# ESc201 : Introduction to Electronics

## Operational Amplifier

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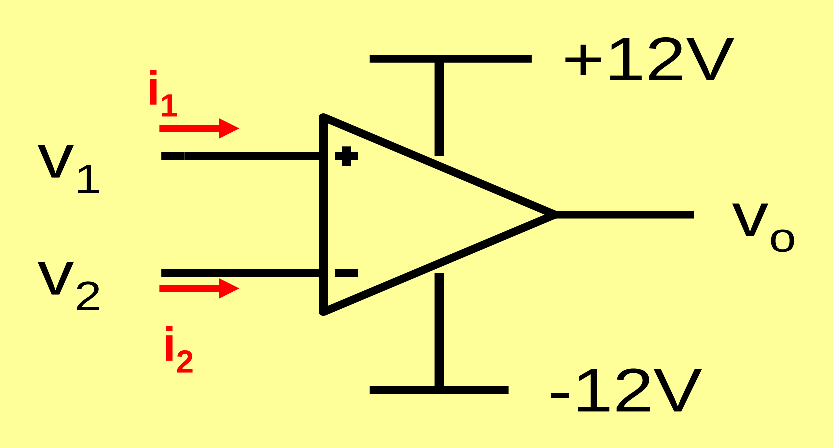
**Recap**

Two important property for analyzing ideal opamp circuits under negative feedback



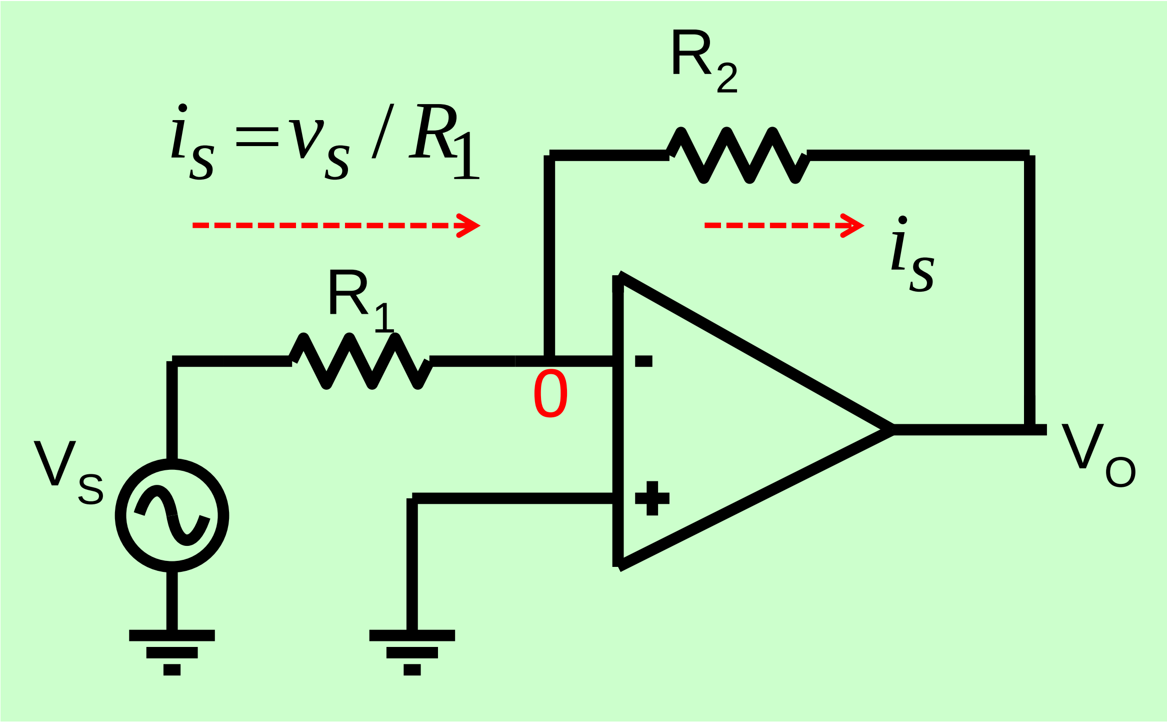

$$1. \quad v_1 = v_2$$

$$2. \quad i_1 = i_2 = 0$$

At the input side opamp appears to be like a short and an open circuit simultaneously !

# Recap: Inverting amplifier

Re-analyze inverting amplifier with these properties



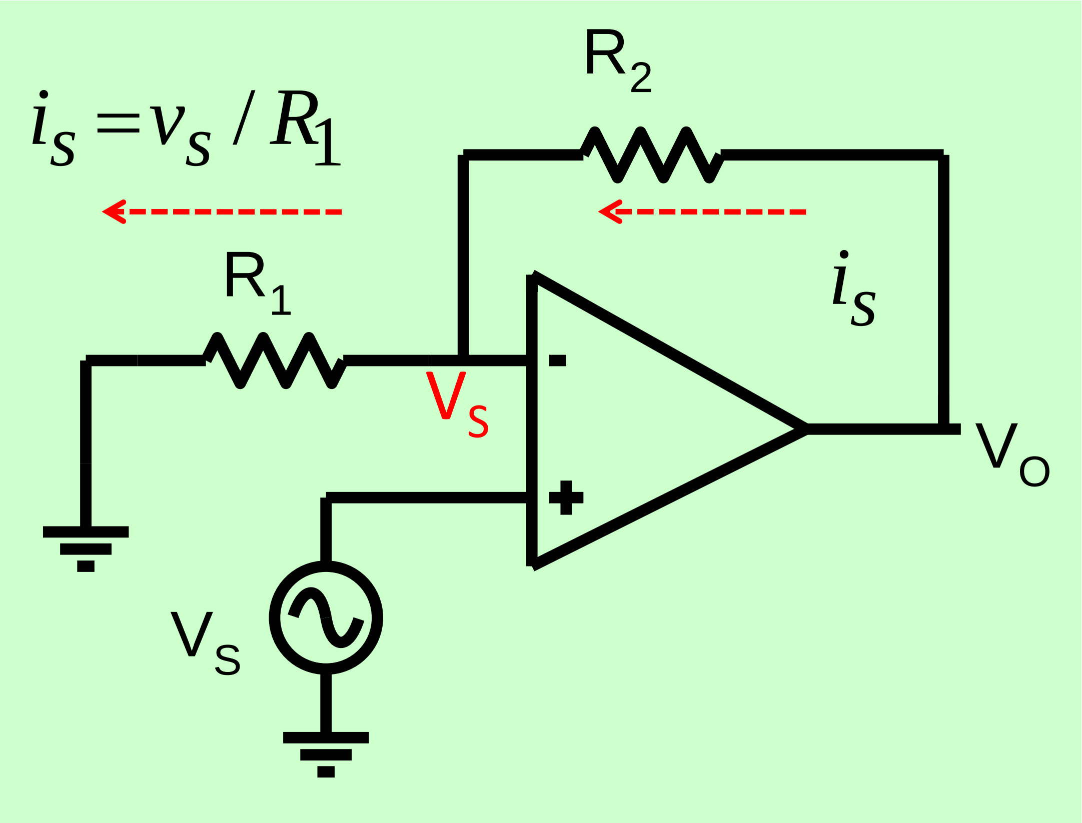

$$\frac{0 - v_O}{R_2} = i_S = \frac{v_S}{R_1}$$

$$\frac{v_O}{v_S} = -\frac{R_2}{R_1}$$

# Recap: Non-Inverting Amplifier

1. $v_1 = v_2$

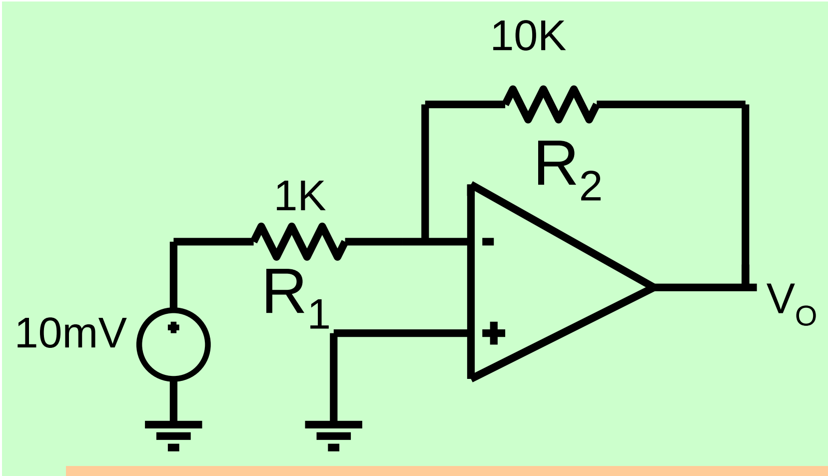
2. $i_1 = i_2 = 0$



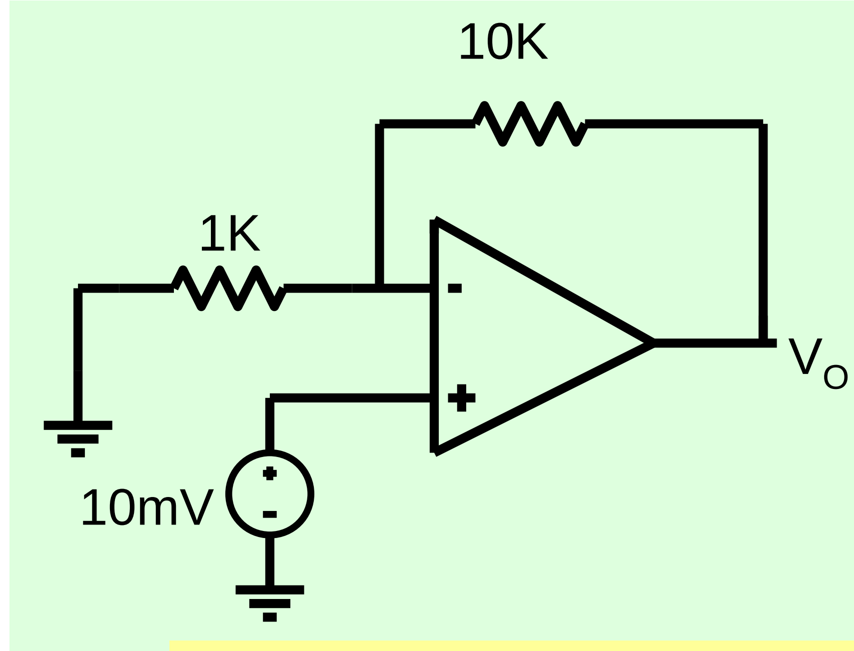

$$\frac{v_o - v_s}{R_2} = i_s = \frac{v_s}{R_1}$$

$$\frac{v_o}{v_s} = 1 + \frac{R_2}{R_1}$$

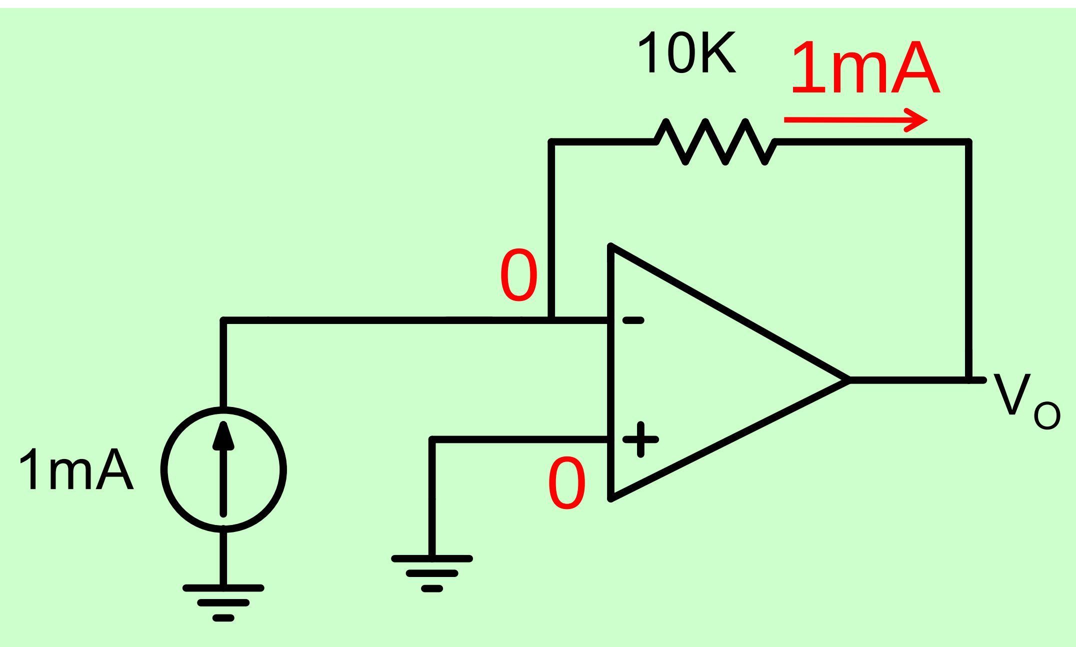
# Examples




$$\frac{v_o}{v_S} = -\frac{R_2}{R_1} \Rightarrow v_o = -100mV$$



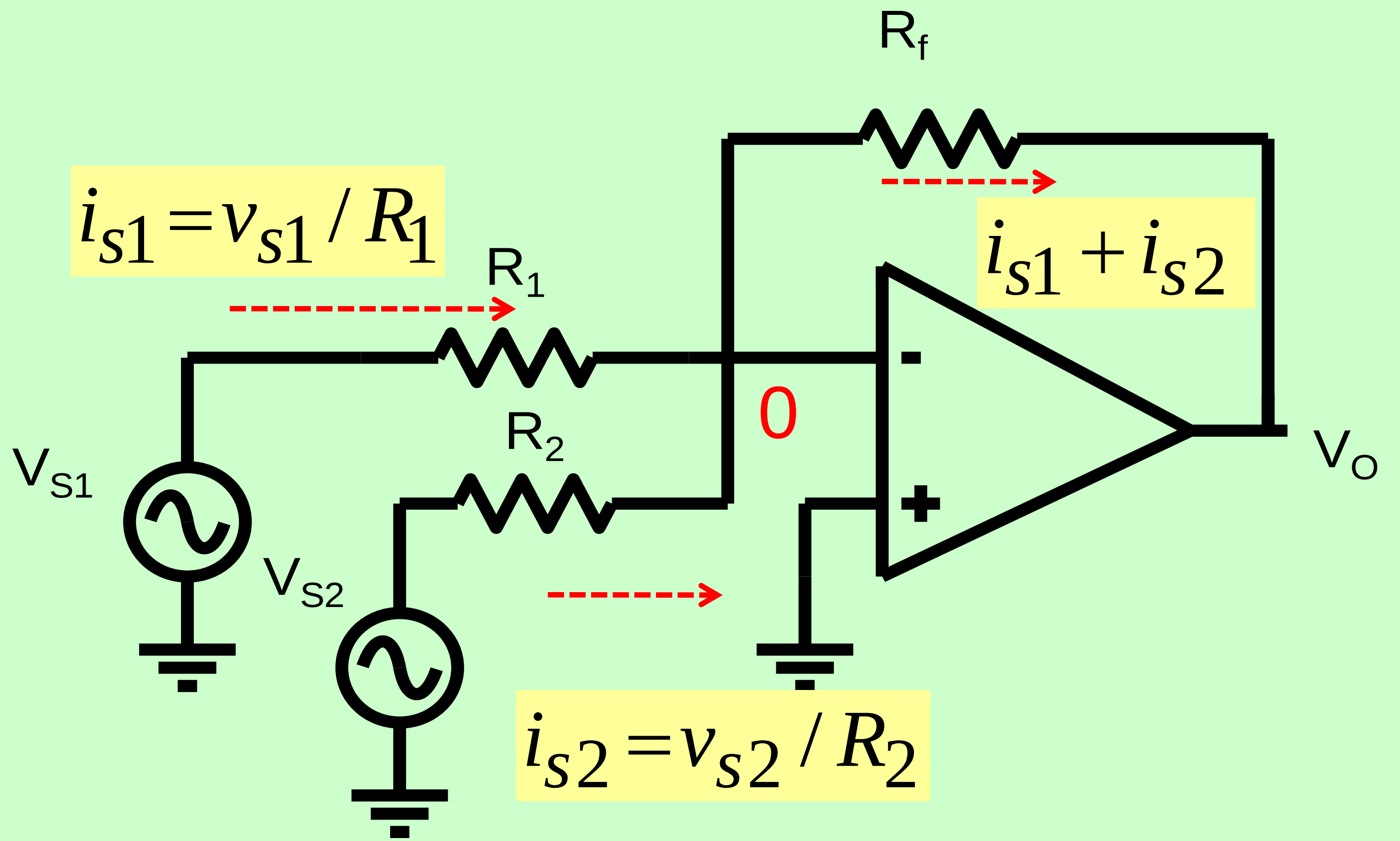

$$\frac{v_o}{v_S} = 1 + \frac{R_2}{R_1} \Rightarrow v_o = 110mV$$




$$\frac{0 - v_o}{10K} = 1mA$$

$$v_o = -10V$$

# Adder

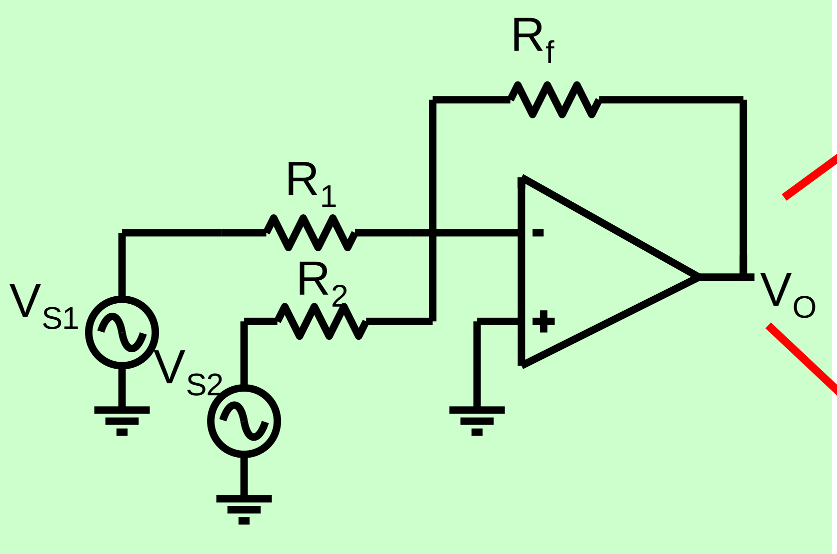


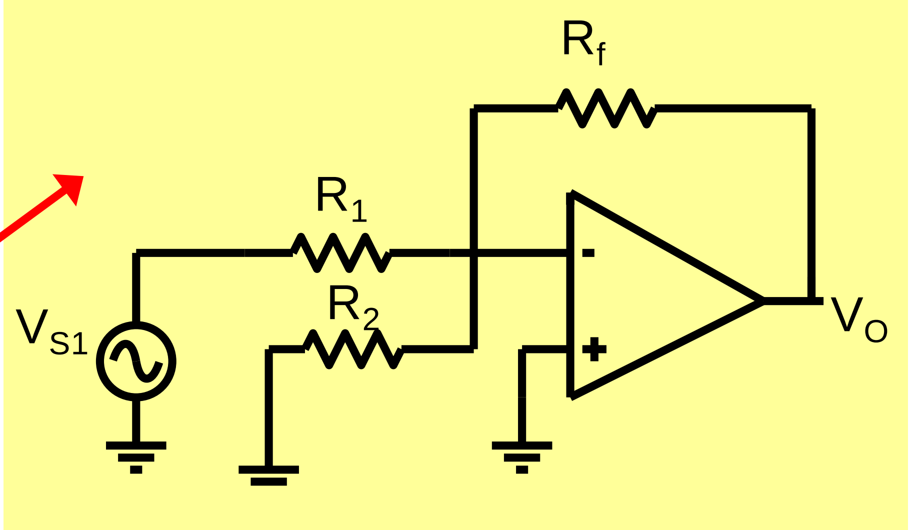

$$\frac{0 - v_o}{R_f} = i_{s1} + i_{s2} = \frac{v_{s1}}{R_1} + \frac{v_{s2}}{R_2}$$

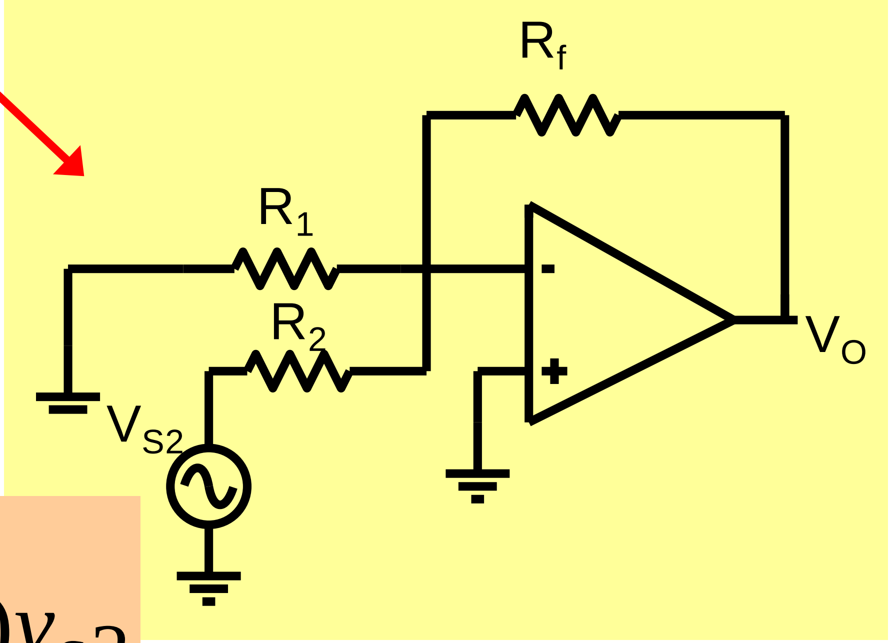
$$v_o = -\left(\frac{R_f}{R_1} v_{s1} + \frac{R_f}{R_2} v_{s2}\right)$$

$$\text{For } \mathrm{R_1 = R_2 = R} \quad v_o = -\frac{R_f}{R}(v_{s1} + v_{s2})$$

# Alternative Analysis



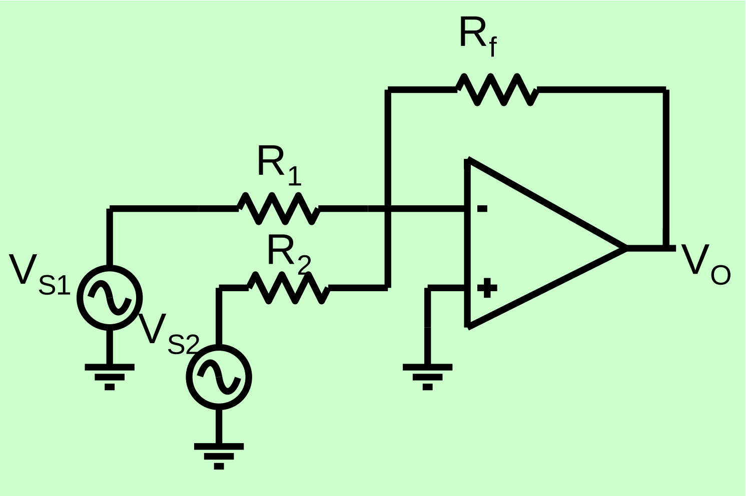




$$v_o = -(\frac{R_f}{R_1})v_{s1} + \quad -(\frac{R_f}{R_2})v_{s2}$$

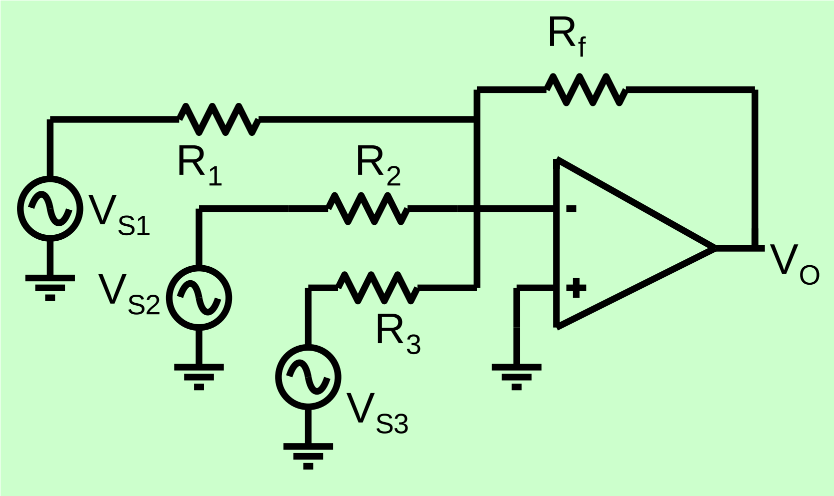
# Design Example

Design a circuit that would generate the following output given three input voltages vs1, vs2 and vs3.

$$v_o = -10v_{s1} - 4v_{s2} - 5v_{s3}$$




$$v_o = -\frac{R_f}{R_1}v_{s1} - \frac{R_f}{R_2}v_{s2}$$



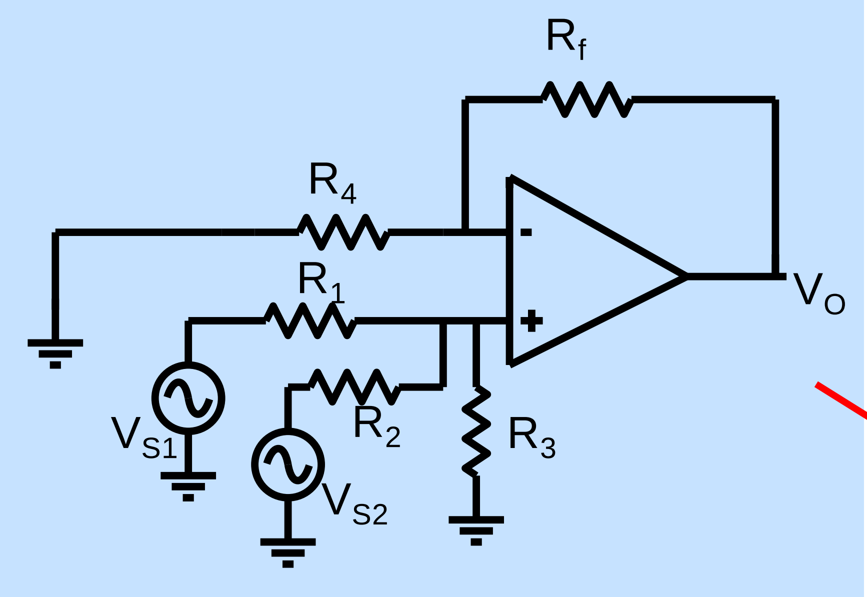

$$v_o = -\frac{R_f}{R_1}v_{s1} - \frac{R_f}{R_2}v_{s2} - \frac{R_f}{R_3}v_{s3}$$

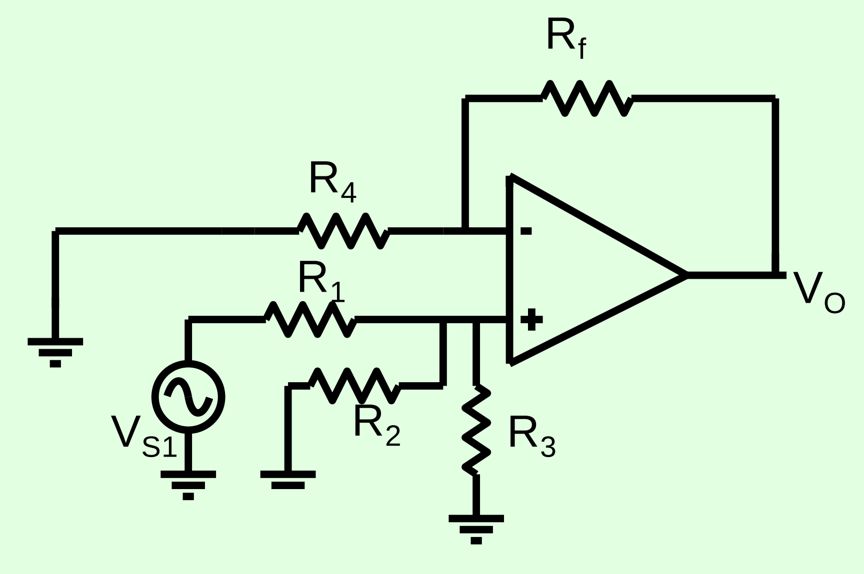
**Choose :**

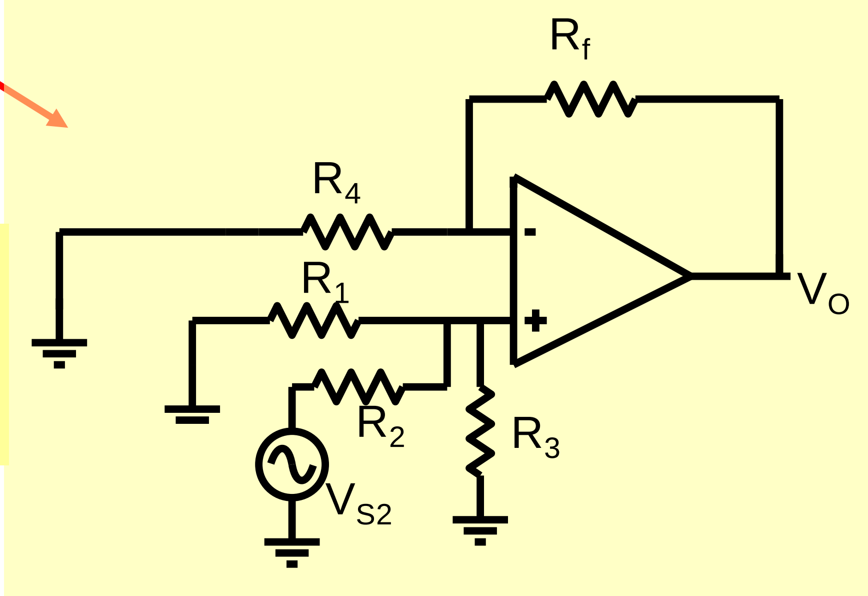
$$R_f = 10K \qquad \Rightarrow R_1 = 1K$$

$$\Rightarrow R_2 = 2.5K \qquad \Rightarrow R_3 = 2K$$

# Adder



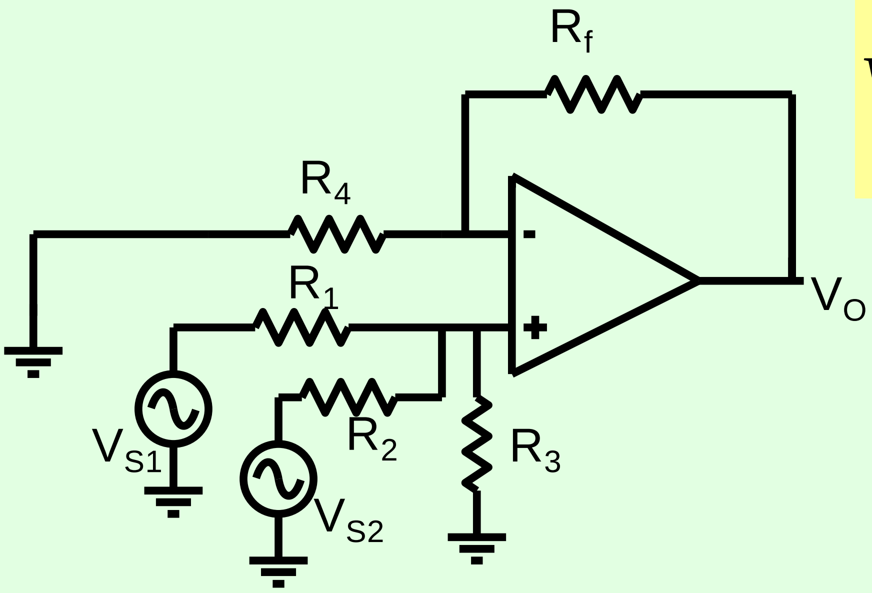




$$v_o = v_{s1}\frac{R_2 \| R_3}{R_2 \| R_3 + R_1} \times \left(1 + \frac{R_f}{R_4}\right)$$

$$+ v_{s2}\frac{R_1 \| R_3}{R_1 \| R_3 + R_2} \times \left(1 + \frac{R_f}{R_4}\right)$$

# Adder




$$v_o = v_{s1}\frac{R_2 \| R_3}{R_2 \| R_3 + R_1} \times (1 + \frac{R_f}{R_4})$$

$$+ v_{s2}\frac{R_1 \| R_3}{R_1 \| R_3 + R_2} \times (1 + \frac{R_f}{R_4})$$

**Complicated expression !!!!!**

$$R_P = R_1 \| R_2 \| R_3$$

$$v_o = (\frac{R_p}{R_1} v_{s1} + \frac{R_p}{R_2} v_{s2}) \times (1 + \frac{R_f}{R_4})$$

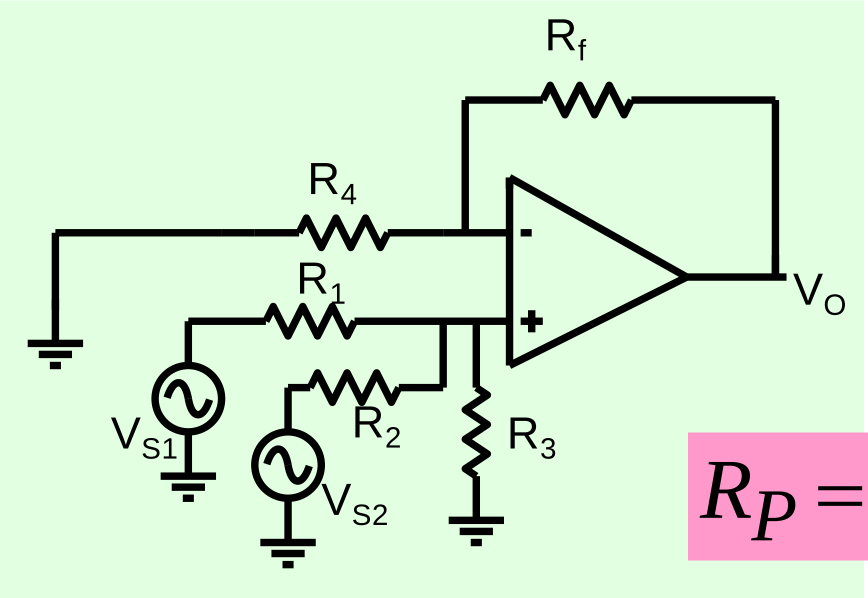
$$R_p = \frac{R_1 \left( R_2 \| R_3 \right)}{R_2 \| R_3 + R_1} = R_1 \| R_2 \| R_3$$

**Simple expression !**

# Design Example

Design a circuit that would generate the following output given two input voltages vs1and vs2.

$$v_o = 10v_{s1} + 4v_{s2}$$




$$v_o = \left(\frac{R_p}{R_1}v_{s1} + \frac{R_p}{R_2}v_{s2}\right)\times\left(1+\frac{R_f}{R_4}\right)$$

$$R_P = R_1 \| R_2 \| R_3$$

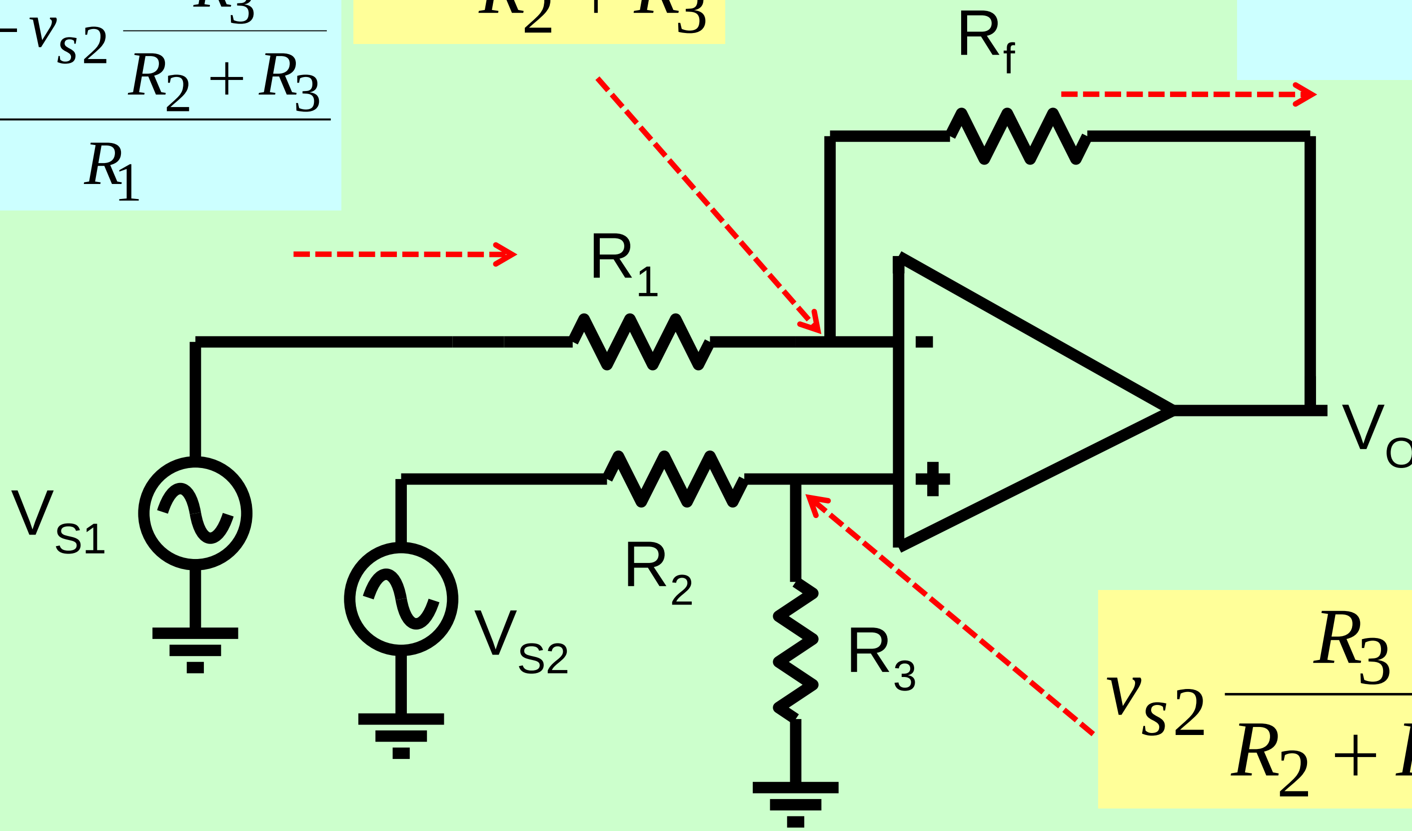
DIY!

# Subtractor

$$v_{s2}\frac{R_3}{R_2+R_3}$$

$$\frac{v_{s1}-v_{s2}\dfrac{R_3}{R_2+R_3}}{R_1}$$

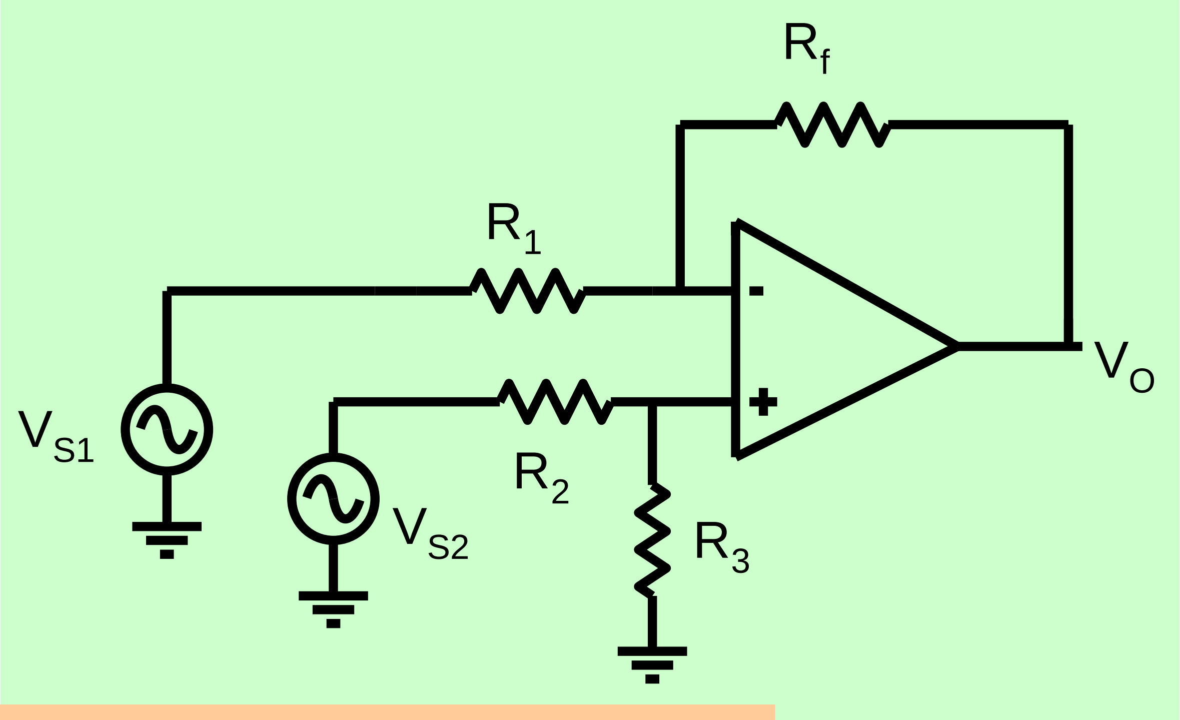
$$\frac{v_{s1}-v_{s2}\dfrac{R_3}{R_2+R_3}}{R_1}$$




$$v_{s2}\frac{R_3}{R_2+R_3}$$

$$\frac{v_{s2}\dfrac{R_3}{R_2+R_3}-v_o}{R_f}=\frac{v_{s1}-v_{s2}\dfrac{R_3}{R_2+R_3}}{R_1}$$

$$v_o=v_{s2}\frac{\dfrac{R_3}{R_2}}{(1+\dfrac{R_3}{R_2})}(1+\frac{R_f}{R_1})-(\frac{R_f}{R_1})v_{s1}$$

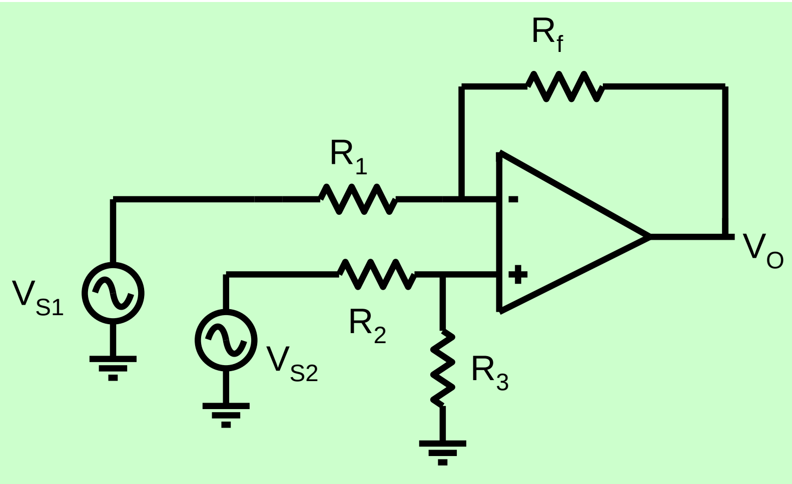


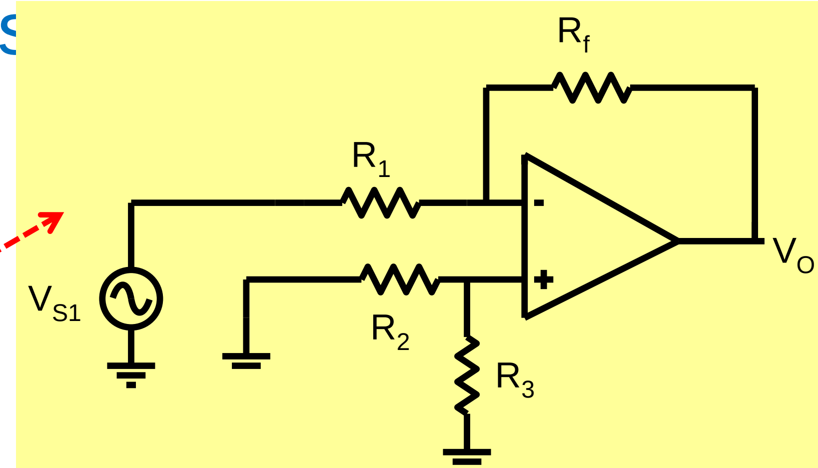

$$v_o = v_{s2}\frac{\frac{R_3}{R_2}}{(1+\frac{R_3}{R_2})}(1+\frac{R_f}{R_1}) - (\frac{R_f}{R_1})v_{s1}$$

Choose $\frac{R_3}{R_2} = \frac{R_f}{R_1}$

$$v_o = \frac{R_f}{R_1}(v_{s2} - v_{s1})$$

# **Subtractor:** Alternative Analysis

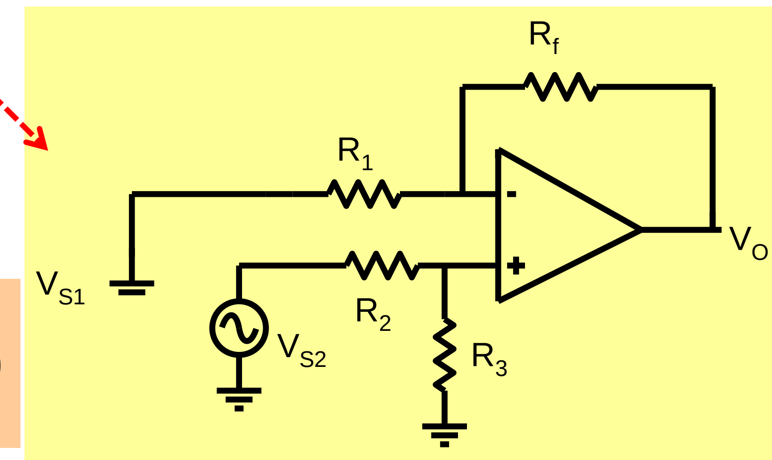



Inverting amplifier

Use superposition theorem

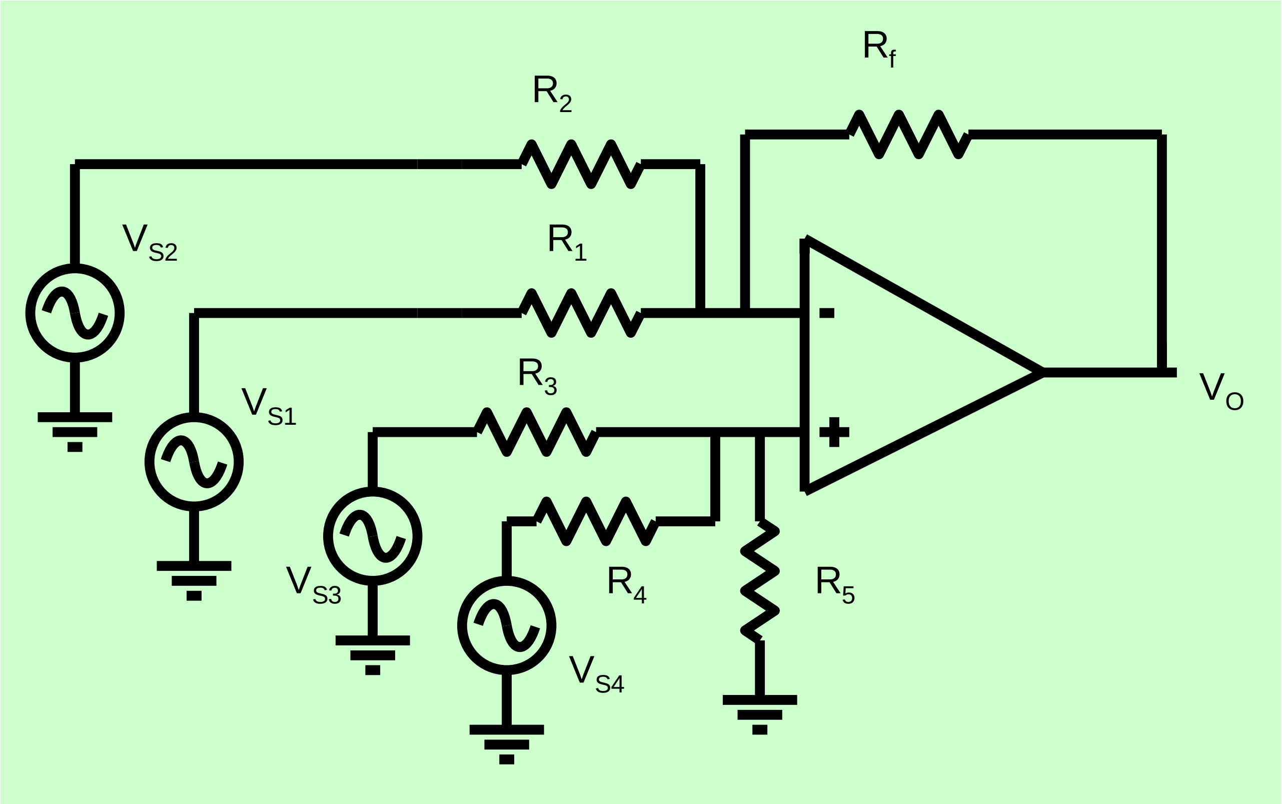
$$v_o = -\left(\frac{R_f}{R_1}\right)v_{s1} \quad + \quad v_{s2}\frac{R_3}{(R_3+R_2)} \times \left(1+\frac{R_f}{R_1}\right)$$




Non-inverting amplifier

Analysis is made simpler by **Re-Using** results derived earlier

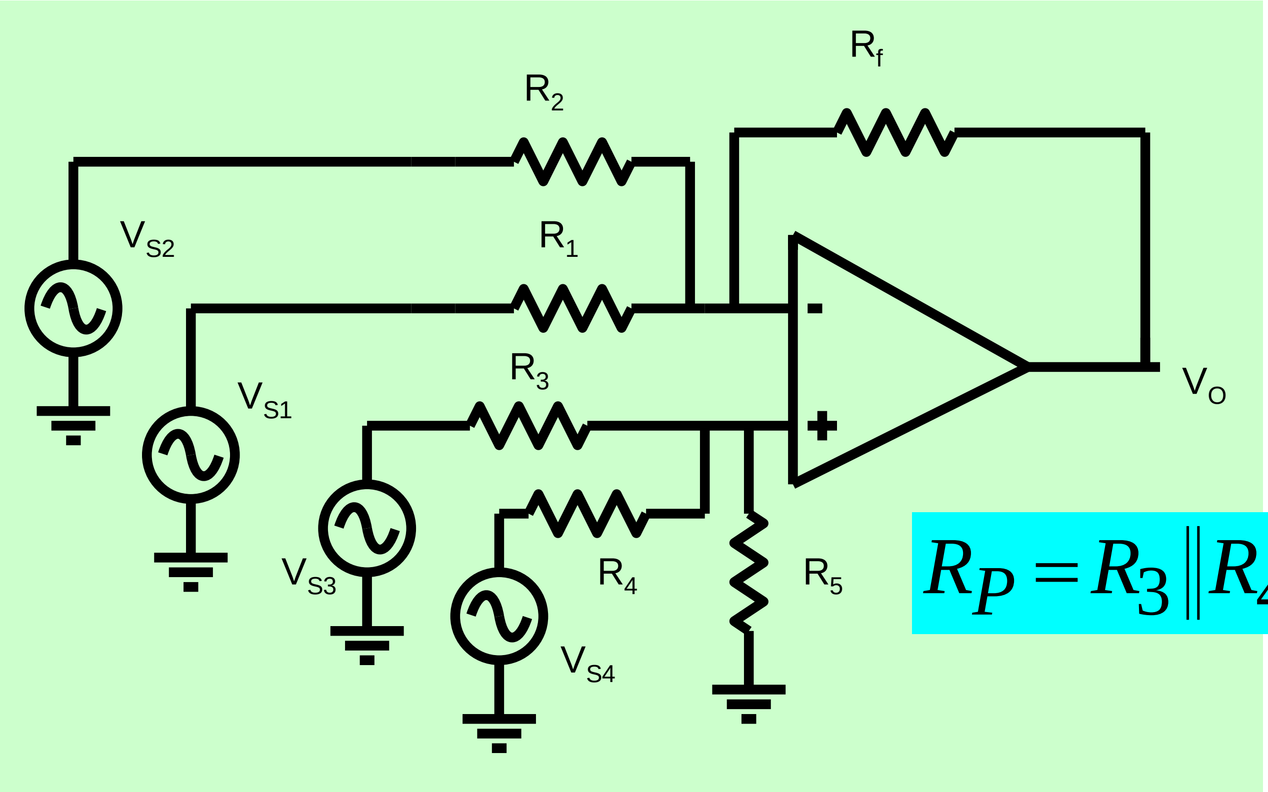
# Adder/Subtractor




$$v_o = -(\frac{R_f}{R_1})v_{s1} + -(\frac{R_f}{R_2})v_{s2} + v_{s3}\frac{R_5 \| R_4}{R_5 \| R_4 + R_3} \times (1 + \frac{R_f}{R_1 \| R_2})$$

$$+ v_{s4}\frac{R_5 \| R_3}{R_5 \| R_3 + R_4} \times (1 + \frac{R_f}{R_1 \| R_2})$$

# Adder/Subtractor

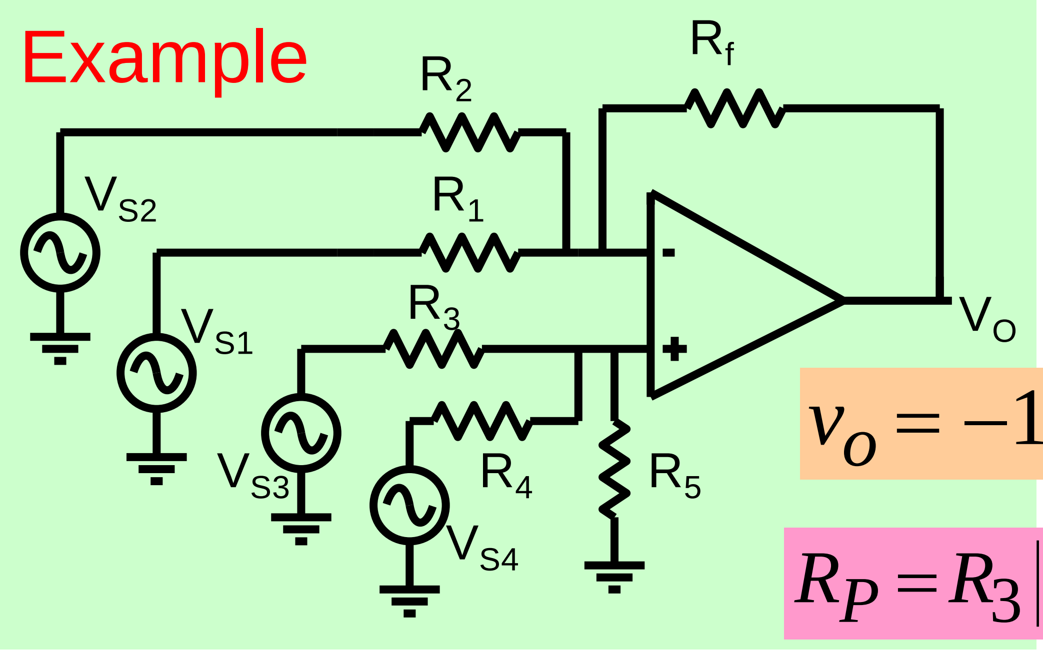



$$R_P = R_3 \| R_4 \| R_5$$

$$v_o = -\left(\frac{R_f}{R_1}\right)v_{s1} + -\left(\frac{R_f}{R_2}\right)v_{s2} + v_{s3}\frac{R_P}{R_3}\times\left(1+\frac{R_f}{R_1 \| R_2}\right)$$

$$+ \; v_{s4}\frac{R_P}{R_4}\times\left(1+\frac{R_f}{R_1 \| R_2}\right)$$

# Example




$$v_o = -10v_{s1} - 4v_{s2} + 5v_{s3} + 2v_{s4}$$

$$R_P = R_3 \| R_4 \| R_5$$

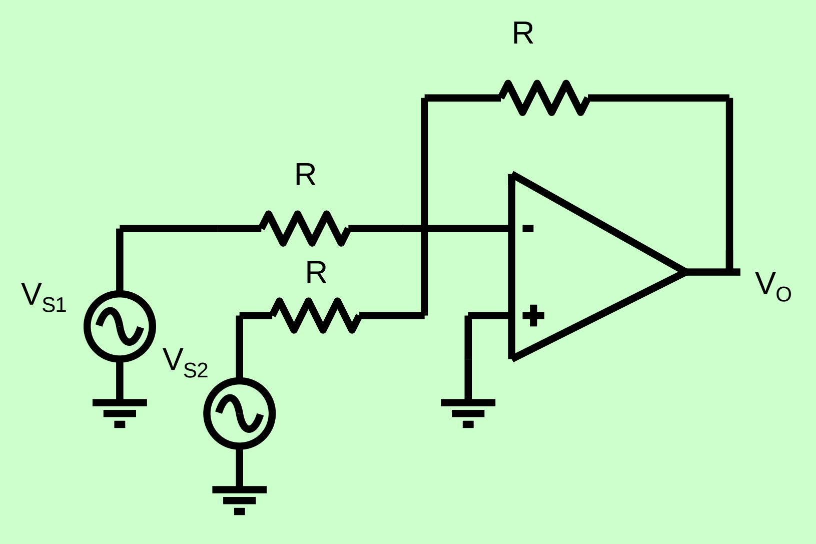
$$v_o = -\left(\frac{R_f}{R_1}\right)v_{s1} - \left(\frac{R_f}{R_2}\right)v_{s2} + \left(1 + \frac{R_f}{R_1 \| R_2}\right) \times \frac{R_P}{R_3} v_{s3} + \left(1 + \frac{R_f}{R_1 \| R_2}\right) \times \frac{R_P}{R_4} v_{s4}$$

Choose : $R_f = 10K$ $\Rightarrow R_1 = 1K$ $\Rightarrow R_2 = 2.5K$

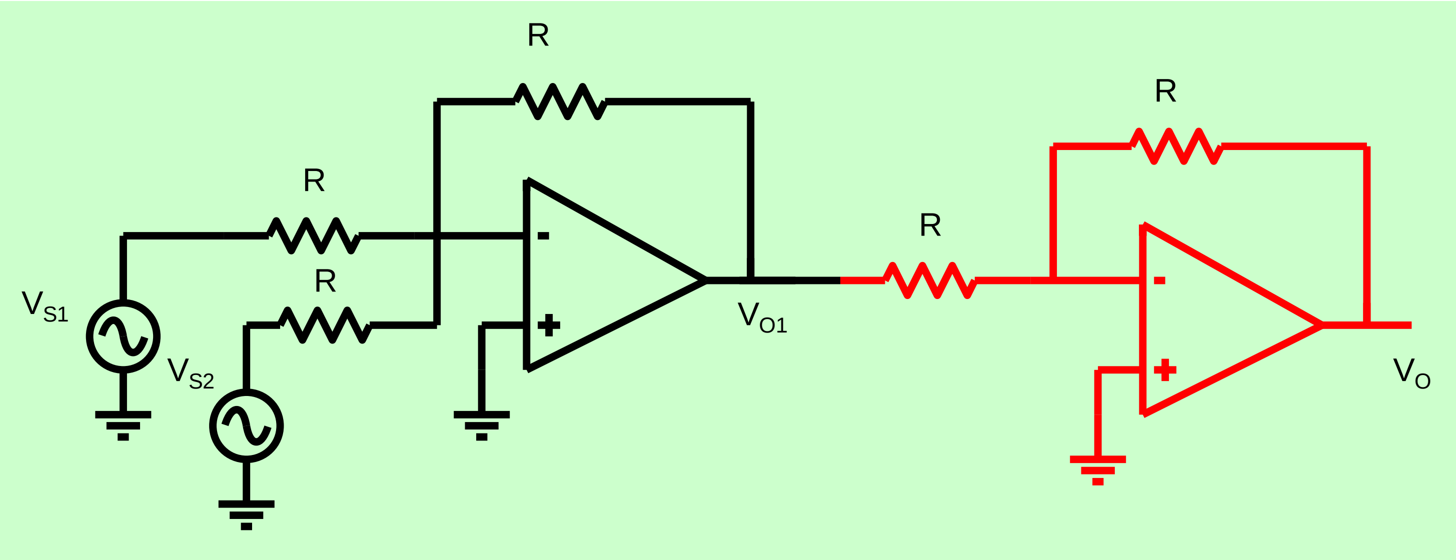
$\Rightarrow \frac{R_P}{R_3} = 0.33$ $\Rightarrow \frac{R_P}{R_4} = 0.133$ $\Rightarrow \frac{R_4}{R_3} = 2.5$

Choose : $R_3 = 1K$ $\Rightarrow R_4 = 2.5K$ $\Rightarrow R_P = 0.33K$ $\Rightarrow R_5 = 0.625K$

# Discussion on loading effect




$$v_o = -(v_{s1} + v_{s2})$$



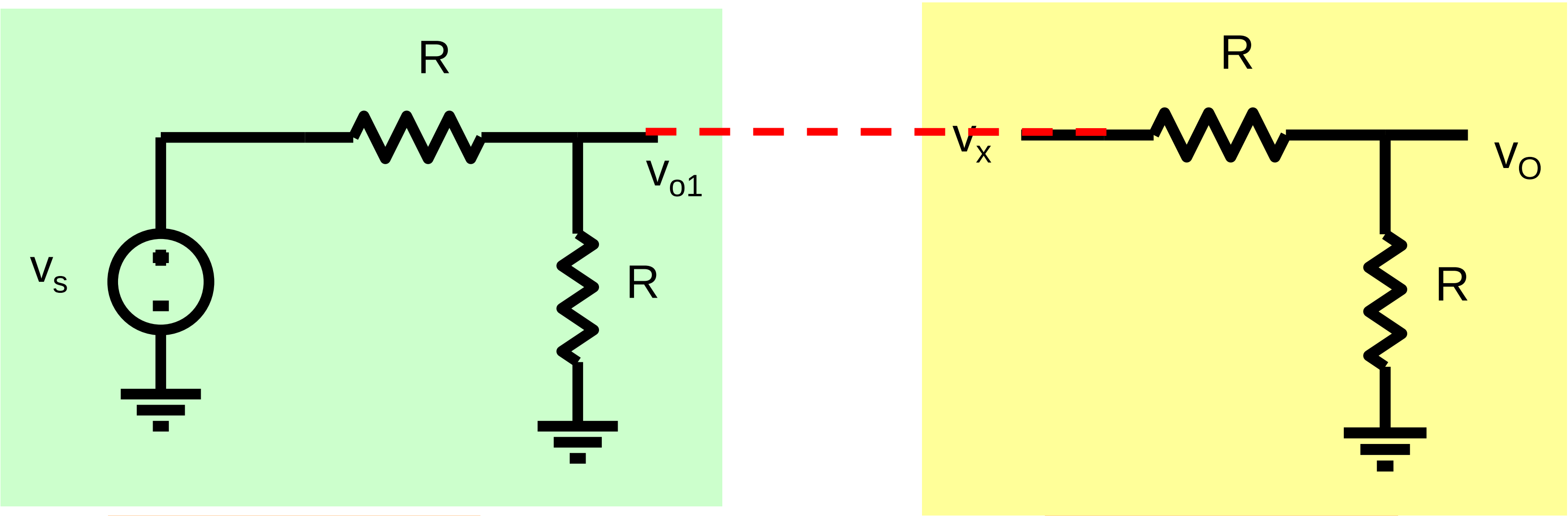

$$v_{o1} = -(v_{s1} + v_{s2})$$

$$v_o = -v_{o1}$$

$$v_o = (v_{s1} + v_{s2})$$

Have we made some assumption here ?

# Example




$$\frac{v_{o1}}{v_s} = 0.5$$

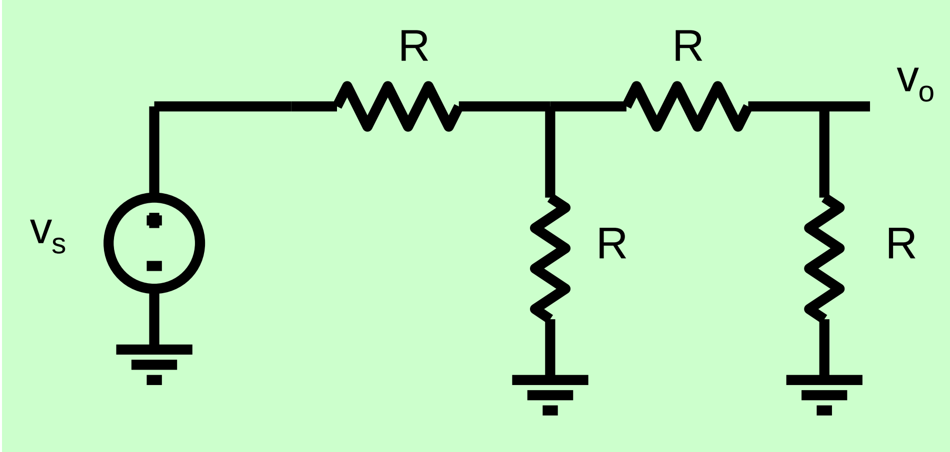
$$\frac{v_o}{v_x} = 0.5$$

$$v_{o1} = v_x$$

$$\frac{v_o}{v_x} = \frac{v_o}{v_{o1}} = 0.5$$

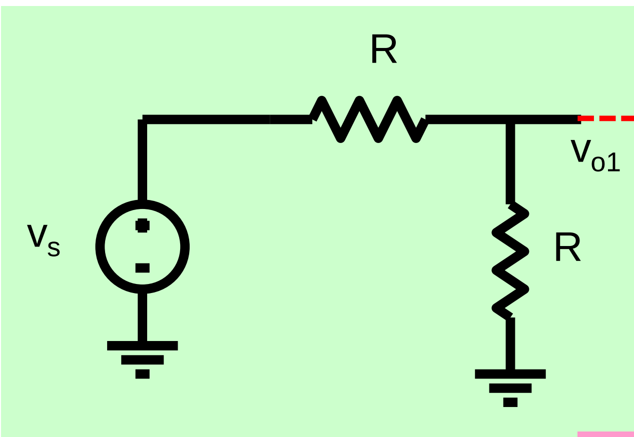
$$\frac{v_o}{v_s} = \frac{v_o}{v_{o1}} \times \frac{v_{o1}}{v_s} = 0.5 \times 0.5 = 0.25$$

BUT

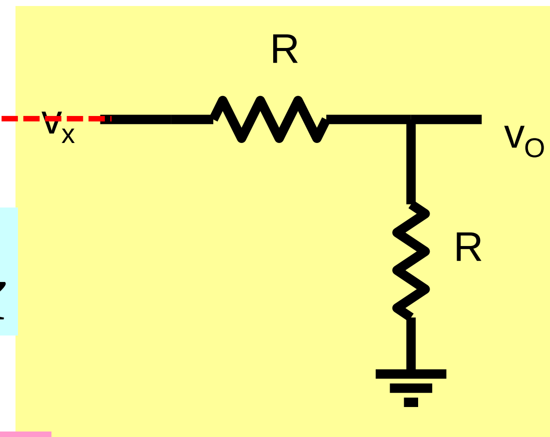



$$\frac{v_o}{v_s} = 0.2$$

Where is the error ?




$$v_{o1} = v_x$$



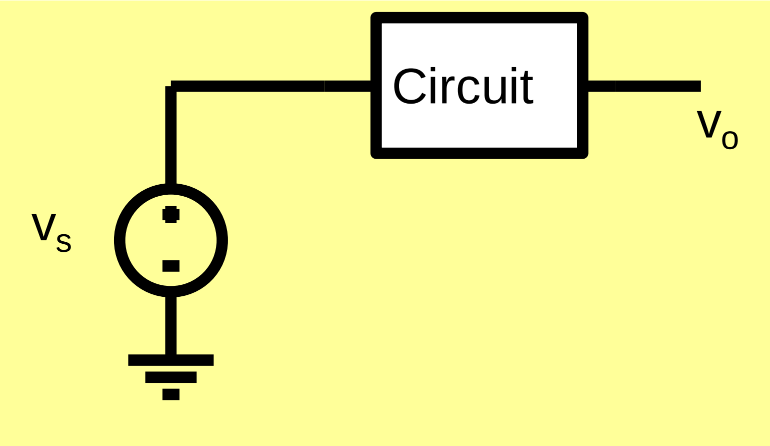

$$\frac{v_{o1}}{v_s} = 0.5$$

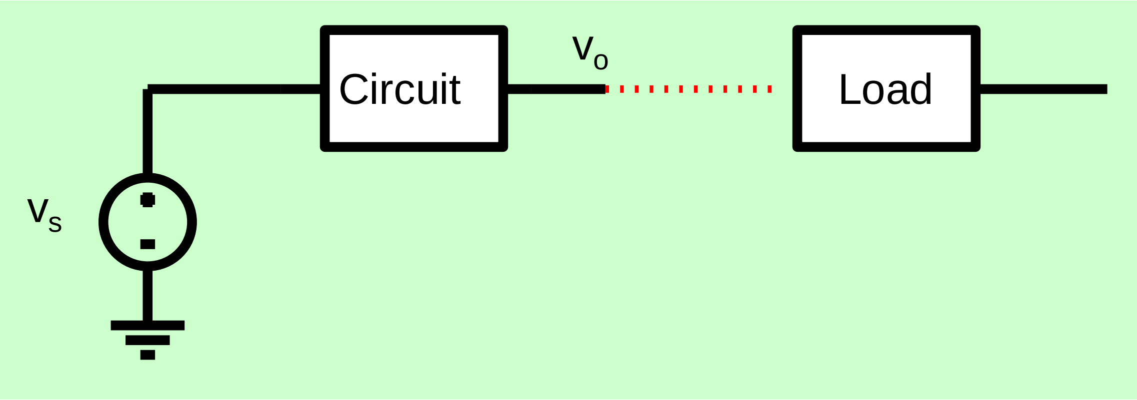
$$\frac{v_{o1}}{v_s} \neq 0.5$$

$$\frac{v_o}{v_x} = 0.5$$

Circuit-1 gets ‘loaded’ by circuit-2 and its output vs. input characteristics get modified.

# Loading Effect

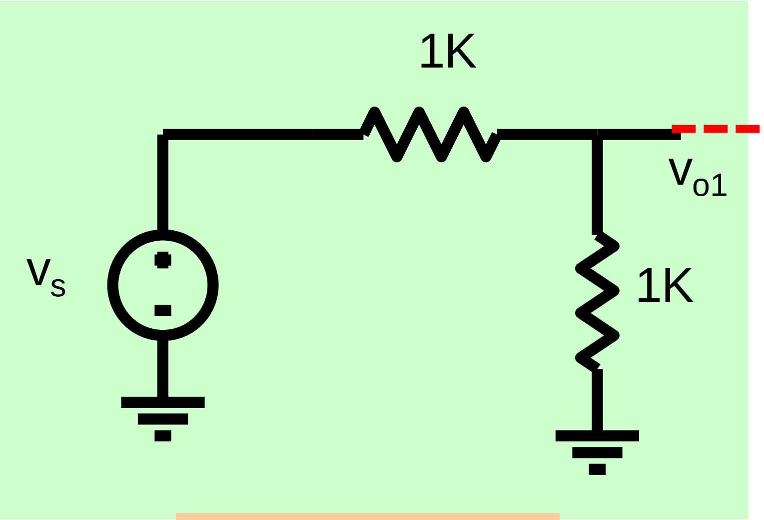


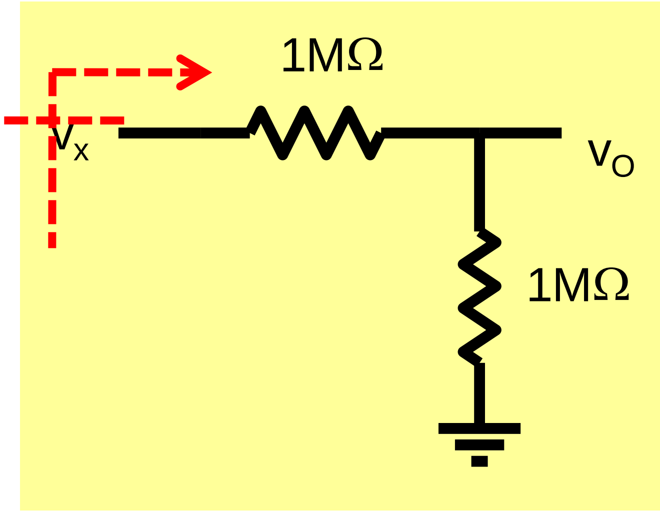




$V_o$ in general gets altered when we connect a load to it

Under what conditions is change in $V_O$ small upon connection of a load ?

# Example

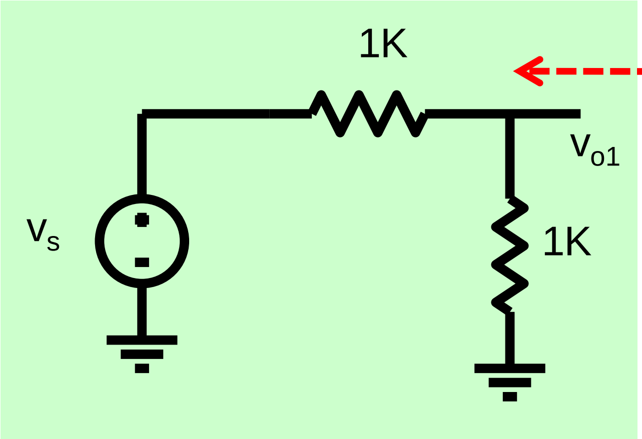



$$\frac{v_{o1}}{v_s} = 0.5$$

After connection of load: $\frac{v_{o1}}{v_s} \cong 0.5$

We can describe this effect in terms of output resistance

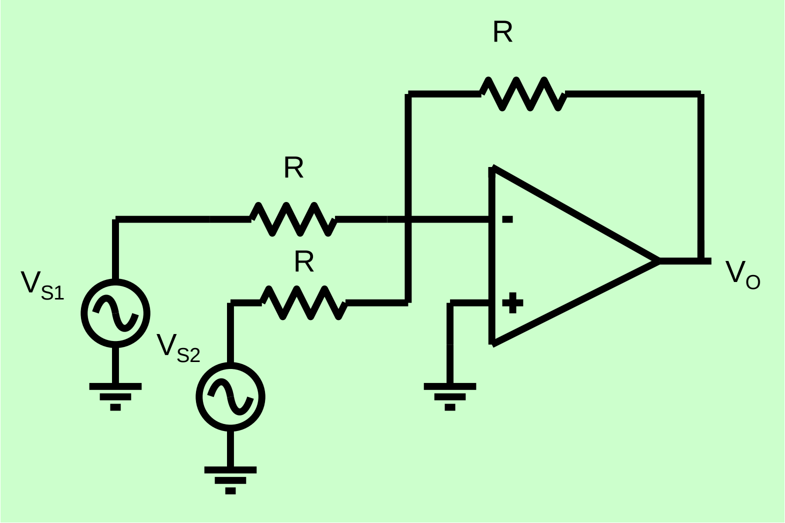



$$R_o = 0.5K \qquad R_L = 2M\Omega$$

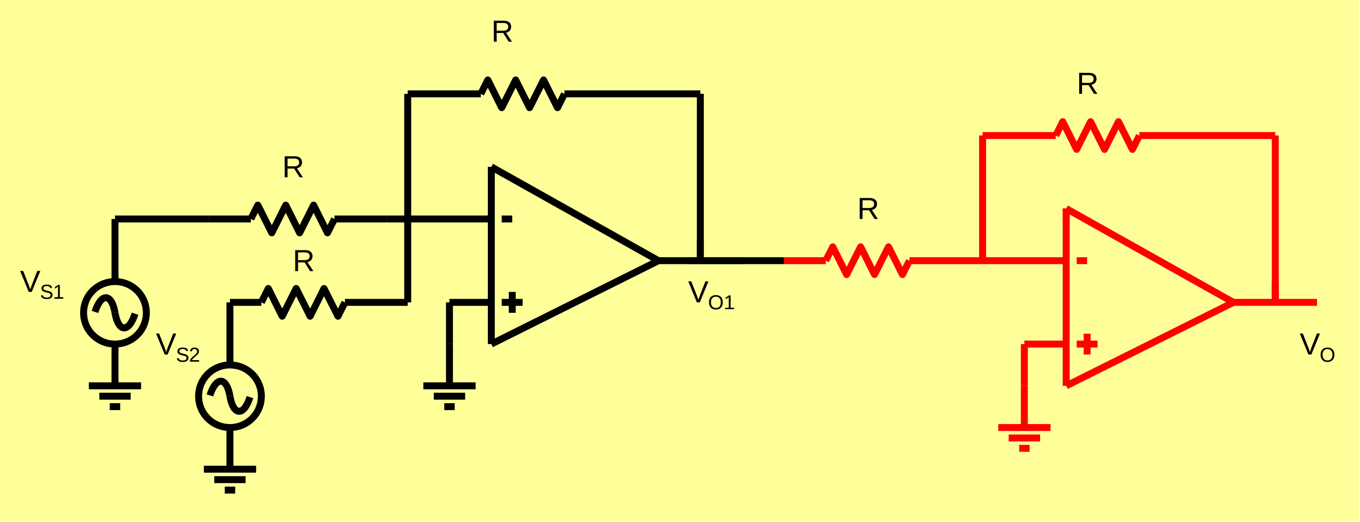
# Loading Effect

Whenever output resistance of a circuit is much smaller than the load resistance, the loading effect is minimal.

$$R_o << R_L$$




$$v_o = -(v_{s1} + v_{s2})$$



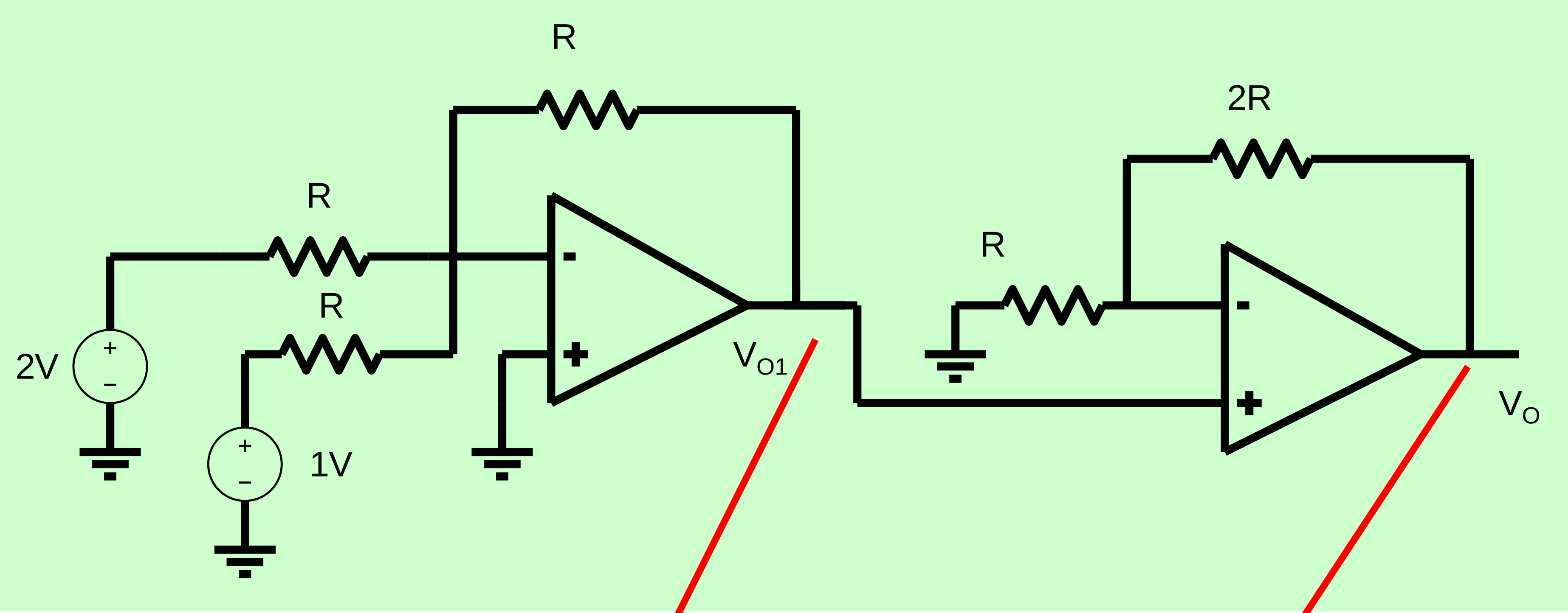

$$v_{o1} = -(v_{s1} + v_{s2})$$

$$v_o = -v_{o1}$$

$$v_o = (v_{s1} + v_{s2})$$

The assumption made here is that there is no loading which is reasonable because opamps have very low o/p resistance

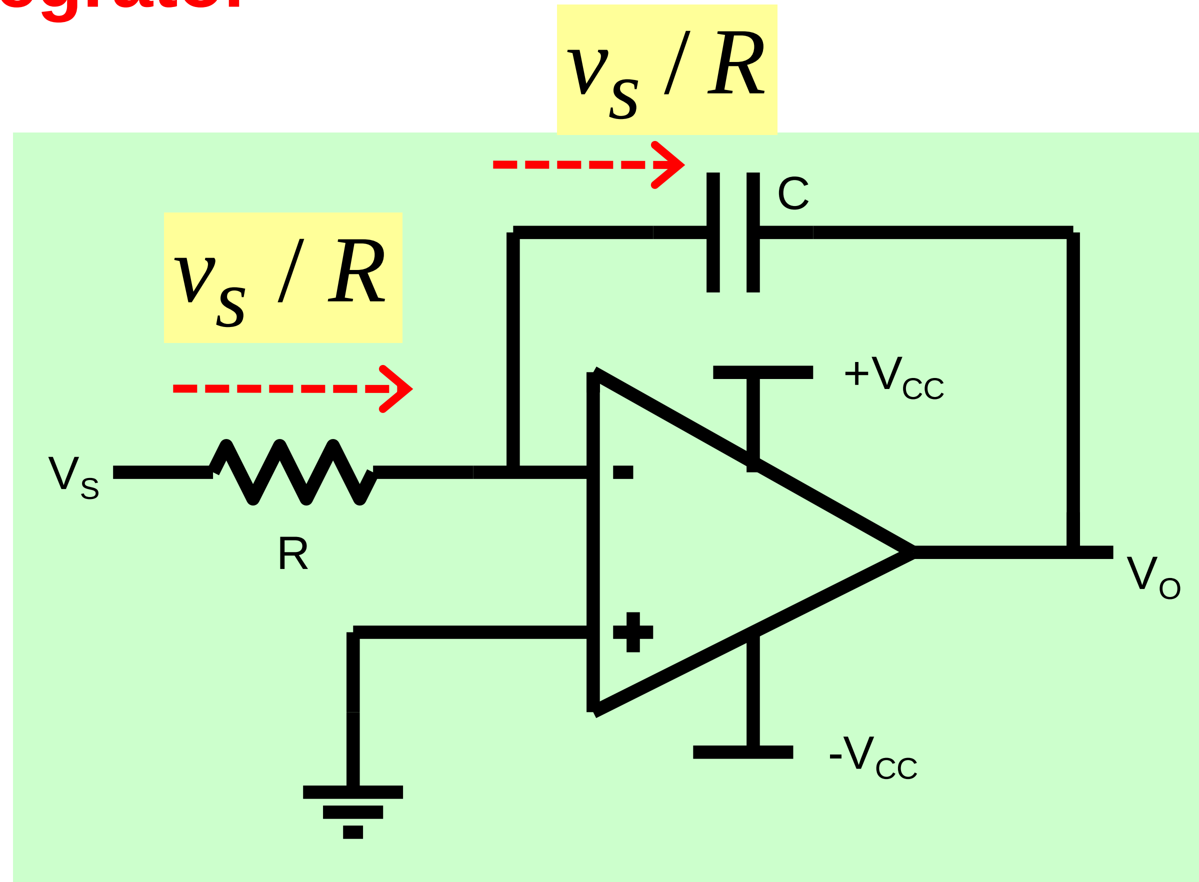
# Example




$$v_{o1} = -\{\frac{R}{R}\times 1 + \frac{R}{R}\times 2\} = -3V$$

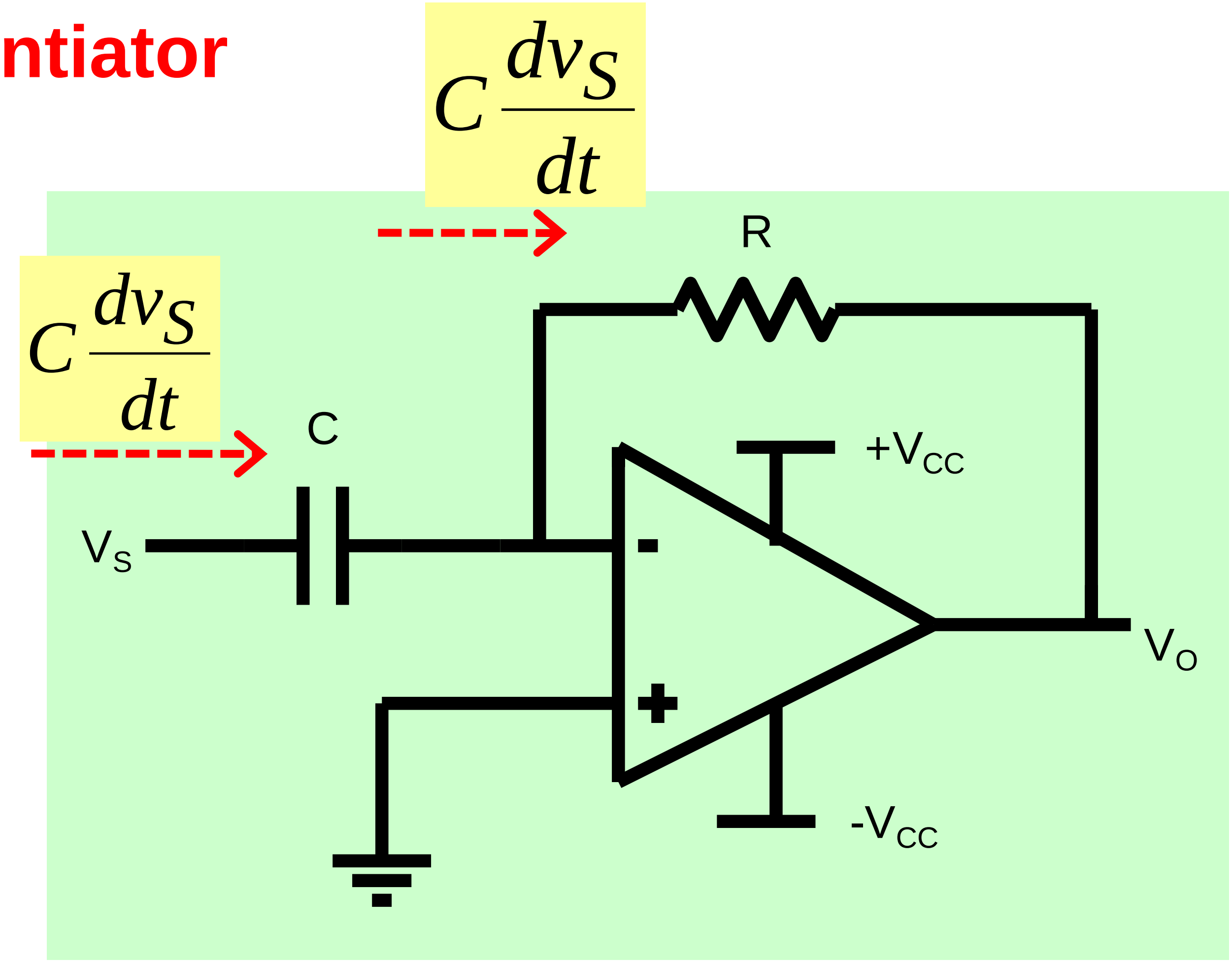
$$\frac{v_o}{v_{o1}} = 1 + \frac{2R}{R} \Rightarrow v_o = -9V$$

# Integrator



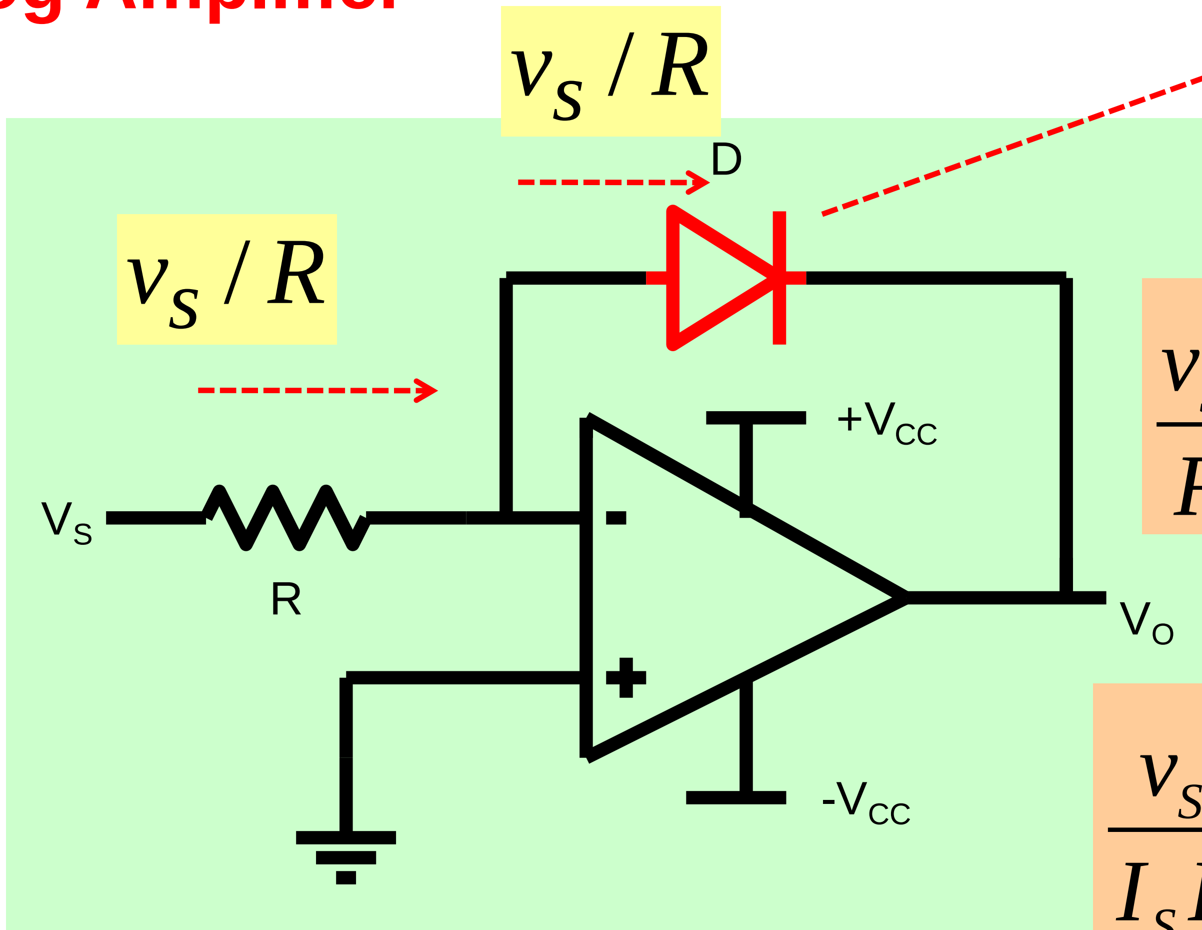

$$\frac{v_S}{R} = -C\frac{dV_O}{dt} \Rightarrow V_O(t) = -\frac{1}{RC}\int v_S\,dt$$

# Differentiator




$$-\frac{V_O}{R} = C\frac{dv_S}{dt} \Rightarrow V_O(t) = -RC\frac{dv_S}{dt}$$

# Log Amplifier



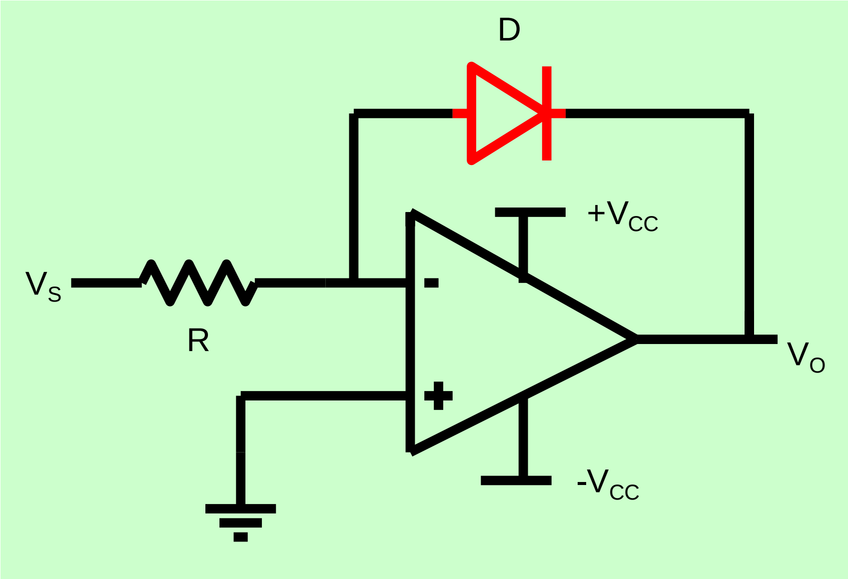

$$I = I_S(e^{\frac{V}{V_T}} - 1)$$

$$\frac{v_S}{R} = I_s(e^{-\frac{V_O}{V_T}} - 1)$$

$$\frac{v_S}{I_S R} + 1 = e^{-\frac{V_O}{V_T}}$$

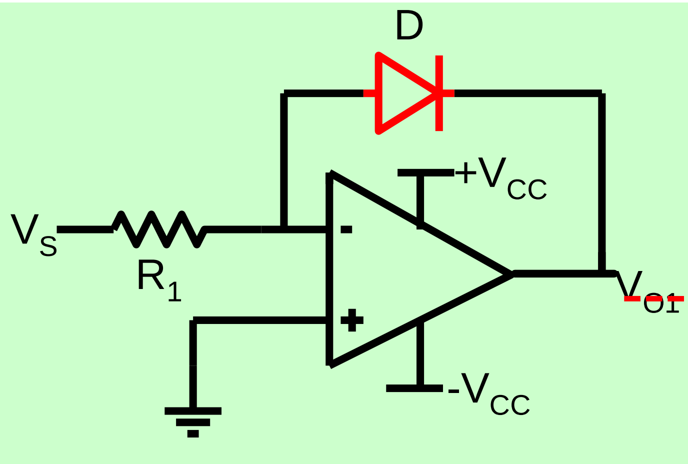
$$\Rightarrow \; -V_O = V_T \times \ln(1 + \frac{v_S}{RI_S}) \cong V_T \times \ln(\frac{v_S}{RI_S})$$

# Temperature Sensor ?

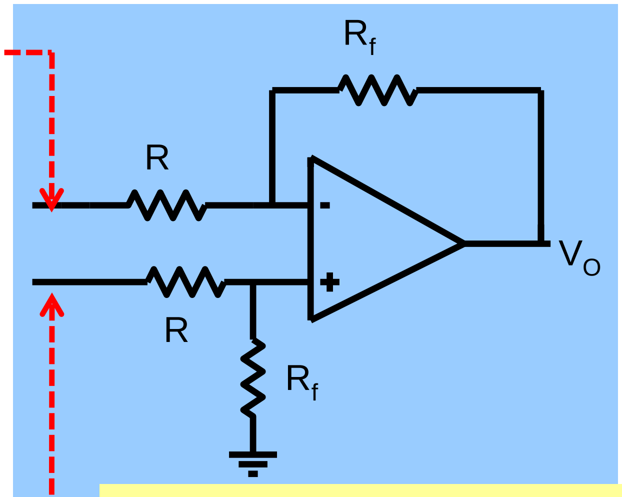



$$V_O = -V_T \times \ln(\frac{V_S}{RI_S}); V_T = \frac{k_B T}{q}$$

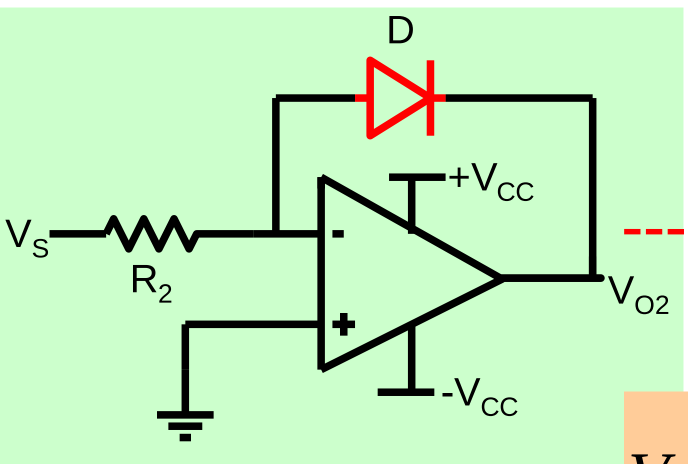
But $I_S$ is a function of temperature as well.




$$V_{O1} = -V_T \times \ln\left(\frac{V_S}{R_1 I_S}\right)$$



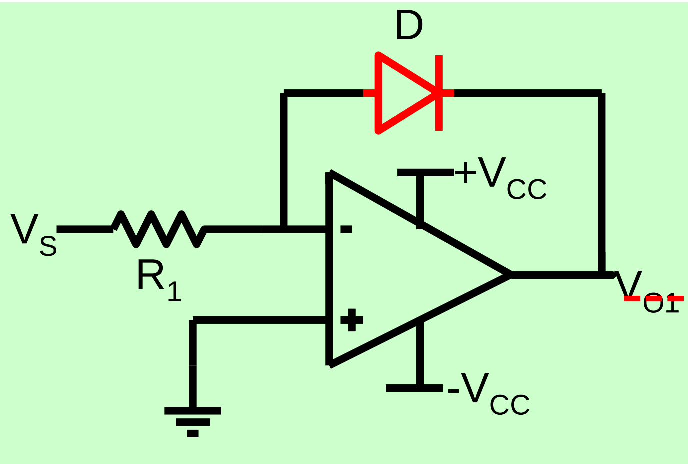

$$V_O = -\frac{R_f}{R} V_{01} + \left(1 + \frac{R_f}{R}\right) \times \frac{R_f}{R_f + R} V_{02}$$



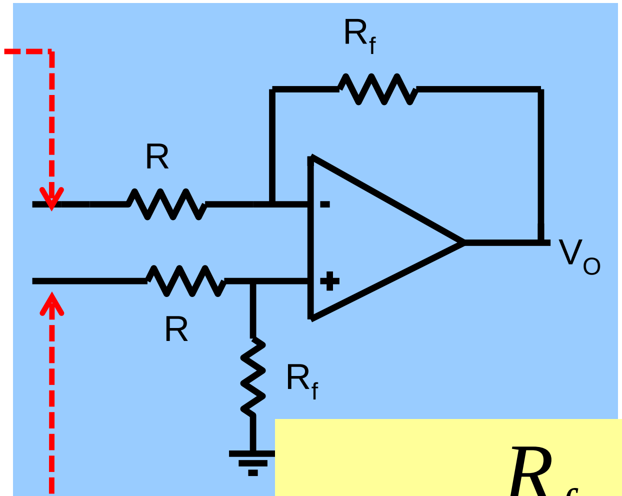

$$V_{O2} = -V_T \times \ln\left(\frac{V_S}{R_2 I_S}\right)$$

$$V_O = \frac{R_f}{R}\left(V_{02} - V_{01}\right)$$

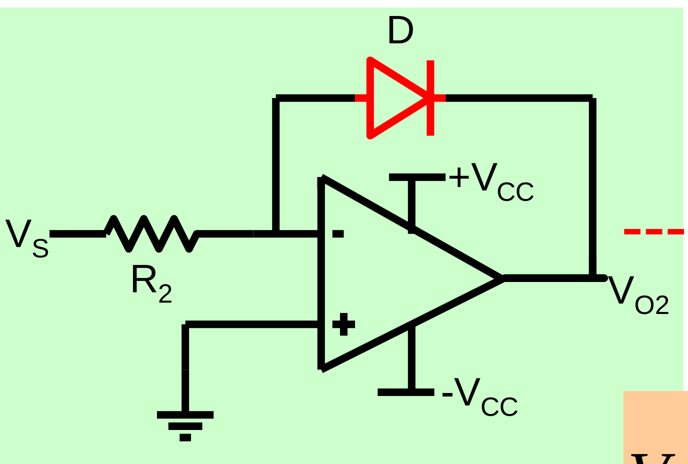
$$V_O = \frac{R_f}{R} V_T \left( -\ln\left(\frac{V_S}{R_2 I_S}\right) + \ln\left(\frac{V_S}{R_1 I_S}\right) \right)$$




$$V_{O1} = -V_T \times \ln\left(\frac{V_S}{R_1 I_S}\right)$$




$$V_O = \frac{R_f}{R} V_T \times \ln\left(\frac{R_2}{R_1}\right)$$

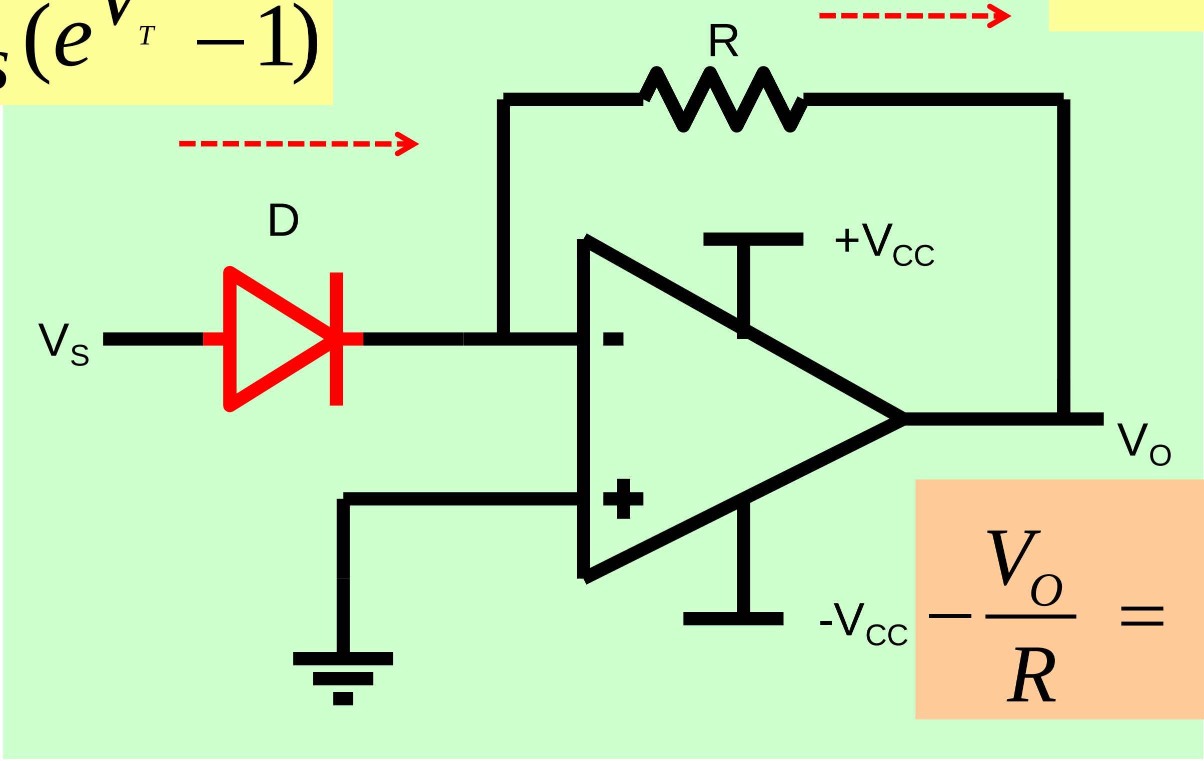



$$V_{O2} = -V_T \times \ln\left(\frac{V_S}{R_2 I_S}\right)$$

Output voltage is directly proportional to temperature

# AntiLog Amplifier

$$I = I_S(e^{\frac{V_s}{V_T}} - 1)$$



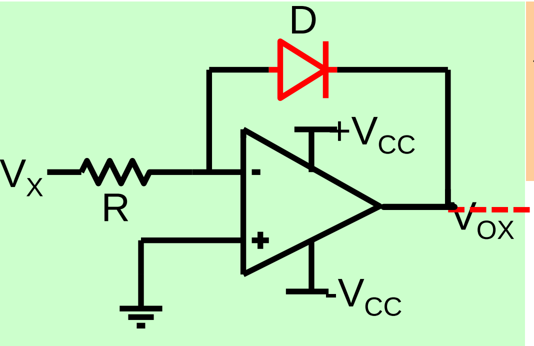

$$I = I_S(e^{\frac{V_s}{V_T}} - 1)$$

$$-\frac{V_O}{R} = I_S(e^{\frac{V_S}{V_T}} - 1)$$

$$\Rightarrow \; V_O = -RI_S(e^{\frac{V_S}{V_T}} - 1) \cong -RI_S \times e^{\frac{V_S}{V_T}}$$

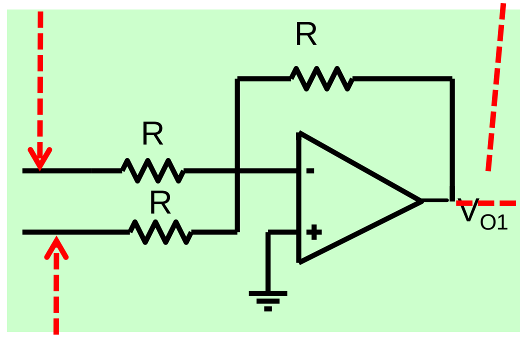
# Multiplier

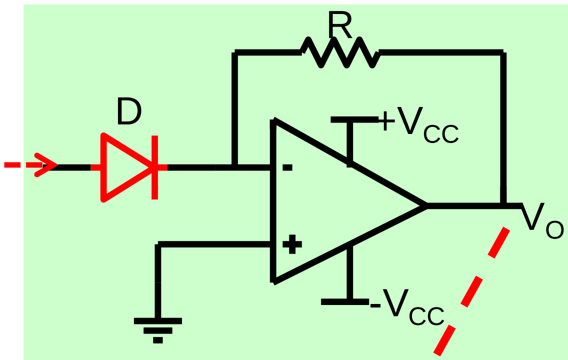
$$V_{O1} = V_T \times (\ln(\frac{V_X}{RI_S}) + \ln(\frac{V_Y}{RI_S}))$$

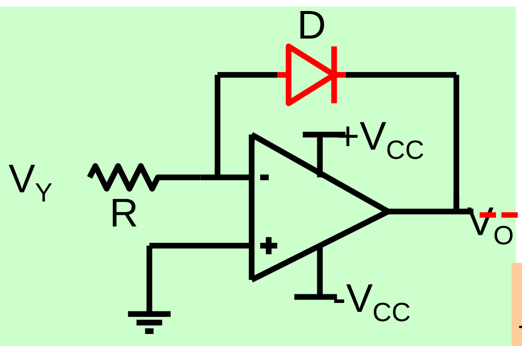



$$V_{OX} = -V_T \times \ln(\frac{V_X}{RI_S})$$

$$V_{O1} = V_T \times \ln(\frac{V_X V_Y}{R^2 I_S^2})$$



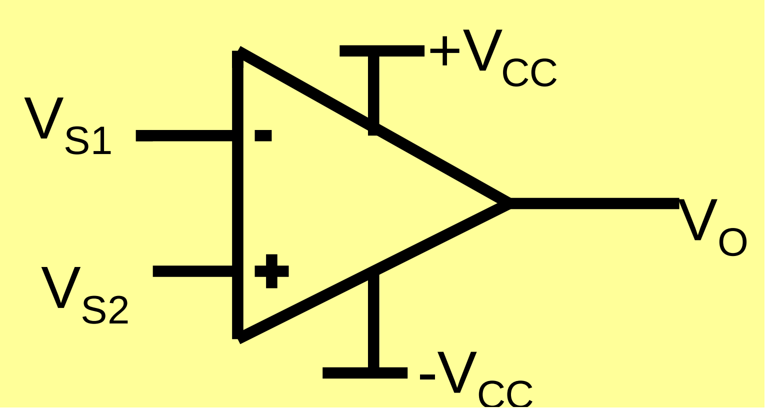



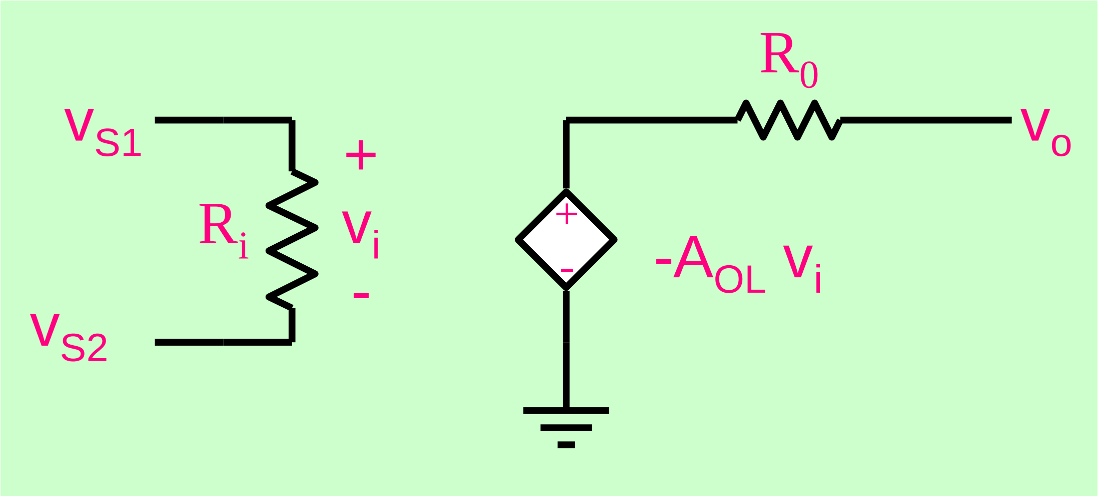



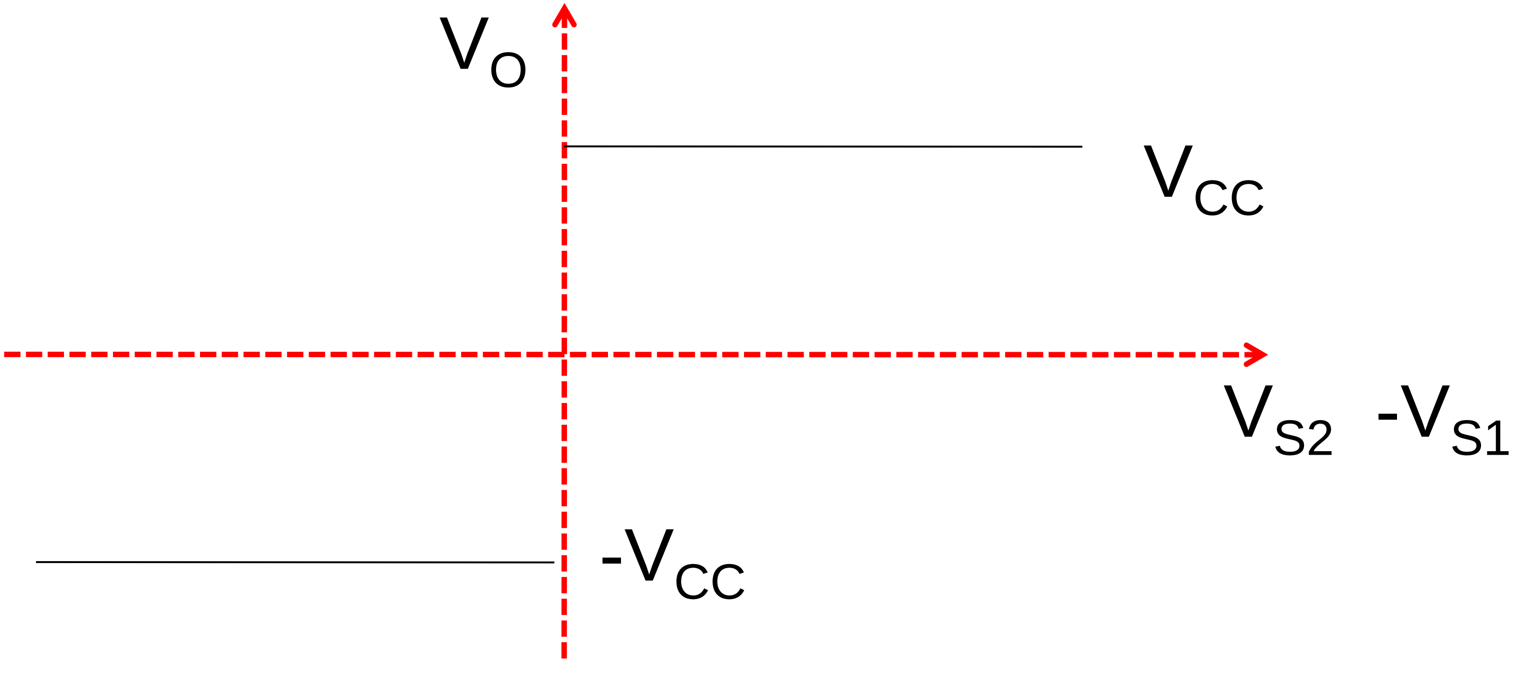

$$V_{OY} = -V_T \times \ln(\frac{V_Y}{RI_S})$$

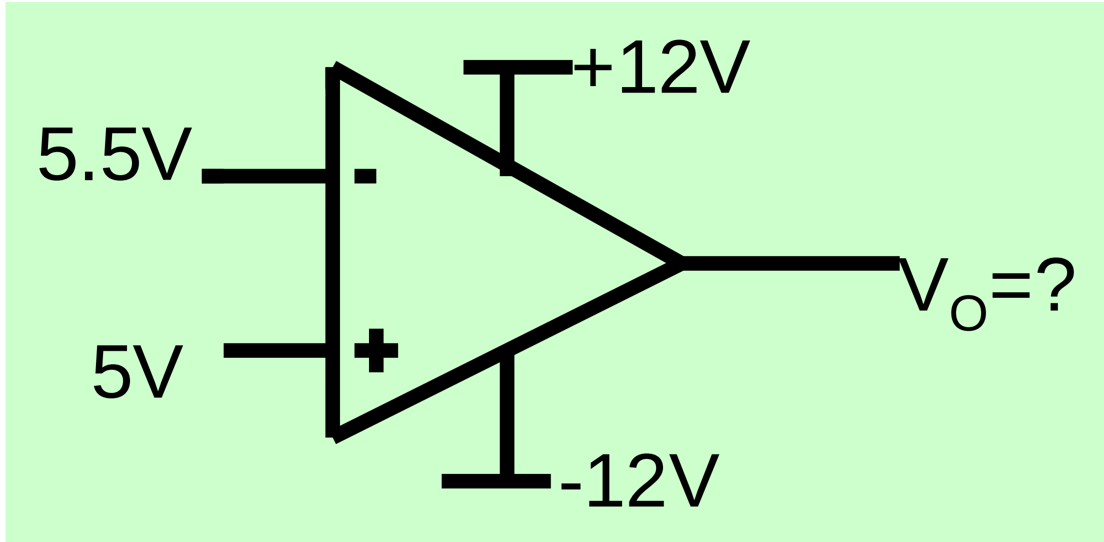
$$V_O \cong -RI_S \times e^{\frac{V_{O1}}{V_T}} = -\frac{V_X V_Y}{RI_S}$$

# Comparator: Opamp under open Loop condition

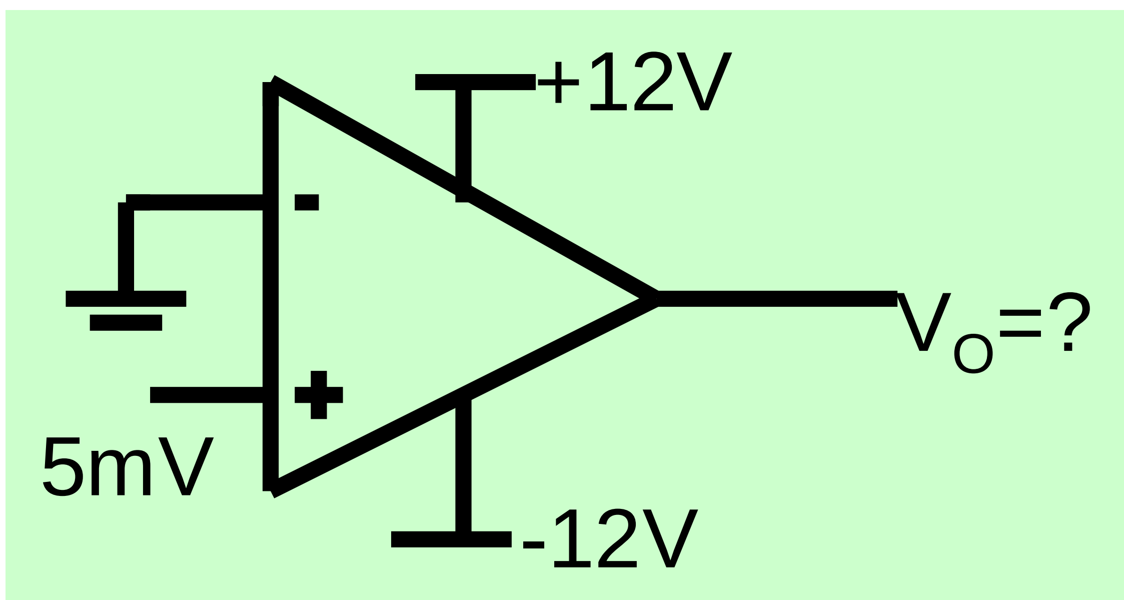




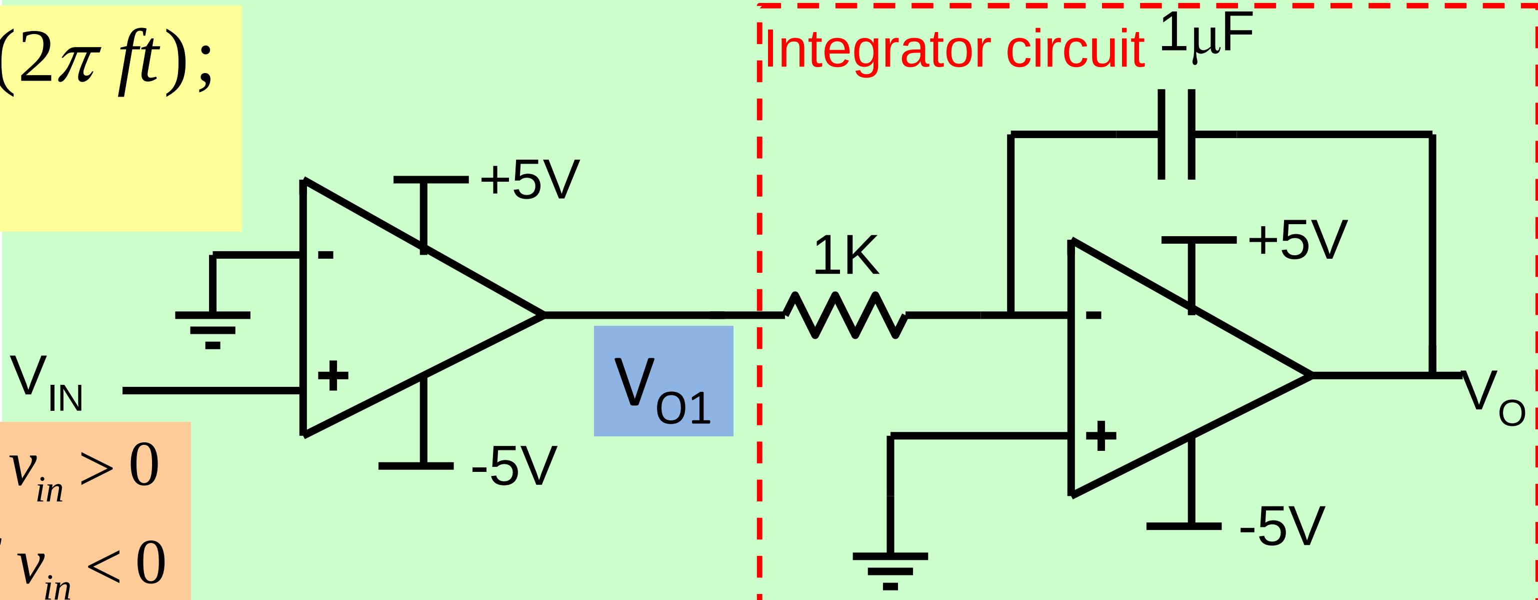

**~ -12V**




**~ +12V**

# Example 1: Plot Vo1 and Vo as a function of time

$$v_{in} = 1V\ Sin(2\pi ft);$$
$$f = 1KHz$$




$$V_{O1} = +5V\ if\ v_{in} > 0$$
$$= -5V\ if\ v_{in} < 0$$

$$\frac{v_S}{R} = -C\frac{dV_O}{dt} \Rightarrow V_O(t) = -\frac{1}{RC}\int v_S dt$$