Here we formalize our intuitive understanding of the way that a Turing machine computes.

Say that configuration C1 **yields** configuration C2 if the Turing machine can legally go from C1 to C2 in a single step. We define this notion formally as follows:

* Suppose that we have *a*, *b*, and *c* in Γ, as well as *u* and *v* in Γ∗ and states *qi* and *qj*. In that case,

*ua qi bv* and *u qj acv* are two configurations. Say that

*ua qi bv* **yields** *u qj acv*

if in the transition function ***δ (qi, b) = (qj, c, L)***.

That handles the case where the Turing machine moves leftward. For a rightward move, say that

*ua qi bv* **yields** *uac qj v*

if ***δ (qi, b) = (qj, c, R)****.*

**Example 3.7:**

In this state diagram, the label 0→␣, R appears on the transition from q1 to q2.

This label signifies that when in state q1 with the head reading 0, the machine goes to state q2, writes ␣, and moves the head to the right. In other words, **δ (q1, 0) = (q2, ␣, R)**.

For clarity we use the shorthand 0→R in the transition from q3 to q4, to mean that the machine moves to the right when reading 0 in state q3 but doesn’t alter the tape, so **δ (q3,0) = (q4, 0, R)**.

**Example 3.9:**

The following is a formal description of M1 = (Q, Σ, Γ, δ, q1, qaccept, qreject), the Turing machine that we informally described (page 167) for deciding the language B = {w#w| w ∈ {0, 1} \*}.

* Q = {q1, ..., q8, qaccept, qreject},
* Σ = {0, 1, #}, and Γ = {0, 1, #, x, ␣}.
* We describe δ with a state diagram (see the following figure).
* The start, accept, and reject states are q1, qaccept, and qreject, respectively

In Figure 3.10, which depicts the state diagram of TM M1, you will find the label 0,1→R on the transition going from q3 to itself. That label means that the machine stays in q3 and moves to the right when it reads a 0 or a 1 in state q3. It doesn’t change the symbol on the tape.

Stage 1 is implemented by states q1 through q7, and stage 2 by the remaining states. To simplify the figure, we don’t show the reject state or the transitions going to the reject state. Those transitions occur implicitly whenever a state lacks an outgoing transition for a particular symbol. Thus because in state q5 no outgoing arrow with a # is present, if a # occurs under the head when the machine is in state q5, it goes to state qreject. For completeness, we say that the head moves right in each of these transitions to the reject state.