bremsstrahlung can be detected in visible, near-infrared and ultraviolet but when the temperature increase we move from the non-relativistic to the relativistic case and we cna detect Bremsstrahlung at high energies. We can then use X-ray observations to detect free-free emission from a plasma, for example in a cluster of galaxies.

7.2 Synchrotron

When a particle moves inside a magnetic and electric field it is exposed to the Lorentz force and the magnetic field $\vec{B}$ changes the direction of the particle, causing an helical motion around the lines of the uniform magnetic field.
The frequency of the circular motion in the relativistic case, also called synchrotron frequency is:

$$\omega_L = \frac{qB}{\gamma mc}$$

During this motion the particle emits radiation and because of it electrons are losing energy. Therefore their temperature decreases and we can evaluate the cooling time.

7.3 Compton and Inverse Compton

In classical physics the interaction between a particle and an electromagnetic wave is described by the Thomson scattering, an elastic scattering process at work when the energy of the photon is much less of the mass energy of the particle. In this process neither the kinetic energy of the particle nor the energy of the photon are modified by the interaction. If this condition is not satisfied then there is no more elastic scattering. The interaction is based on the quantum physics assumption that the electromagnetic wave behaves like a particle and is described by the Compton scattering, which is an heating mechanism.
When an electron is movign at relativistic speed and hits a low energy photon we pbserve the inverse compton