

Preliminar notes on maximum-flow problem solved in FCPP.

Notation:

- \mathcal{C} will denote the directed weighted graph of capacities.
- s and t will denote respectively source and sink of our graph.
- \mathcal{R} will denote the directed weighted graph of residual capacities.
- For a directed graph G , $d_G(\delta, \delta')$ will be the distance from node δ to node δ' . It takes values in $\mathbb{N} \cup \{\infty\}$.
- For a acyclic directed graph G , $D_G(\delta, \delta')$ will be the maximum of the lenghts of all paths from node δ to node δ' .
- For a path \mathfrak{p} we will denote with $|\mathfrak{p}|$ its lenght.

When it will be convenient, we will treat a directed weighted graph G as a function $G : V \times V \rightarrow \mathbb{R}_+$, where V is its set of vertices. With a little abuse of notation we will say that the edge from a node δ to a node δ' is in G , or $(\delta, \delta') \in G$, in place of $G(\delta, \delta') \neq 0$.

Likewise we will say that a path \mathfrak{p} is contained in G , or $\mathfrak{p} \subset G$, if every edge in \mathfrak{p} is in G .

Observation 1. \mathcal{C} is acyclic and

$$\forall \delta, \delta' : \mathcal{C}(\delta, \delta') \neq 0 \Rightarrow \mathcal{C}(\delta', \delta) = 0 \quad (1)$$

i.e. the capacity between any two nodes can be nonzero in almost one direction. \mathcal{R} , has the same set of vertices than \mathcal{C} , it can have cycles, and has the property that

$$\forall \delta, \delta' : \mathcal{R}(\delta, \delta') + \mathcal{R}(\delta', \delta) = \mathcal{C}(\delta, \delta') + \mathcal{C}(\delta', \delta) . \quad (2)$$