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Assignment 9

(1) $A = \{a_1, a_2, a_3\}$, $B = \{b_1, b_2, b_3\}$

$P = [[b_1]_A \; [b_2]_A \; [b_3]_A]$

$\Rightarrow [x]_A = {}_A\underset{}{P}_B [x]_B = [[b_1]_A \; [b_2]_A \; [b_3]_A] [x]_B$

$\rightarrow [x]_B = \left({}_A\underset{}{P}_B\right)^{-1} [x]_A$

(2) $B = \left\{\begin{bmatrix}0\\-1\\3\end{bmatrix}, \begin{bmatrix}-3\\2\\-5\end{bmatrix}, \begin{bmatrix}5\\-5\\0\end{bmatrix}\right\}$, $C = \left\{\begin{bmatrix}4\\7\\9\end{bmatrix}, \begin{bmatrix}1\\-7\\-7\end{bmatrix}, \begin{bmatrix}-5\\6\\5\end{bmatrix}\right\}$

and $[x]_B = \left\{\begin{bmatrix}1\\3\\5\end{bmatrix}\right\}$

we have $[x]_C = {}_C\underset{}{P}_B [x]_B$

we know that

$[c_1 \; c_2 \; c_3 \; | \; b_1 \; b_2 \; b_3] = [I \; | \; {}_C\underset{}{P}_B]$

$\rightarrow \left[\begin{array}{ccc|ccc} 4 & 1 & -5 & 0 & -3 & 5 \\ 7 & -7 & 6 & -1 & 2 & -5 \\ 9 & -7 & -5 & 3 & -5 & 0 \end{array}\right] \times 1/4 \rightarrow \left[\begin{array}{ccc|ccc} 1 & 1/4 & -5/4 & 0 & -3/4 & 5/4 \\ 7 & -7 & 6 & -1 & 2 & -5 \\ 9 & -7 & -5 & 3 & -5 & 0 \end{array}\right] - 7\,eq1$

$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 1/4 & -5/4 & 0 & -3/4 & 5/4 \\ 0 & -35/4 & 59/4 & -1 & 29/4 & -55/4 \\ 9 & -7 & -5 & 3 & -5 & 0 \end{array}\right] - 9\,eq1 \rightarrow \left[\begin{array}{ccc|ccc} 1 & 1/4 & -5/4 & 0 & -3/4 & 5/4 \\ 0 & -35/4 & 59/4 & -1 & 29/4 & -55/4 \\ 0 & -37/4 & 25/4 & 3 & 7/4 & -45/4 \end{array}\right]$

$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 1/4 & -5/4 & 0 & -3/4 & 5/4 \\ 0 & 1 & -\frac{59}{35} & \frac{4}{35} & -\frac{29}{35} & \frac{11}{7} \\ 0 & -\frac{37}{4} & 25/4 & 3 & 7/4 & -45/4 \end{array}\right] - 1/4\,eq2 \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{29}{35} & -\frac{1}{35} & -\frac{19}{35} & \frac{6}{7} \\ 0 & 1 & -59/35 & \frac{4}{35} & -\frac{29}{35} & \frac{11}{7} \\ 0 & -\frac{37}{4} & \frac{25}{4} & 3 & 7/4 & -\frac{45}{4} \end{array}\right]$

$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{29}{35} & -\frac{1}{35} & -\frac{19}{35} & \frac{6}{7} \\ 0 & 1 & -59/35 & 4/35 & -29/35 & 11/7 \\ 0 & -1 & 25/37 & 12/37 & 7/37 & -\frac{45}{37} \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{29}{35} & -1/35 & -19/35 & 6/7 \\ 0 & 1 & -59/35 & 4/35 & -29/35 & 11/7 \\ 0 & 0 & -\frac{1308}{1295} & \frac{568}{1295} & -\frac{828}{1295} & \frac{92}{259} \end{array}\right]$

$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & -29/35 & -1/35 & -19/35 & 6/7 \\ 0 & 1 & -59/35 & 4/35 & -29/35 & 11/7 \\ 0 & 0 & 1 & -\frac{142}{327} & \frac{69}{109} & -\frac{115}{327} \end{array}\right] + 29/35\,eq3$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -127/327 & -2/109 & 185/327 \\ 0 & 1 & -59/35 & 4/35 & -29/35 & 11/7 \\ 0 & 0 & 1 & -142/327 & 69/109 & -115/327 \end{array}\right] + 59/35\ eq3$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -127/327 & -2/109 & 185/327 \\ 0 & 1 & 0 & -202/327 & 26/109 & 820/327 \\ 0 & 0 & 1 & -142/327 & 69/109 & -115/327 \end{array}\right]$$

$$\Rightarrow [x]_C = {}^{\beta}_{C\leftarrow\beta}[x]_\beta \Rightarrow [x]_C = \begin{bmatrix} -127/327 & -2/109 & 185/327 \\ -202/327 & 26/109 & 820/327 \\ -142/327 & 69/109 & -115/327 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

$$\rightarrow [x]_C = \begin{bmatrix} 260/109 \\ 544/109 \\ -32/109 \end{bmatrix}$$

(3.) $B = \left\{ \begin{bmatrix} 6 \\ -7 \end{bmatrix}, \begin{bmatrix} 8 \\ -3 \end{bmatrix} \right\}$, $C = \left\{ \begin{bmatrix} -6 \\ -5 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \end{bmatrix} \right\}$ and $[x]_\beta = \begin{Bmatrix} 1 \\ 2 \end{Bmatrix}$

(b_1, b_2; c_1, c_2)

$$[x]_C = {}^{\beta}_{C\leftarrow\beta}[x]_\beta$$

$$[c_1\ c_2 \mid b_1\ b_2] \rightarrow \left[\begin{array}{cc|cc} -6 & 2 & 6 & 8 \\ -5 & -1 & -5 & -3 \end{array}\right] - eq1 \rightarrow \left[\begin{array}{cc|cc} -6 & 2 & 6 & 8 \\ 1 & -3 & -11 & -11 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{cc|cc} -3 & 1 & 3 & 9 \\ 1 & -3 & -11 & -11 \end{array}\right] + 3eq2 \rightarrow \left[\begin{array}{cc|cc} 0 & -8 & -30 & -29 \\ 1 & -3 & -11 & -11 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{cc|cc} 1 & -3 & -11 & -11 \\ 0 & -8 & -30 & -29 \end{array}\right] \rightarrow \left[\begin{array}{cc|cc} 1 & -3 & -11 & -11 \\ 0 & 1 & 15/4 & 29/8 \end{array}\right] + 3eq2 \rightarrow \left[\begin{array}{cc|cc} 1 & 0 & 1/4 & -1/8 \\ 0 & 1 & 15/4 & 29/8 \end{array}\right]$$

$$\Rightarrow {}^{\beta}_{C\leftarrow\beta} = \begin{bmatrix} 1/4 & -1/8 \\ 15/4 & 29/8 \end{bmatrix} \Rightarrow [x]_C = \begin{bmatrix} 1/4 & -1/8 \\ 15/4 & 29/8 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 11 \end{bmatrix}$$

(4.) $B = \left\{ \begin{bmatrix} -3 \\ 6 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 8 \\ -9 \end{bmatrix}, \begin{bmatrix} 5 \\ 5 \\ -2 \end{bmatrix} \right\}$, $C = \left\{ \begin{bmatrix} 1 \\ -8 \\ -8 \end{bmatrix}, \begin{bmatrix} 1 \\ 5 \\ 8 \end{bmatrix}, \begin{bmatrix} -7 \\ 1 \\ -1 \end{bmatrix} \right\}$

→ Changes from B to C

$$\rightarrow [c_1\ c_2\ c_3 \mid b_1\ b_2\ b_3] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 1 & -7 & -3 & 1 & 5 \\ -8 & 5 & 1 & 6 & 8 & 5 \\ -8 & 8 & -1 & 2 & -9 & -2 \end{array}\right] - eq5$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & -7 & -3 & 1 & 5 \\ 0 & -3 & 2 & 4 & 12 & 7 \\ -8 & 8 & -1 & 2 & -4 & -2 \end{array}\right] + 8\,eq1 \rightarrow \left[\begin{array}{ccc|ccc} 1 & 1 & -7 & -3 & 1 & 5 \\ 0 & -3 & 2 & 4 & 12 & 7 \\ 0 & 16 & -57 & -22 & 4 & 38 \end{array}\right] + 5\,eq2$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & -7 & -3 & 1 & 5 \\ 0 & -3 & 2 & 4 & 12 & 7 \\ 0 & 1 & -47 & -2 & 64 & 73 \end{array}\right] + 3\,eq3 \rightarrow \left[\begin{array}{ccc|ccc} 1 & 1 & -7 & -3 & 1 & 5 \\ 0 & 0 & -139 & -2 & 204 & 226 \\ 0 & 1 & -47 & -2 & 64 & 73 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 1 & -7 & -3 & 1 & 5 \\ 0 & 1 & -47 & -2 & 64 & 73 \\ 0 & 0 & -139 & -2 & 204 & 226 \end{array}\right] -eq2 \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 40 & -1 & -63 & -68 \\ 0 & 1 & -47 & -2 & 64 & 73 \\ 0 & 0 & 1 & 2/139 & -204/139 & -226/139 \end{array}\right] \begin{array}{l} -40\,eq3 \\ +47\,eq3 \end{array}$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -219/139 & -597/139 & -412/139 \\ 0 & 1 & 0 & -184/139 & -692/139 & -475/139 \\ 0 & 0 & 1 & 2/139 & -204/139 & -226/139 \end{array}\right] \quad \begin{matrix} \beta \\ \Rightarrow C \in \beta \end{matrix} = \begin{bmatrix} -219/139 & -597/139 & -412/139 \\ -184/139 & -692/139 & -475/139 \\ 2/139 & -204/139 & -226/139 \end{bmatrix}$$

Transformation from C to β

$$\left[\begin{array}{ccc|ccc} b_1 & b_2 & b_3 & c_1 & c_2 & c_3 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|ccc} -3 & 1 & 5 & 1 & 1 & -7 \\ 6 & 8 & 5 & -8 & 5 & 1 \\ 2 & -4 & -2 & -8 & 8 & -1 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & -1/3 & -5/3 & -1/3 & -1/3 & 7/3 \\ 6 & 8 & 5 & -8 & 5 & 1 \\ 2 & -4 & -2 & -8 & 8 & -1 \end{array}\right] \begin{array}{l} -6\,eq1 \\ -2\,eq1 \end{array} \rightarrow \left[\begin{array}{ccc|ccc} 1 & -1/3 & -5/3 & -1/3 & -1/3 & 7/3 \\ 0 & 10 & 15 & -6 & 7 & -13 \\ 0 & -10/3 & 4/3 & -22/3 & 26/3 & -17/3 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -1/3 & -5/3 & -1/3 & -1/3 & 7/3 \\ 0 & 1 & 3/2 & -3/5 & 7/10 & -13/10 \\ 0 & -10/3 & 4/3 & -22/3 & 26/3 & -17/3 \end{array}\right] \begin{array}{l} + 1/3\,eq2 \\ \Rightarrow \\ + 10/3\,eq2 \end{array} \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & -7/6 & -8/15 & -1/10 & 19/10 \\ 0 & 1 & 3/2 & -3/5 & 7/10 & -13/10 \\ 0 & 0 & 19/3 & -28/3 & 11 & -10 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & -7/6 & -8/15 & -1/10 & 19/10 \\ 0 & 1 & 3/2 & -3/5 & 7/10 & -13/10 \\ 0 & 0 & 1 & -28/19 & 33/19 & -30/19 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -214/95 & 183/95 & 11/190 \\ 0 & 1 & 0 & 153/95 & -181/95 & 203/190 \\ 0 & 0 & 1 & -28/19 & 33/19 & -30/19 \end{array}\right]$$

$$\rightarrow \begin{bmatrix} -214/95 & 183/95 & 11/190 \\ 153/95 & -181/95 & 203/190 \\ -28/19 & 33/19 & -30/19 \end{bmatrix} = {}_{\beta}\mathcal{S}_{C}$$

5. $P\leftarrow_I A = \begin{bmatrix} 5 & 0 & -2 \\ 6 & -2 & -5 \\ 0 & 5 & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} [a_1]_b & [a_2]_b & [a_3]_b \end{bmatrix} = P\leftarrow_B A$

$\Rightarrow [a_1]_b = \begin{bmatrix} 5 \\ 6 \\ 0 \end{bmatrix} \Rightarrow a_1 = 5b_1 + 6b_2 + 0b_3 \quad ; b_1 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}$

$[a_2]_b = \begin{bmatrix} 0 \\ -2 \\ 5 \end{bmatrix} \Rightarrow a_2 = 0b_1 + (-2)b_2 + 5b_3 \quad b_2 = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}$

$[a_3]_b = \begin{bmatrix} -2 \\ -5 \\ -4 \end{bmatrix} \Rightarrow a_3 = -2b_1 - 5b_2 - 4b_3 \quad b_3 = \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} -2 \\ -5 \\ -2 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \\ 0 \end{bmatrix} \Rightarrow \begin{matrix} 5x_1 + 6x_2 + 0x_3 = -2 \\ 5y_1 + 6y_2 + 0y_3 = -5 \\ 5z_1 + 6z_2 + 0z_3 = -2 \end{matrix}$ (1)

$\begin{bmatrix} -8 \\ -7 \\ 8 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{bmatrix} \begin{bmatrix} 0 \\ -2 \\ 5 \end{bmatrix} \rightarrow \begin{matrix} 0x_1 + (-2)x_2 + 5x_3 = -8 \\ 0y_1 + (-2)y_2 + 5y_3 = -7 \\ 0z_1 + (-2)z_2 + 5z_3 = 8 \end{matrix}$ (2)

$\begin{bmatrix} 9 \\ 1 \\ -8 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{bmatrix} \begin{bmatrix} -2 \\ -5 \\ 4 \end{bmatrix} \Rightarrow \begin{matrix} -2x_1 - 5x_2 + 4x_3 = 9 \\ -2y_1 - 5y_2 + 4y_3 = 1 \\ -2z_1 - 5z_2 + 4z_3 = -8 \end{matrix}$ (3)

From (1), (2), (3), Solve for 3 matrixes

$\begin{matrix} 5x_1 + 6x_2 + 0x_3 = -2 \\ 0x_1 + (-2)x_2 + 5x_3 = -8 \\ -2x_1 - 5x_2 + 4x_3 = 9 \end{matrix} \rightarrow \begin{bmatrix} 5 & 6 & 0 & -2 \\ 0 & -2 & 5 & -8 \\ -2 & -5 & 4 & 9 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 6/5 & 0 & -2/5 \\ 0 & 1 & -5/2 & 4 \\ -2 & -5 & 4 & 9 \end{bmatrix} +2\,eq1$

$\rightarrow \begin{bmatrix} 1 & 6/5 & 0 & -2/5 \\ 0 & 1 & -5/2 & 4 \\ 0 & -13/5 & 4 & 41/5 \end{bmatrix} +13/5\,eq2 \rightarrow \begin{bmatrix} 1 & 6/5 & 0 & -2/5 \\ 0 & 1 & -5/2 & 5/2 \\ 0 & 0 & -5/2 & 93/5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 6/5 & 0 & -2/5 \\ 0 & 1 & -5/2 & 5/2 \\ 0 & 0 & 1 & \frac{-186}{25} \end{bmatrix}$

$\rightarrow x_3 = \frac{-186}{25}, \; x_2 = -73/5, \; x_1 = \frac{428}{25}$

$\begin{matrix} 5y_1 + 6y_2 + 0y_3 = -5 \\ 0y_1 + (-2)y_2 + 5y_3 = -7 \\ -2z_1 - 5z_2 + 4z_3 = 1 \end{matrix} \rightarrow \begin{bmatrix} 5 & 6 & 0 & -5 \\ 0 & -2 & 5 & -7 \\ -2 & -5 & 4 & 1 \end{bmatrix} \Rightarrow \begin{matrix} y_1 = 113/25 \\ y_2 = -23/5 \\ y_3 = -81/25 \end{matrix}$

$\begin{matrix} 5z_1 + 6z_2 + 0z_3 = -2 \\ 0z_1 + (-2)z_2 + 5z_3 = 8 \\ -2z_1 - 5z_2 + 4z_3 = -8 \end{matrix} \rightarrow \begin{bmatrix} 5 & 6 & 0 & -2 \\ 0 & -2 & 5 & 8 \\ -2 & -5 & 4 & -8 \end{bmatrix} \Rightarrow \begin{matrix} z_1 = -466/25 \\ z_2 = 761/5 \\ z_3 = 192/25 \end{matrix}$

$\Rightarrow b_1 = \begin{bmatrix} \frac{428}{25} \\ 113/25 \\ -466/25 \end{bmatrix}, \; b_2 = \begin{bmatrix} -73/5 \\ -23/5 \\ 761/5 \end{bmatrix}, \; b_3 = \begin{bmatrix} -186/25 \\ -81/25 \\ 192/25 \end{bmatrix}$

(6)

a) Mapped B to R_2 ; we have

$$x = P_B [x]_B$$

Resolution for B is 1440 x 1080 ; Choose $x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} b_1 & b_2 \end{bmatrix} \begin{bmatrix} 1440 \\ 1080 \end{bmatrix}$$

$$\Rightarrow b_1 = \begin{bmatrix} 1/1440 \\ 0 \end{bmatrix}, \Rightarrow b_2 = \begin{bmatrix} 0 \\ 1/1080 \end{bmatrix}$$

b) Mapped C to R_2, we have (choose C = 1920 x 1080)

$$x = P_C [x]_C$$

$$\Rightarrow \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} c_1 & c_2 \end{bmatrix} \begin{bmatrix} 1920 \\ 1080 \end{bmatrix}$$

$$\Rightarrow c_1 = \begin{bmatrix} 1/1920 \\ 0 \end{bmatrix}, c_2 = \begin{bmatrix} 0 \\ 1/1080 \end{bmatrix}$$

→ Augmented B and C to find transform matrix from B to C

$$\Rightarrow [c_1 \; c_2 \; : \; b_1 \; b_2] = \left[\begin{array}{cc:cc} 1/1920 & 0 & 1/1440 & 0 \\ 0 & 1/1080 & 0 & 1/1080 \end{array}\right]$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 4/3 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \Rightarrow {}_{C \leftarrow B}P = \begin{bmatrix} 4/3 & 0 \\ 0 & 1 \end{bmatrix}$$

$${}_{B \leftarrow C}P = ({}_{C \leftarrow B}P)^{-1} = \begin{bmatrix} 3/4 & 0 \\ 0 & 1 \end{bmatrix}$$

c) $[x]_C = \begin{bmatrix} 854 \\ 1001 \end{bmatrix} \Rightarrow [x]_B = \begin{bmatrix} 3/4 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 864 \\ 1001 \end{bmatrix} = \begin{bmatrix} 1281/2 \\ 1001 \end{bmatrix}$

$$[x]_C = {}_{C \leftarrow B}P \, [x]_B = \begin{bmatrix} 4/3 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 100 \\ 200 \end{bmatrix} = \begin{bmatrix} 400/3 \\ 200 \end{bmatrix}$$