Cheat Sheet for 2320 Updated: 4/7/2020

1	2	3	4
$\log_a x = \dfrac{\log_b x}{\log_b a}$	$a^{\log_b x} = x^{\log_b a}$	$(x^a)^b = (x^b)^a$ $x^a * x^b = x^{a+b}$	$\lg(x^a) = a(\lg x)$ $\lg(xy) = lgx + lgy$

a) Summation of consecutive values: *1+2+3+ … +n* = $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$

b) Summation of squares: *1+2²+3²+ … +n²* = $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

c) Summation of Arithmetic series (where $a_i = a_1 + (i-1)d$): $\sum_{i=1}^{n} a_i = n\frac{(a_1 + a_n)}{2} = \frac{n}{2}[2a_1 + (n-1)d]$

d) Summation of Geometric Series: $1+x+x^2+ \dots x^n$

0<x<1	***x>1***	***x=1***
$\sum_{k=0}^{n} x^k = \frac{1-x^{n+1}}{1-x} < \frac{1}{1-x}$	$\sum_{k=0}^{n} x^k = \frac{x^{n+1}-1}{x-1}$	$\sum_{k=0}^{n} 1^k = n+1$

e) Harmonic series: $\ln(n+1) \le \sum_{k=1}^{n} \frac{1}{k} \le \ln n + 1$

e2) $\sum_{k=0}^{\infty} \frac{1}{k!} = e = 2.718281\ldots \cong 2.72$

f) $\sum_{k=0}^{\infty} kx^k = \frac{x}{(1-x)^2}, \, for \, |x| < 1$ (CLRS pg.1148)

g) Approximation by integrals (CLRS, 1154):

f(x) monotonically **increasing or decreasing** (but not both)	***f(x)*** monotonically **increasing:** $x \le y \Rightarrow f(x) \le f(y)$	***f(x)*** is monotonically **decreasing:** $x \le y \Rightarrow f(x) \ge f(y)$
$\sum_{k=S}^{N} f(k) = \Theta(\,F(N) - F(S)\,)$ Where F is the antiderivative of f. (unofficial formula)	$\int_{m-1}^{n} f(x)dx \le \sum_{k=m}^{n} f(k) \le \int_{m}^{n+1} f(x)dx$	$\int_{m}^{n+1} f(x)dx \le \sum_{k=m}^{n} f(k) \le \int_{m-1}^{n} f(x)dx$

h) Radix sort (optimal r): r = min{b, floor(lg N)}

i) Master Theorem (CLRS): Let *a ≥ 1* and *b > 1*, let *f(n)* be a function, and let *T(n)* be defined on the nonnegative integers by the recurrence: ***T(n) = aT(n/b) + f(n)*** , where we interpret n/b to mean either $\lfloor n/b \rfloor$ or $\lceil n/b \rceil$.

Let $c_{crit} = \log_b a$.

Then *T(n)* has the following asymptotic bounds:

1. If *f(n) = O*(n^c) for some constant c < c_{crit}, then *T(n) = Θ*($n^{c_{crit}}$).
2. If *f(n) = Θ*(($n^{c_{crit}} log^k n$), for some constant k≥0 then *T(n) = Θ*($n^{c_{crit}} log^{(k+1)} n$).
   - 2a) if k>-1 , then *T(n) = Θ*($n^{c_{crit}} log^{(k+1)} n$).
   - *2b) if k==-1, then T(n) = Θ*($n^{c_{crit}} \log\log n$).
   - *2c) if k<-1, then T(n) = Θ*($n^{c_{crit}}$).
3. If *f(n) = Ω*(n^c), for some c > c_{crit}, and if *af(n/b) ≤ kf(n)* for some *k < 1* and all sufficiently large *n*, then *T(n) = Θ*($f(n)$).

j) $L'Hospital\ rule: if \ \lim_{n\to\infty} f(n) \ and \ \lim_{n\to\infty} g(n) \ are \ both \ 0 \ or \ \pm\infty, and \ if \ \lim_{n\to\infty} \frac{f'(n)}{g'(n)} \ is \ a \ constant \ or \ \pm\infty,$

$then \quad \lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{f'(n)}{g'(n)}.$

k)

	Notation	Limit theorem	Definition with constants
1	$f(n) = \Theta(g(n))$	$\lim_{n\to\infty} \frac{f(n)}{g(n)} = c \ne 0$	There exist <u>positive</u> constants c_0, c_1 and n_0 s.t.: $c_0\, g(n) \le$ ***f(n)*** $\le c_1\, g(n)$ for all $n \ge n_0$
2	$f(n) = O(g(n))$	$\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0 \ or \ c$	There exist <u>positive</u> constants c_1 and n_0 such that: ***f(n)*** $\le c_1\, g(n)$ for all $n \ge n_0$
3	$f(n) = \Omega(g(n))$	$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \infty \ or \ c$	There exist <u>positive</u> constants c_0 and n_0 such that: $c_0\, g(n) \le$ ***f(n)*** for all $n \ge n_0$
4	$f(n) = o(g(n))$	$\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$	For any <u>positive</u> constant c_1, there exists n_0 s.t.: ***f(n)*** $< c_1\, g(n)$ for all $n \ge n_0$
6	$f(n) = \omega(g(n))$	$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \infty$	For any <u>positive</u> constant c_0, there exist n_0 s.t.: $c_0\, g(n) <$ ***f(n)*** for all $n \ge n_0$