ESc201: Introduction to Electronics

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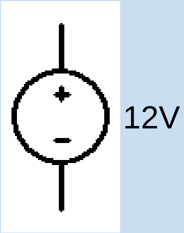
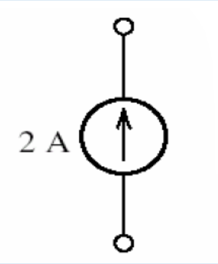
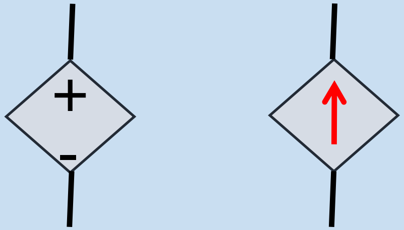
Recap

Independent Sources





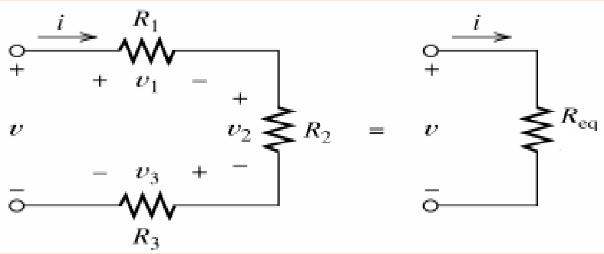
Dependent Sources



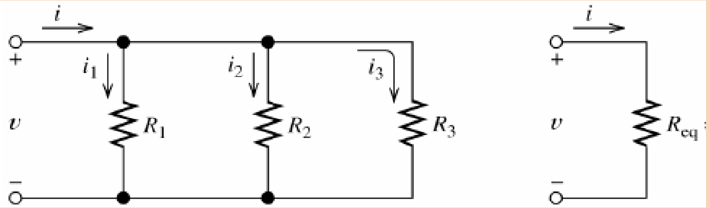
Concept of equivalent circuits

Two circuits are equivalent if they have the same current-voltage behavior

Series combination



Parallel combination



Voltage Division

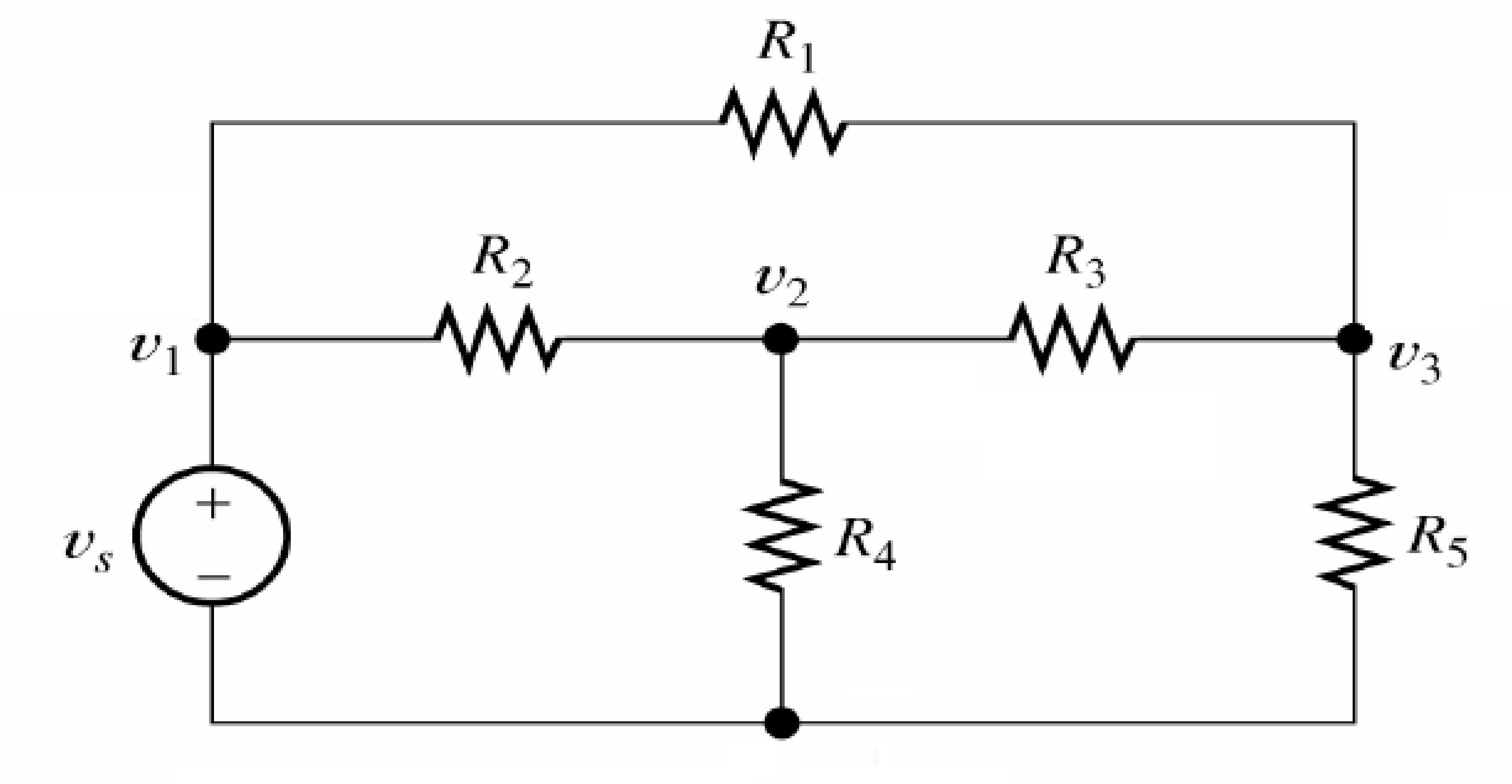
A voltage applied to resistors connected in series is divided among them

Current Division

The total current flowing into a parallel combination of resistors is divided among them

Limitations

Although series/parallel equivalents and the current/voltage division principles are very important concepts, yet they are not sufficient to solve all circuits !!



Circuit Analysis

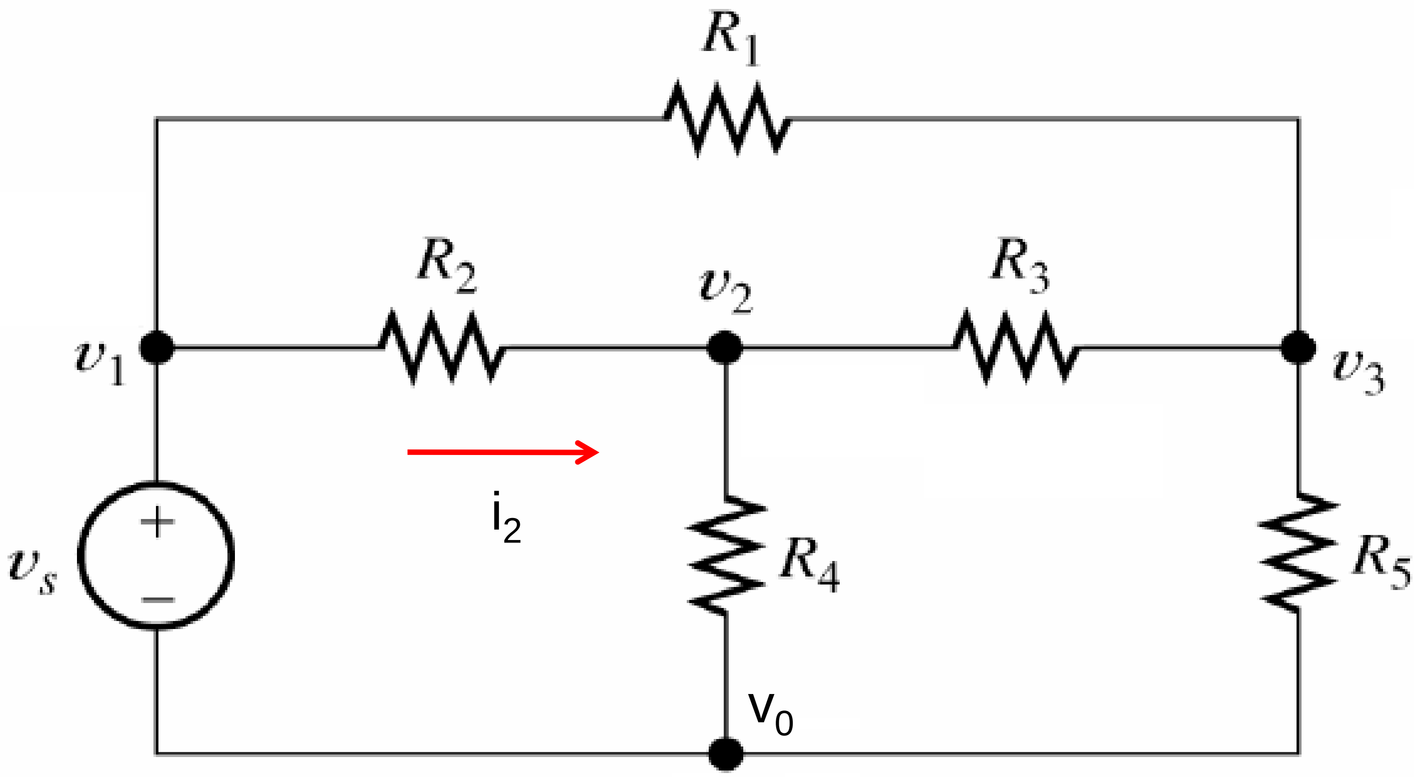
Goal is to find voltages, currents and power in the circuit

If we know voltage and current then power can be easily determined

$$P(t) = v(t) \times i(t)$$

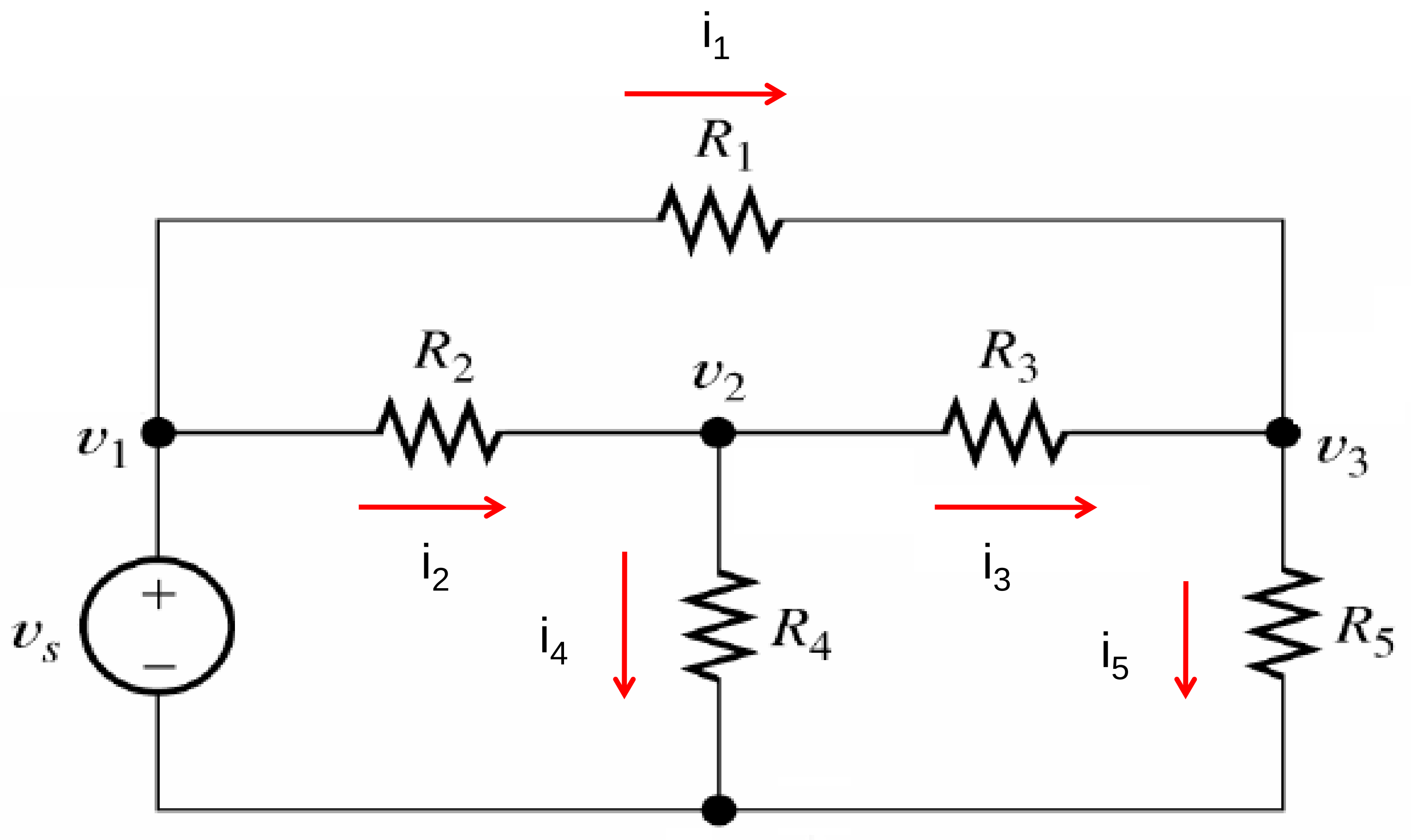
If we determine the voltages, then we can determine the currents using the models of circuit elements

Or if we determine the currents, then we can determine the voltages using the models of circuit elements



If we determine the voltages v_1, v_2,… then we can determine the currents as well

$$v_1 - v_2 = i_2 \times R_2$$



If we determine all the currents, then we can determine the voltages as well

$$v_1 - v_2 = i_2 \times R_2$$

General Circuit Analysis Method: Nodal Analysis

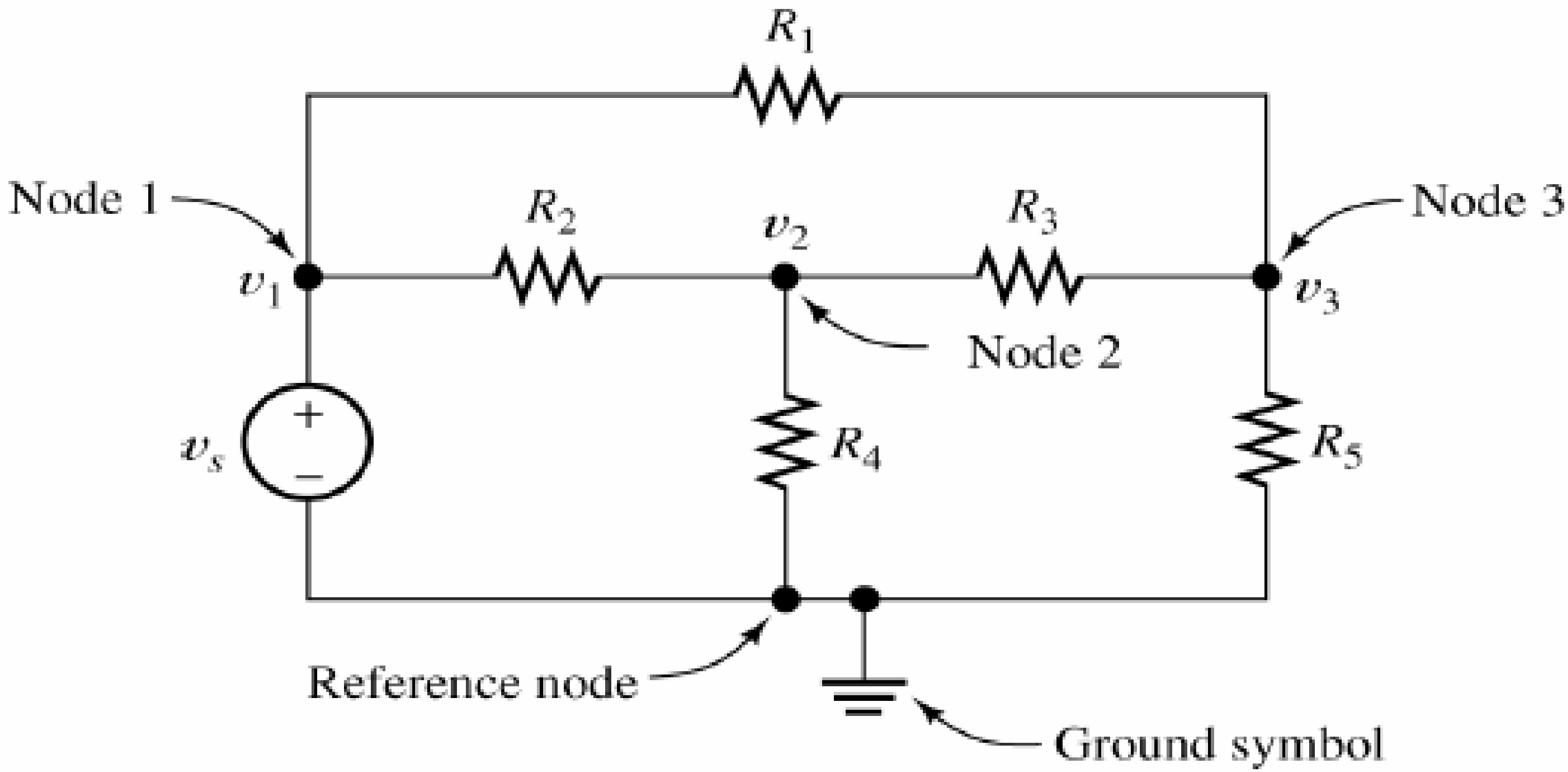
In nodal analysis, the variables used to describe the circuit will be “Node Voltages” (Recall Nodes!)

– Nodal voltage are the voltages of each node with respect to a pre-selected reference node

– Usually the reference node has many branches connected to it

– Reference node is also called ground

– Node voltages are selected as being positive with respect to the reference node

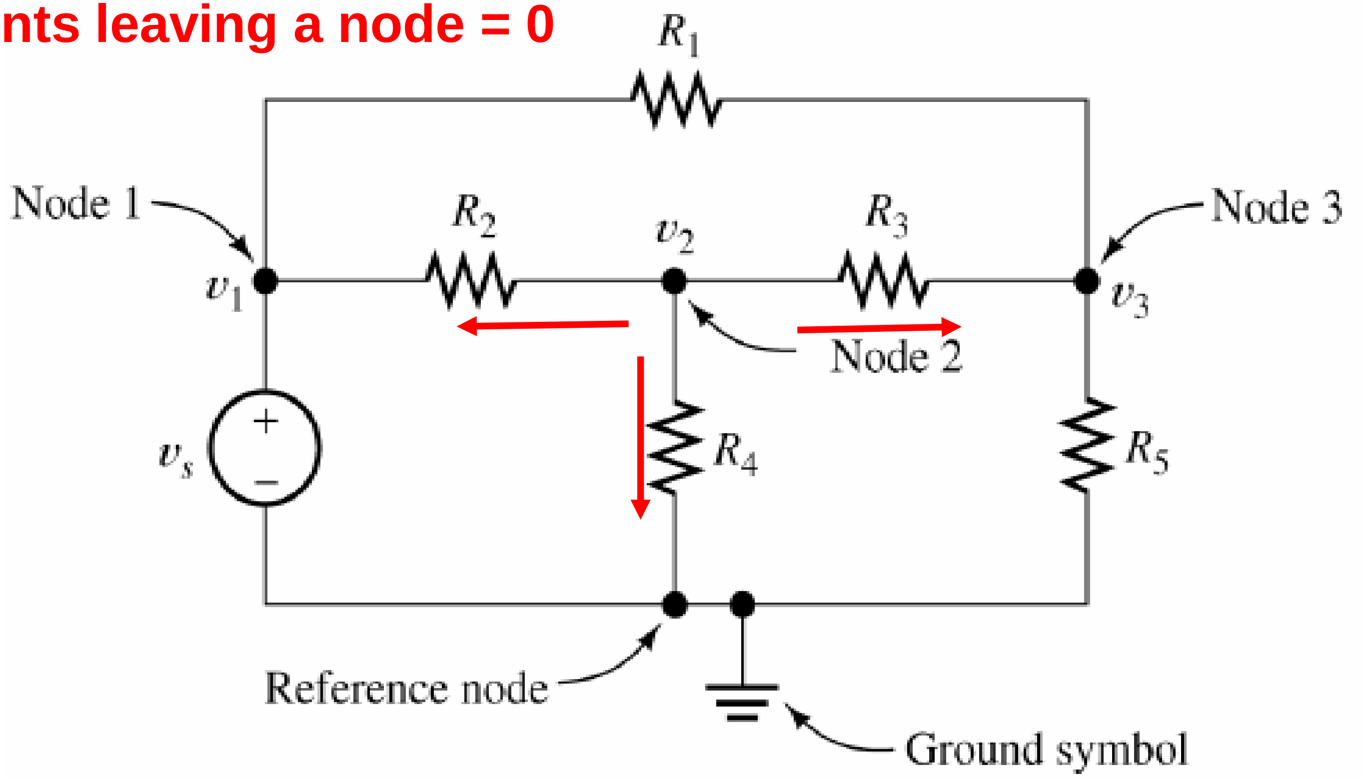


Nodal analysis will give values of node voltages v_1, v_2 and v_3 with respect to the reference node

How to calculate node voltages?

1. Identify and number the nodes
2. Writing KCL Equations in Terms of the Node Voltages

Sum of currents leaving a node = 0

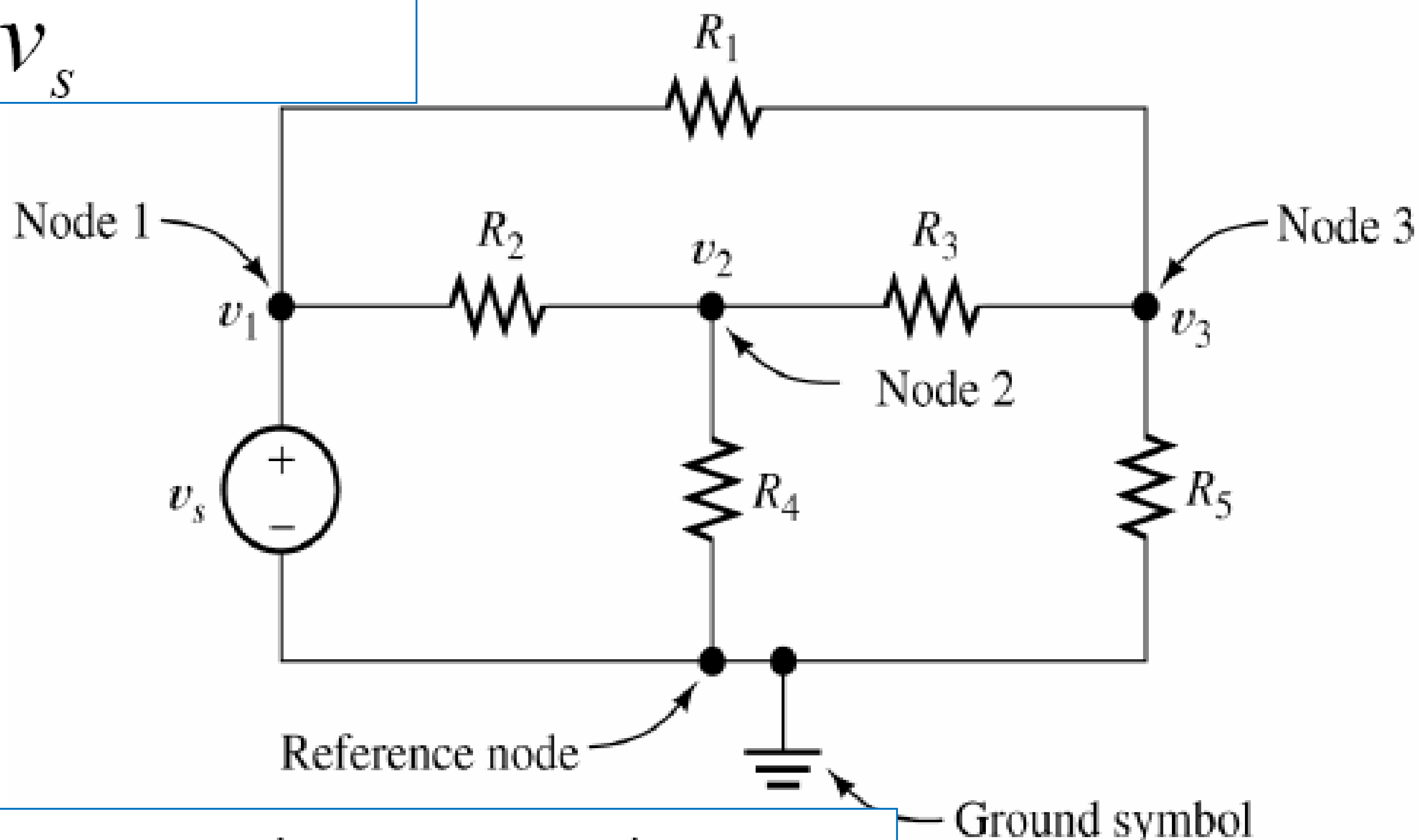


Sum of currents leaving node $2 = 0$

$$\frac{v_2 - v_1}{R_2} + \frac{v_2}{R_4} + \frac{v_2 - v_3}{R_3} = 0$$

Voltage for node 1 is known

$$v_1 = v_s$$

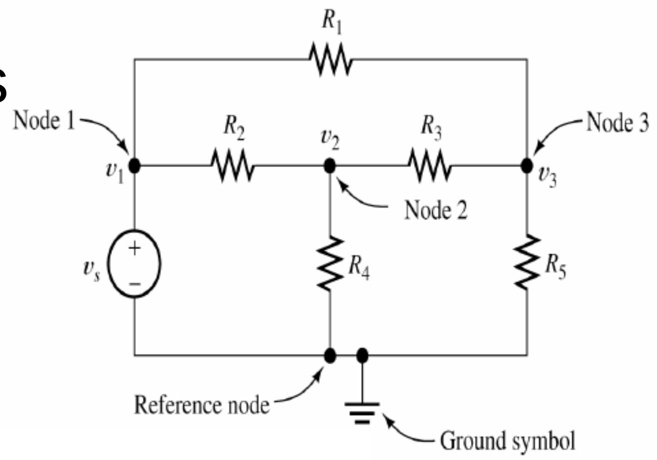


Sum of currents leaving node $3 = 0$

$$\frac{v_3 - v_1}{R_1} + \frac{v_3}{R_5} + \frac{v_3 - v_2}{R_3} = 0$$

Circuit Analysis

- Transformation of circuit into equations
- Solution of equations



<u>Node 2</u>

$$\frac{v_2 - v_1}{R_2} + \frac{v_2}{R_4} + \frac{v_2 - v_3}{R_3} = 0$$

<u>Node 3</u>

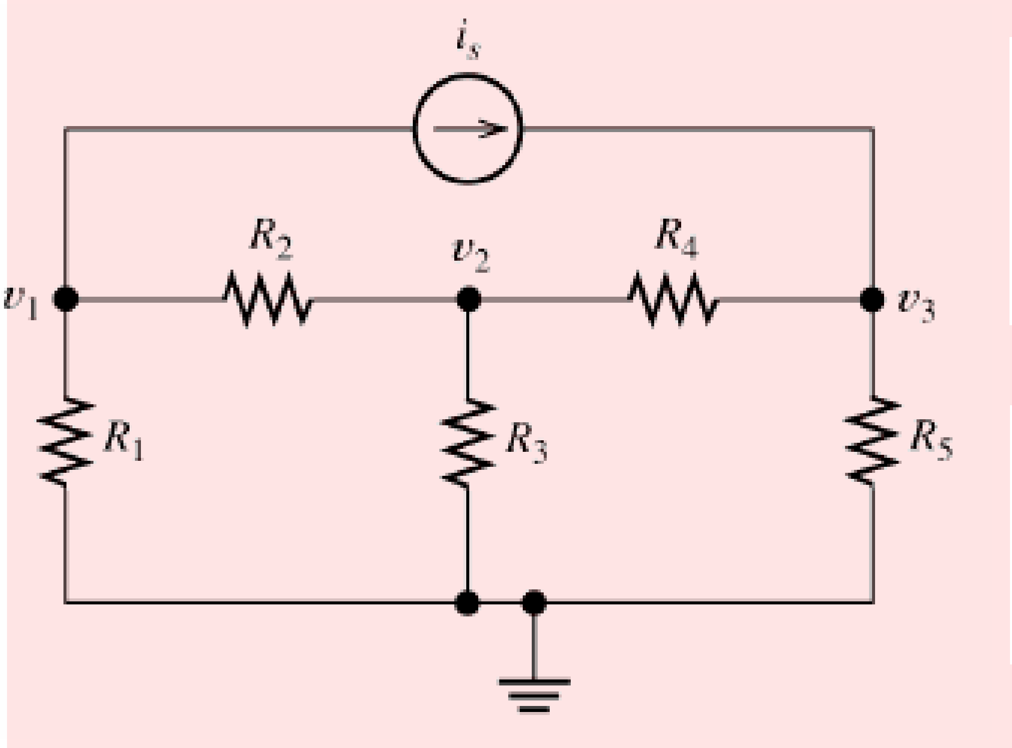
$$\frac{v_3 - v_1}{R_1} + \frac{v_3}{R_5} + \frac{v_3 - v_2}{R_3} = 0$$

<u>Node 1</u>

$$v_1 = v_s$$

Circuits with Independent Current Sources

Sum of currents leaving a node $= 0$



Node 1:

$$\frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2} + i_s = 0$$

Node 2:

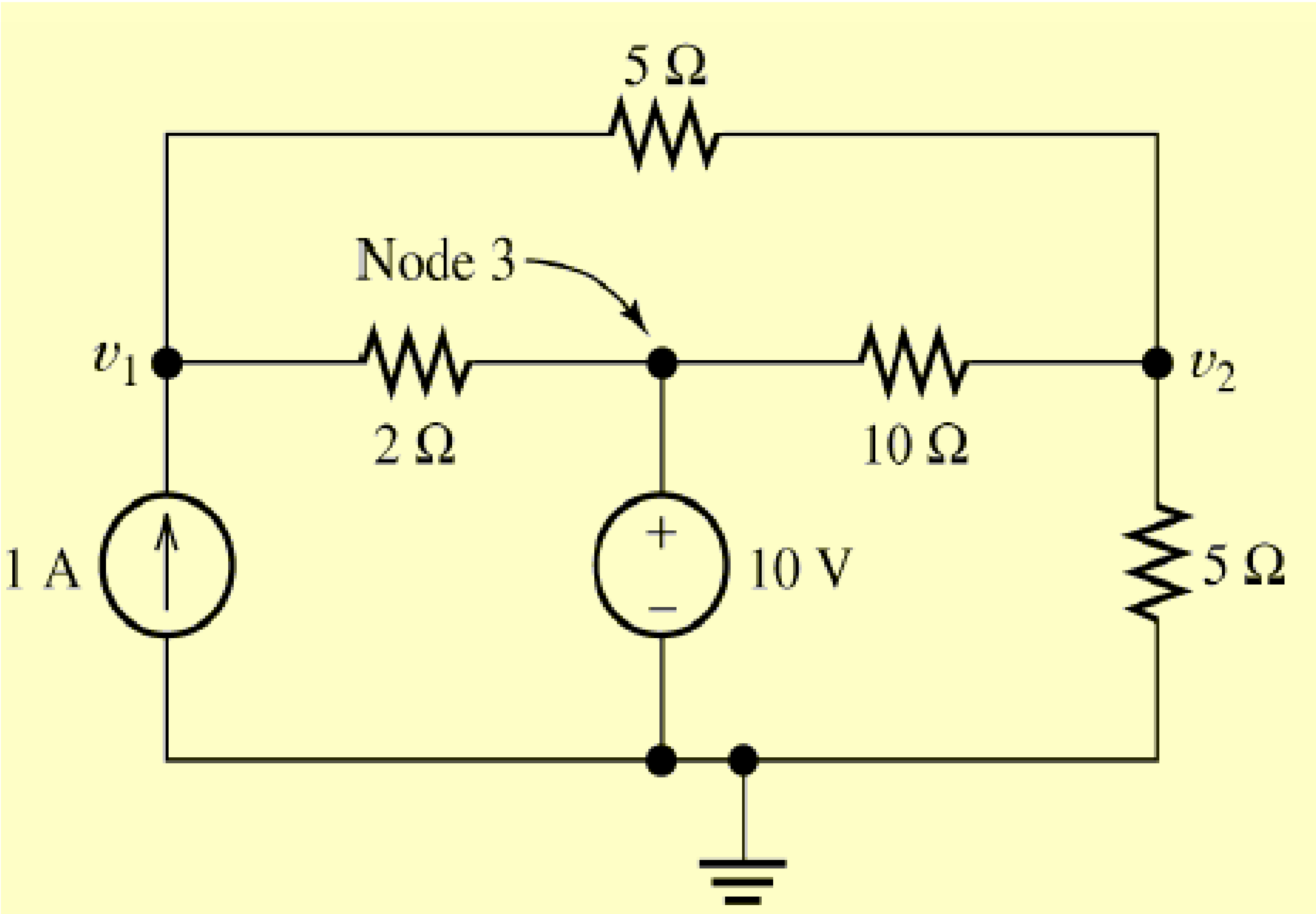
$$\frac{v_2 - v_1}{R_2} + \frac{v_2}{R_3} + \frac{v_2 - v_3}{R_4} = 0$$

$$Node\,3:$$

$$\frac{v_3}{R_5} + \frac{v_3 - v_2}{R_4} - i_s = 0$$

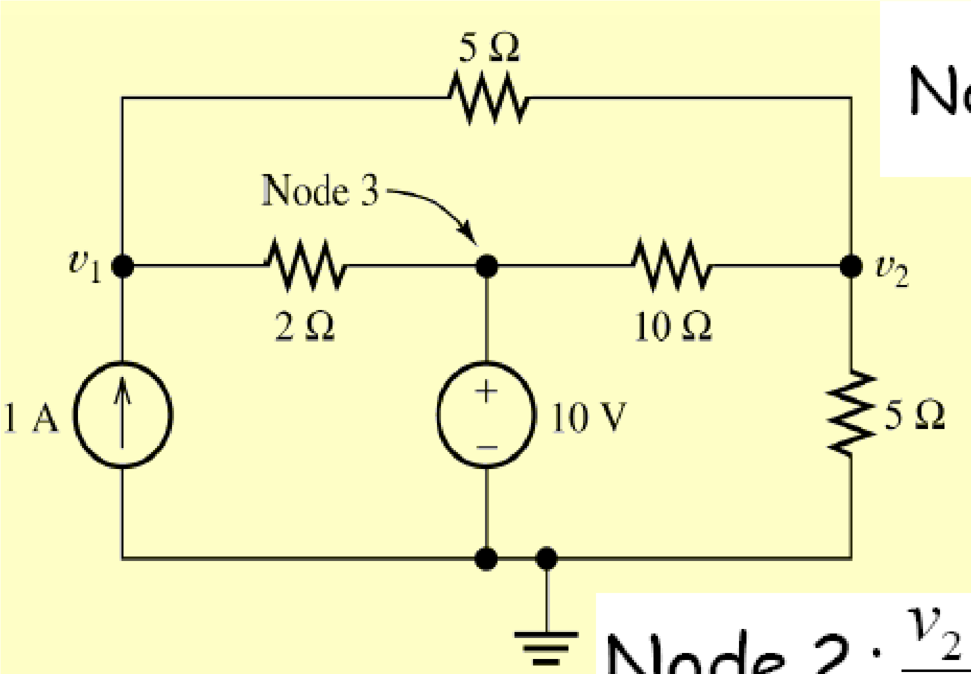
Example

Find the node voltages v_1 and v_2



Example contd..

How many unknown node-voltages?



Node 3: v_3 = 10 V

$$\text{Node1}: \frac{v_1 - v_2}{5} + \frac{v_1 - 10}{2} = 1 \quad : \text{x}\,10$$

$$2v_1 - 2v_2 + 5v_1 - 50 = 10$$

$$0.7v_1 - 0.2v_2 = 6 \qquad (1)$$

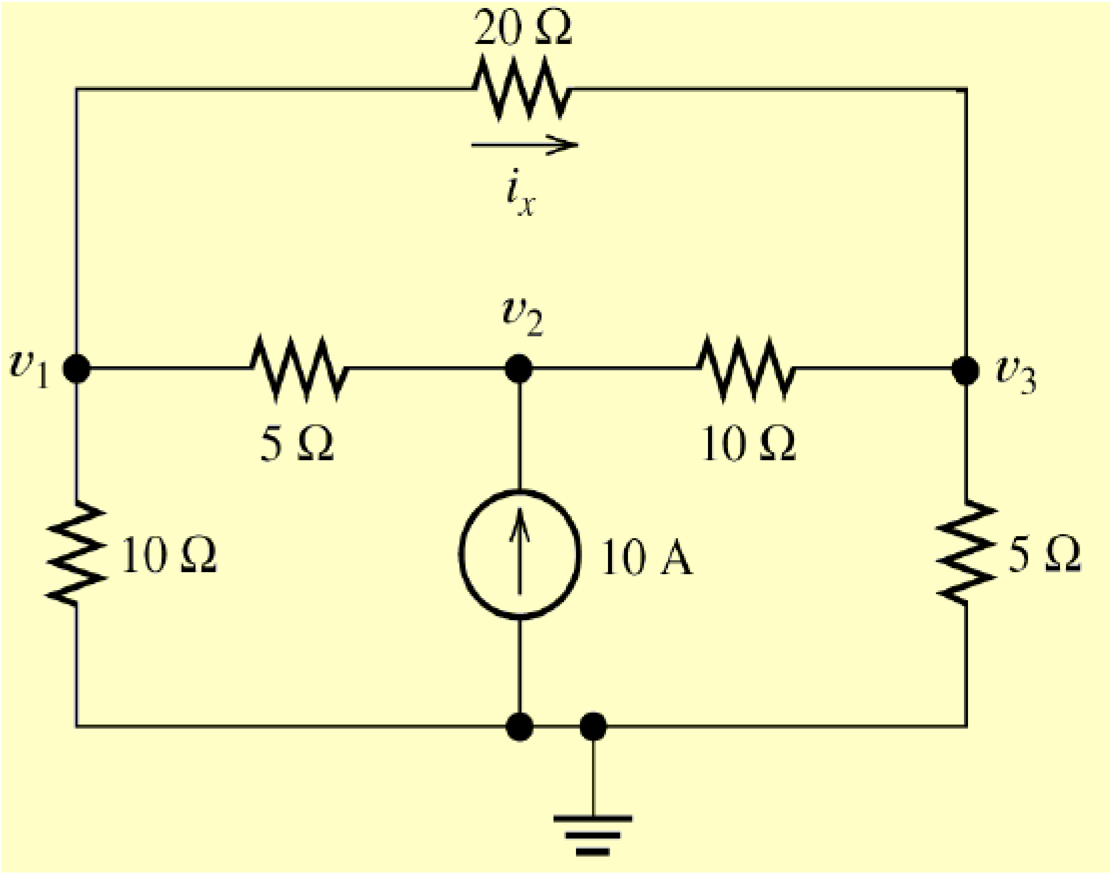
$$\text{Node 2}: \frac{v_2}{5} + \frac{v_2 - 10}{10} + \frac{v_2 - v_1}{5} = 0 \quad : \text{x}\,10$$

$$2v_2 + v_2 - 10 + 2v_2 - 2v_1 = 0$$

$$-0.2v_1 + 0.5v_2 = 1 \qquad (2)$$

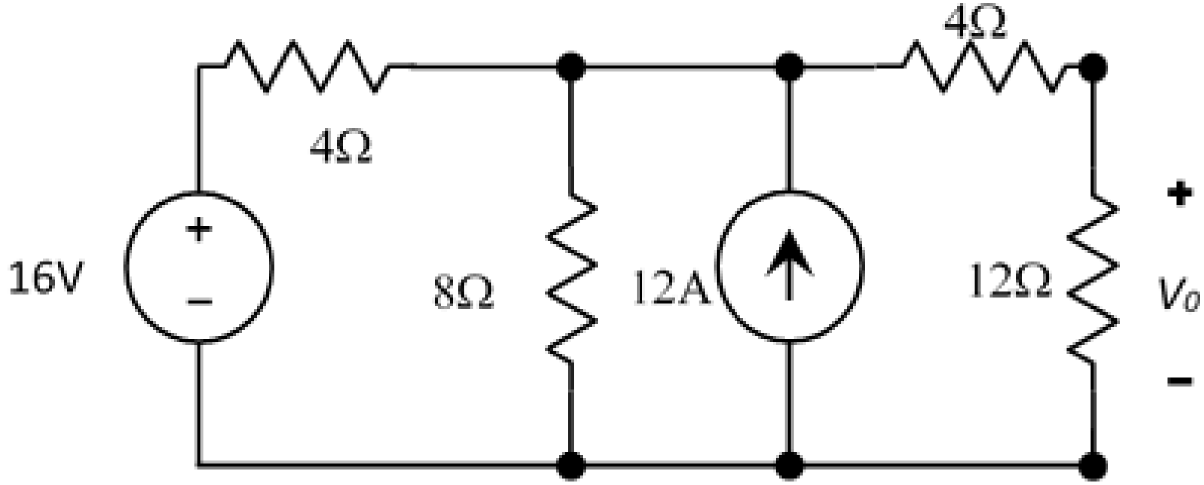
Solving (1) and (2) $v_1 = 10.32\,V \qquad v_2 = 6.129\,V$

Exercise: Find i_x using nodal analysis

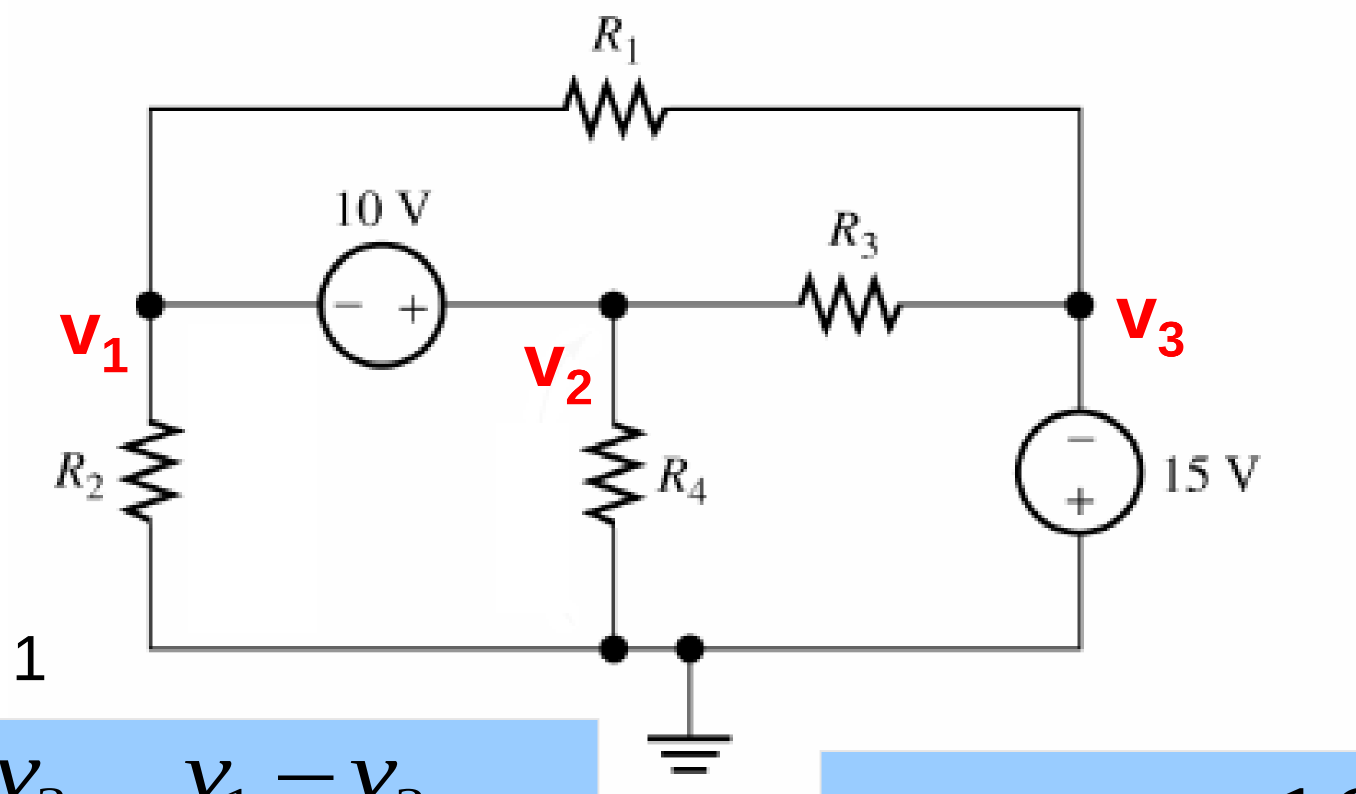


Assignment 1
Problem 6

Find V_0 using Nodal Method



Circuit with Voltage Sources



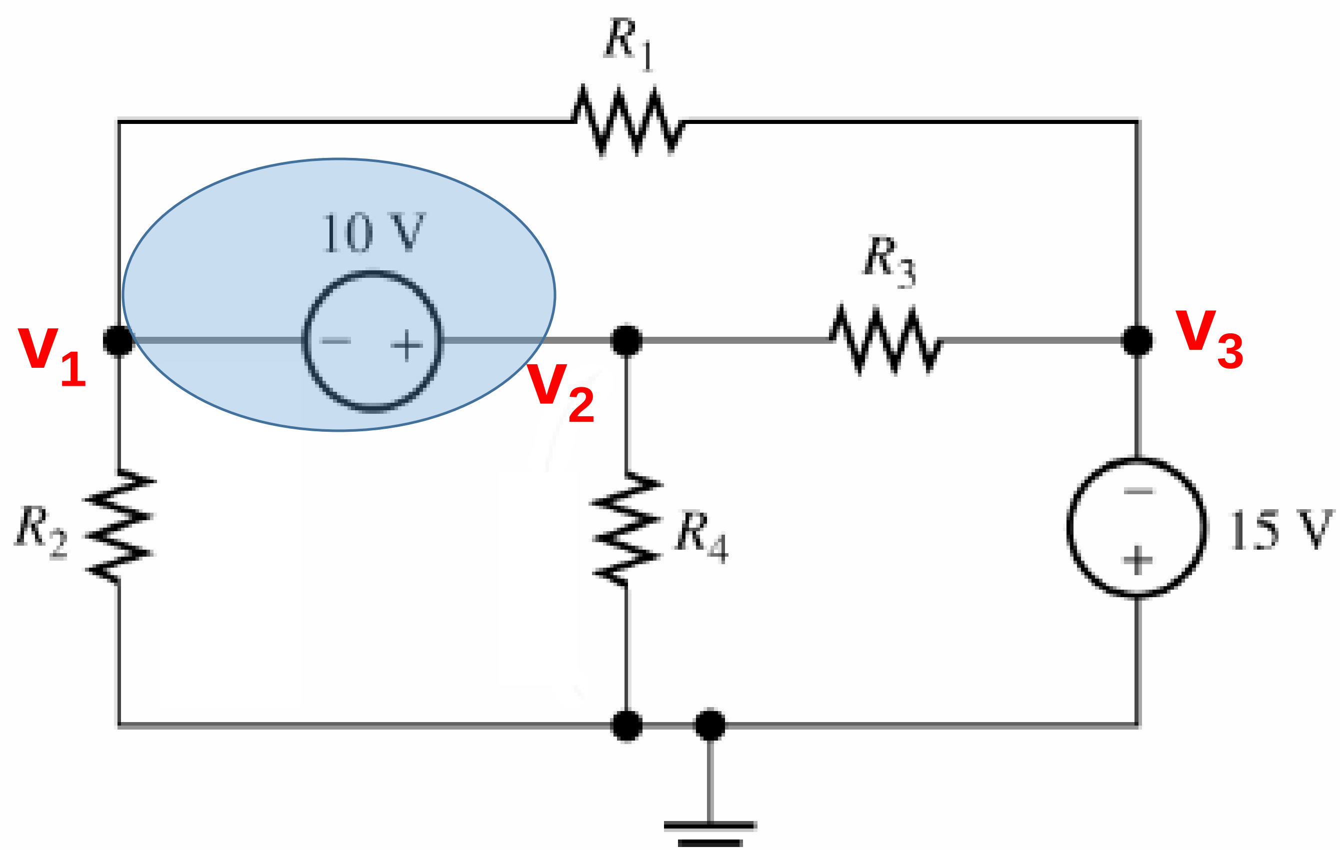
KCL at node 1

$$\frac{v_1}{R_2}+\frac{v_1-v_3}{R_1}+\frac{v_1-v_2}{?}=0$$

$$v_2-v_1=10$$

KCL at node 2

$$\frac{v_2}{R_4}+\frac{v_2-v_3}{R_3}+\frac{v_2-v_1}{?}=0$$

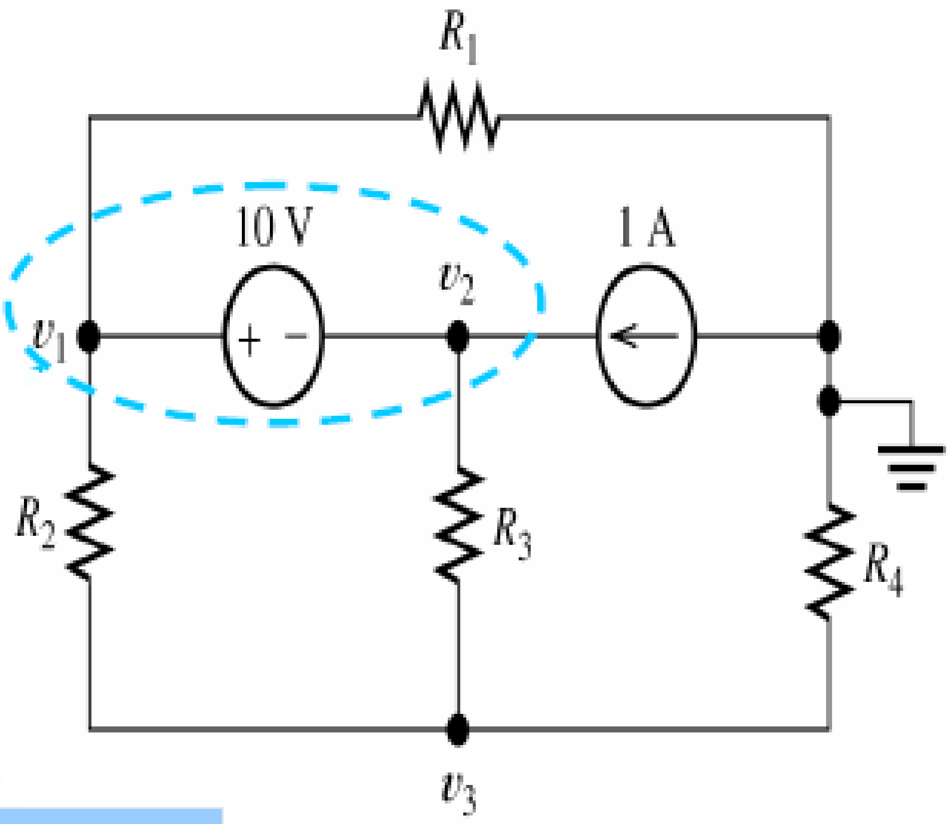


Node 1 and node 2 are merged together into a super node. KCL is applied to the super node

Sum of currents leaving a super node is zero

$$\frac{v_1}{R_2} + \frac{v_1 - v_3}{R_1} + \frac{v_2 - v_3}{R_3} + \frac{v_2}{R_4} = 0$$

Example



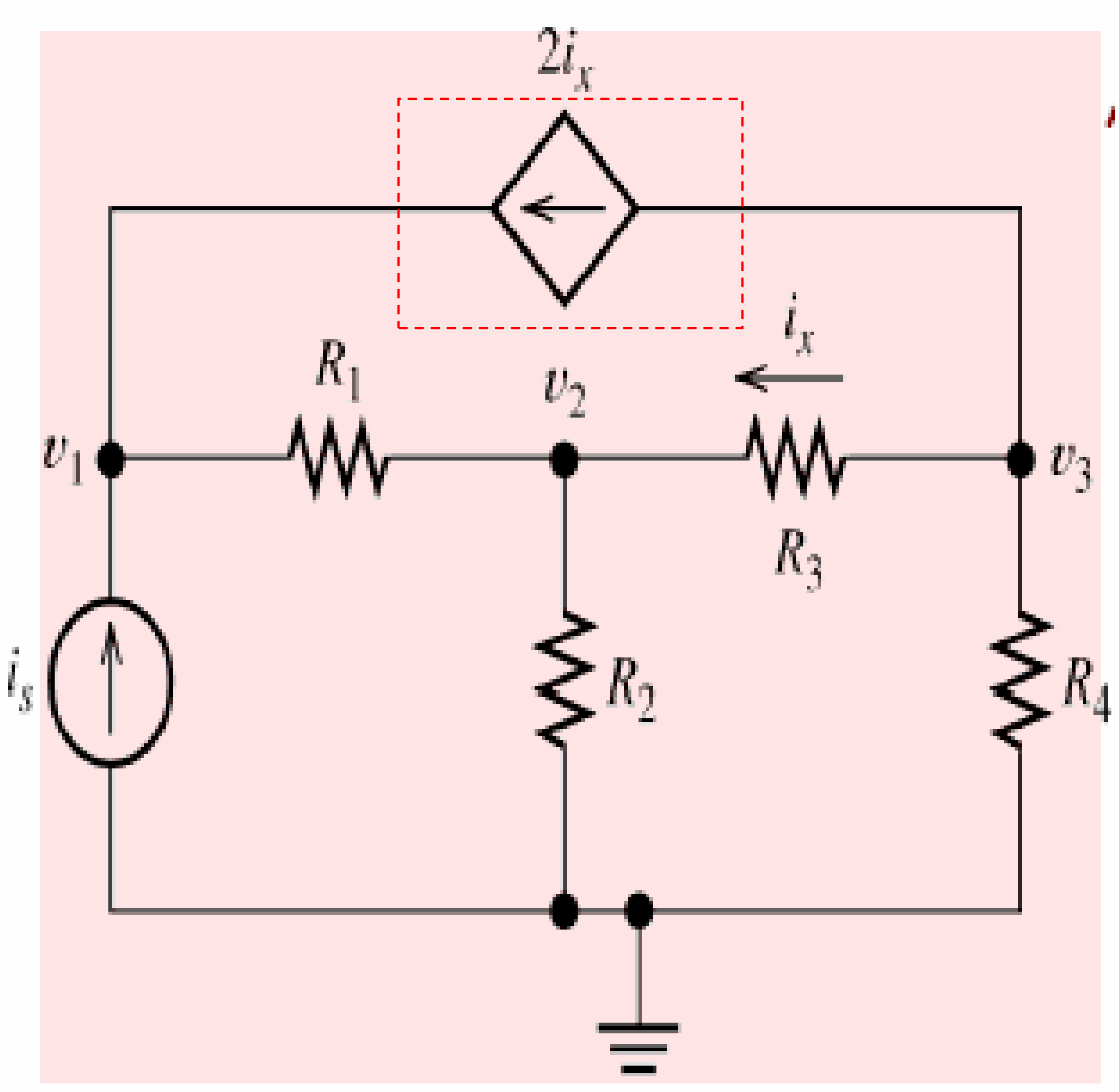
$$v_1 - v_2 = 10$$

$$\frac{v_1}{R_1} + \frac{v_1 - v_3}{R_2} + \frac{v_2 - v_3}{R_3} - 1 = 0$$

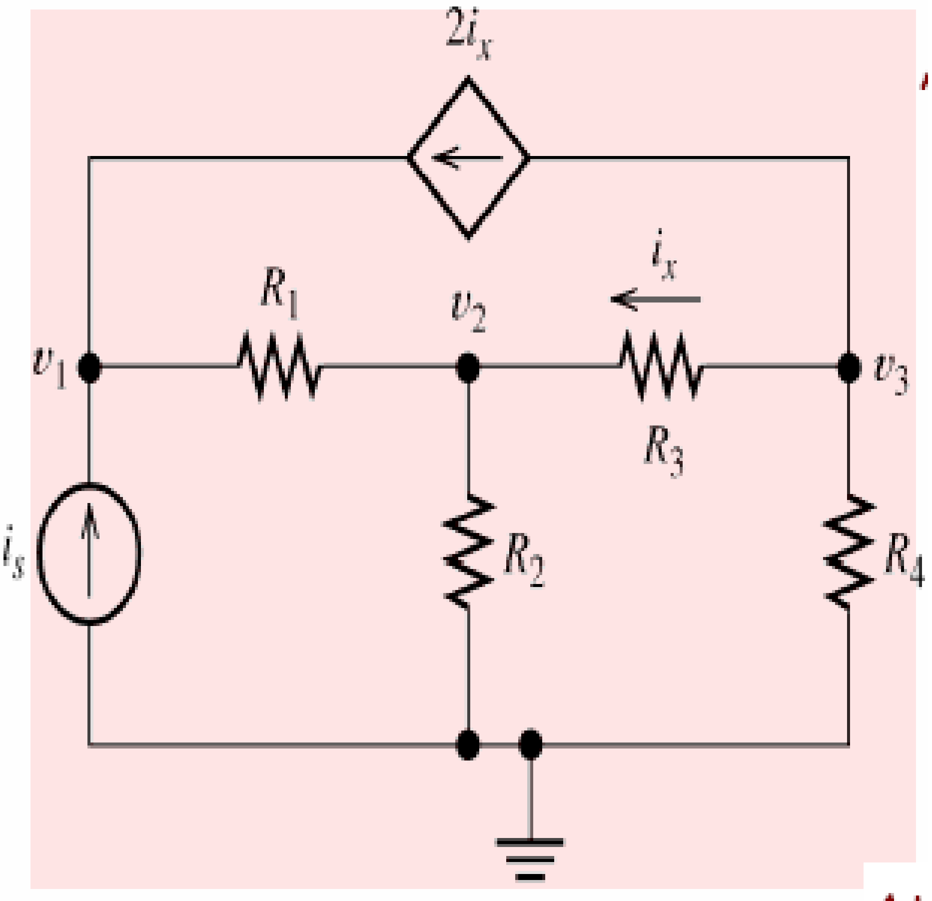
At node 3:

$$\frac{v_3 - v_1}{R_2} + \frac{v_3 - v_2}{R_3} + \frac{v_3}{R_4} = 0$$

Node-Voltage Analysis with a Dependent Source



Node-Voltage Analysis with a Dependent Source



At Node 1

$$\frac{v_1 - v_2}{R_1} - i_s - 2i_x = 0$$

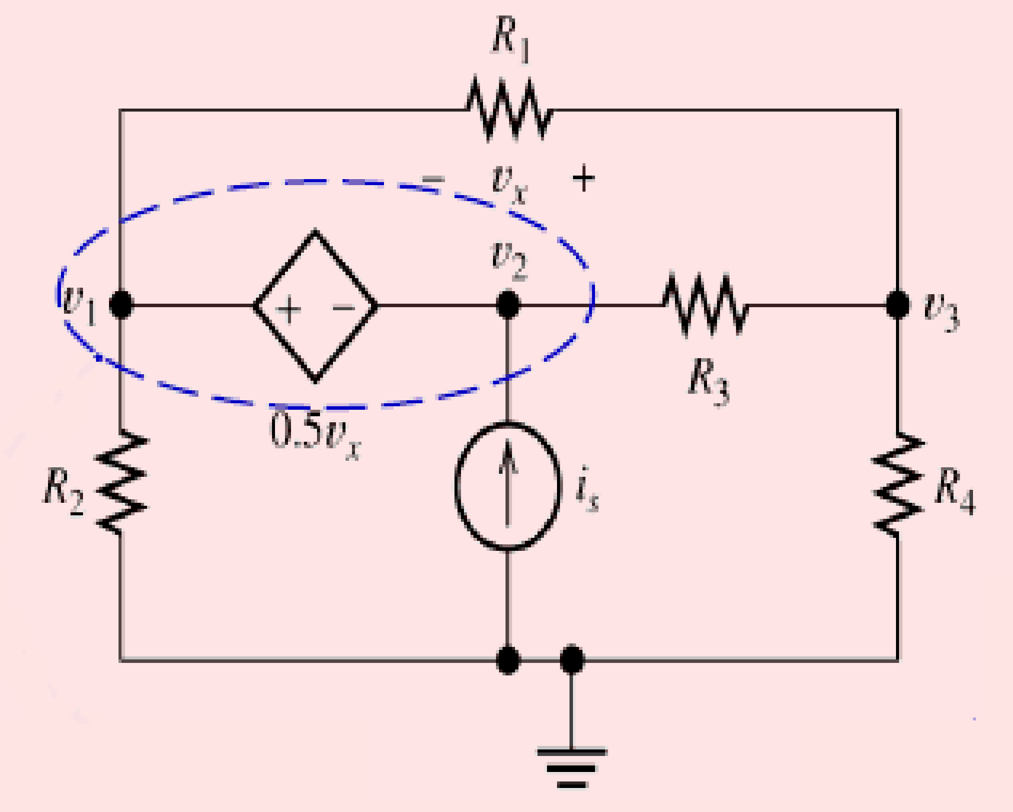
At node 2:

$$\frac{v_2 - v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_2 - v_3}{R_3} = 0$$

At node 3:

$$\frac{v_3 - v_2}{R_3} + \frac{v_3}{R_4} + 2i_x = 0$$

$$i_x = \frac{v_3 - v_2}{R_3}$$



Dependent voltage source :

$$v_1 - v_2 = 0.5v_x$$

Node 3

$$\frac{v_3}{R_4} + \frac{v_3 - v_2}{R_3} + \frac{v_3 - v_1}{R_1} = 0$$

supernode (nodes 1 and 2)

$$\frac{v_1}{R_2} + \frac{v_1 - v_3}{R_1} + \frac{v_2 - v_3}{R_3} = i_s$$

Contolling variable

$$v_x = v_3 - v_1$$

Summary: Node-Voltage Analysis

1. Select a reference node and assign variables for the unknown node voltages

2. Write network equations
- First, use KCL to write current equations for nodes and super nodes
- Then, if you do not have enough equations because of voltage sources connected between nodes, use KVL to write additional equations

3. If the circuit contains dependent sources
- Find expressions for the controlling variables in terms of the node voltages
- Substitute into the network equations, and obtain equations having only the node voltages as unknowns

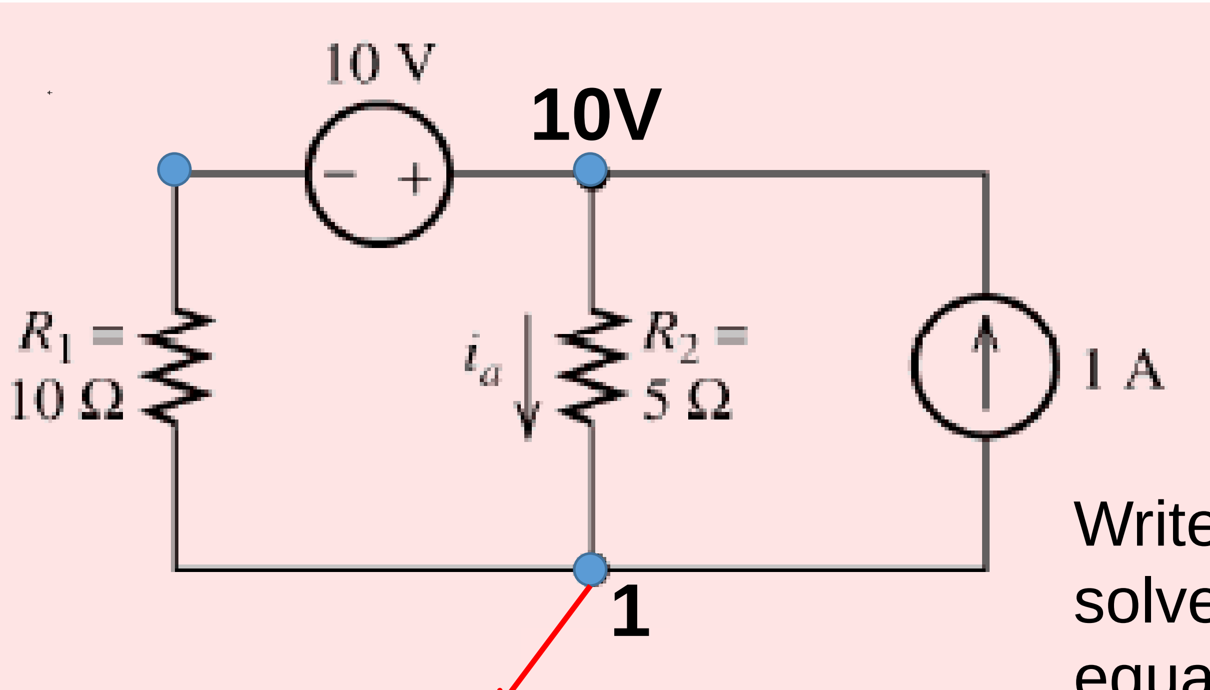
4. Put the equations into standard form and solve for the node voltages

5. Use the values found for the node voltages to calculate any other currents or voltages of interest

Example

Find i_a *using nodal analysis*

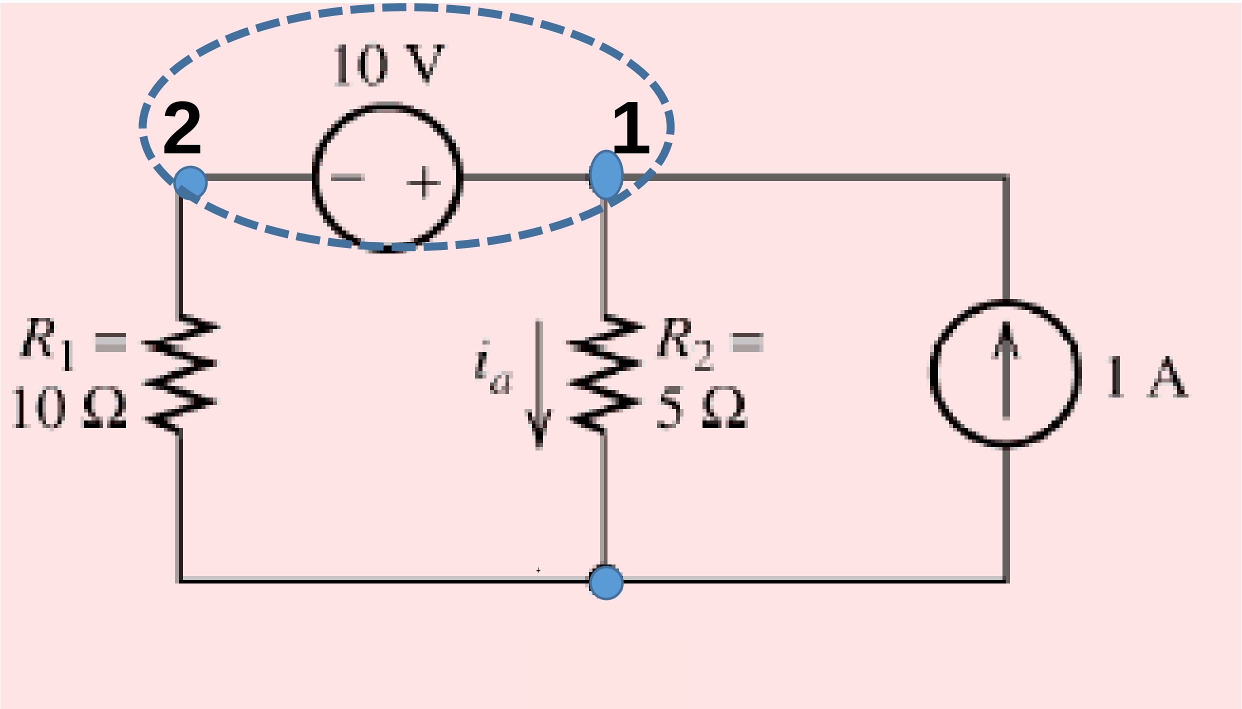
Choose a reference node wisely



Write KCL at nodes and solve the resulting equations

$$\frac{v_1}{10} + \frac{v_1 - 10}{5} + 1 = 0 \Rightarrow v_1 = \frac{10}{3}$$

$$i_a = \frac{10 - v_1}{5} = 1.33A$$

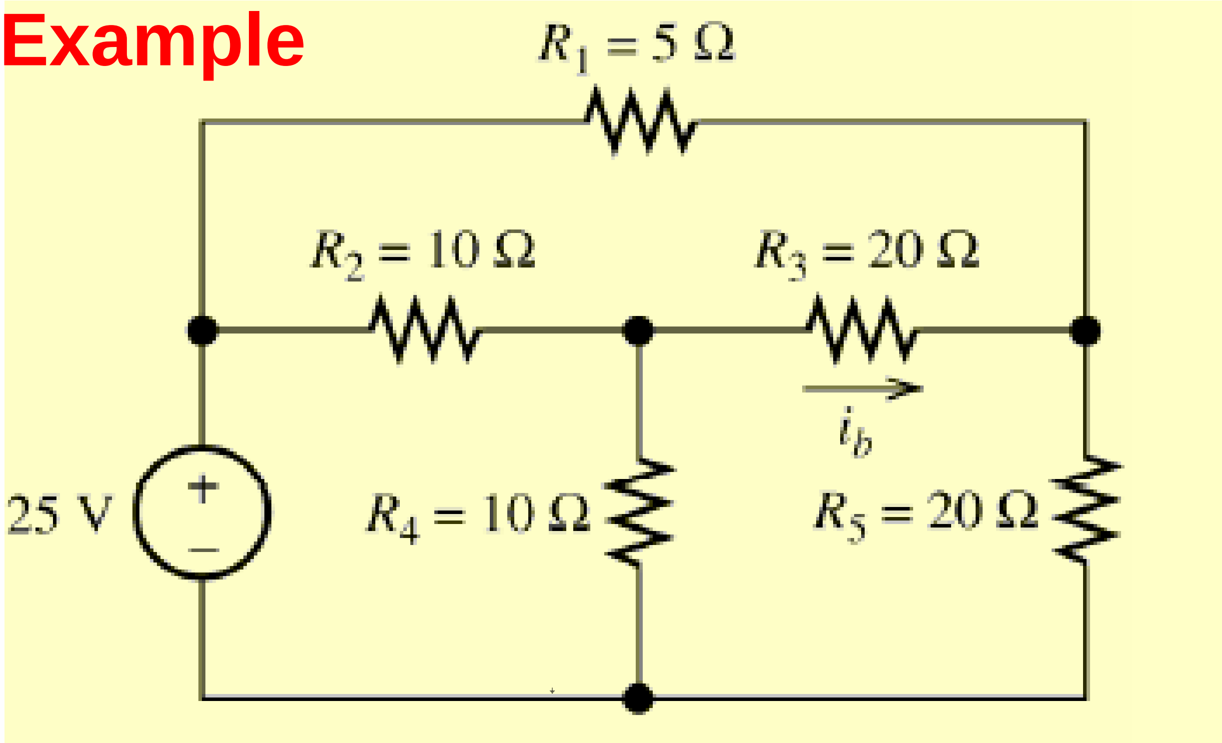


$$v_1 - v_2 = 10V$$

KCL at super node:

$$\frac{v_1}{5} - 1 + \frac{v_2}{10} = 0$$

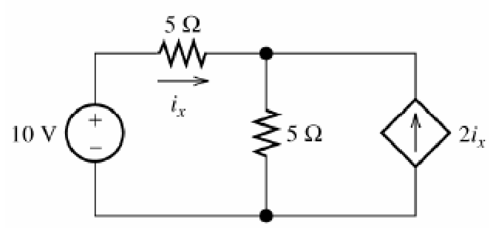
Example



Which should be the reference node?

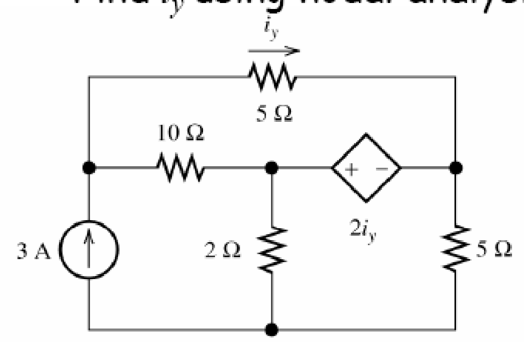
Exercises

Find i_x using nodal analysis



(a)

Find i_y using nodal analysis



(b)