ESC201A Assignment 4

Instructor Abhishek Gupta

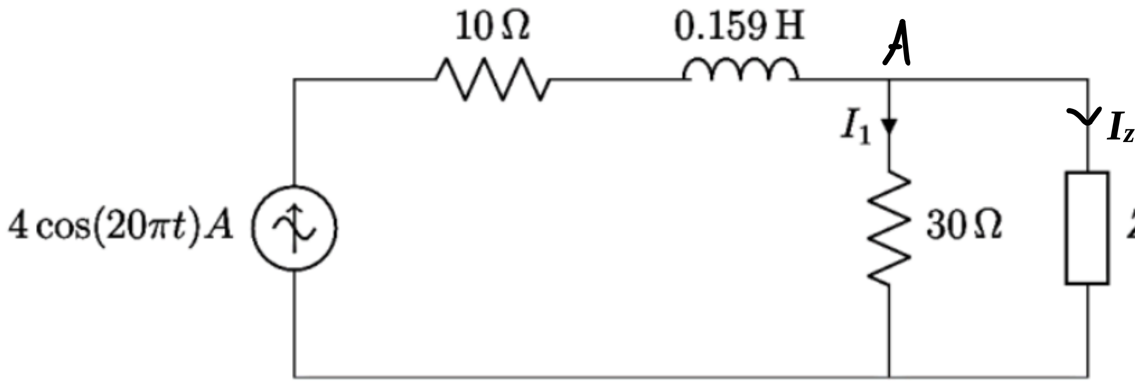
2023-2024 Semester I

Topics

Sinusoidal sources, Phasors, Impedance model, frequency response

Questions

1. Determine the value of the impedance Z in the following circuit if the current $I_1 = (2.56+j1.92)$A.



Solution:

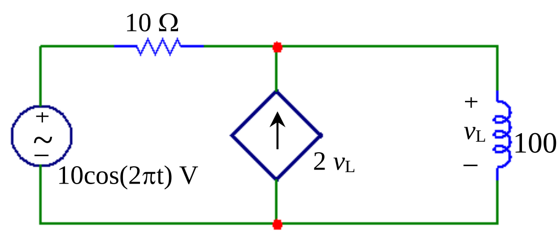
Applying KCL at node A,

$$4 = I_1 + I_z$$
$$\implies I_z = 4 - (2.56 + j1.92)A = (1.44 - j1.92)A$$

Since 30Ω and Z are in parallel, $I_z \times Z = I_1 \times 30\Omega$
Therefore,

$$Z = \frac{I_1 \times 30}{I_z} = \frac{(2.56 + j1.92) \times 30}{(1.44 - j1.92)}\Omega = \frac{30 \times 3.2\angle 36.87^\circ}{2.4\angle - 53.13^\circ}\Omega = 40\angle 90^\circ\Omega = j40\Omega$$

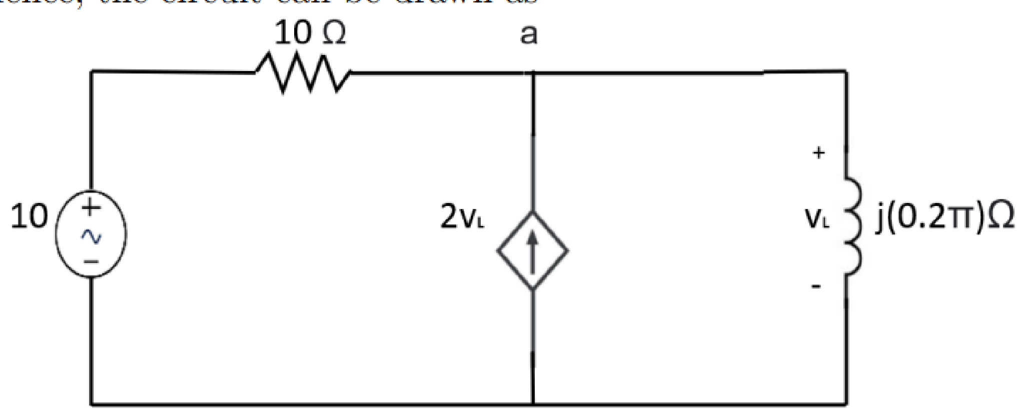
2. Determine the voltage v_L across the inductor in the following circuit, and the average power supplied by the dependent current source.



Solution:

$$\omega = 2\pi \implies X_L = \frac{j \times 2\pi \times 100}{1000}\Omega = j(0.2\pi)\Omega$$

Hence, the circuit can be drawn as



Applying nodal analysis at node a:

$$\frac{V_L - 10}{10} - 2V_L + \frac{V_L}{j(0.2\pi)} = 0$$

$$\implies \frac{V_L}{10} - 2V_L + \frac{-jV_L}{0.2\pi} = 1$$

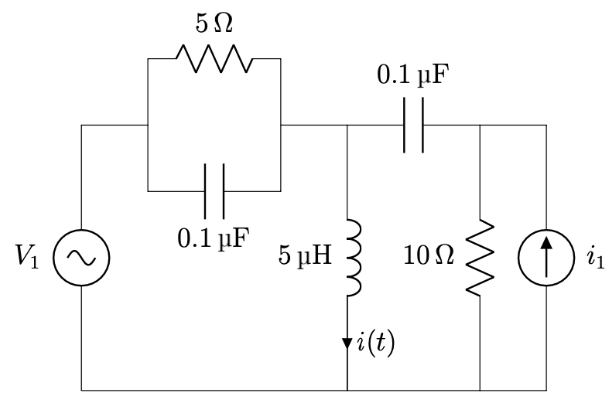
$$\implies V_L = \frac{-1\angle 0^\circ}{2.477\angle 40^\circ}V = 0.404\angle 140^\circ V$$

$$\implies v_L = 0.404cos(\omega t + 140^\circ)V.$$

The peak voltage at dependent source is $V_M = 0.404V$ and the peak current is $I_M = 2V_M$. Average power supplied by the dependent current source :

$$P = 0.5 \times 2V_M \times V_M = v_M^2 = 163mW.$$

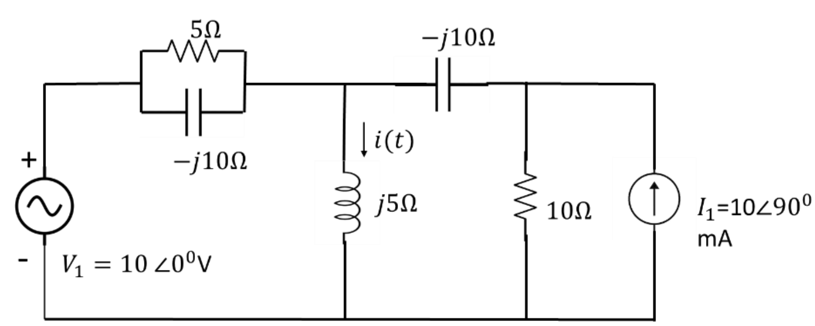
3. For the circuit shown below, $V = 10\angle 0^{o}$V and $I = 10\angle 90^{o}$ mA at $\omega = 10^{5}$ rad/s. If the circuit is in steady state, find the current $i(t)$ through the inductor.



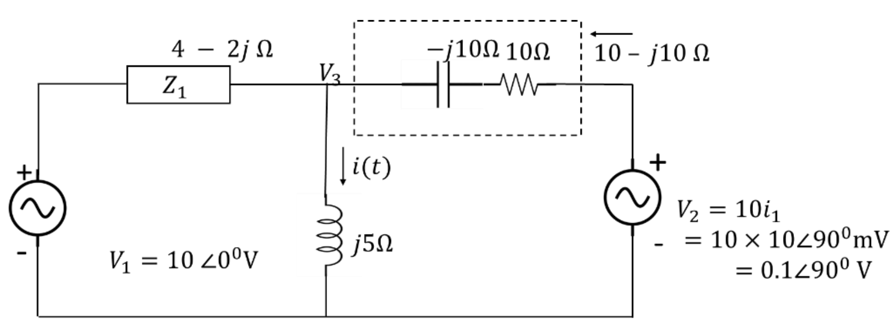
Solution:

$$\omega = 10^{6}\ rad/s$$
$$X_L = \omega L = 10^{6} \cdot 5 \cdot 10^{-6} = 5\Omega$$
$$X_{C_i} = \frac{1}{\omega C_i} = \frac{1}{10^{6} \times 0.1 \times 10^{-6}} = 10\ \Omega \ \ (\text{for i} = 1,2)$$

Then the circuit reduces to



From source transformation theorem



$$Z_1 = 5\ \|\ (-j10\,) = \frac{5 \times -10j}{5 - 10j} = 4 - 2j\ \Omega$$

Using the nodal analysis for the above circuit, we write that

$$\frac{(V_3 - 10\angle 0^0)}{4-2j} + \frac{V_3}{5j} + \frac{(V_3 \; - \; 0.1\angle 90^0)}{10-10j} = 0$$

$$V3\left[\frac{1}{4-2j} + \frac{1}{5j} + \frac{1}{10-10j}\right] = \frac{10\angle 0^0}{4-j2} + \frac{0.1\angle 90^0}{10-j10}$$

$$V_3\ [\,0.2 + 0.1j - 0.2j + 0.05 + 0.05j] = 2 + j1 - 5\times 10^{-3} + j5\times 10^{-3}$$

$$V_3\ (0.25 - 0.05j) = 1.995 + j1.005$$

$$V_3 = \frac{1.995 + j1.005}{0.25 - 0.05j} = 6.9 + j5.4\ volts$$

Now,

$$i(t) = \frac{V_3}{j5} = \frac{6.9 + j5.4}{j5}$$

$$= 1.08 - j1.38$$

$$= 1.753\angle -51.95^0\ A$$

$$\therefore \quad i(t) = 1.753\angle -51.95^0\ A$$

Now,

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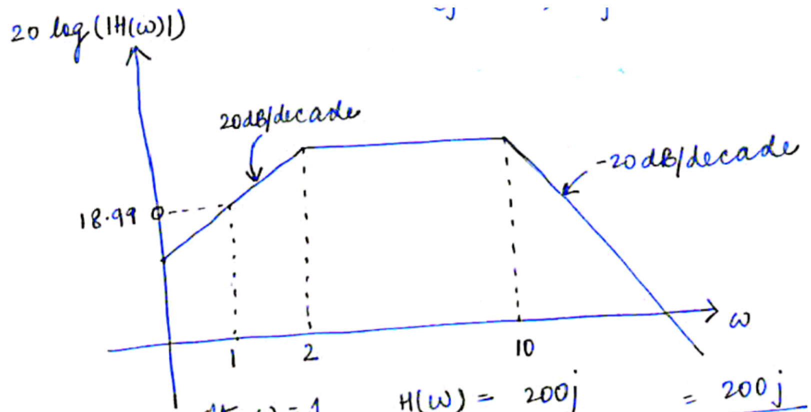
4. Draw the Bode magnitude plot for the following transfer functions.

(a) $H(j\omega) = \dfrac{200j\omega}{(j\omega+2)(j\omega+10)}$

(b) $H(j\omega) = \dfrac{(j\omega)^2(j\omega+100)}{(j\omega+1)(j\omega+10)(j\omega+1000)}$

Solutions:

(a) a) $H(j\omega) = \dfrac{200j\omega}{(j\omega+2)(j\omega+10)}$



At $\omega = 1$, $\quad H(\omega) = \dfrac{200j}{(2+j)(10+j)} = \dfrac{200j}{19+j12}$

$$|H(\omega)| = \frac{200}{\sqrt{19^2+12^2}} = 8.9$$

$$20\log|H(\omega)| = 20\log(8.9) = 18.99$$

(b) $H(j\omega) = \dfrac{(j\omega)^2(j\omega+100)}{(j\omega+1)(j\omega+10)(j\omega+1000)}$

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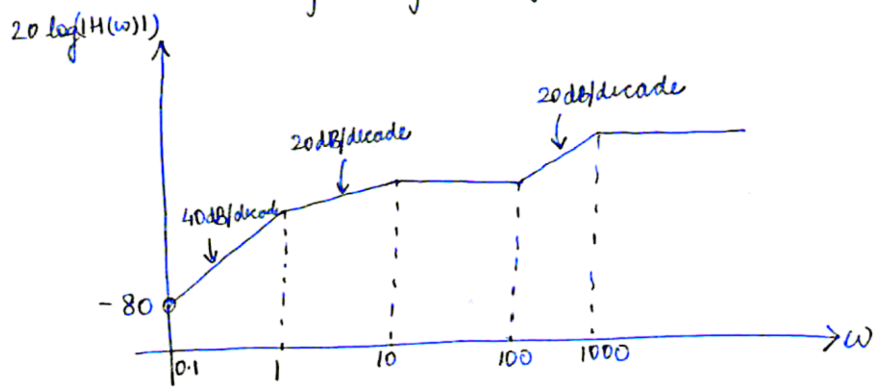
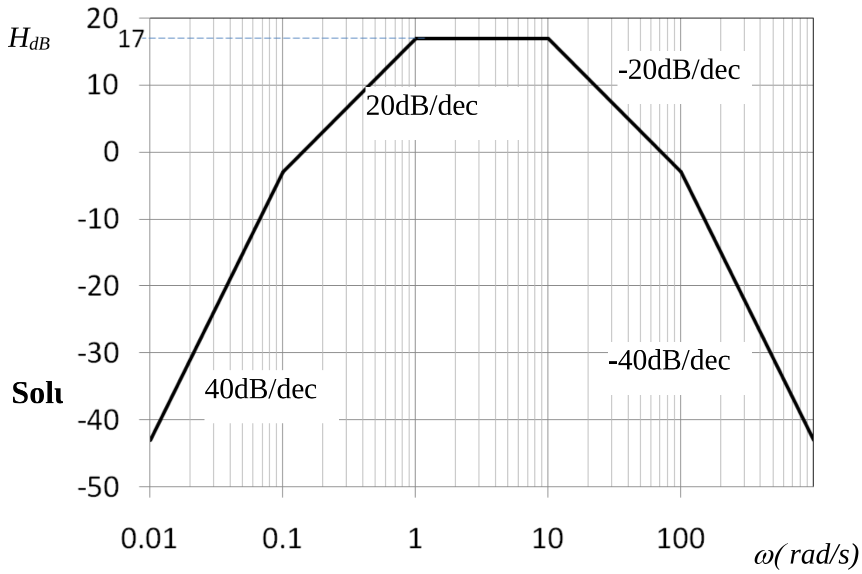


At $\omega = 0.1$,

$$|H(\omega)| = \left|\frac{(j(0.1))^2\,(j0.1+100)}{(j0.1+1)(j0.1+10)(j0.1+1000)}\right| = 10^{-4}$$

$$20\log(|H(\omega)|) = -80\log(10) = -80$$

5. Find the transfer function for the following Bode plot.



Solu

$$H(\omega) = \frac{A(j\omega)^2}{\left(1+\frac{j\omega}{0.1}\right)\left(1+j\omega\right)\left(1+\frac{j\omega}{10}\right)\left(1+\frac{j\omega}{100}\right)}$$

$$H(\omega=1) = 17\ dB$$

$$\Rightarrow 17 = 20\log\left|\frac{A}{(1+j10)(1+j)(1+0.1j)(1+0.01j)}\right|$$

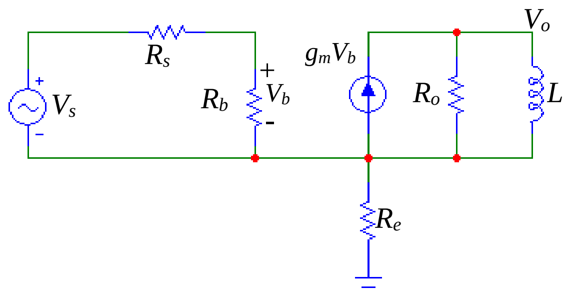
$$= 20\log\left(\frac{A}{10\times1\times1\times1}\right)$$

$$\Rightarrow 20\log A = 17+20 = 37$$

$$\therefore\ A = 10^{37/20} = 70.79$$

$$\therefore\ H(\omega) = \frac{7079\,(j\omega)^2}{(j\omega+0.1)(j\omega+1)(j\omega+10)(j\omega+100)}$$

6. Determine the transfer function (V_o/V_s) for the following circuit.



Solution:

$$V_b = \frac{R_b V_s}{R_b + R_s}$$

$$I_b = \frac{V_b}{R_b} = \frac{V_s}{R_s + R_b}$$

Applying nodal

$$-gmV_b + \frac{V_0}{R_0} + \frac{V_0}{jwL} = 0$$

$$V_0\left(\frac{1}{R_0} + \frac{1}{j\omega L}\right) = gmV_b$$

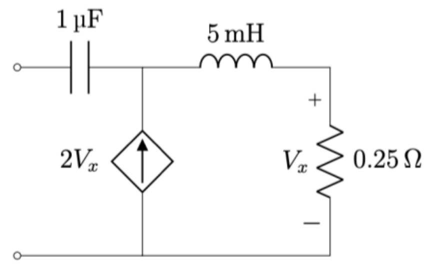
$$V_0\left[\frac{j\omega L + R_0}{j\omega L R_0}\right] = g_m V_b$$

$$V_0 = \frac{j\omega g_m R_0 L}{j\omega L + R_0} \cdot V_b$$

$$V_0 = \frac{j\omega L g_m R_0}{j\omega L + R_0} * \frac{R_b V_s}{R_b + R_s}$$

$$\boxed{\therefore \frac{V_0}{V_s} = \frac{j\omega g_m R_0 R_b L}{(jwL+R_0)(R_b+R_s)}}$$

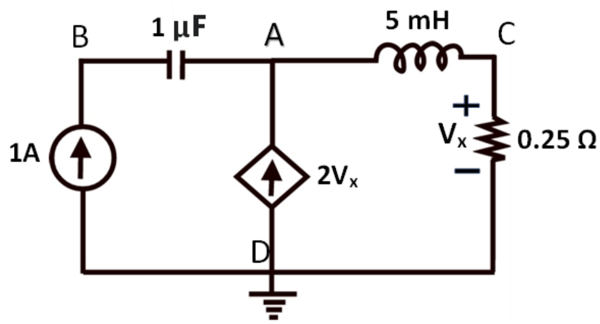
7. Determine the resonant frequency for circuit shown below.



Solution:

Calculate the Z_{eq} of the circuit and make imaginary part to be zero to calculate the resonant frequency.

To calculate the impedance, we connect a current source.



<u>Applying KCL at node A</u>:

$$-1 - 2V_x + \frac{V_x}{0.25} = 0$$

$$\Rightarrow -1 - 2V_x + 4V_x = 0$$

$$\Rightarrow 2V_x = 1$$

$$\Rightarrow V_x = 0.5V$$

Let say the voltage drop across the current source is V_1.

Then applying KVL in BACDB,

$$-V_1 + \frac{1}{j\omega C}.1 + j\omega L(1 + 2V_x) + V_x = 0$$

$$\Rightarrow V_1 = \frac{1}{j\omega C} + j\omega L\,(1 + 2 \times 0.5) + 0.5$$

$$\Rightarrow V_1 = \frac{1}{j\omega C} + j\omega L \times 2 + 0.5$$

Hence, the impedance is

$$Z_{eq} = \frac{V_1}{1} = \frac{1}{j\omega C} + j\omega L * 2 + 0.5.$$

At resonance Im $(Z_{eq}) = 0$

$$-\frac{1}{\omega_0 C} + 2\omega_0 L = 0$$

$$\Rightarrow \frac{1}{\omega_0 C} = 2\omega_0 L$$

$$\omega_0 = \frac{1}{\sqrt{2}LC} = \frac{1}{\sqrt{2}\cdot 5\cdot 10^{-3}\cdot 10^{-6}}$$
$$\omega_0 = 10^4\ rad/sec$$
$$f_0 = \frac{\omega_0}{2\pi} = \frac{10^4}{2\pi} = 1591.5\ Hz$$

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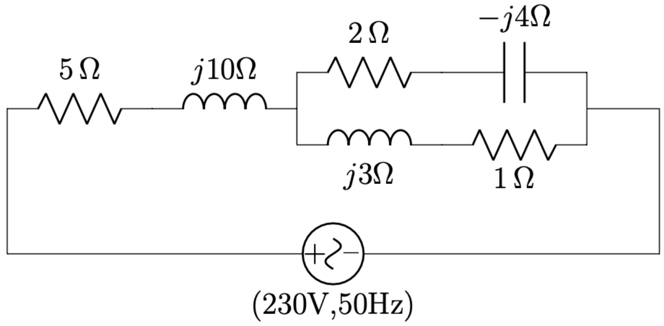
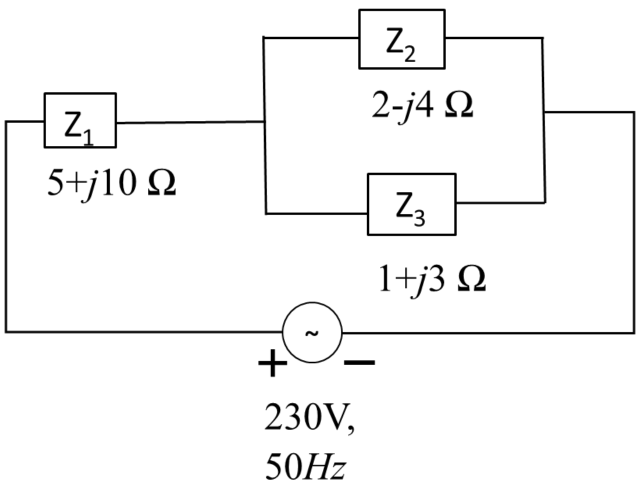
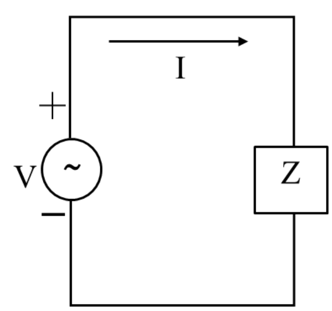
8. Find the average power and reactive power, in the network shown in figure below



Solution

The resultant circuit is





$Here,$

$$Z = Z_1 + (Z_2\,||\,Z_3) = Z_1 + \frac{Z_2 Z_3}{Z_2 + Z_3}$$
$$= 5 + j10 + \frac{(2-j4)(1+j3)}{2-j4+1+j3}$$
$$= 5 + j10 + \frac{14+j2}{3-j} = 5 + 10j + \frac{(14+2j)(3+j)}{10}$$
$$= 5 + 10j + \frac{1}{10}(42 + 6j + 14j - 2)$$

$= 5 + 10j + 4 + 2j = 9 + j12\ \Omega$

& $V_{rms} = 230\ Volts.$
Then

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{230}{9 + j12} = \frac{230(9 - j12)}{225} = 9.2 - j12.27 = 15.33\angle - 53.13°\ A$$

$\Longrightarrow\ \theta = 53.13°$

∴ Average Power,

$$P_{avg} = V_{rms} I_{rms} cos\theta = 230 * 15.33 * cos(53.13°) \qquad = 2115.54\ W$$

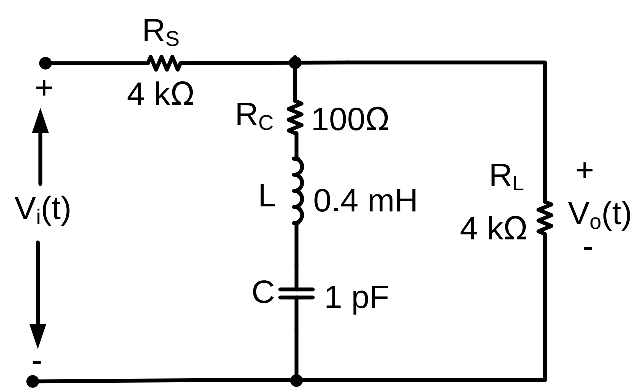
∴ Reactive Power,

$$P_{react} = V_{rms} I_{rms} sin\theta = 230 * 15.33 * sin(53.13°) = 2820.72\ W$$

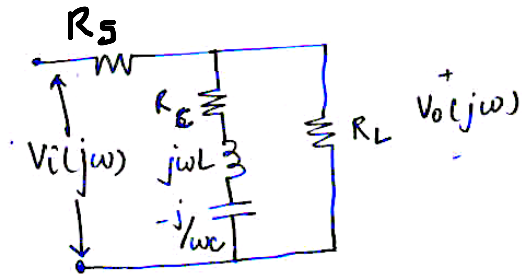
9. A band–reject (notch) filter is shown below. Derive the expression of its transfer function H in the form

$$H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega} = K\left[\frac{(1 + ja)}{(1 + jb)}\right].$$

Find out the expressions for the coefficients K, a and b. Determine the magnitudes of this transfer function at very low and very high frequencies from physical arguments. What is the resonance frequency of this circuit? What is the magnitude of the transfer function at this resonance frequency? Also calculate the level of rejection (in dB) at resonance frequency.



Converting to phasor domain,



Let the equivalent impedance of the two parallel branches be Z_1.

$$\therefore Z_1 = R_L \,||\, \left[R_C + j\left(\omega L - \frac{1}{\omega C}\right)\right]$$

$$\Rightarrow Z_1 = \frac{R_L\left[R_C + j\left(\omega L - \frac{1}{\omega C}\right)\right]}{R_L + R_C + j\left(\omega L - \frac{1}{\omega C}\right)}$$

Applying voltage division,

$$V_o(j\omega) = \frac{Z_1 \times V_i(j\omega)}{Z_1 + R_S}$$

$$\Rightarrow \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{Z_1}{Z_1 + R_S}$$

$$\Rightarrow \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{\dfrac{R_L\left[R_C + j\left(\omega L - \frac{1}{\omega C}\right)\right]}{R_L + R_C + j\left(\omega L - \frac{1}{\omega C}\right)}}{\dfrac{R_L\left[R_C + j\left(\omega L - \frac{1}{\omega C}\right)\right]}{R_L + R_C + j\left(\omega L - \frac{1}{\omega C}\right)} + R_S}$$

$$\Rightarrow \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{R_L R_C + jR_L\left(\omega L - \frac{1}{\omega C}\right)}{R_L R_C + jR_L\left(\omega L - \frac{1}{\omega C}\right) + R_S R_L + R_S R_C + jR_S\left(\omega L - \frac{1}{\omega C}\right)}$$

$$\Rightarrow \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{R_L R_C + jR_L\left(\omega L - \frac{1}{\omega C}\right)}{R_L R_C + R_S R_L + R_S R_C + j\left(\omega L - \frac{1}{\omega C}\right)(R_S + R_L)}$$

converting to the desired form,

$$\Rightarrow \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{R_L R_C\left[1 + \frac{j}{R_C}\left(\omega L - \frac{1}{\omega C}\right)\right]}{R_L R_C + R_S R_L + R_S R_C\left[1 + \frac{j(R_S+R_L)}{(R_L R_C + R_S R_L + R_S R_C)}\left(\omega L - \frac{1}{\omega C}\right)\right]}$$

$$\therefore K = \frac{R_L R_C}{R_L R_C + R_S R_L + R_S R_C}$$

Substituting the values of R_L, R_S, R_C, $K = \frac{1}{42}$.

$$a = \frac{1}{RC}\left(\omega L - \frac{1}{\omega C}\right), \quad b = \frac{(R_S+R_L)\left(\omega L - \frac{1}{\omega C}\right)}{(R_L R_C + R_S R_L + R_S R_C)}$$

For very low frequencies, $\omega \to 0$, $X_L \to 0$ and $X_C \to \infty$.
Therefore, the ~~R~~ R-L-C branch is open circuited.

$$|H(j\omega)|_{\omega \to 0} = \frac{R_L}{R_L + R_S} = \frac{4k}{4k+4k} = 0.5$$

For very high frequencies, $\omega \to \infty$, $X_L \to \infty$, $X_C \to 0$.
~~Agin~~ Again, the R-L-C branch is open circuited.

$$\therefore |H(j\omega)|_{\omega \to \infty} = \frac{R_L}{R_S + R_L} = 0.5$$

For resonance, $\mathrm{Im}[H(j\omega)] = 0$

$$\therefore K\left[\frac{1+ja}{1+jb} \times \frac{1-jb}{1-jb}\right] = K\left[\frac{1 - jb + ja + ab}{1+b^2}\right] = K\left[\frac{1 + j(a-b) + ab}{1+b^2}\right]$$

$$\Rightarrow a - b = 0$$

$$\Rightarrow \omega_0 = \frac{1}{\sqrt{LC}} \quad \Rightarrow f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$f_r = \frac{1}{2\pi\sqrt{0.4\times10^{-3}\times10^{-12}}}\ \text{Hz}$$

$$f_r = 7.96\,\text{MHz}$$

$$|H(j\omega)|_{\omega=2\pi f_r} = k\left|\frac{1+ab}{1+b^2}\right|$$

Substituting values,

$$(a)_{f_r} = -0.08961$$

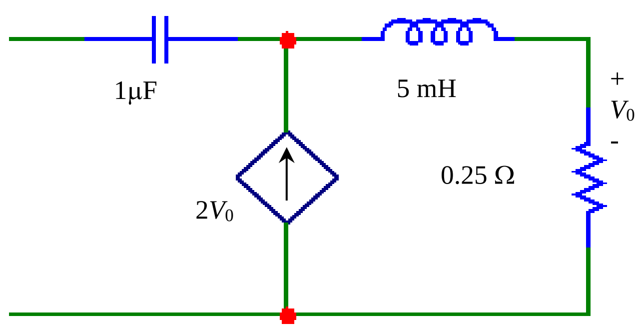
$$(b)_{f_r} = -0.00189$$

$$= \frac{1}{42}\left[\frac{1+(-0.08961)(-0.00189)}{1+(-0.00189)^2}\right]$$

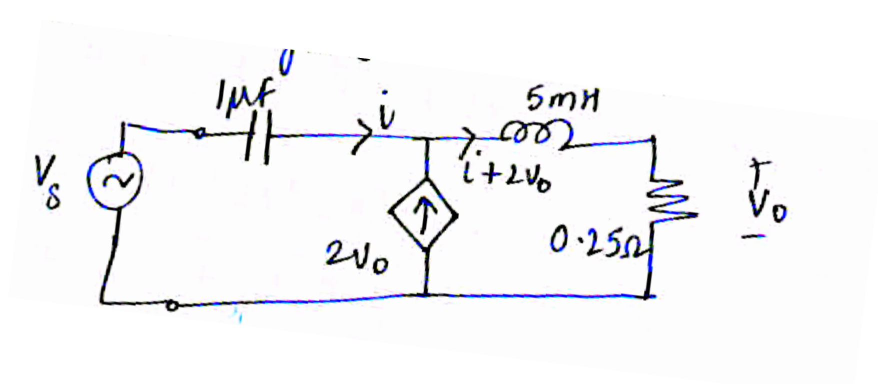
$$= 23.809\times10^{-3}$$

$$\text{Rejection} = 20\log_{10}(23.809\times10^{-3})\,\text{dB} = -32.465\,\text{dB}.$$

10. Determine the resonant frequency of the following circuit



Solution:



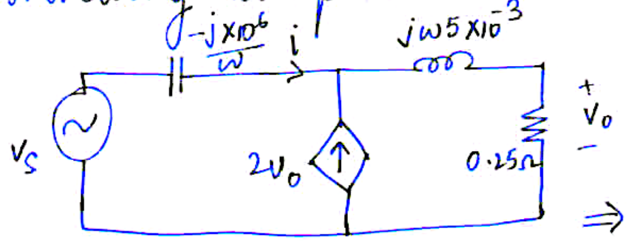
$$V_0 = (i + 2V_0)\,0.25$$

$$\therefore V_0 = 0.25i + 0.5V_0$$

$$\Rightarrow 0.5V_0 = 0.25i$$

$$\text{or } \frac{V_0}{i} = \frac{0.25}{0.5} = \frac{1}{2}$$

Converting to phasor domain,



$$V_s + j\frac{\times 10^6}{\omega}i - j\omega 5\times 10^{-3}(i + 2V_0) - V_0 = 0$$

$$\Rightarrow V_s + j\frac{10^6}{\omega}i - j\omega 5\times 10^{-3} i - j\omega V_0 10^{-2} - V_0 = 0$$

$$\Rightarrow V_s = \left(j\omega 5\times 10^{-3} - j\frac{10^6}{\omega}\right)(2V_0) + j\omega V_0 10^{-2} + V_0$$

$$\Rightarrow \frac{V_0}{V_s} = \frac{1}{1 + j\left(\omega \times 2\times 10^{-2} - \frac{2\times 10^6}{\omega}\right)}$$

At resonance, $X_L = X_C$

$$\therefore \omega_r = 10^4 \text{ rad/sec}$$