Object tracking assignment report

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Question 1:

Prediction step

$$\hat{\boldsymbol{x}}_{k|k-1} = \boldsymbol{\Phi}\hat{\boldsymbol{x}}_{k-1},$$
$$\boldsymbol{P}_{k|k-1} = \boldsymbol{\Phi}\boldsymbol{P}_{k-1}\boldsymbol{\Phi}^T + \boldsymbol{Q},$$

where the transition matrix is as follows:

$$\boldsymbol{\Phi} = \begin{bmatrix} 1 & 0 & T_s & 0 \\ 0 & 1 & 0 & T_s \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

and the process noise covariance matrix is

$$\boldsymbol{Q} = q\begin{bmatrix} T_s^3/3 & 0 & T_s^2/2 & 0 \\ 0 & T_s^3/3 & 0 & T_s^2/2 \\ T_s^2/2 & 0 & T_s & 0 \\ 0 & T_s^2/2 & 0 & T_s \end{bmatrix},$$

where q is the acceleration power density power (PSD) in m^2/s^3 which models the dynamics of an object.

Update step

$$\hat{\boldsymbol{x}}_k = \hat{\boldsymbol{x}}_{k|k-1} + \boldsymbol{K}_k\left(\boldsymbol{z}_k - \boldsymbol{h}(\hat{\boldsymbol{x}}_{k|k-1})\right),$$
$$\boldsymbol{P}_k = \boldsymbol{P}_{k|k-1} - \boldsymbol{K}_k(\boldsymbol{H}_k\boldsymbol{P}_{k|k-1}),$$

where the Kalman is given as:

$$\boldsymbol{K}_k = \boldsymbol{P}_{k|k-1}\boldsymbol{H}_k^T\left(\boldsymbol{H}_k\boldsymbol{P}_{k|k-1}\boldsymbol{H}_k^T + \boldsymbol{R}_k\right)^{-1},$$

the sonar model including range and azimuth measurements is as follow:

$$\boldsymbol{h}_k(\boldsymbol{x}_k) = \begin{bmatrix} \sqrt{x_k^2 + y_k^2} \\ atan2\left(\frac{y_k}{x_k}\right) \end{bmatrix},$$

and due to the nonlinearity of the measurement model with respect to the state vector, the measurement matrix linearized about the state vector is

$$\boldsymbol{H}_k = \begin{bmatrix} \dfrac{x_k}{\sqrt{x_k^2 + y_k^2}} & \dfrac{y_k}{\sqrt{x_k^2 + y_k^2}} & 0 & 0 \\ \dfrac{-y_k}{x_k^2 + y_k^2} & \dfrac{x_k}{x_k^2 + y_k^2} & 0 & 0 \end{bmatrix},$$

and finally, the measurement noise covariance matrix is simply

$$\boldsymbol{R} = \begin{bmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_\theta^2 \end{bmatrix},$$

where σ_r^2 and σ_θ^2 are range and azimuth measurement noise variances respectively.

Question 2:

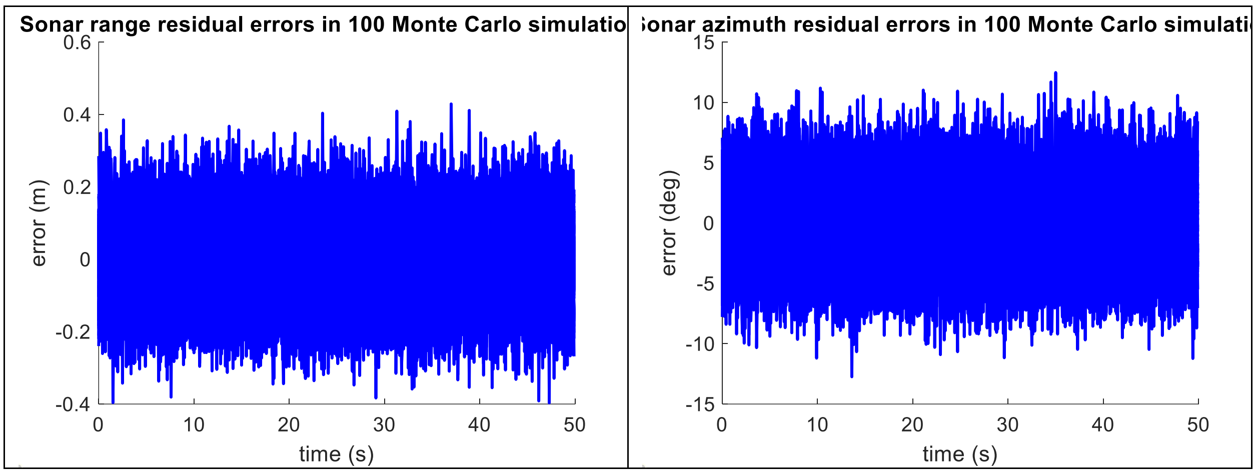

Figure 1: 100 sonar measurement residual error sequences over the entire trajectories. Sonar range measurement sequences are contaminated by Gaussian noise with 0.1 m standard deviation while azimuth measurement sequences are contaminated by Gaussian noise with 3 deg standard deviation.

Question 3

An extended Kalman filter (EKF) is implemented with initial guess acceleration PSD $q = 1$.

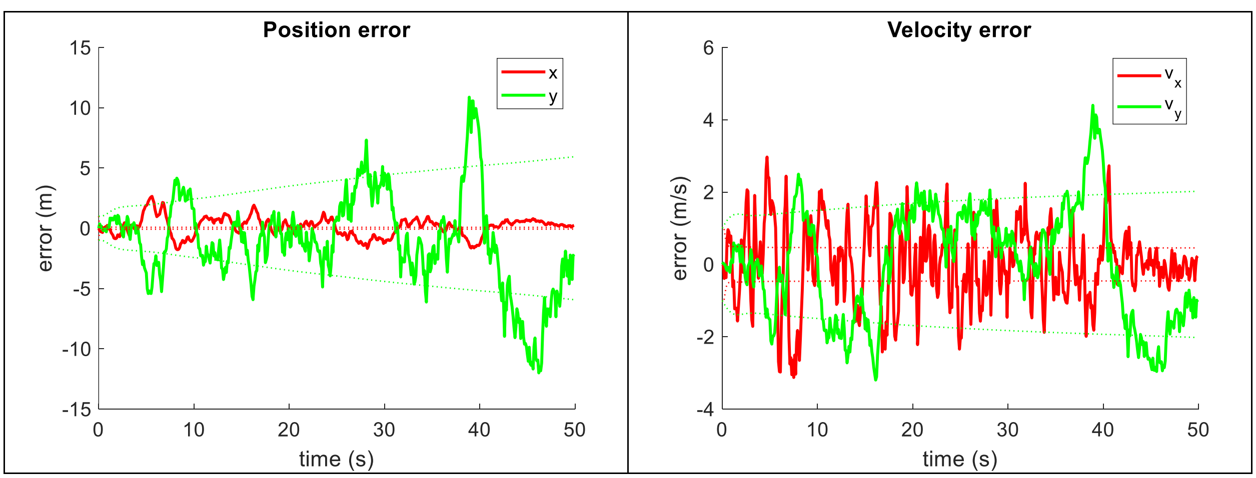

Figure 2: Position and velocity errors as a function of time (the confidence intervals are represented by dotted curves).

The performance in x axis is much better than that of y axis because the object is mostly moving along the x axis. As a result, the range measurement increases the information (thus reduces the error) in the direction formed by the positions of the object and the sonar. In addition, poor azimuth sonar measurements (3 deg std) causes this y axis accuracy degradation.

The sonar range and azimuth measurements correct the velocity indirectly through the correlation between the position and velocity in the state covariance matrix. The assignment gives a hint to initialize the velocity with 0 but the initial velocity covariance should not be set to large value for the goodness of EKF linearization.

To verify the EKF implementation, the residual analysis is showed in the following figures. The innovation sequences are centered and matches with the innovation covariances $\boldsymbol{H}_k \boldsymbol{P}_{k|k-1} \boldsymbol{H}_k^T + \boldsymbol{R}_k$.

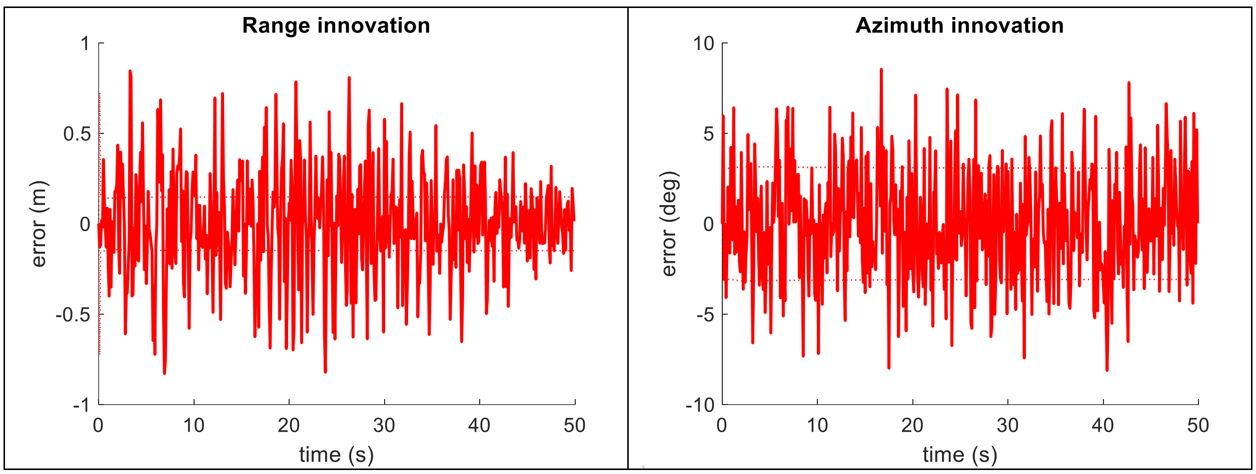

Figure 3: Innovation for sonar range and azimuth measurements (the confidence intervals are represented by dotted curves).

Question 4

With the acceleration PSD of 1 in Question 3, we evaluate the performance over 1000 Monte Carlo simulation.

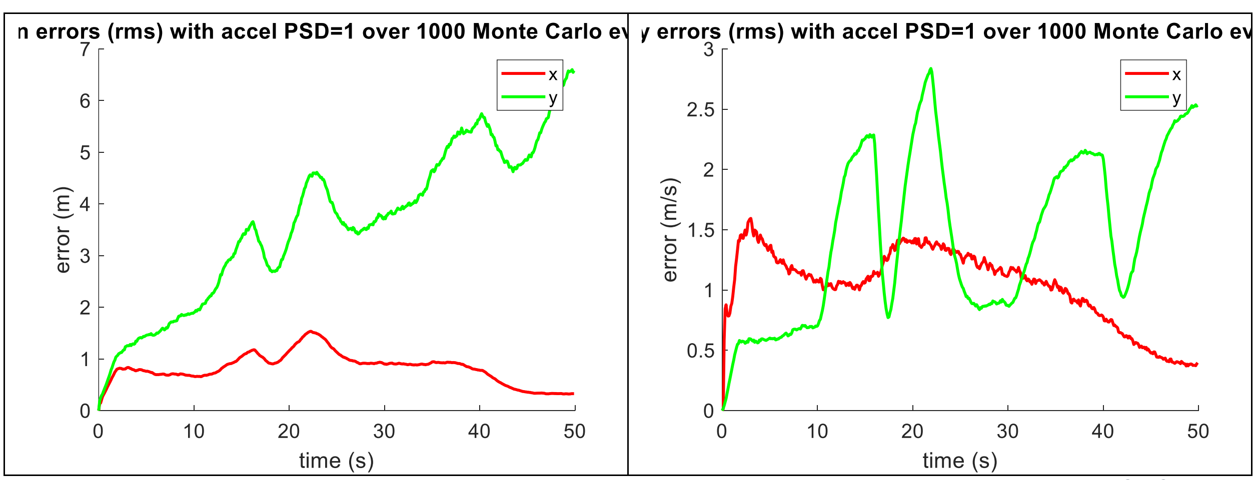

Figure 4: Position and velocity errors (rms) over 1000 Monte Carlo evaluations with acceleration PSD of 1 m^2/s^3.

Now the acceleration PSD is increased 10 times.

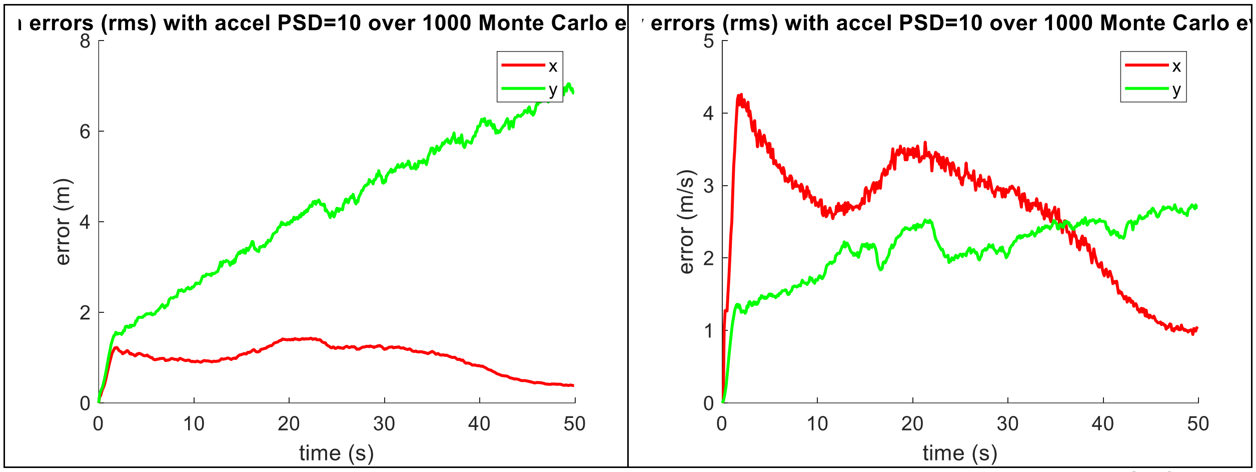

Figure 5: Position and velocity errors (rms) over 1000 Monte Carlo evaluations with acceleration PSD of 10 m^2/s^3.

It can be seen that the performances in both position and velocity are slightly decreased since more uncertainty is added to the prediction model. This observation is confirmed by the following figures with very large acceleration PSD of 100.

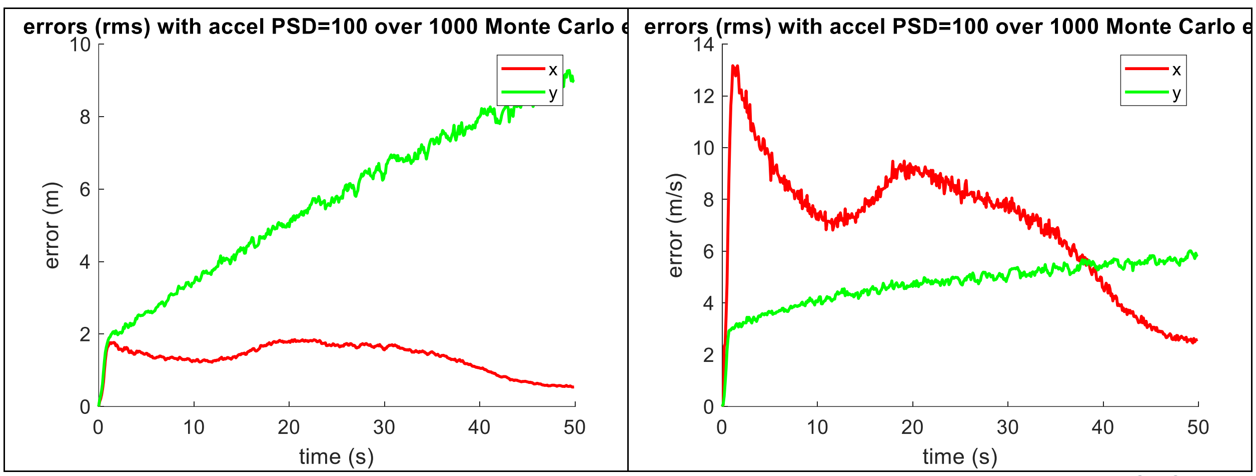

Figure 6: Position and velocity errors (rms) over 1000 Monte Carlo evaluations with acceleration PSD of 100 m^2/s^3.

However, very small process noise is also not good. Let $q = 0.1$, the prediction model is considered to be accurate but practically, the object dynamic model is unknown so the prediction model should include a specific level of uncertainty. Otherwise, the prediction model can not track the object while put lower weights to the measurements (through Kalman gain) leading to poor performance.

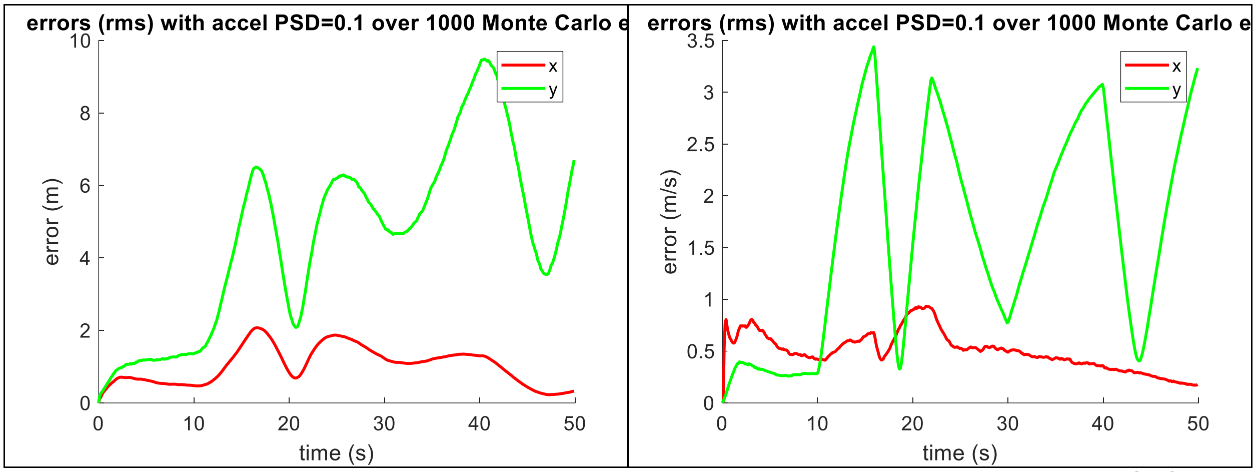

Figure 7: Position and velocity errors (rms) over 1000 Monte Carlo evaluations with acceleration PSD of 0.1 m^2/s^3.

Question 5

Filter tuning (especially process noise covariance matrix Q) is critical in Kalman filter to optimize the performance due to incorrectly modelled effects (prediction and measurement models) and dynamics and/or errors which are intentionally ignored to simplify the filter design (the constant velocity model is here in this assignment). Besides, tuning includes other parameters such as measurement noise R, initial uncertainty P_0. In principles, a couple of method are used as follows:

- Monte Carlo method samples the high-dimensional tuning parameters and finds the best configuration.
- Adaptive method requires to store the innovation in a time window then computes the matrices Q and R based on this innovation sequences.