ESc201 : Introduction to Electronics

Frequency Domain Response

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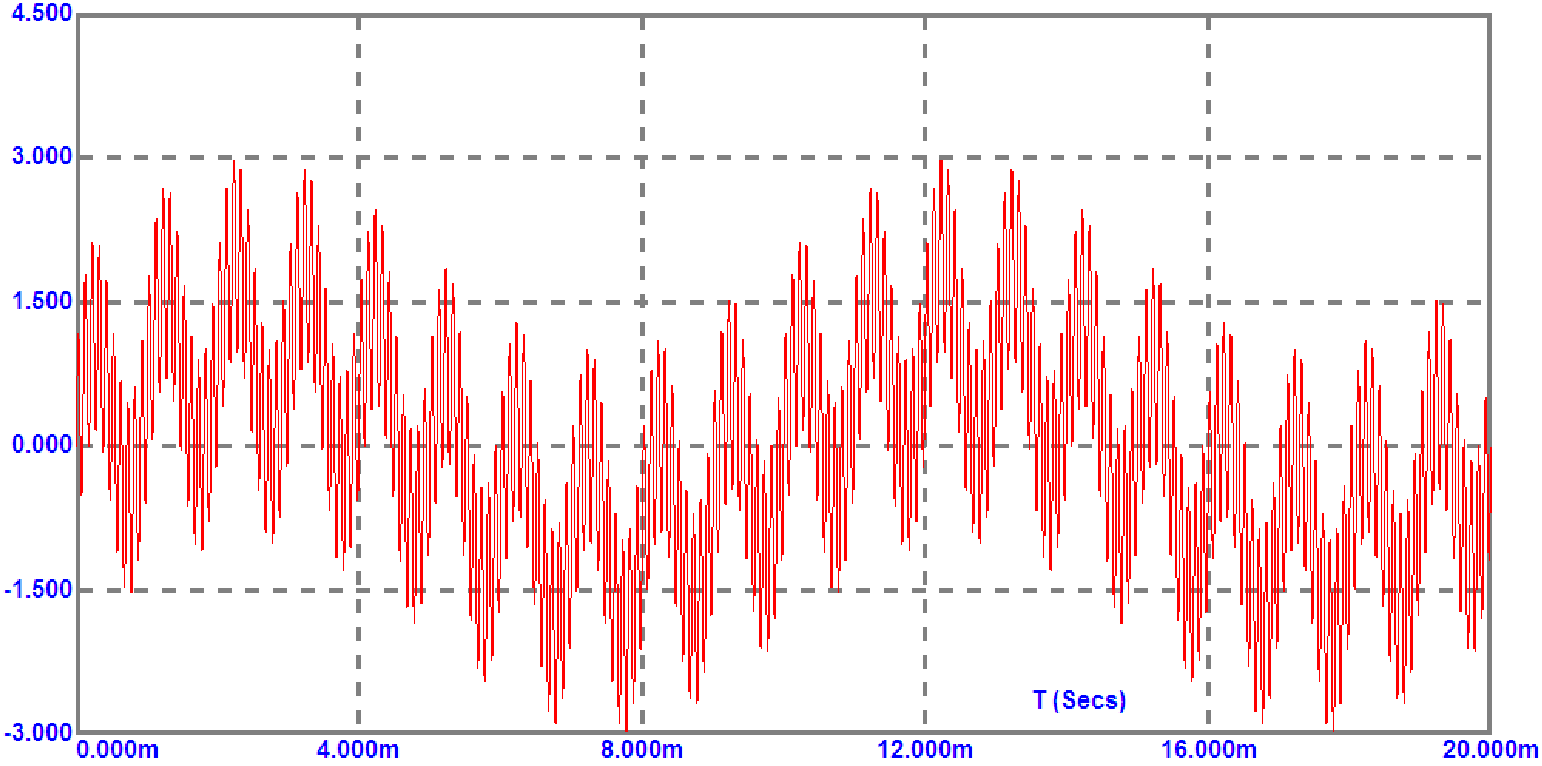
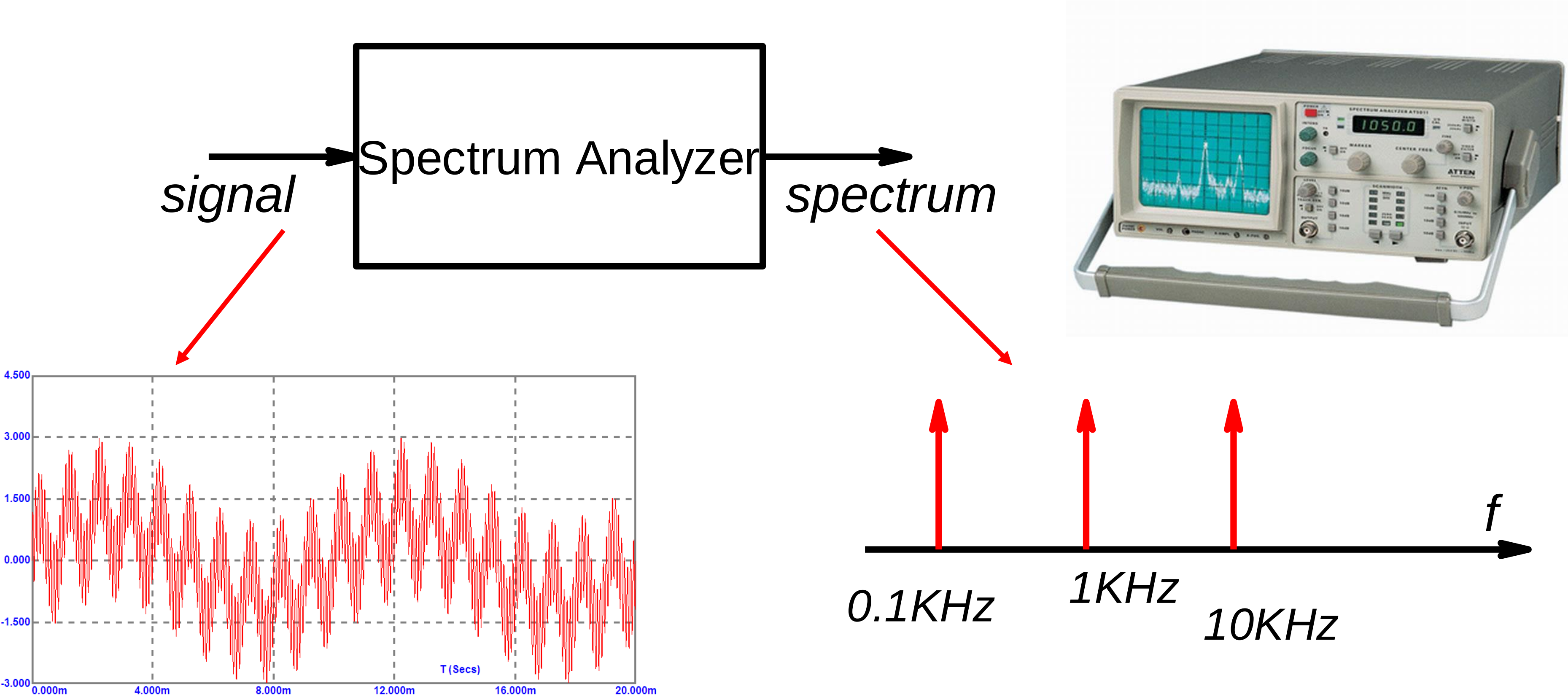
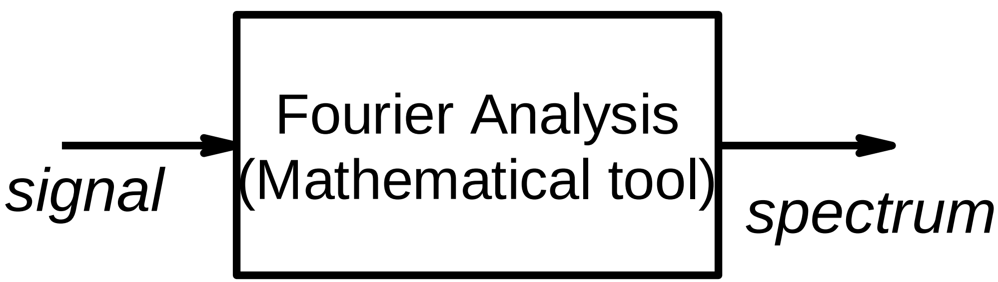
Time domain vs. Frequency domain analysis

Signal



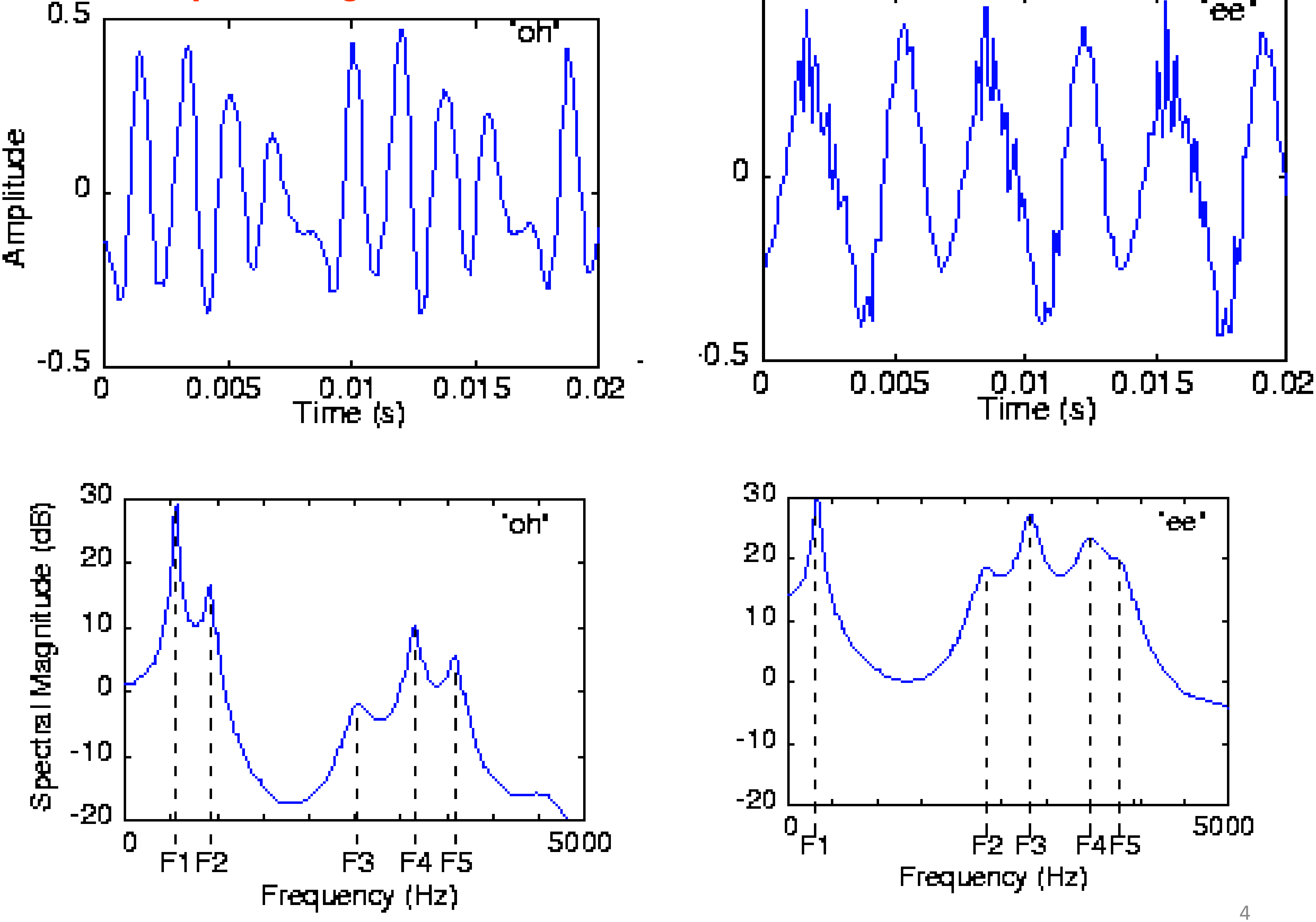


Spectrum Analyzer
signal
spectrum
4.500
3.000
1.500
0.000
-1.500
-3.000
0.000m
4.000m
8.000m
12.000m
16.000m
20.000m
T (Secs)
f
0.1KHz
1KHz
10KHz



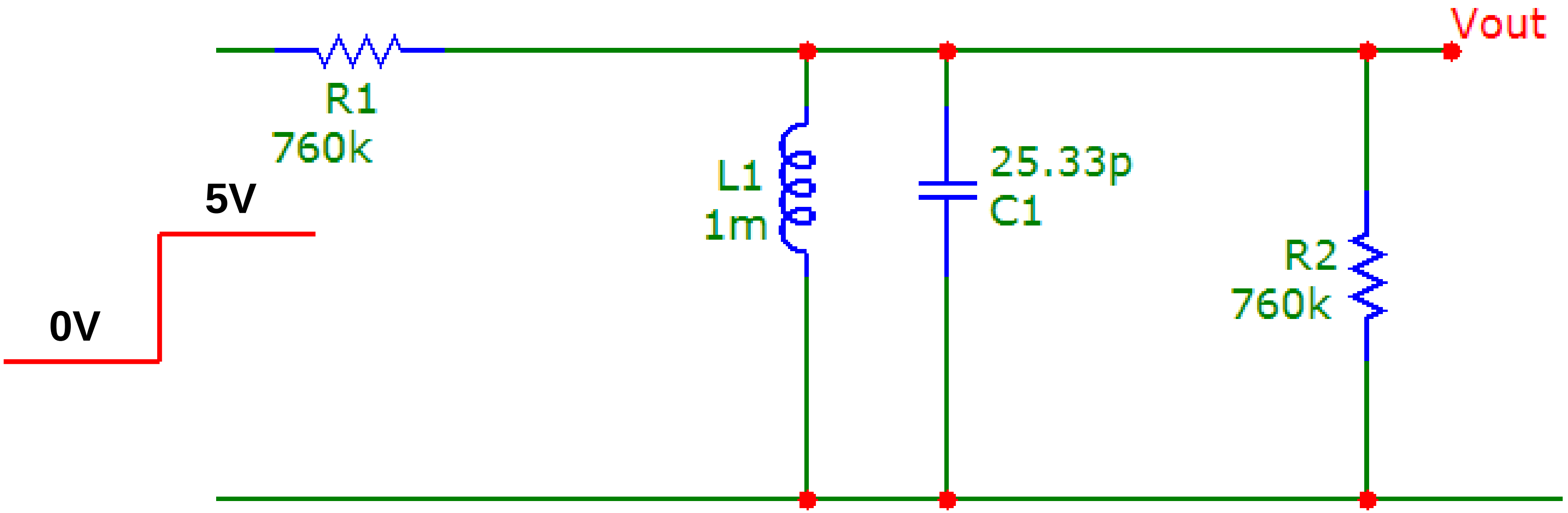
signal
Fourier Analysis
(Mathematical tool)
spectrum

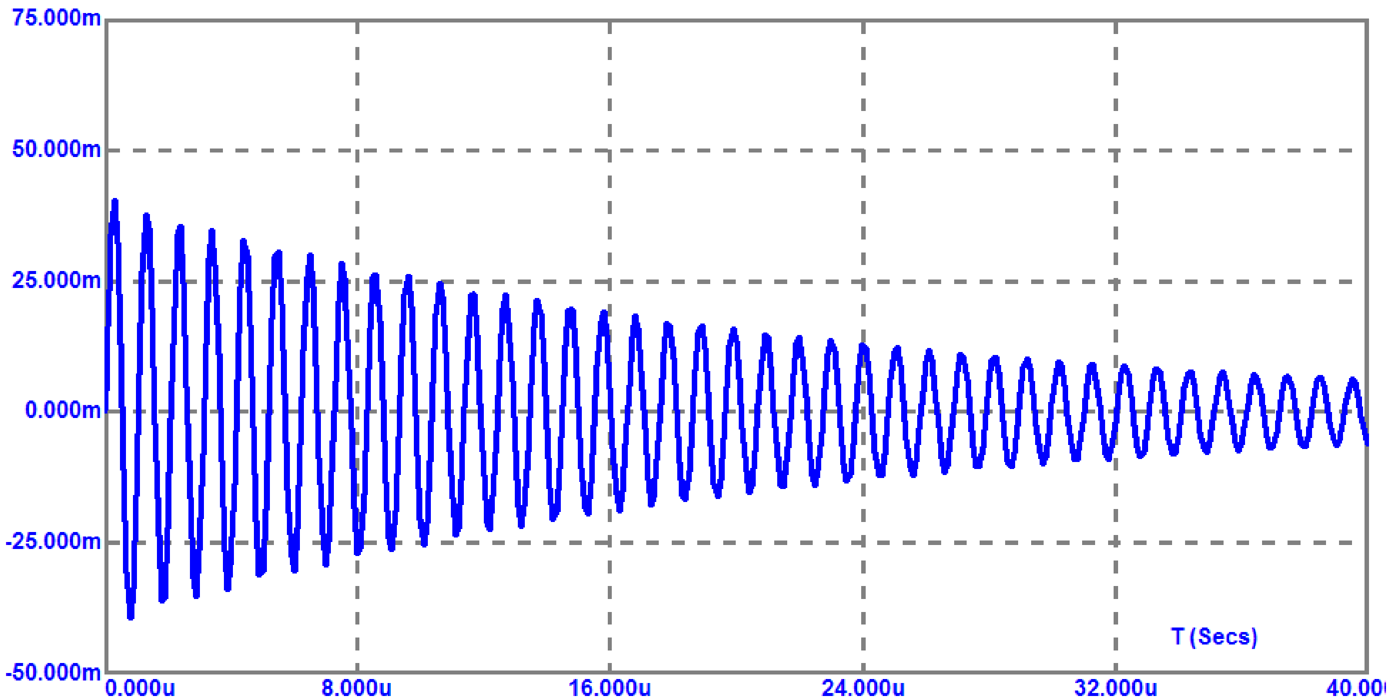
Speech signal



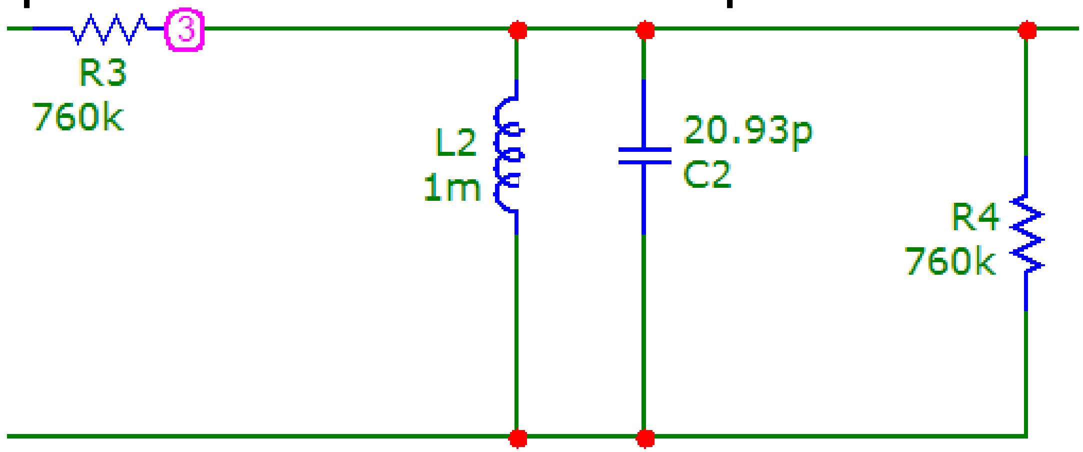
System

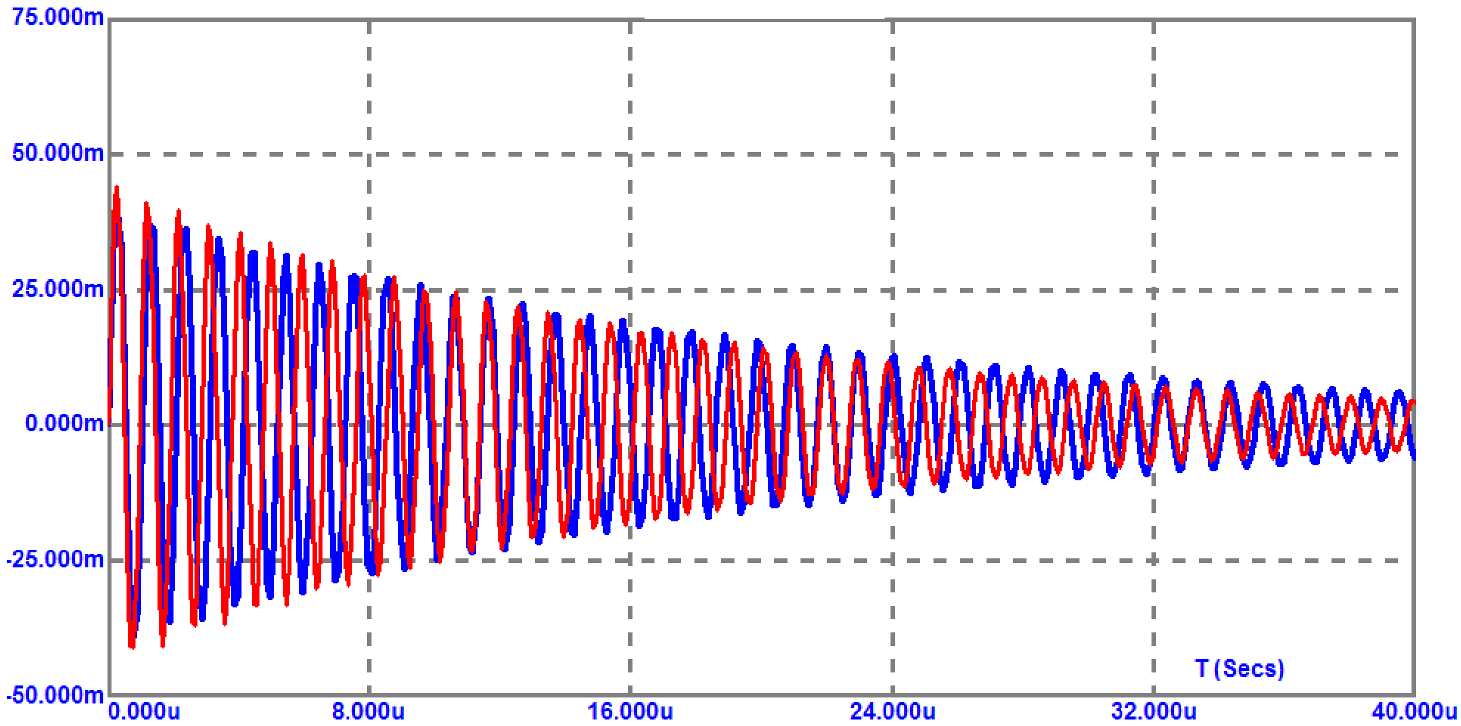
What does this circuit do ?





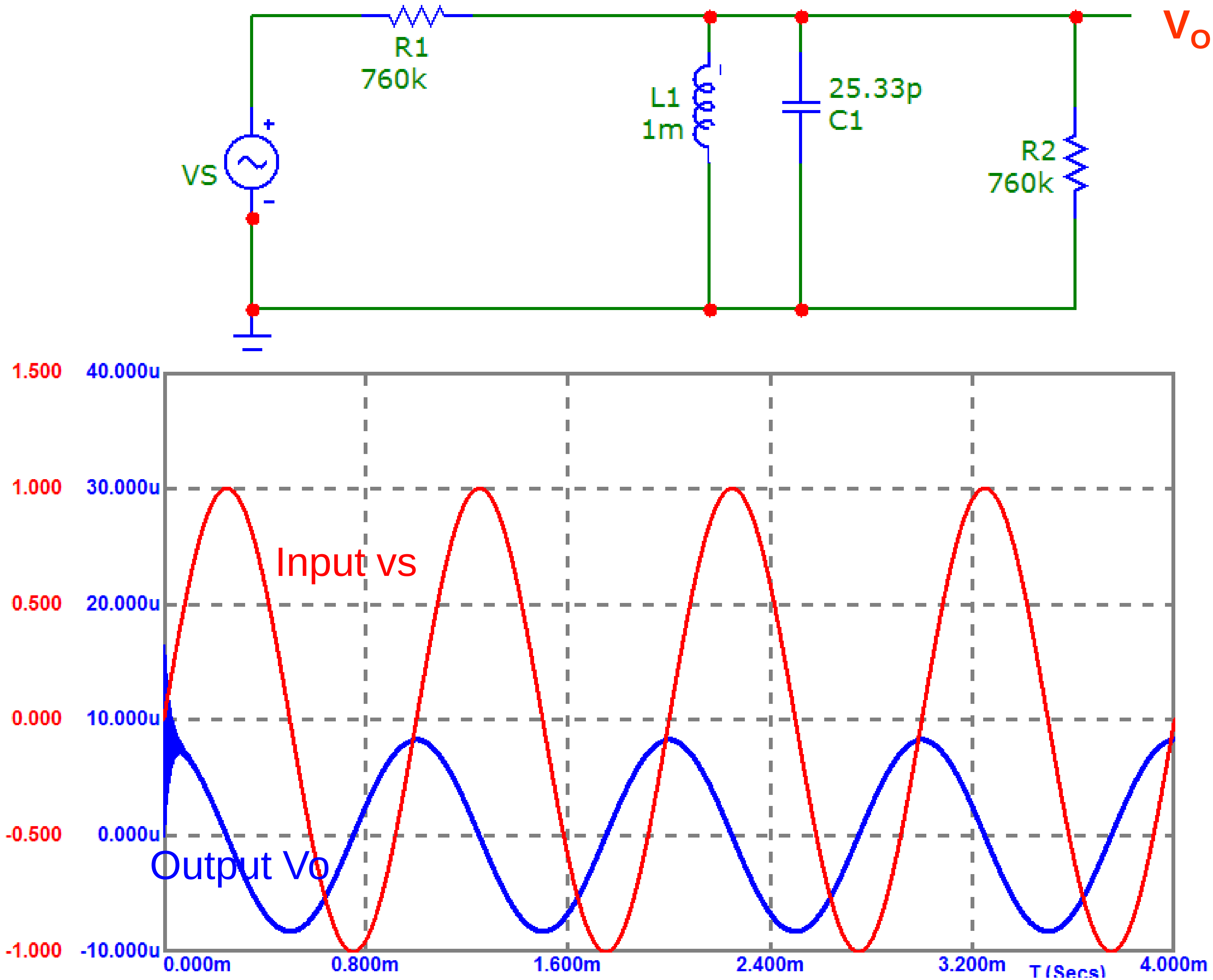
Suppose the capacitor is reduced to ~21pF.



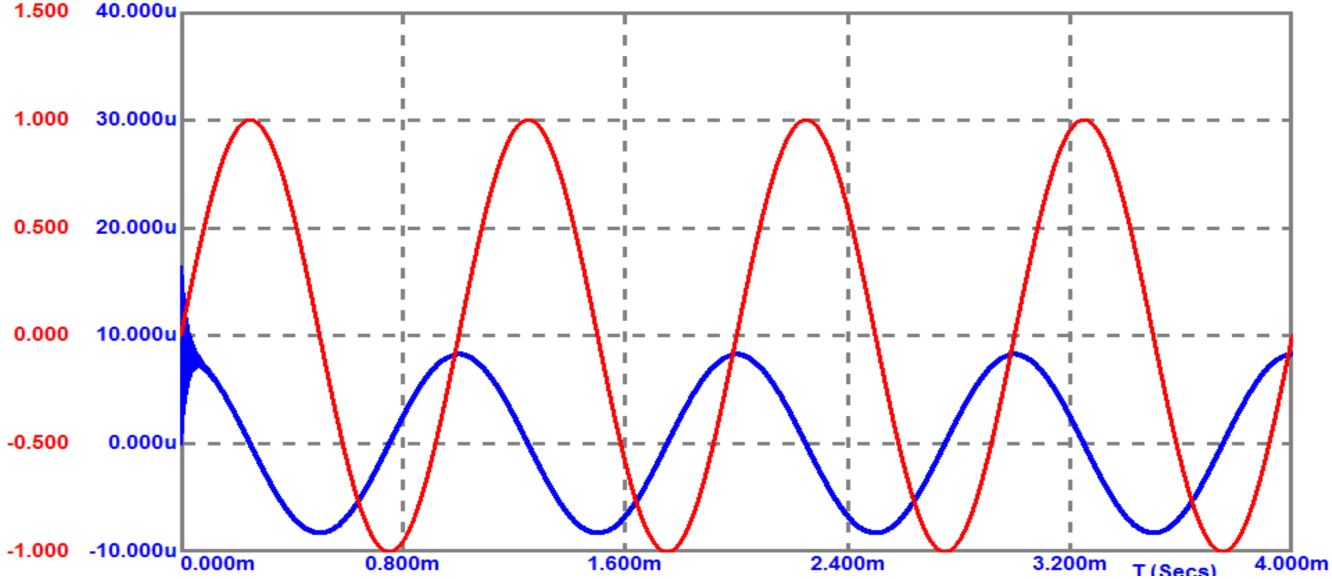


It is hard to find out what impact the change in capacitor has on circuit behavior

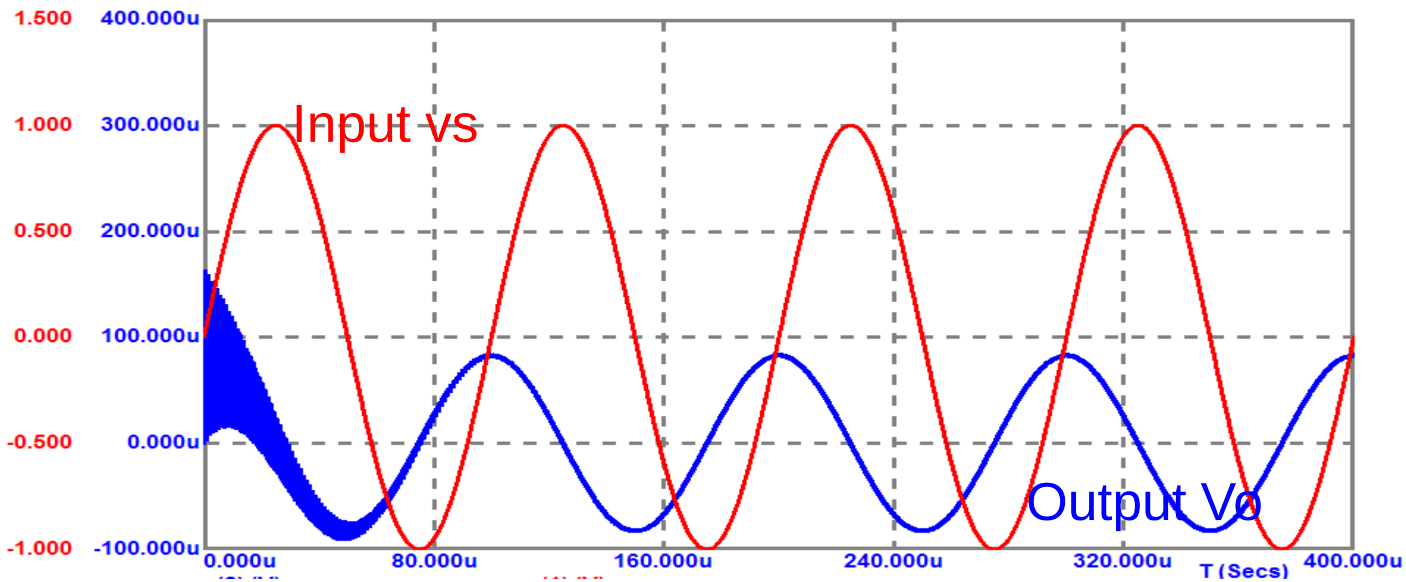
Frequency domain analysis



Measure response at many different frequencies for a constant input amplitude

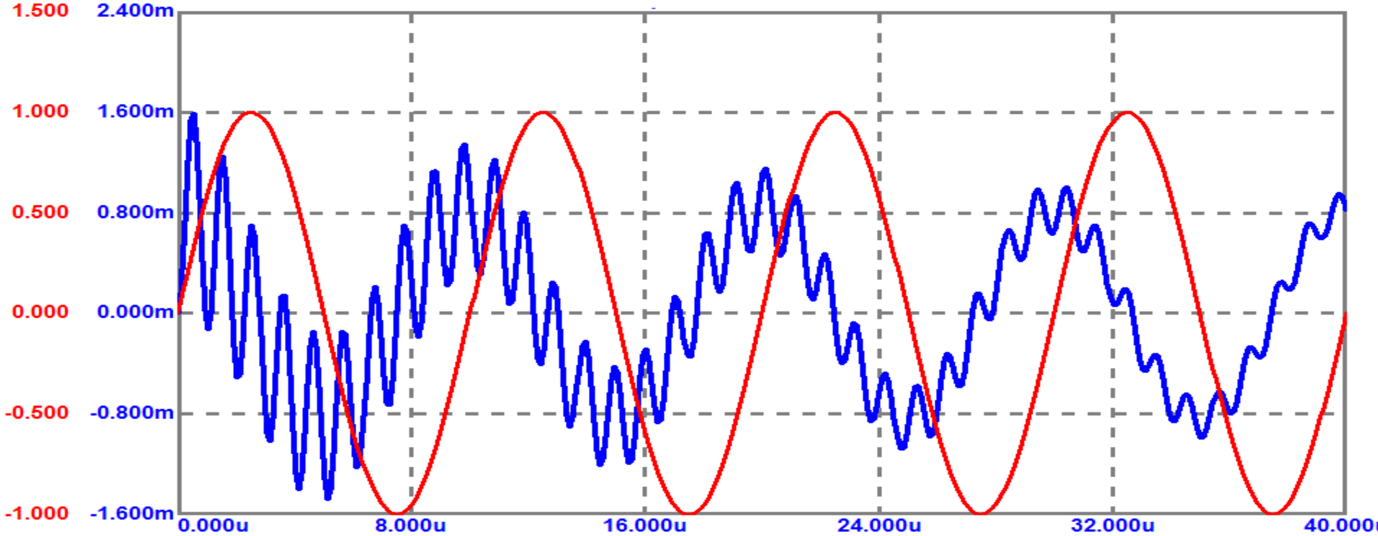


f=1KHz

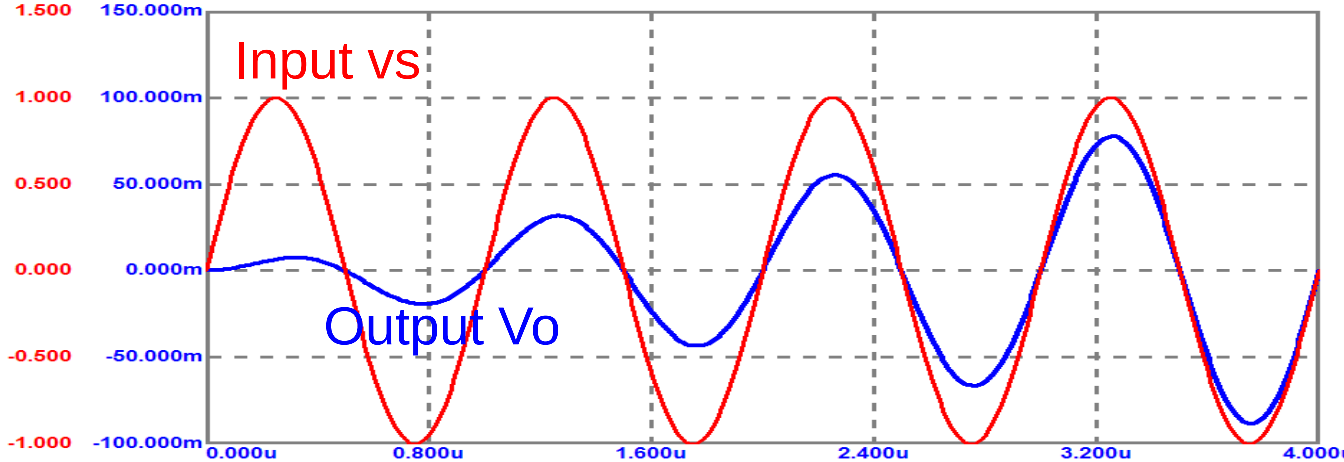


f=10KHz

Measure response at many different frequencies for a constant input amplitude



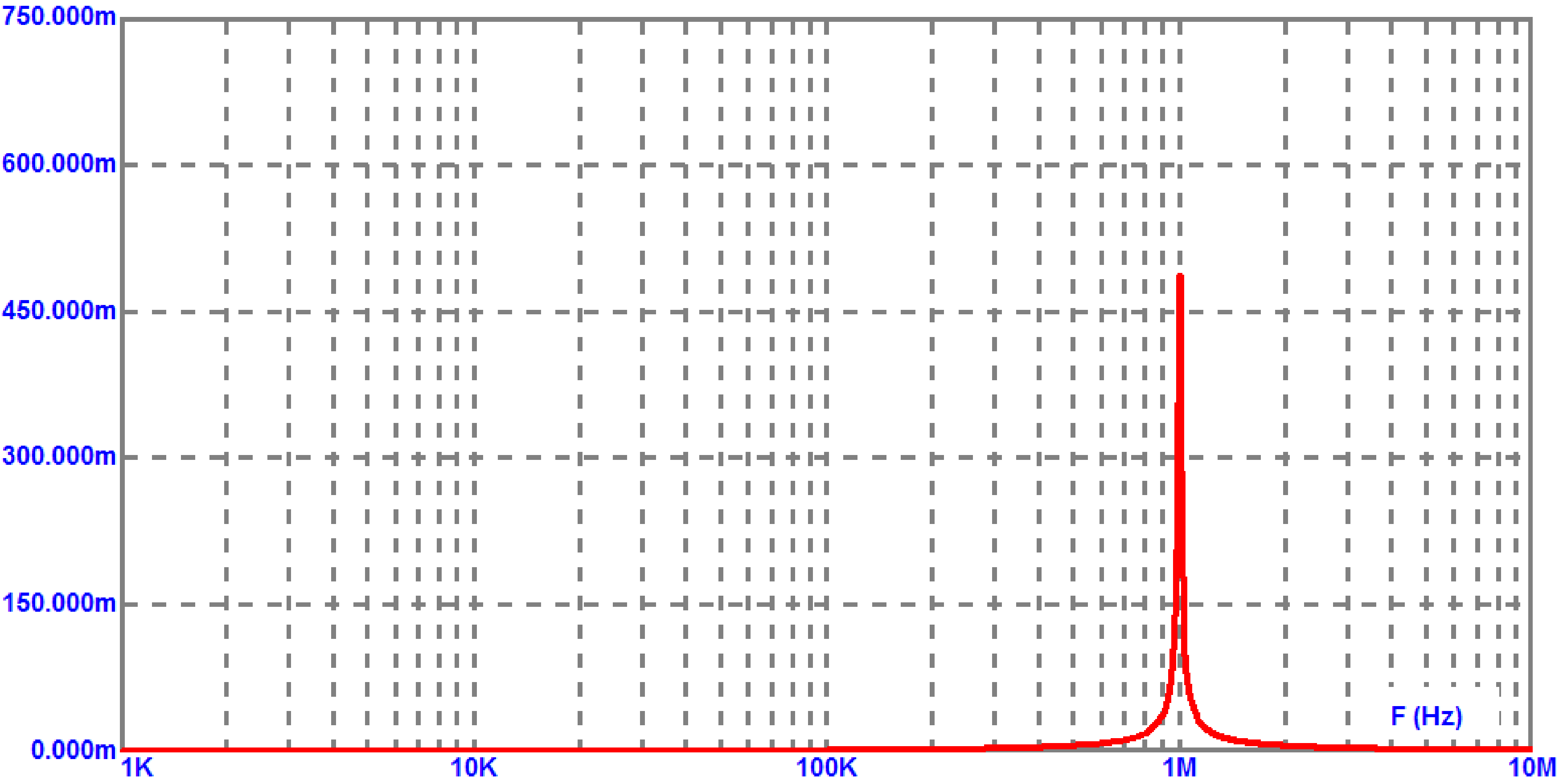
f=100KHz



f=1000KHz

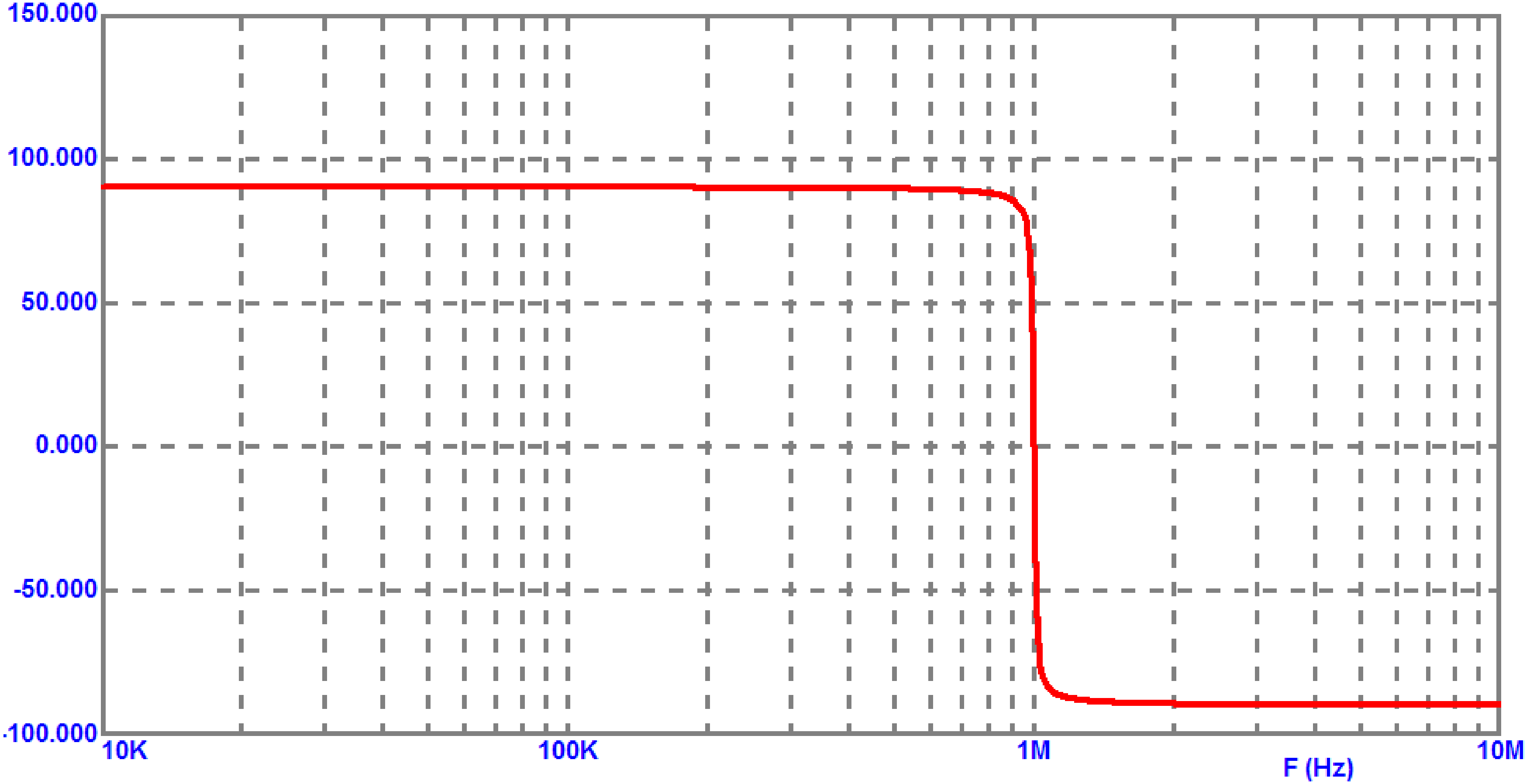
Plot the amplitude and phase as a function of frequency

Amplitude as a function of frequency

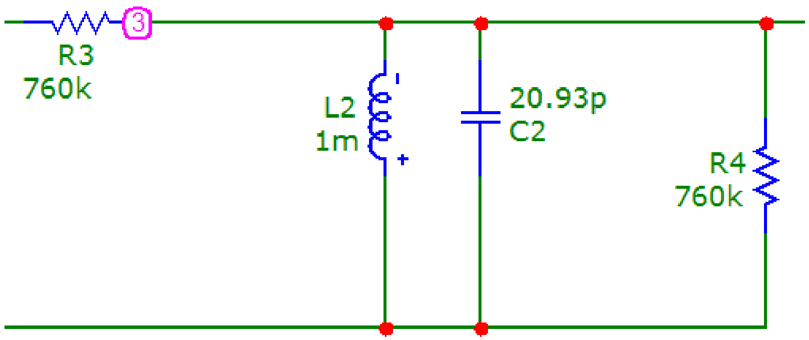


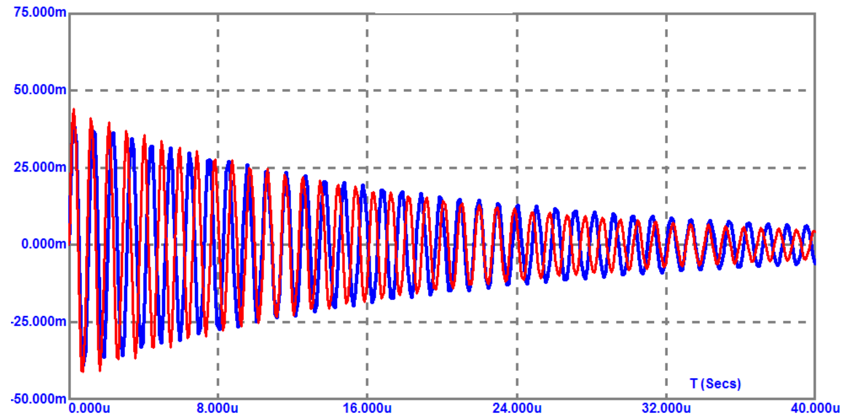
One can clearly see the frequency selective (often called a filter) nature of the circuit

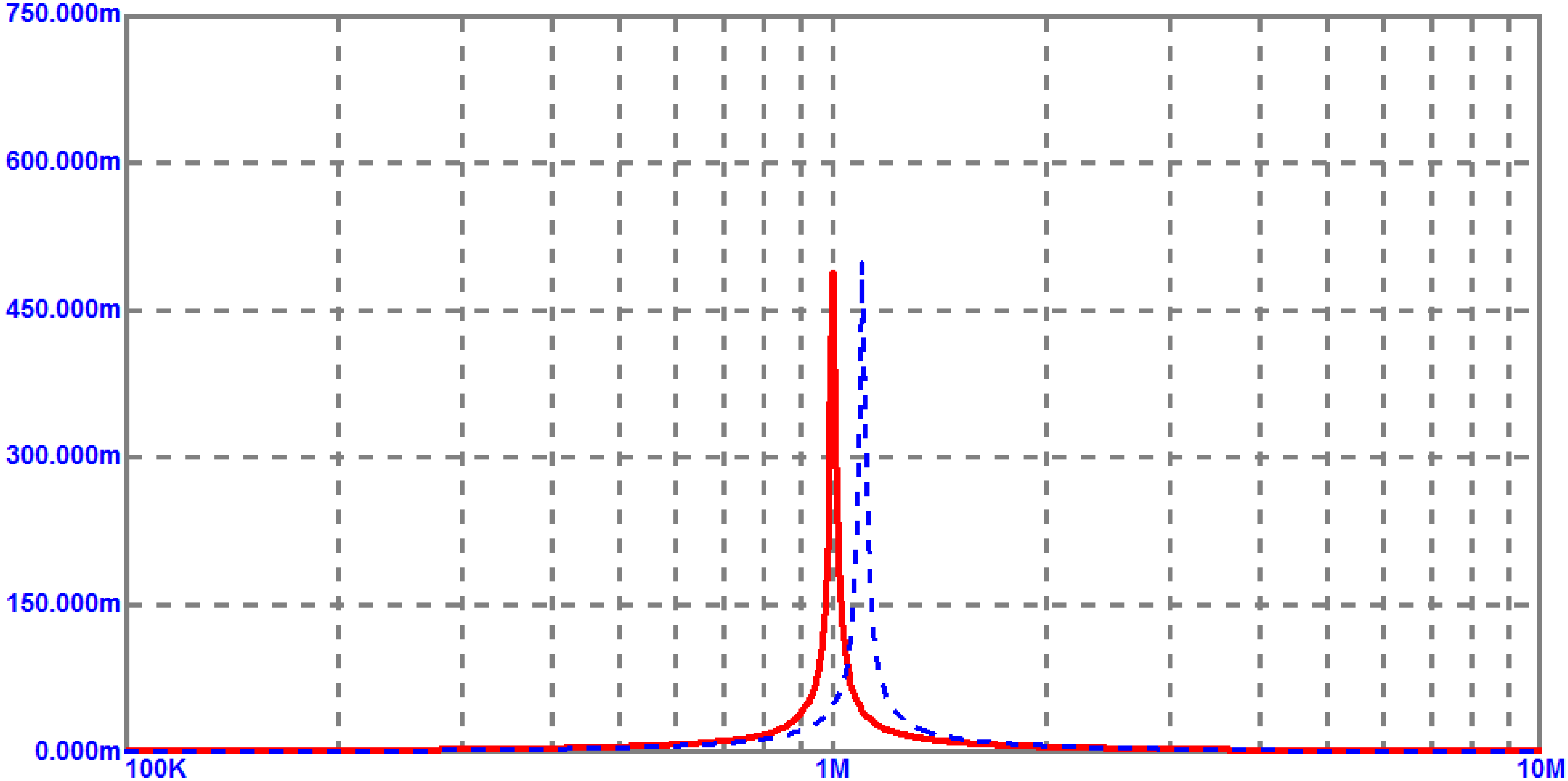
Phase as a function of frequency



Suppose the capacitor is reduced to ~21pF.



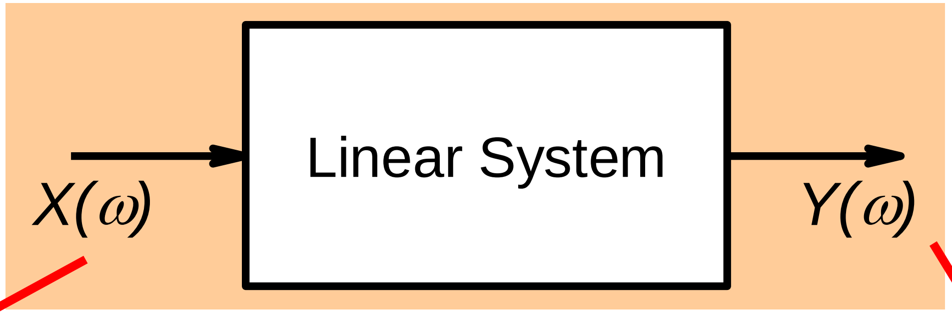




Analysis of signals and systems in frequency domain often provides useful insight into their behavior.

Frequency domain analysis

Transfer function is a useful tool for finding the frequency response of a system



Phasor (V or I)

Phasor (V or I)

$$\text{Transfer Function:} \quad H(\omega) = \frac{Y(\omega)}{X(\omega)}$$

Transfer function has a magnitude and a phase

$$H(\omega) = \frac{Y(\omega)}{X(\omega)}$$

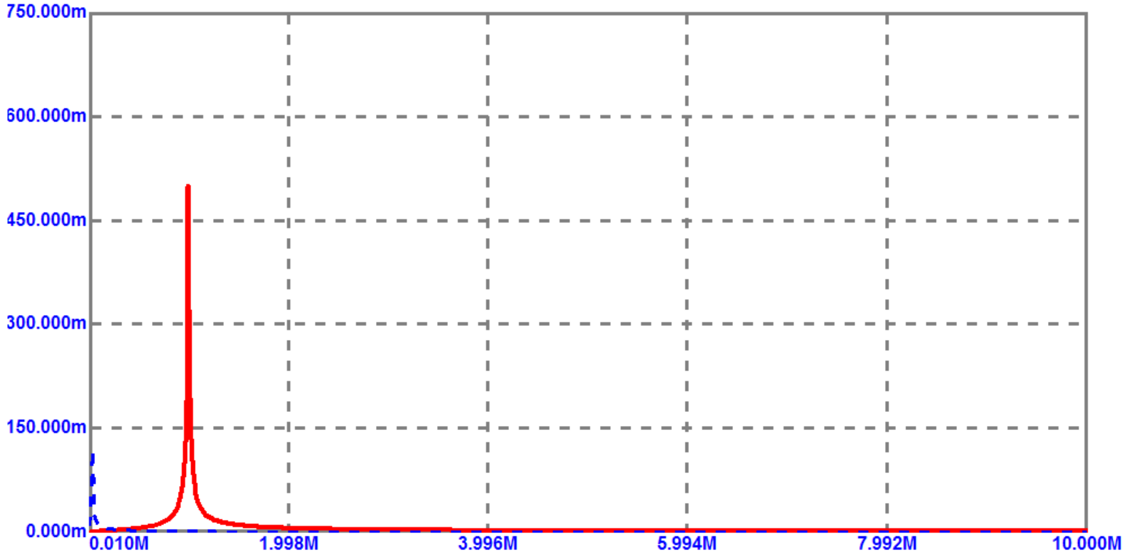
$$\mathbf{H}(\omega) = \text{Voltage gain} = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$$

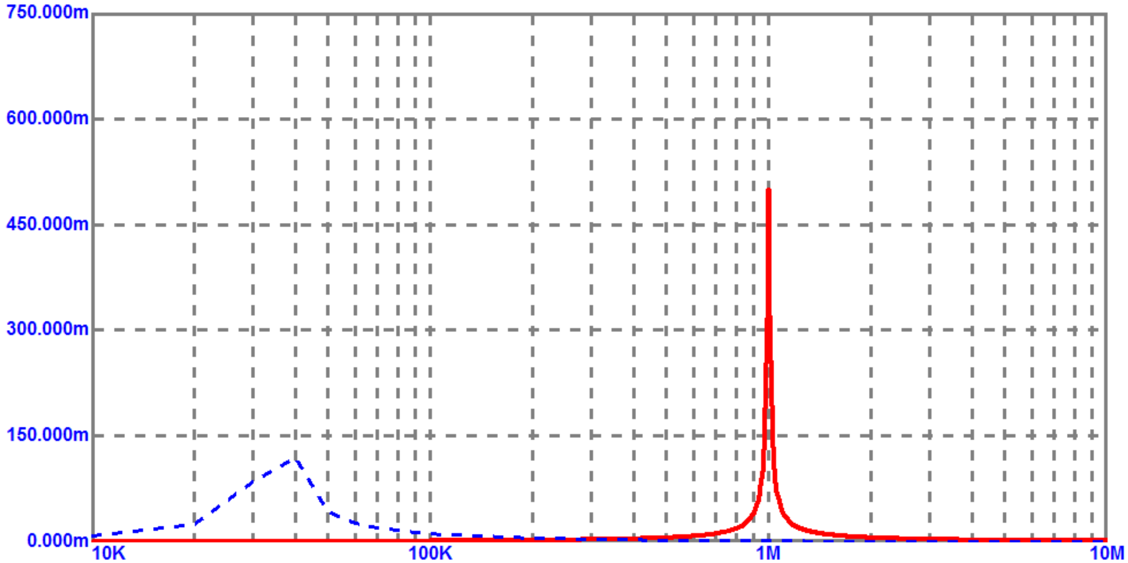
$$\mathbf{H}(\omega) = \text{Current gain} = \frac{\mathbf{I}_o(\omega)}{\mathbf{I}_i(\omega)}$$

$$\mathbf{H}(\omega) = \text{Transfer Impedance} = \frac{\mathbf{V}_o(\omega)}{\mathbf{I}_i(\omega)}$$

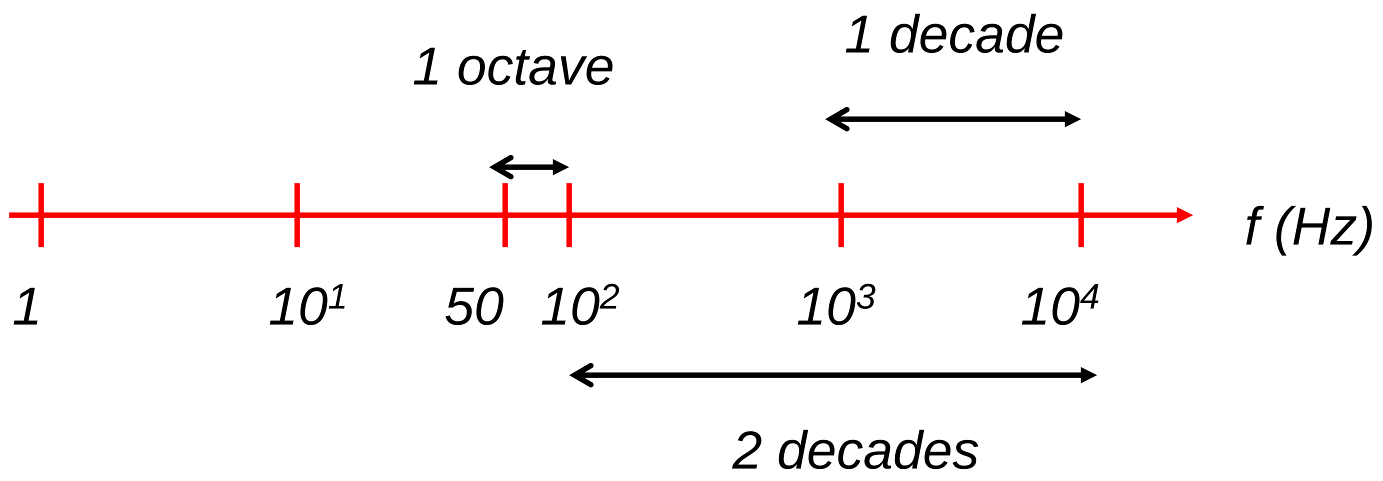
$$\mathbf{H}(\omega) = \text{Transfer Admittance} = \frac{\mathbf{I}_o(\omega)}{\mathbf{V}_i(\omega)}$$

Because of the wide dynamic range of frequency, plotting frequency on log axis is often more revealing !





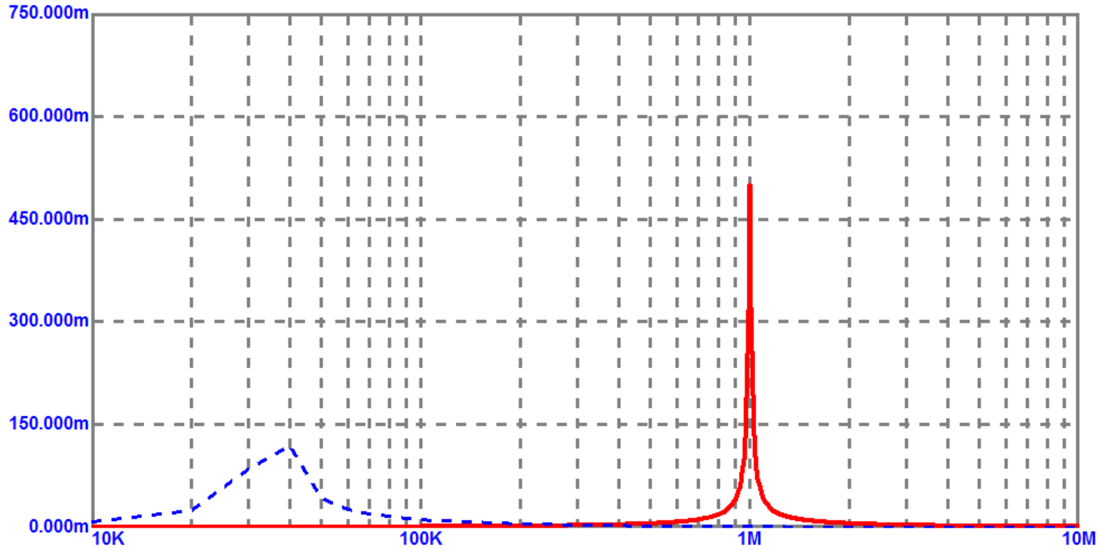
Logarithmic frequency scale

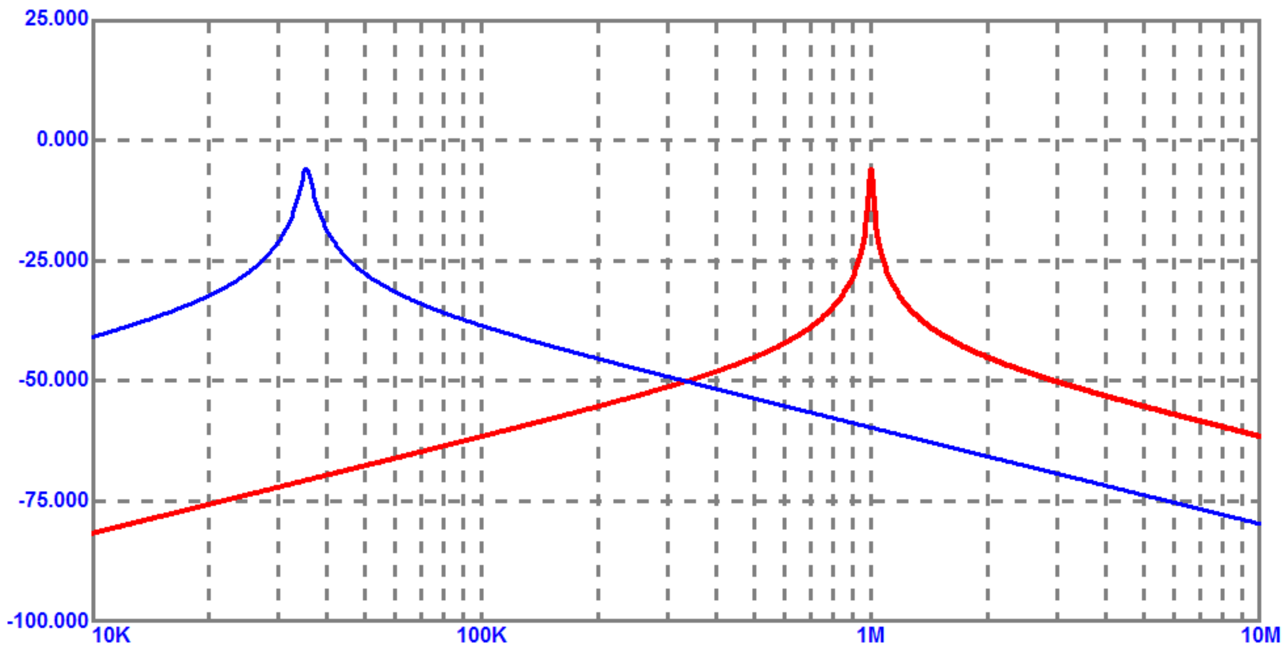


$$\text{No. of decades} = \log_{10}\left(\frac{f_2}{f_1}\right)$$

$$\text{No. of octaves} = \log_{2}\left(\frac{f_2}{f_1}\right) = \frac{\log_{10}\left(\frac{f_2}{f_1}\right)}{\log_{10}(2)}$$

Decibel scale often reveals more information about behavior





The magnitude of transfer function is often specified in **decibels**

$$G_{dB} = 10\log_{10}\left(\frac{P_2}{P_1}\right)$$

Because power is proportional to V^2 or I^2, voltage gain and current gain in decibels is specified as

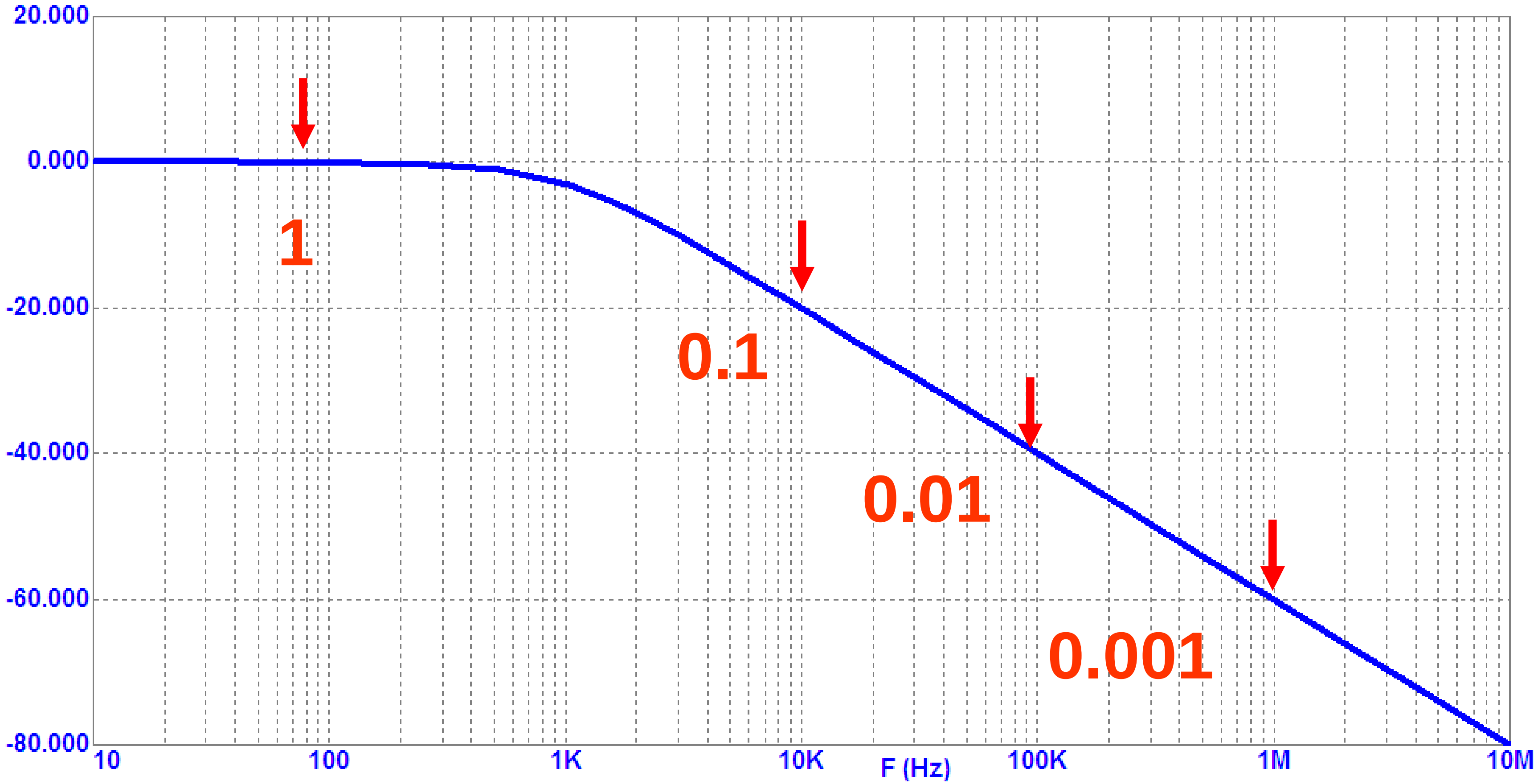
$$G_{dB} = 20\log_{10}\left(\frac{V_2}{V_1}\right)$$

$$G_{dB} = 20\log_{10}\left(\frac{I_2}{I_1}\right)$$

Decibel Scale

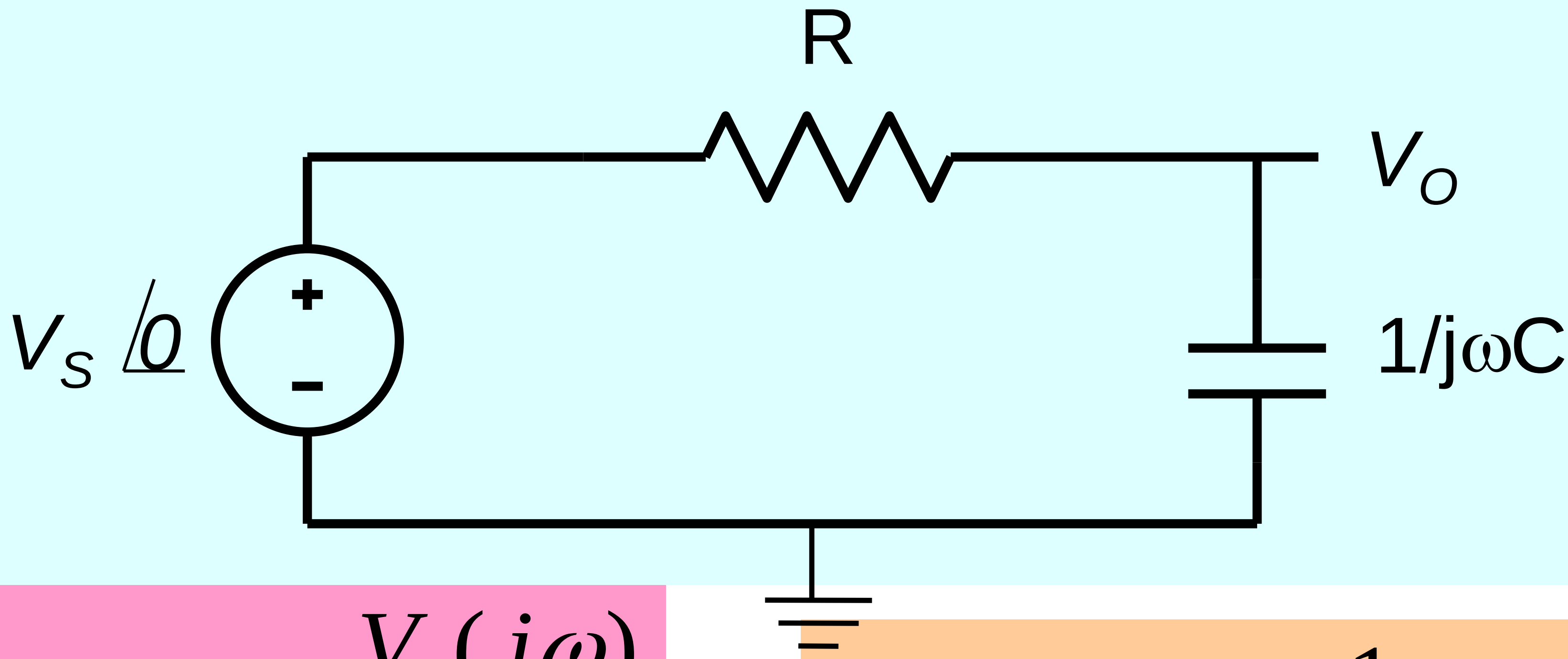
$\|H\|$	$20\mathrm{Log}_{10}(\|H\|)$
1000	60
100	40
10	20
2	6
$\sqrt{2}$	3
1	0
$1/\sqrt{2}$	-3
0.5	-6
0.1	-20
0.01	-40

dB Scale



A plot of the decibel magnitude of transfer function versus frequency using a logarihmic scale for frequency is called a **Bode plot**

How to determine the transfer function?



$$H(j\omega) = \frac{V_0(j\omega)}{V_S(j\omega)}$$

$$H(j\omega) = \frac{1}{1 + j\omega RC}$$

$$\left|H(j\omega)\right| = \frac{1}{\sqrt{1+\left(\omega RC\right)^2}}$$

$$\phi(\omega) = -\tan^{-1}(\omega CR)$$

Plot Magnitude

$$|H(j\omega)| = \frac{1}{\sqrt{1+(\omega RC)^2}}$$

$$H_{dB} = -20\log_{10}\sqrt{1+(\omega RC)^2}$$

$$\omega_{3dB} = \frac{1}{RC}$$

$$H_{dB} = -20\log_{10}\sqrt{1+\left(\frac{\omega}{\omega_{3dB}}\right)^2}$$

$$\omega << \omega_{3dB}$$

$$H_{dB} \approx -20\log_{10}(1) = 0$$

$$\omega >> \omega_{3dB}$$

$$H_{dB} \approx -20\log_{10}\left(\frac{\omega}{\omega_{3dB}}\right)$$

$$\omega >> \omega_{3dB}$$

$$H_{dB} \approx -20\log_{10}\left(\frac{\omega}{\omega_{3dB}}\right)$$

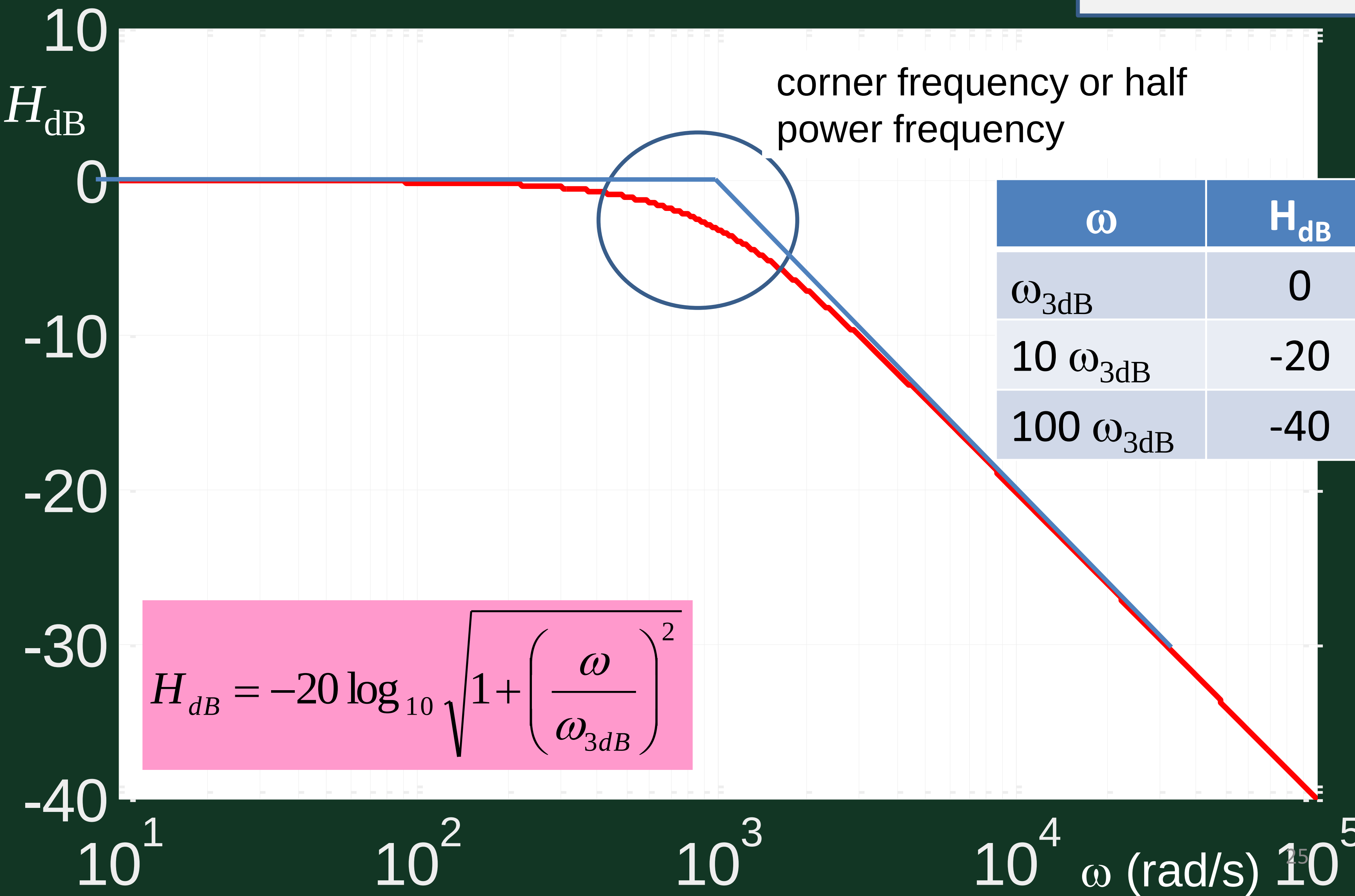
ω	H_{dB}
ω_{3dB}	0
10 ω_{3dB}	-20
100 ω_{3dB}	-40

-20 dB per decade

$$\log_{10} 10 = 1$$

3dB point

$$\omega_{3dB} = \frac{1}{RC}$$



ω	H_{dB}
ω_{3dB}	0
10 ω_{3dB}	-20
100 ω_{3dB}	-40

$$H_{dB} = -20 \log_{10} \sqrt{1 + \left(\frac{\omega}{\omega_{3dB}} \right)^2}$$

Example

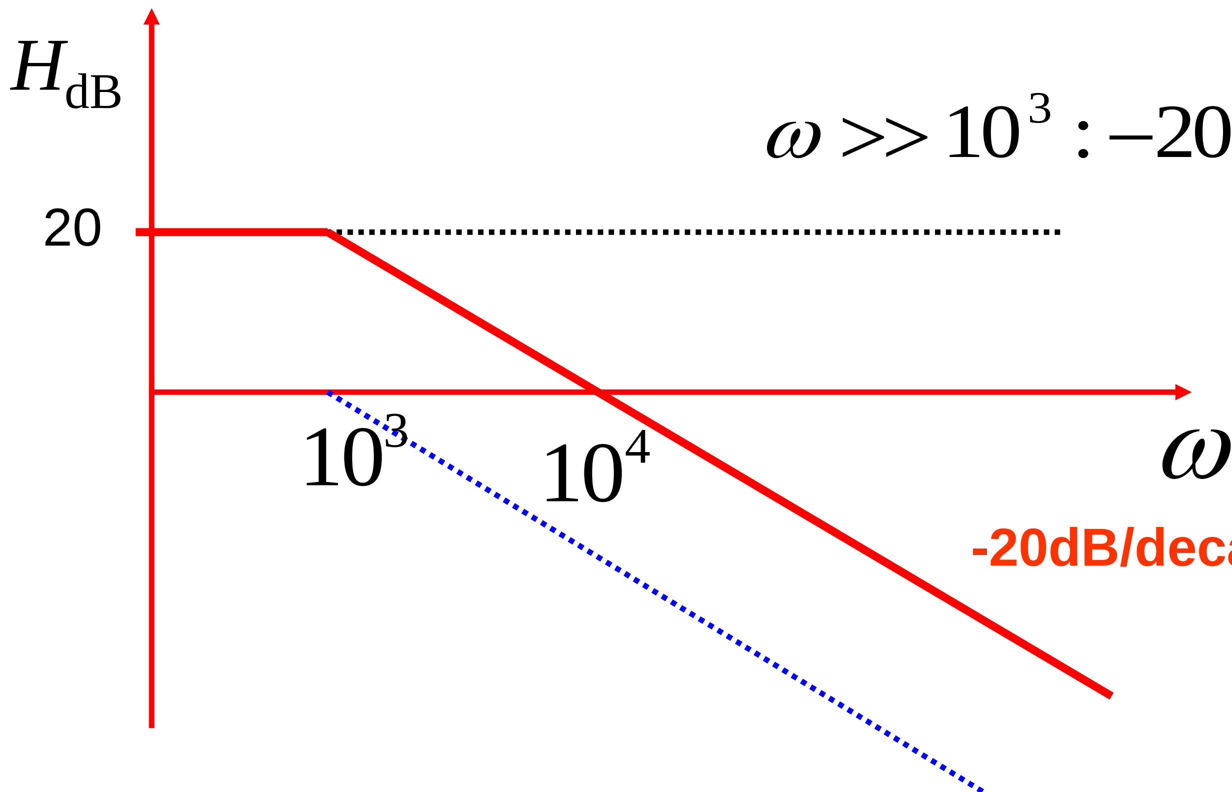
$$H(j\omega) = \frac{10}{1 + j\omega 10^{-3}}$$

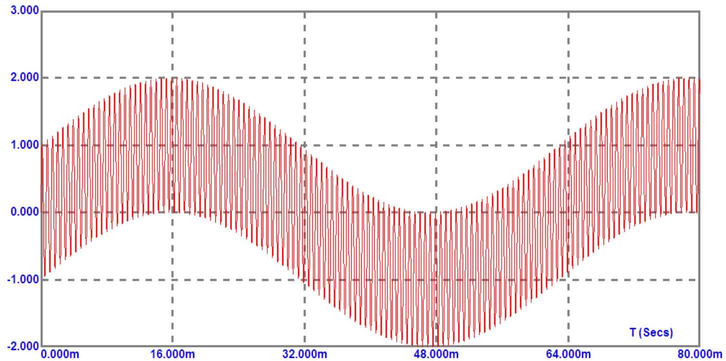
$$\omega_{3dB} = 10^3$$

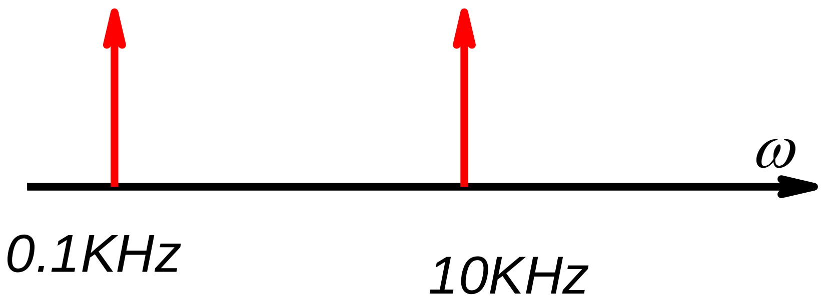
$$H_{dB} = 20 - 20\log_{10}\sqrt{1 + \left(\frac{\omega}{10^3}\right)^2}$$

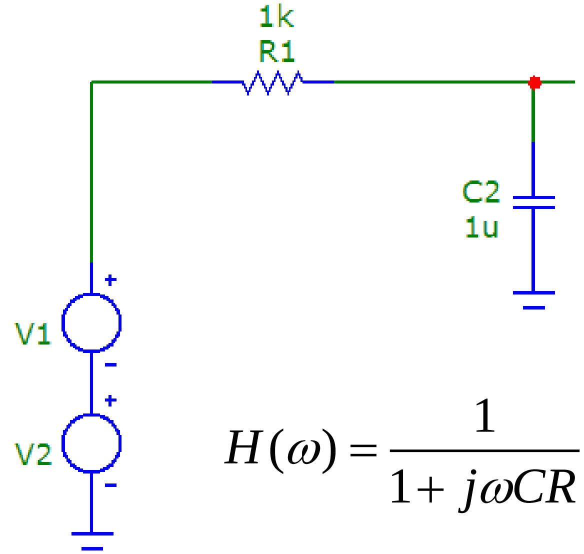
$$\omega << 10^3 : 0dB$$

$$\omega >> 10^3 : -20\log_{10}\frac{\omega}{10^3}$$



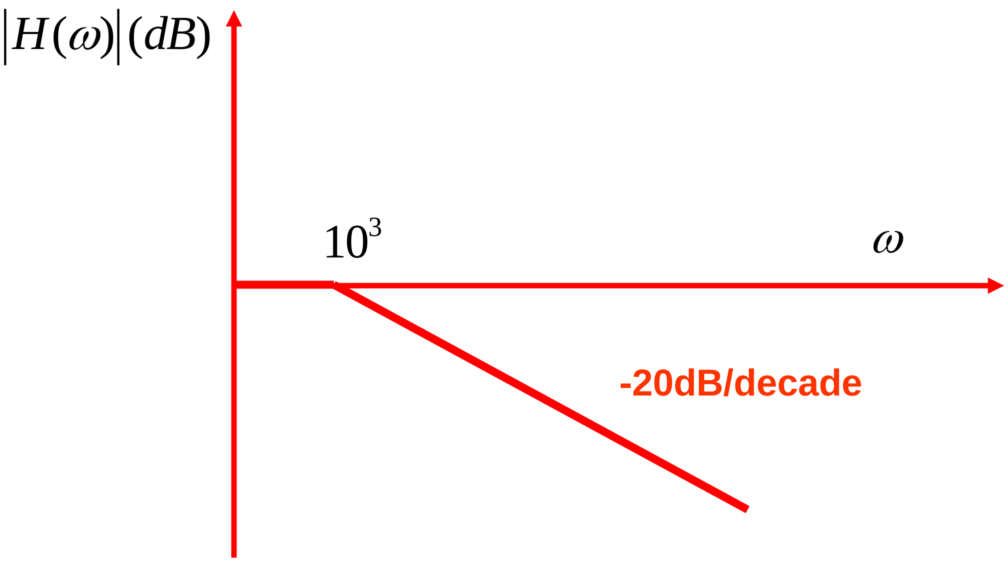




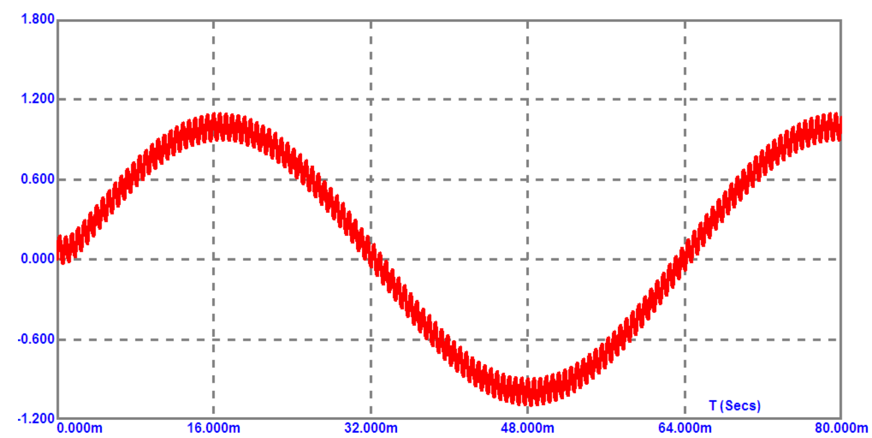


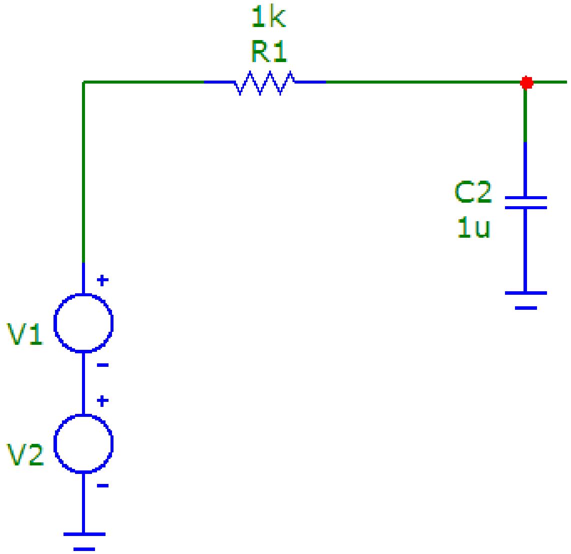
$$H(\omega)=\frac{1}{1+j\omega CR}$$

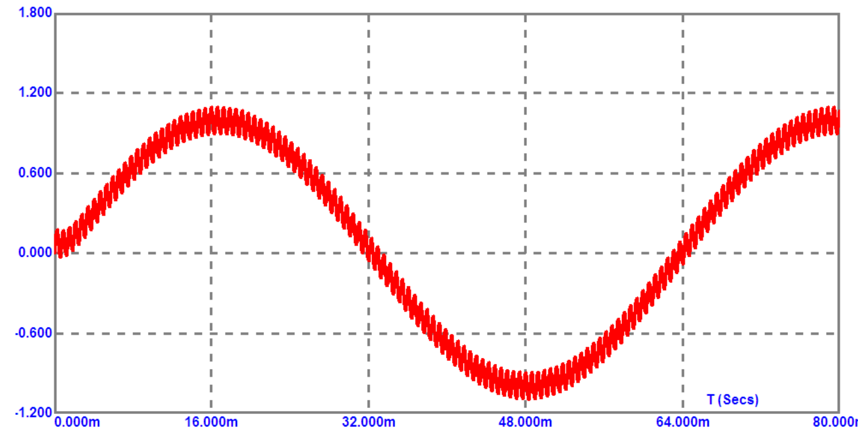
$$H(\omega)=\frac{1}{1+j\omega 10^{-3}}=\frac{1}{1+j\dfrac{\omega}{10^{3}}}$$

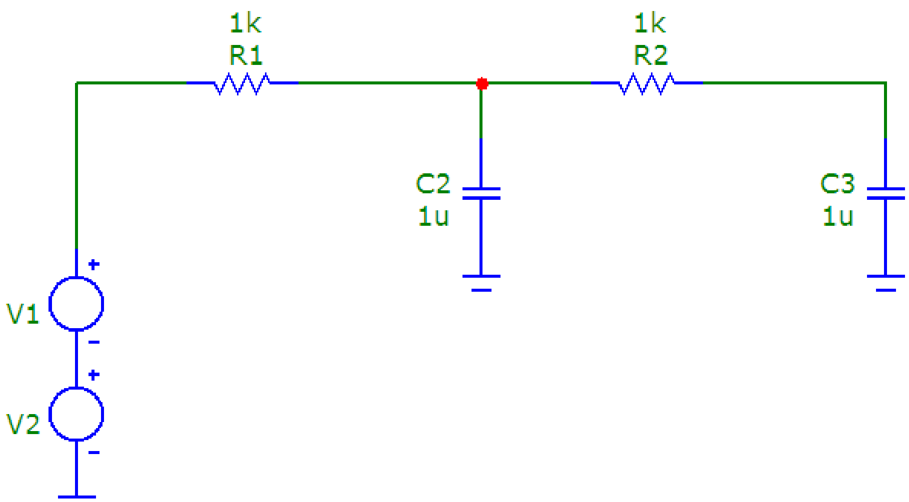


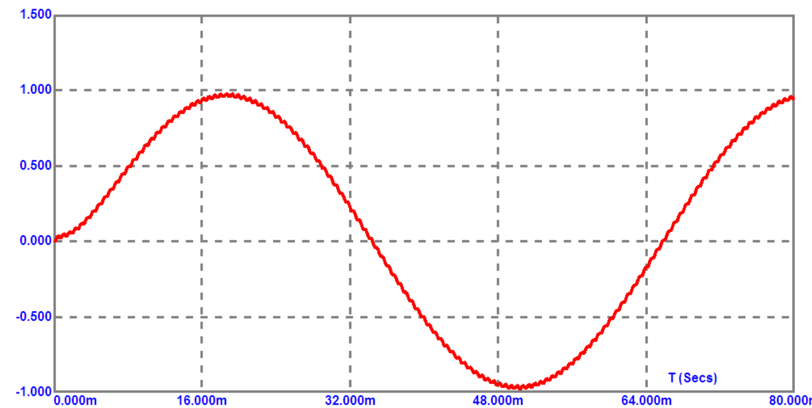
$$V_O(t)=1Sin(100t)+0.1Sin(10^4 t)$$

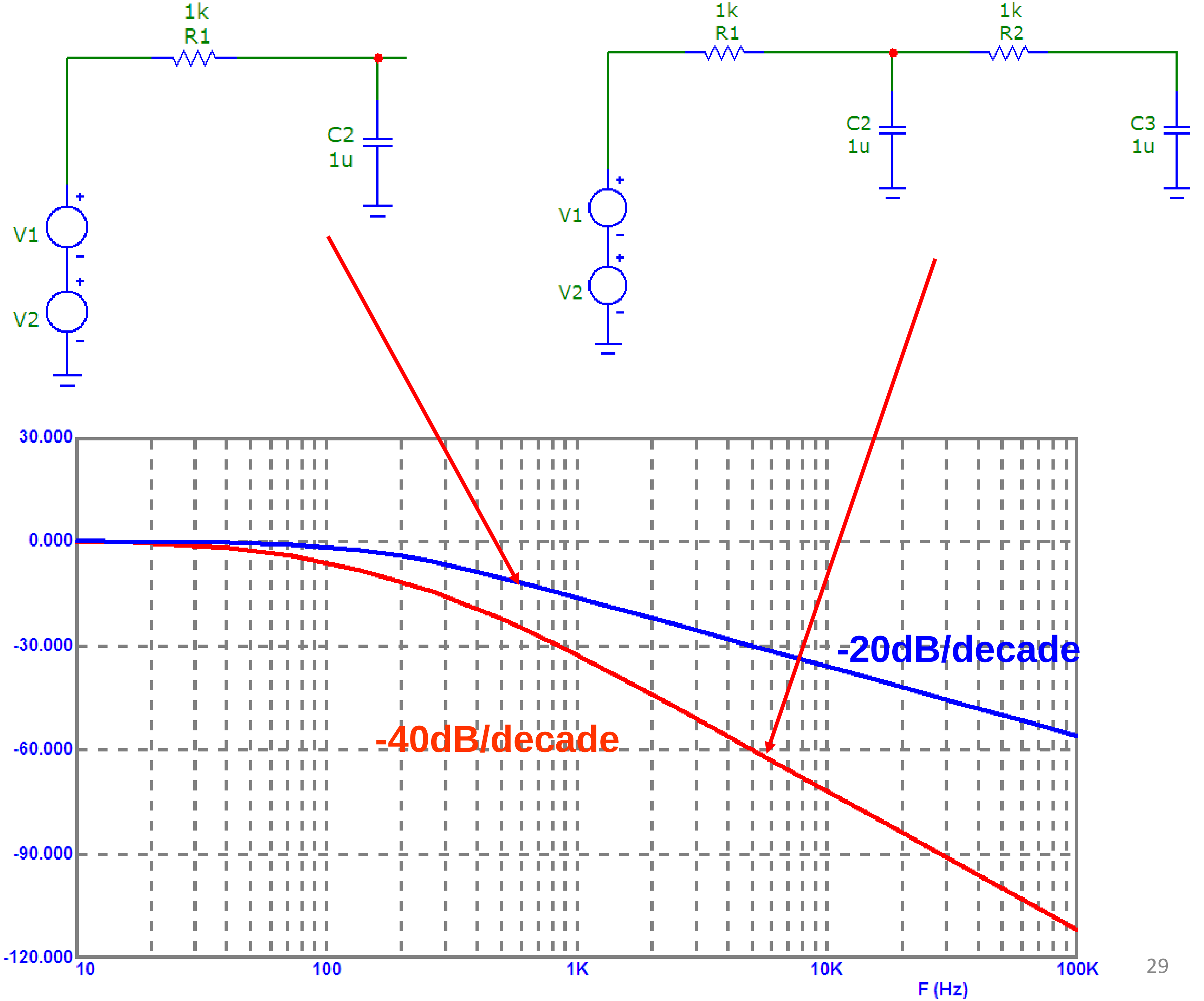



1k
R1
C2
1u
V1
V2


1.800
1.200
0.600
0.000
-0.600
-1.200
0.000m
16.000m
32.000m
48.000m
64.000m
80.000m
T (Secs)


1k
R1
1k
R2
C2
1u
C3
1u
V1
V2


1.500
1.000
0.500
0.000
-0.500
-1.000
0.000m
16.000m
32.000m
48.000m
64.000m
80.000m
T (Secs)



1k
R1
C2
1u
V1
V2
1k
R1
1k
R2
C2
1u
C3
1u
V1
V2
30.000
0.000
-30.000
-60.000
-90.000
-120.000
10
100
1K
10K
100K
F (Hz)
-20dB/decade
-40dB/decade

Adding more RC stages, makes the characteristics sharper