

Preliminar notes on maximum-flow problem solved in FCPP.

Notation:

- $G$  will denote the directed weighted graph of capacities,  $s$  and  $t$  will denote respectively source and sink of our graph.
- We will assume that capacities are nonnegative and symmetrical.
- For a path  $\mathbf{p}$  we will denote with  $|\mathbf{p}|$  its lenght.

When it will be convenient, we will treat a directed weighted graph  $G$  as a function  $G : V \times V \rightarrow \mathbb{R}_+$ , where  $V$  is its set of vertices. With a little abuse of notation we will say that the edge from a node  $\delta$  to a node  $\delta'$  is in  $G$ , or  $(\delta, \delta') \in G$ , in place of  $G(\delta, \delta') \neq 0$ .

Likewise we will say that a path  $\mathbf{p}$  is contained in  $G$ , or  $\mathbf{p} \subset G$ , if every edge

- For a field  $\Phi$ ,  $\mathcal{R}_\Phi$  will denote the directed weighted graph of residual capacities respect to  $\Phi$ :

$$\mathcal{R} = G - \Phi \tag{1}$$

- We define admissible path respect to  $\Phi$  a path  $\gamma$  in  $G$  from source to sink such that

$$\Phi|_\gamma < G|_\gamma$$

- We say that a flow  $\Phi$  is maximal if there are no admissible paths in  $G$  respect to  $\Phi$ .

**Proposizione 0.1.**  *$\Phi$  is a maximum flow iff  $\mathcal{R}_\Phi$  does not have paths from source to sink.*

For a field-value  $f$  in  $\delta$  we define  $|f| := \sum_{\delta \sim \delta'} f(\delta')$ . We now formalize functions involved in the algorithm. Let  $\Phi$  be a field and  $\delta \sim \delta'$  be devices, we define

$\Phi^*(\delta, \delta') := \Phi(\delta', \delta)$ . By co-induction:

$$\begin{aligned}
\Phi_0 &:= \Phi \\
\mathcal{R}_n &:= \mathcal{R}_{\Phi_n} \\
e_n(\delta) &:= \begin{cases} \infty & \text{if } \delta = s \\ -\infty & \text{if } \delta = t \\ |\Phi_n(\delta)| & \text{otherwise} \end{cases} \\
d_0(\delta) &:= \begin{cases} 0 & \text{if } \delta = t \\ \infty & \text{otherwise} \end{cases} \\
d_n(\delta) &:= \begin{cases} 0 & \text{if } \delta = t \\ \min\{d_{n-1}(\delta') + 1 \mid \mathcal{R}_{n-1}(\delta, \delta') > 0\} & \text{otherwise} \end{cases} \\
I_{n+1} &:= \text{trunc}((G + \Phi_n^*) \cdot (d_{n-1}^* < d_n), e(\Phi_n^*)) \\
\Phi_{n+1} &:= -\Phi_n^* + I_{n+1} + \text{trunc}(\Phi_n^*, e(\Phi_n^* - I_{n+1})) \\
X_n &:= \{\delta \mid d_n(\delta) = \infty\}
\end{aligned}$$