Algorithmic Game Theory - Università della Calabria

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ASSIGNMENT

The system must select the user who will perform the tour, must define the places that the tour will visit and must define the payments charged to the users. The only constraint we have is the maximum payout. Costs proportional to kilometers should be equally distributed and fixed costs should be based on declared utilities, so we must implement mechanisms that lead to truthful declarations of utilities.

HYPOTHETICAL MECHANISM CHOICE

according to what game theory says, how we arrived to the appropriate choice of a mechanism

APPLICATION THEORY

the choices made to shape this type of problem, rules and variables

APPLICATION WORKFLOW

graphical representation of the problem, based on the application example, to improve the understandability

APPLICATION EXAMPLE

an example to illustrate the practical use of the mechanism

HYPOTHETICAL MECHANISM CHOICE

A Mechanism (for a Bayesian game setting (N, O, Θ, p, u)) is a pair (A,M), where

- $A = A_1 \times \ldots A_n$, where A_i is the set of actions available to agent $i \in N$; and
- $M : A \mapsto \Pi(O)$ maps each action profile to a distribution outcomes.

we start from the definition of what is a Mechanism

then we want to define a direct mechanism in which the only action to each agent is to announce is private information (his true type), so we can exploit the quasilinear preferences to define the Quasilinear mechanism

A mechanism in the quasilinear setting (for a Bayesian game setting$(N, O = X \times \mathbb{R}^n, \Theta, p, u)$) is a triple $(A, \chi, \wp)$, where

- $A = A_1 \times \ldots A_n$, where A_i is the set of actions available to agent $i \in N$
- $\chi : A \mapsto \Pi(X)$ maps each action profile to a distribution over choices, and
- $\wp : A \mapsto \mathbb{R}^n$ maps each action profile to a payment for each agent.

HYPOTHETICAL MECHANISM CHOICE

A direct quasilinear mechanism (for a Bayesian game setting$(N, O = X \times \mathbb{R}^n, \Theta, p, u)$) is a pair $(\chi, \wp)$; it defines a standard mechanism in the quasilinear setting, where each i, $A_i = \Theta_i$.

we have split the function in a choice rule and a payment rule; now we can exploit the direct quasilinear mechanism which is the only setting where each agent is asked to state his type

at this point we make again assumption that agents' utilities depend only on their own types exploiting the property of conditional utility independence

A Bayesian game exhibits conditional utility independence if for all agent $i \in N$, for all outcomes $o \in O$ and all pairs of joint types θ and $\theta' \in \Theta$ for which $\theta_i = \theta'_i$ it holds that $u_i(o, \theta) = u_i(o, \theta')$.

HYPOTHETICAL MECHANISM CHOICE

A quasilinear mechanism is truthful if it is direct and $\forall_i \forall v_i$, agent i's equilibrium strategy is to adopt the strategy $\hat{v}_i = v_i$.

and again we back to the definition of truthfulness, in this case for a quasilinear mechanism

established what is truthful for game theory, we can analyze the part relating to costs, in particular the most coherent choice for this type of problem is to look at coalitional games

In coalitional game theory our focus is on what groups of agents, rather than individual agents, can achieve. Given a set of agents, a coalitional game defines how well each group (or coalition) of agents can do for itself. We are not concerned with how the agents make individual choices within a coalition, how they coordinate, or any other such detail; we simply take the payoff (or costs, this definition is valuable also for costs - that is our case) to a coalition as given.

HYPOTHETICAL MECHANISM CHOICE

Given a coalitional game (N,v), the feasible payoff set is defined as

$$\{x \in \mathbb{R}^N | \sum_{i \in N} x_i \leq v(N)\}$$

now we can define some aspects related to the division of cost starting from the definition of feasible payoff; this set contains all payoff vectors that do not distribute more than the worth of the grand coalition. We can view this as requiring the payoffs to be weakly budget balanced.

for this kind of issue we need to think about group of people who wants to maximize utilities and obtain as outcome the most convenient tour, in a fair way, so now we can define the Shapley Value. This will be the formula that we are going to exploit in the application part to verify the validity of a game.

Given a coalitional game (N,v), the Shapley Value of player i is given by:

$$\phi_i(N,v) = \frac{1}{|N|!} \sum_{S \subseteq |N| \setminus \{i\}} |S|!(|N| - |S| - 1)![v(S \cup \{i\}) - v(S)].$$

APPLICATION THEORY

- preferences (utilities) from 1 to 5
  - for the same utility the one with the lowest cost wins; at the same cost I would prefer to visit as many cities as possible, as well as have as many people as possible in the vehicle.
- a city can be visited by a particular user if that user can afford to visit that city or that tour of cities
- list of cities with fixed costs
  - v is the characteristic function that is equivalent to the cost of each city, consequently it will be the cost of the winning tour
- cost of singleton is equal to the cost of the tour with the maximum utility (always if the user can afford it)

- $\sum_{p=1}^{k} \binom{|N|}{p}$ possible games in a maximum of k vehicle seats
  - M is the maximum number of possible games

NOTE:

- stability - definition of core

A payoff vector x is in the core of a coalitional game (N,v) if and only if

$$\forall S \subseteq N, \sum_{i \in S} x_i \geq v(S)$$

Thus, a payoff is in the core if and only if no sub-coalition has an incentive to break away from the grand coalition.

ISSUE:

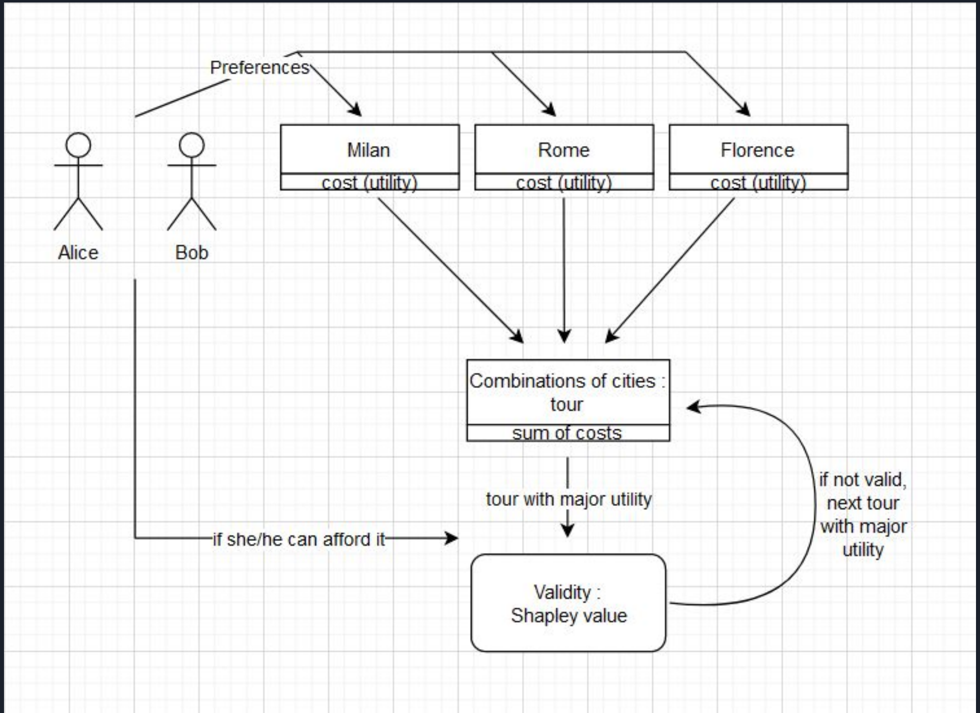
- convex game

Shapley value can be used only in a convex game but we didn't defined the game as convex, so we are going to exploit two theorems:

(1) Every convex game has a nonempty core.
(2) In every convex game, the Shapley value is in the core.

So we are not able to define and be sure that this game is convex, we assume that and we declare that the Shapley value is in the core based on these theorems.

APPLICATION WORKFLOW



APPLICATION EXAMPLE

	M	R	F
Alice	5	4	3
Bob	5	4	2

maximum payment	Alice 250
	Bob 350

M	400
R	300
F	200
MR	500
MF	450
RF	370
MRF	700

$v = 0 \leftarrow$ empty group
$v(\{A\}) = 200$
$v(\{B\}) = 300$
$v(\{A\} \cup \{B\}) = ?$

APPLICATION EXAMPLE

$v(\{A\} \cup \{B\}) = ?$

Shapley value formula:

$$\phi_i(G,v) = \frac{1}{G!} \sum_{S \subseteq G \setminus \{i\}} |S|!(G - |S| - 1)![v(S \cup \{i\}) - v(S)].$$

Winning tour: MR (< maximum payments) → cost = 500

$$\left.\begin{aligned} \mathrm{S}(\{\mathrm{A}\}) &= \frac{500-300}{2} + \frac{200-0}{2} = 200 \\ \mathrm{S}(\{\mathrm{B}\}) &= \frac{500-200}{2} + \frac{300-0}{2} = 300 \end{aligned}\right]$$

budget balance

Both are valid: S({A}) = 200 < 250
S({B}) = 300 < 350

CONCLUSIONS

Of course from a computer science point of view this is not an ideal condition, being recursive, having to calculate everything every time and being able to fall into a very large case is expensive (computationally speaking) but it is still a concrete solution to solve this type of problem.

Thank you for your attention.

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