ESc201 : Introduction to Electronics

Sinusoidal Steady State Analysis (contd)

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Circuit Analysis Using Phasors and Impedances

1. Replace the time descriptions of the voltage and current sources with the corresponding phasors. (All of the sources must have the same frequency)

2. Express components by their complex impedances:

   Replace inductances by their complex impedances

$$Z_L = j\omega L$$

   Replace capacitances by their complex impedances

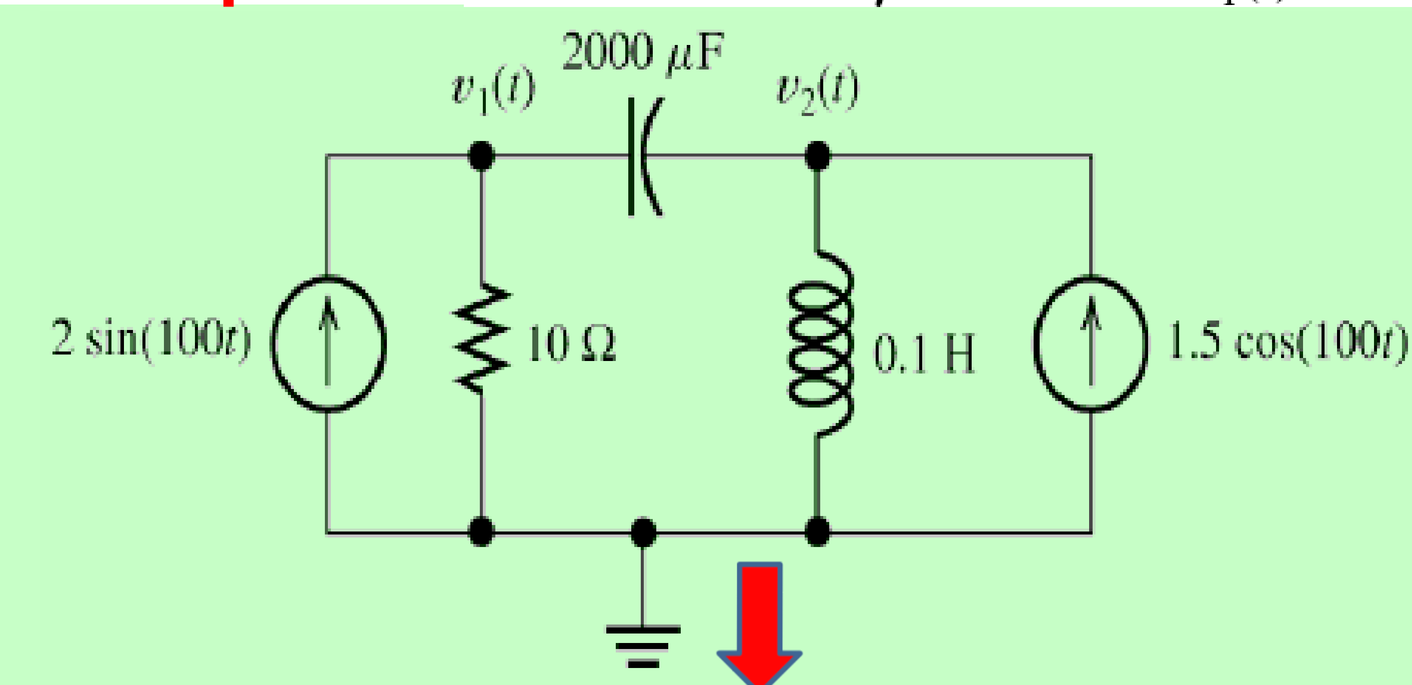
$$Z_C = 1/(j\omega C)$$

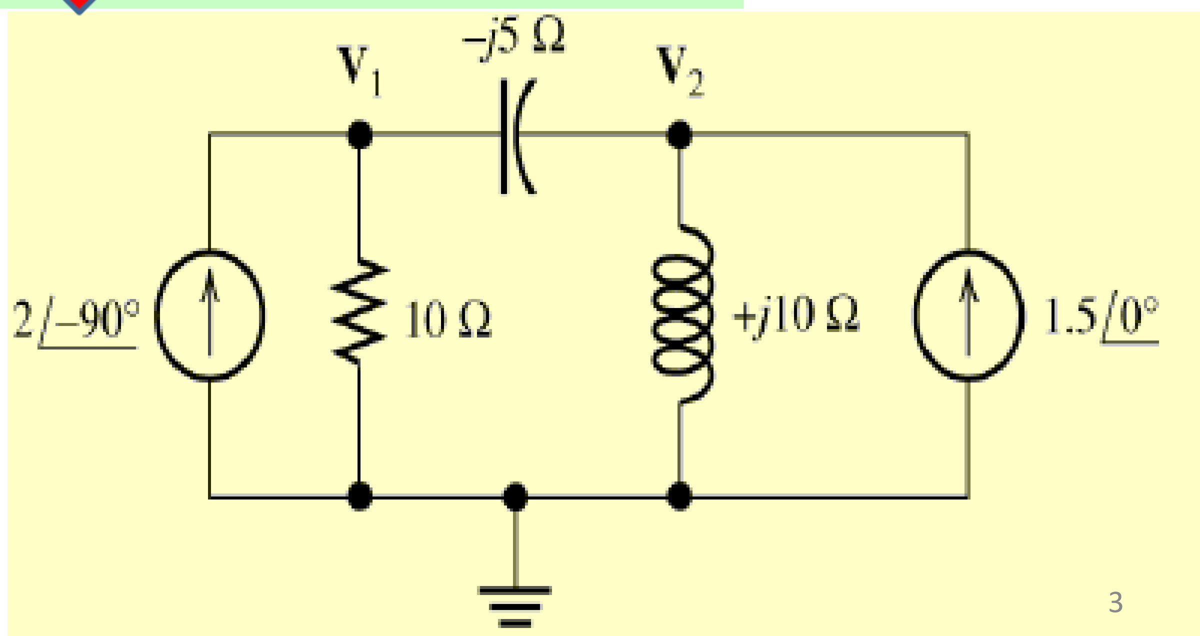
   Resistances have impedances equal to their resistances

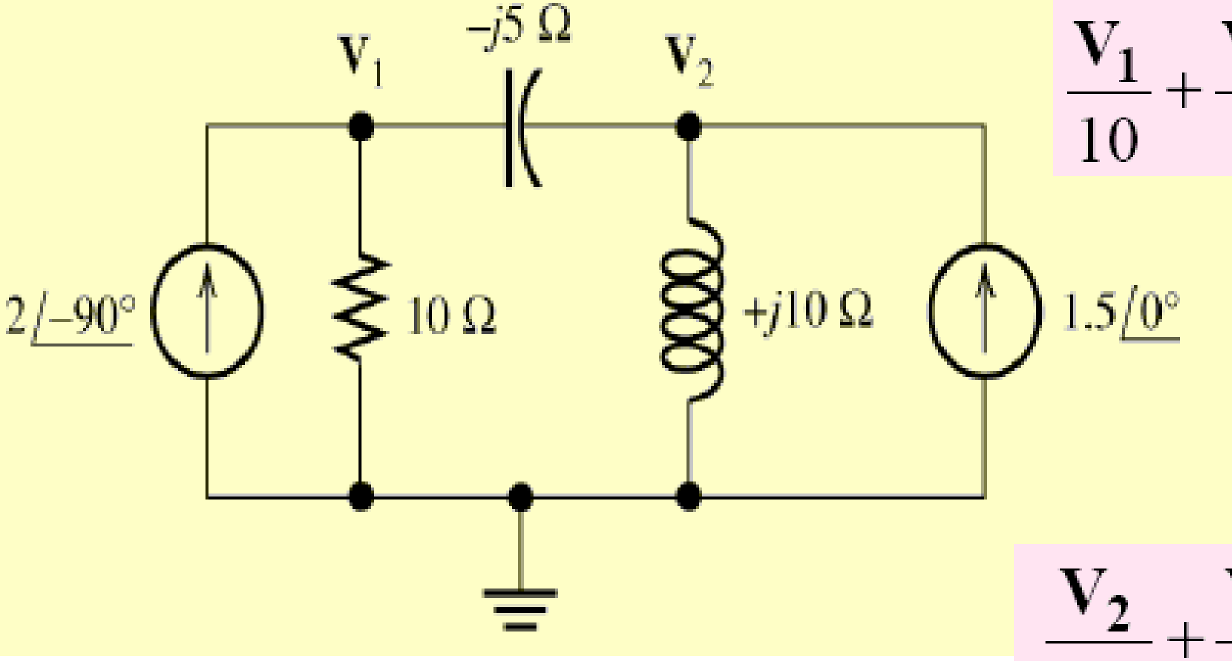
$$Z_R = R$$

3 Analyze the circuit using any of the techniques studied earlier performing the calculations with complex arithmetic

Example-8 Use nodal analysis to find $v_1(t)$ in steady state







$$\frac{\mathbf{V_1}}{10} + \frac{\mathbf{V_1} - \mathbf{V_2}}{-j5} = 2\angle -90^\circ \qquad (1)$$

$$\frac{\mathbf{V_2}}{j10} + \frac{\mathbf{V_2} - \mathbf{V_1}}{-j5} = 1.5\angle 0^\circ \qquad (2)$$

$$(0.1 + j0.2)\mathbf{V_1} - j0.2\mathbf{V_2} = -j2 \quad (1a)$$

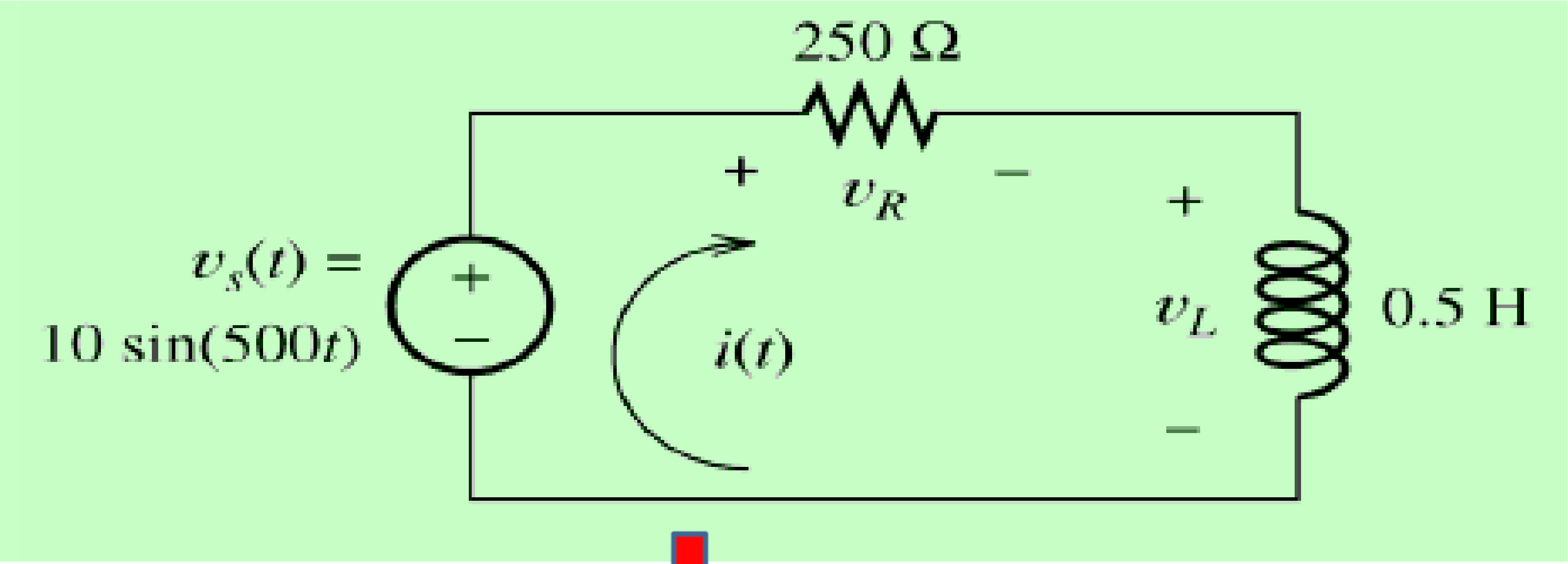
$$-j0.2\mathbf{V_1} + j0.1\mathbf{V_2} = 1.5 \qquad (2a)$$

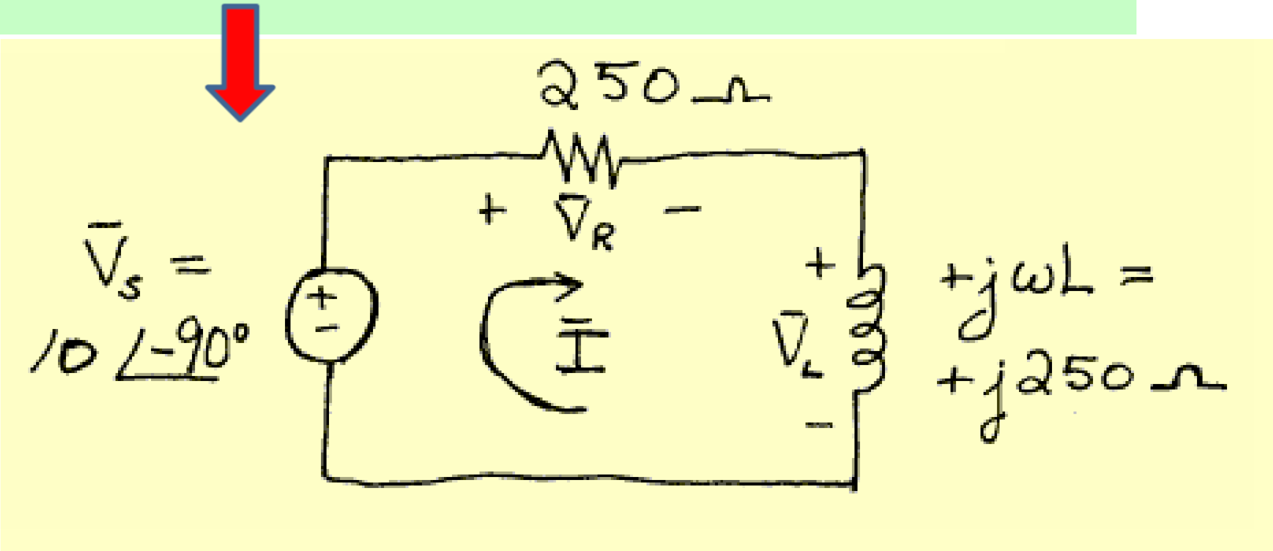
Solving $(1a)$ and $(1b)$ $\quad V_1 = 16.1\angle 29.7^\circ$

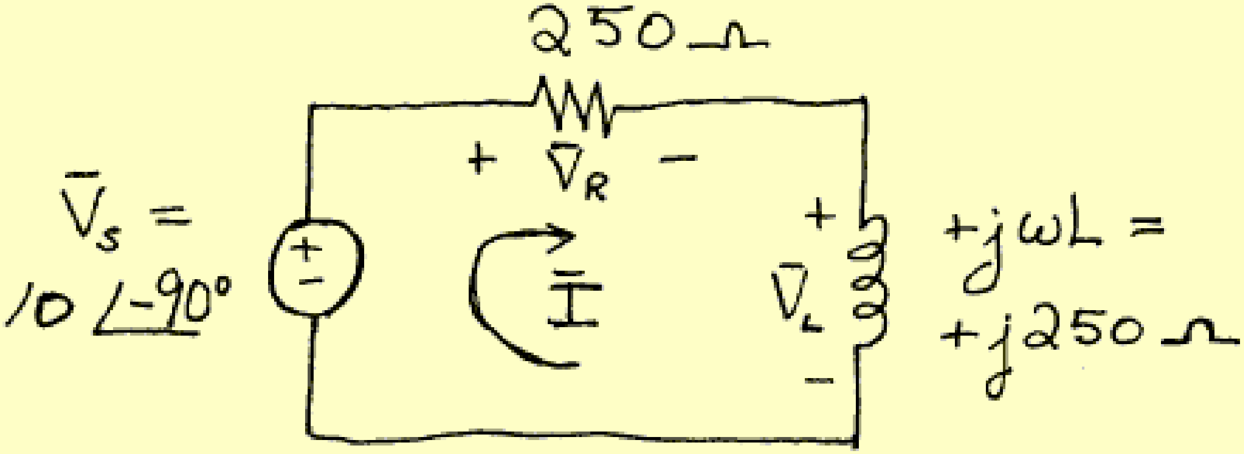
$$v_1(t) = 16.1\cos(100t + 29.7^\circ)$$

Example-9

In the circuit shown, find steady-state current, phasor voltages and construct a phasor diagram





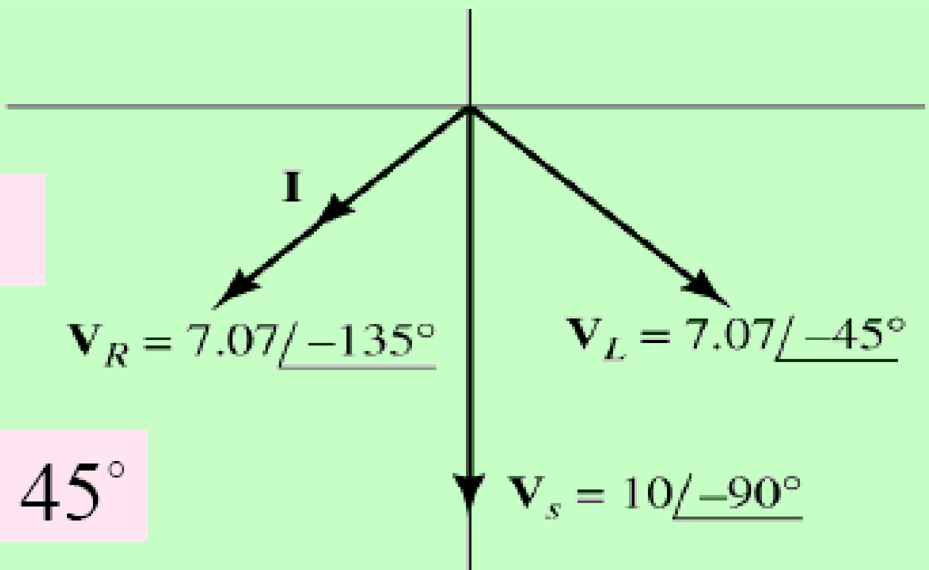


$$\mathbf{I} = \frac{\mathbf{V}_s}{Z} = \frac{10\angle -90^\circ}{250 + j250} = 28.28\angle -135^\circ \text{ mA}$$

$$i(t) = 28.28\cos(500t - 135^\circ) \text{ mA}$$

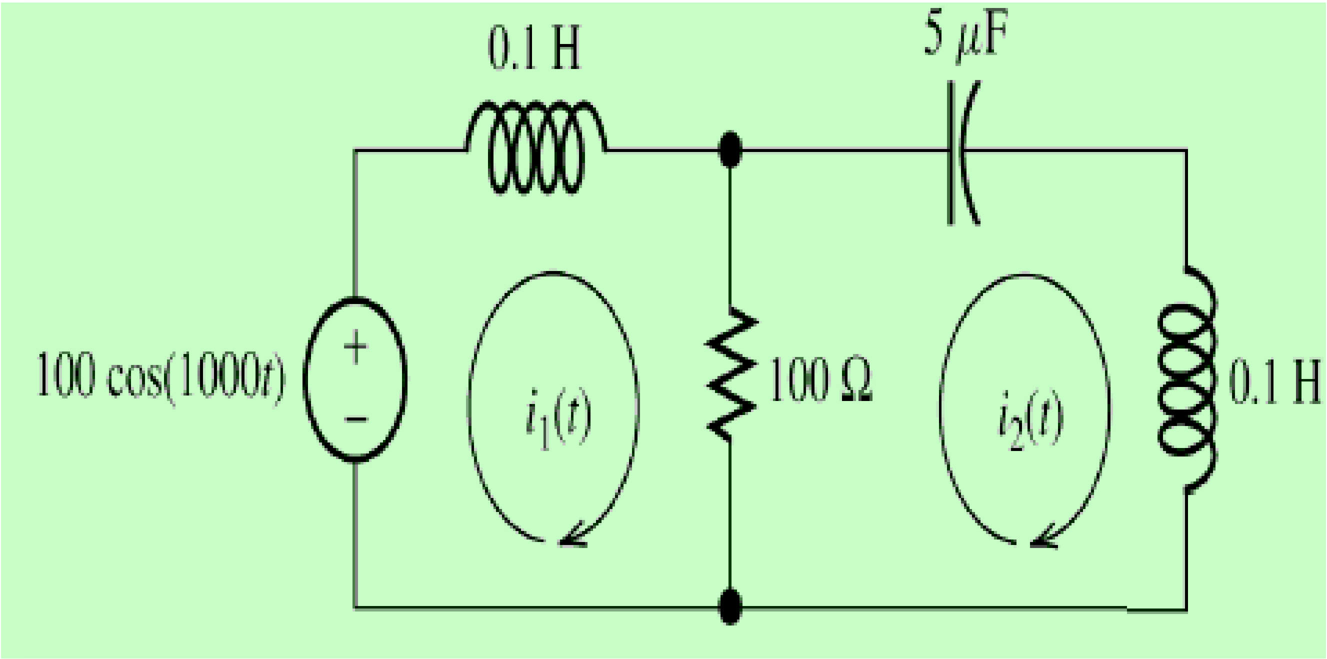
$$\mathbf{V}_R = R\mathbf{I} = 7.07\angle -135^\circ$$

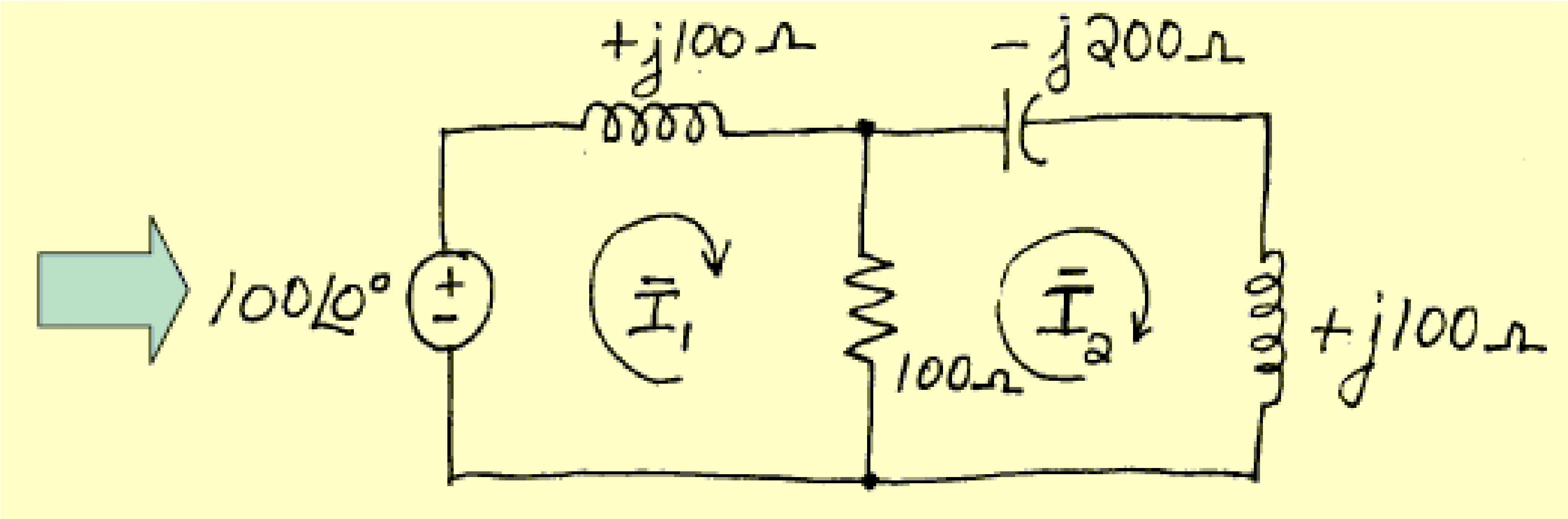
$$\mathbf{V}_L = j\omega L\mathbf{I} = 7.07\angle -45^\circ$$

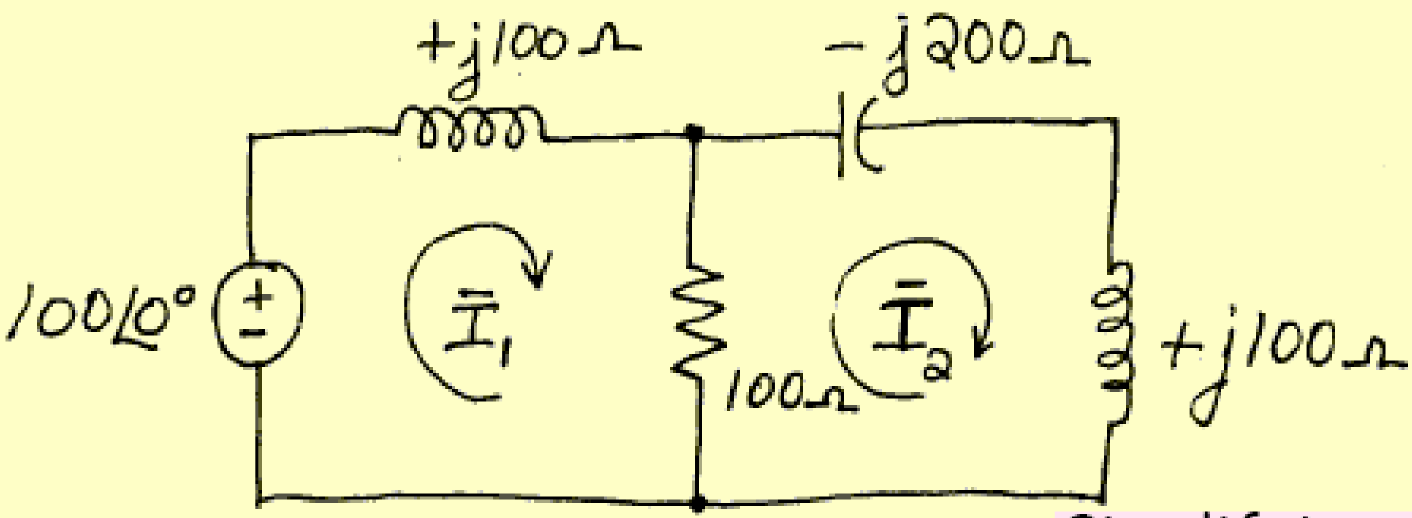


Example-10

Solve for mesh currents:







$$j100I_1 + 100(I_1 - I_2) = 100$$

$$-j200I_2 + j100I_2 + 100(I_2 - I_1) = 0$$

Simplifying, we have

$$(100 + j100)I_1 - 100I_2 = 100$$

$$-100I_1 + (100 - j100)I_2 = 0$$

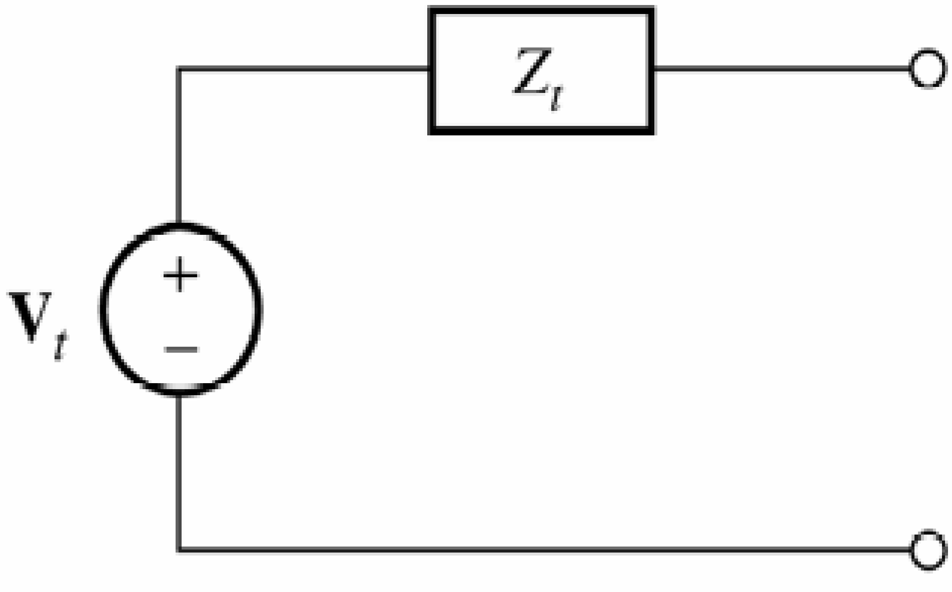
Solving we find

$$I_1 = 1.414\angle -45^\circ \text{ A and } I_2 = 1\angle 0^\circ \text{ A}$$

$$i_1(t) = 1.414\cos(1000t - 45^\circ)$$

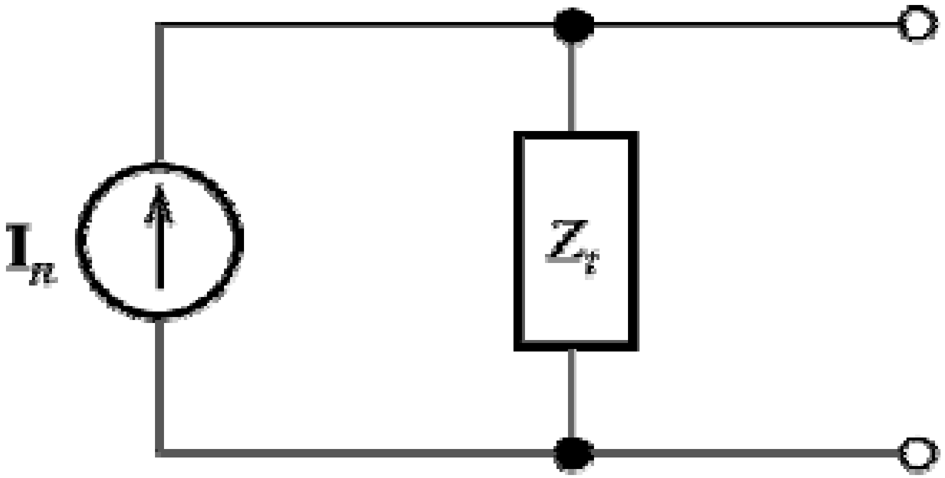
$$i_2(t) = \cos(1000t)$$

Thévenin and Norton Equivalent Circuits



The Thévenin equivalent for an ac circuit consists of a phasor voltage source $\mathbf{V}_t$ in series with a complex impedance Z_t.

Thévenin and Norton Equivalent Circuits



The Norton equivalent circuit consists of a phasor current source $\mathbf{I}_n$ in parallel with the complex impedance Z_t.

Superposition Theorem is also applicable for independent sinusoidal sources

Thévenin and Norton Equivalent Circuits

- The Thévenin voltage is equal to the open-circuit phasor voltage of the original circuit

$$V_t = V_{oc}$$

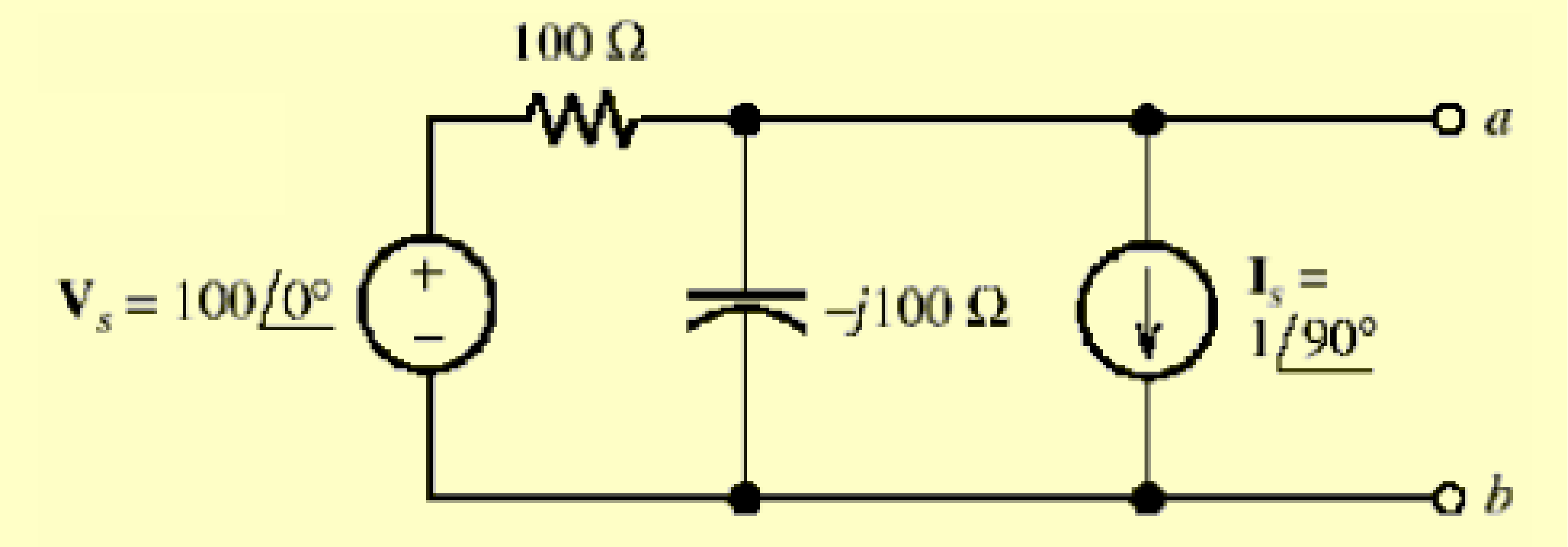
- We can find the Thévenin impedance by zeroing the independent sources and determining the impedance looking into the circuit terminals

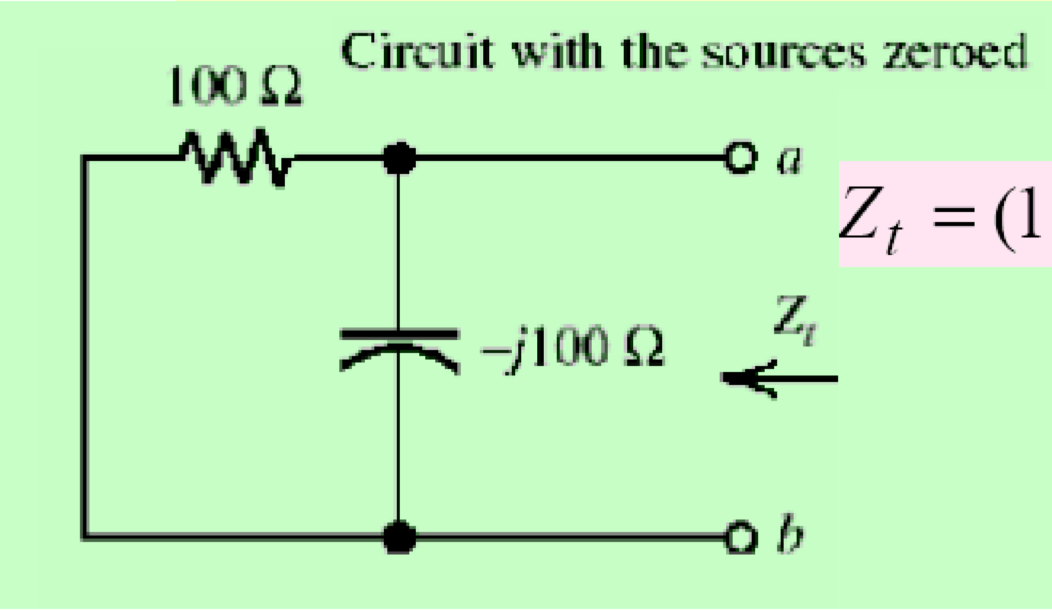
- The Thévenin impedance equals the open-circuit voltage divided by the short-circuit current

$$Z_t = \frac{V_{oc}}{I_{sc}} = \frac{V_t}{I_{sc}} \qquad I_n = I_{sc}$$

Example-11

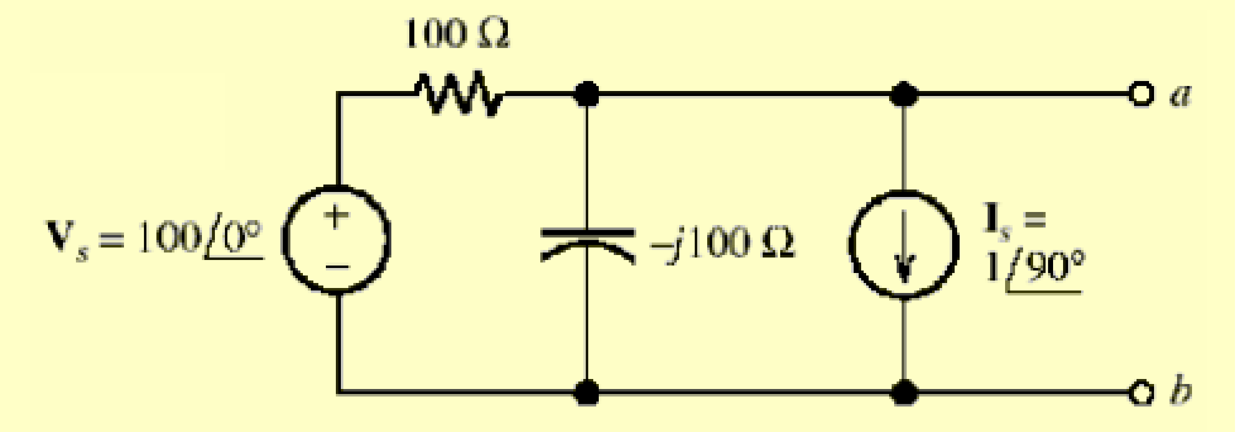
Find Thévenin and Norton equivalents for the following circuit

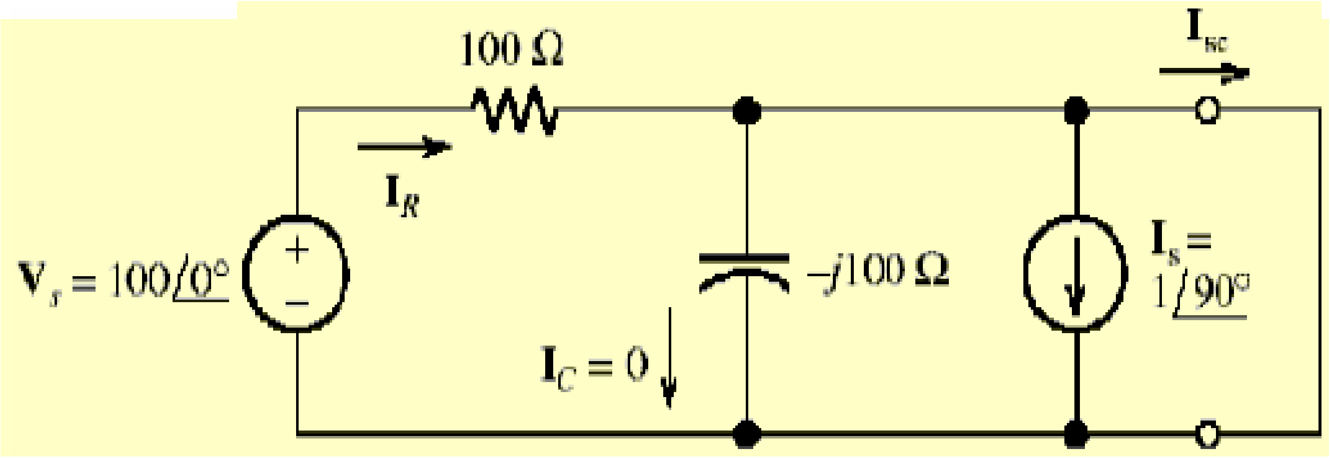




$$Z_t = (100) \| (-j100) = 70.71\angle -45^\circ$$

$$= 50 - j50\Omega$$





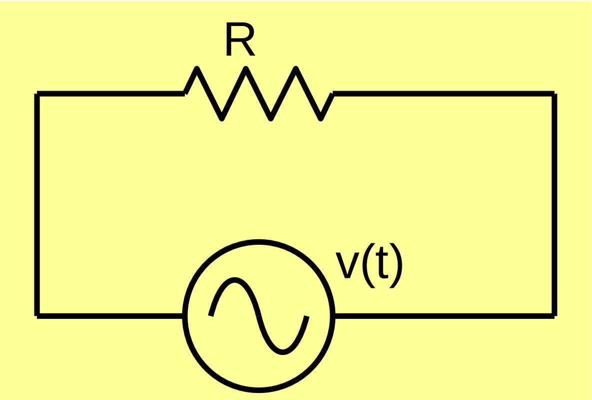
$$I_{sc} = I_R - I_s = \frac{100\angle 0°}{100} - 1\angle 90°$$

$$= 1 - j = 1.414\angle -45°$$

$$V_t = I_{sc} Z_t = 100\angle -90°$$

Power dissipation in RLC Circuits

For Resistance



$$p = \frac{v(t)^2}{R}$$

$$p_{avg} = \frac{1}{T}\int_0^T \frac{v(t)^2}{R}\,dt$$

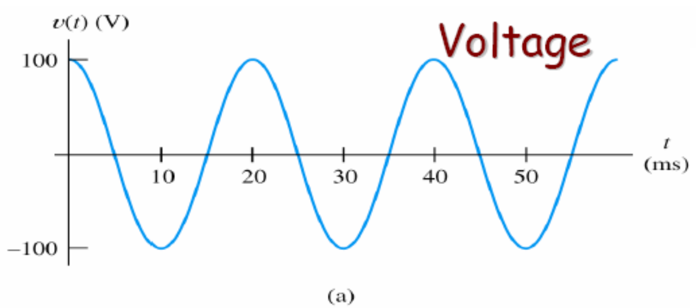
$$p_{avg} = \frac{V_{rms}^2}{R}$$

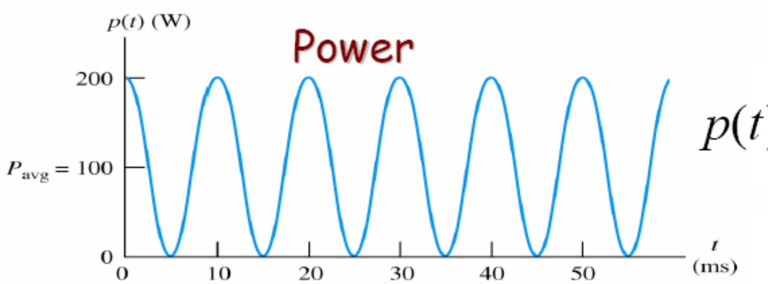
$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

$$p_{avg} = I_{rms}^2 R$$

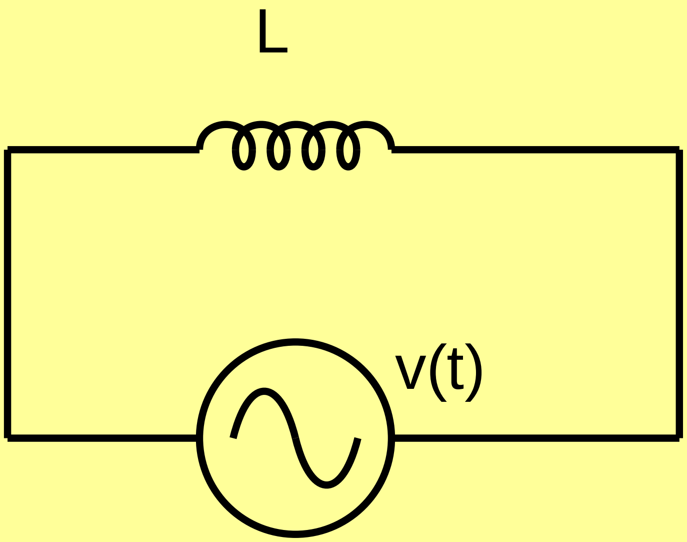
$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

Voltage applied to a 50-Ω resistance





$$p(t) = 200\cos^2(100\pi t)\ \text{W}$$

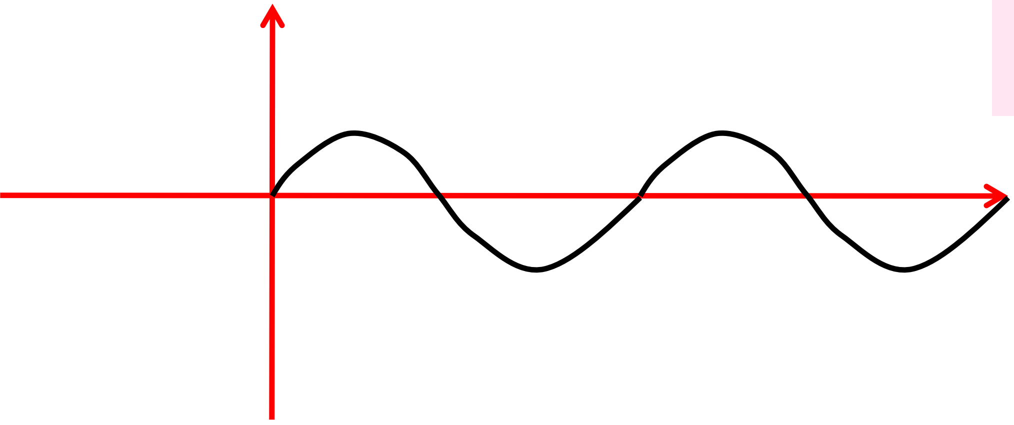


$$v(t) = V_m \cos(\omega t)$$

$$i(t) = I_m \cos(\omega t - 90°) = I_m \sin(\omega t)$$

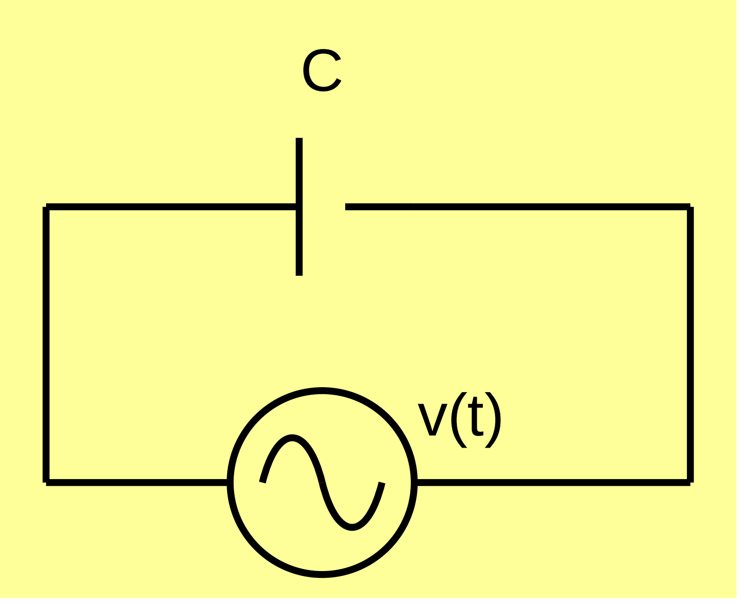
$$p(t) = v(t)i(t) = V_m I_m \cos(\omega t)\sin(\omega t)$$

$$= \frac{V_m I_m}{2} \sin(2\omega t)$$



- Average power absorbed by an inductor is zero

$$p_{avg} = 0$$



$$v(t) = V_m \cos(\omega t)$$

$$i(t) = I_m \cos(\omega t + 90^\circ) = -I_m \sin(\omega t)$$

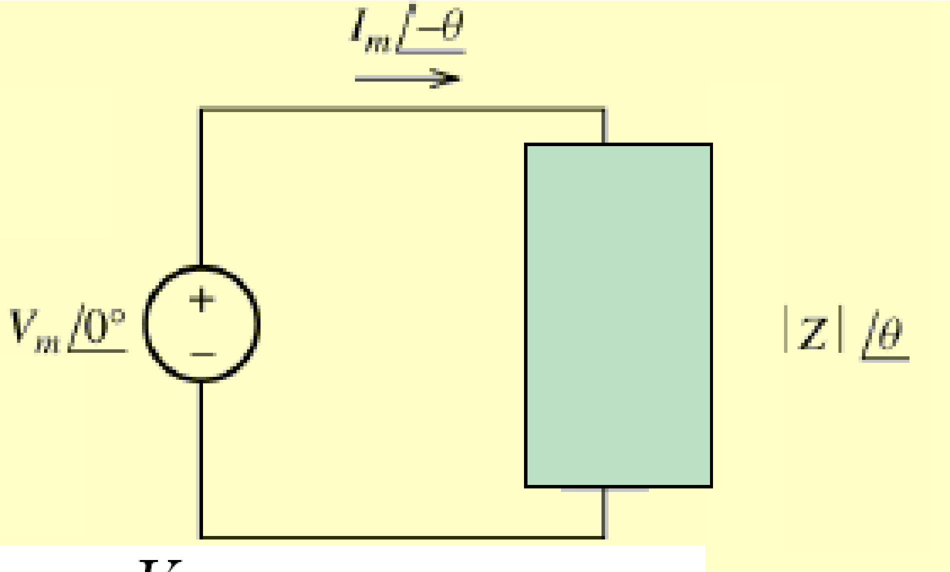
$$p(t) = v(t)i(t) = -V_m I_m \cos(\omega t)\sin(\omega t)$$

$$= -\frac{V_m I_m}{2}\sin(2\omega t)$$

$$p_{avg} = 0$$

- Average power absorbed by a capacitor is zero

General Rule



$$v(t) = V_m Cos(\omega t)$$

$$i(t) = I_m Cos(\omega t - \theta)$$

Average Power:

$$p = \frac{1}{T}\int_0^T v(t) \times i(t)\,dt$$

$$I = \frac{V}{Z} = \frac{V_m \angle 0°}{|Z| \angle \theta} = I_m \angle -\theta$$

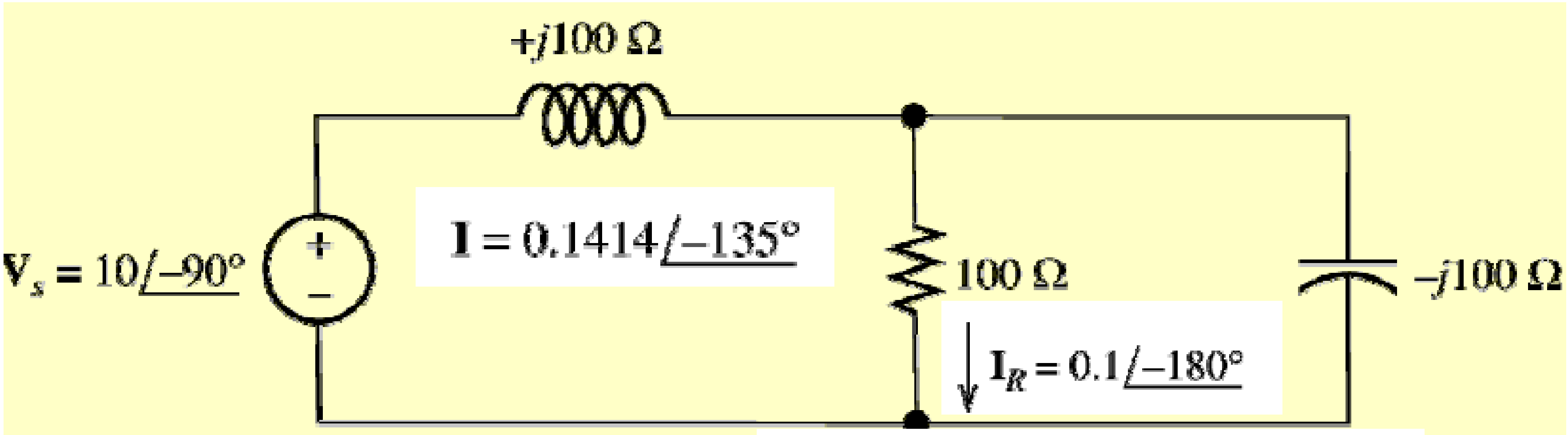
$$P = V_{\text{rms}} I_{\text{rms}} \cos\theta$$

Power Factor $PF = \cos \theta$

For a resistor PF = 1, while for inductor and capacitor it is 0

θ is phase difference between voltage and current

Find the average power drawn from the supply



$$P = V_{srms} I_{rms} \cos(\theta)$$

$$V_{rms} = \frac{|\mathbf{V_s}|}{\sqrt{\mathbf{2}}} = \frac{10}{\sqrt{2}} = 7.071V$$

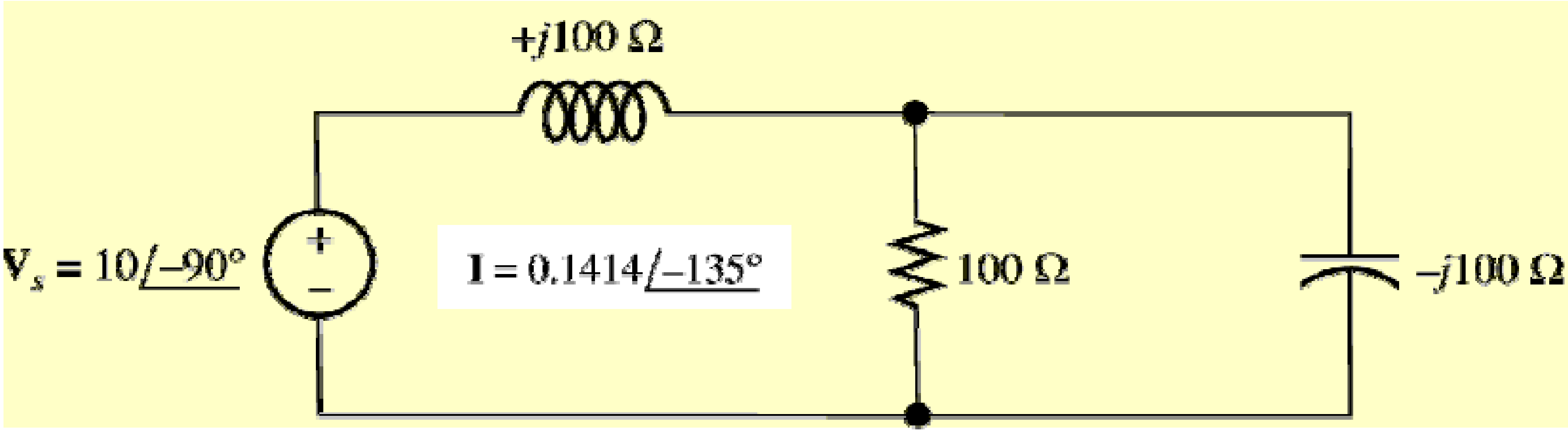
$$P = V_{srms} I_{rms} \cos(\theta)$$
$$= 7.071 \times 0.1 \times \cos(45°)$$
$$= 0.5\ \mathbf{W}$$

$$I_{rms} = \frac{|\mathbf{I}|}{\sqrt{\mathbf{2}}} = \frac{0.1414}{\sqrt{2}} = 0.1A$$

Where is this power dissipated?

$$I_{Rrms} = \frac{0.1}{\sqrt{2}} = 0.071$$

$$P = V_{Rrms} \times I_{Rrms} \cos\theta = I_{Rrms}^2 \times R = 0.5W$$



Average power dissipated through the inductor

$$V_L = j100 \times I = 14.14\angle -45$$

$$\theta = \angle -45 + 135 = \angle 90$$

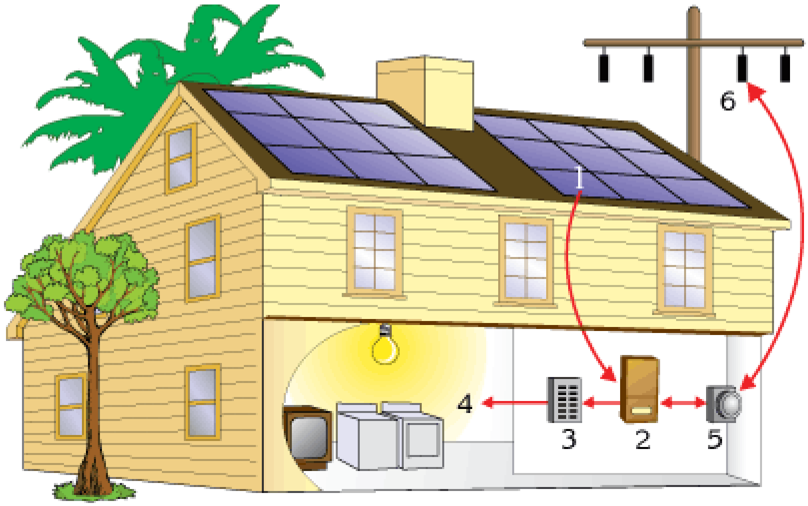
$$P = V_{\text{rms}} I_{\text{rms}} \cos\theta$$

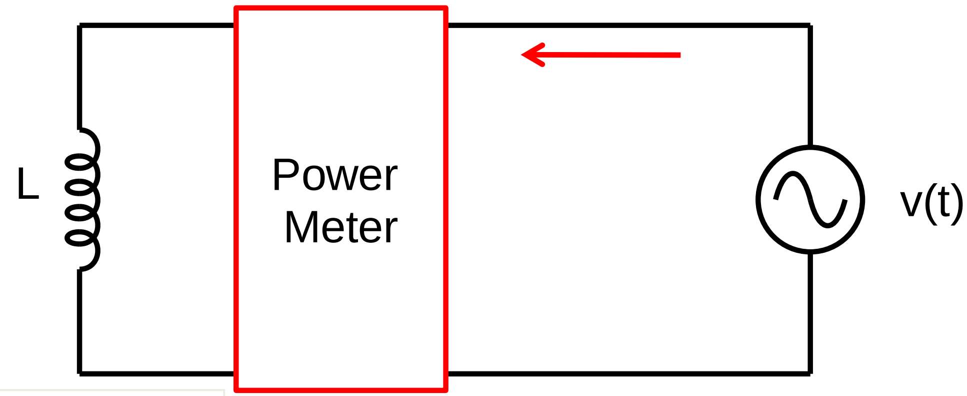
$$P = 0$$

$$P = I_{Rrms}^2 \times R = 0.5W$$

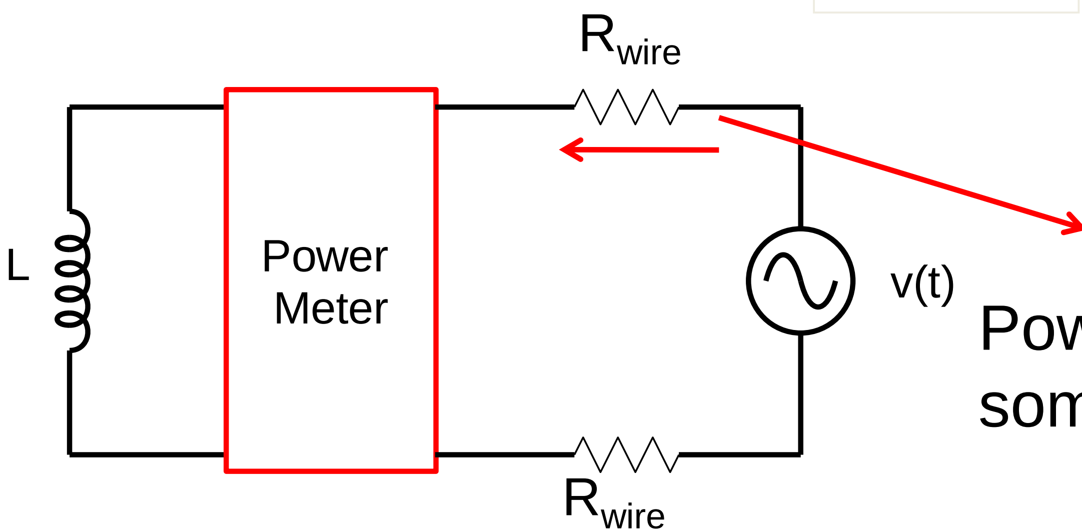
$$P = I_{Rrms}^2 \times R = \frac{1}{2}\left|I_R\right|^2 R$$

Should a Power company charge a person even though power consumed is zero?





$$p_{avg} = 0$$



Power is dissipated and somebody has to pay for it.

Average Power: $P = V_{\text{rms}} I_{\text{rms}} \cos\theta$

Reactive Power:

$$Q = V_{\text{rms}} I_{\text{rms}} \sin\theta$$ (Volt Amperes Reactive (VAR))

- No average power is consumed in a pure inductive or capacitive load

- But, reactive power has current associated with it and causes loss of power in transmission lines and transformers

- Electric-power companies charge their industrial customers for reactive power

Apparent Power

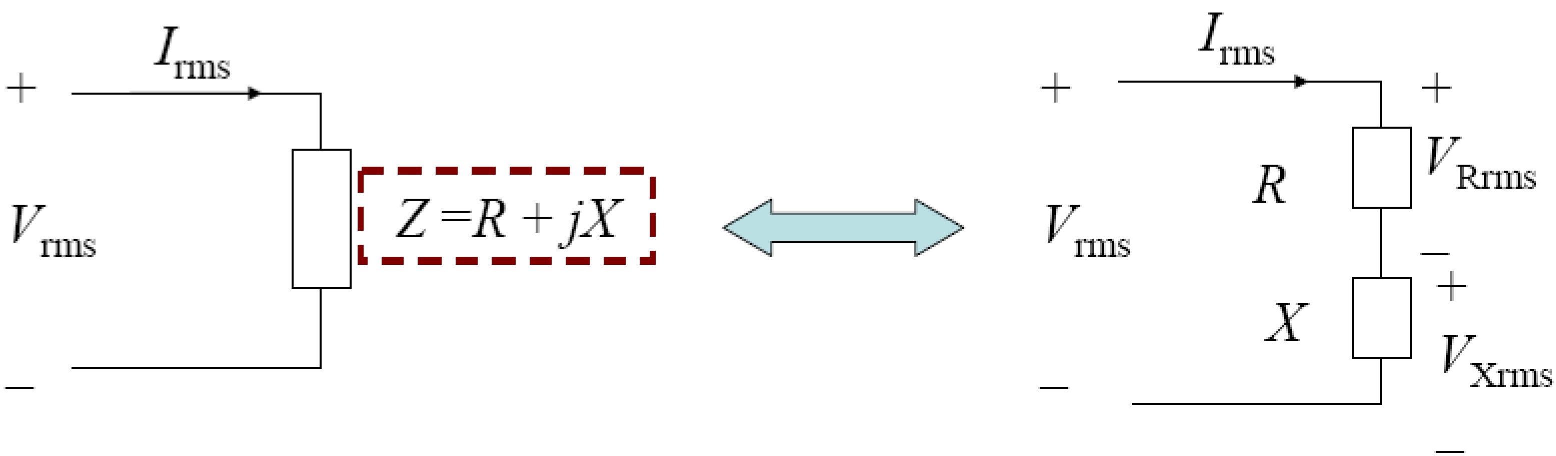
$$\text{Apparent power} = V_{\text{rms}} I_{\text{rms}}$$ Units: volt-amperes (VA)

$$P = V_{\text{rms}} I_{\text{rms}} \cos\theta \qquad Q = V_{\text{rms}} I_{\text{rms}} \sin\theta$$

$$\begin{aligned} P^2 + Q^2 &= V_{rms}^2 I_{rms}^2 \cos^2(\theta) + V_{rms}^2 I_{rms}^2 \sin^2(\theta) \\ &= \left(V_{rms} I_{rms}\right)^2 \\ &= (\text{apparant power})^2 \end{aligned}$$

$$\boxed{\text{Apparent Power} = \sqrt{P^2 + Q^2}}$$

Apparent Power



$$P = I_{\text{rms}}^2 R \quad \text{or} \quad P = \frac{V_{\text{Rrms}}^2}{R}$$

$$Q = I_{\text{rms}}^2 X \quad \text{or} \quad Q = \frac{V_{\text{Xrms}}^2}{X}$$

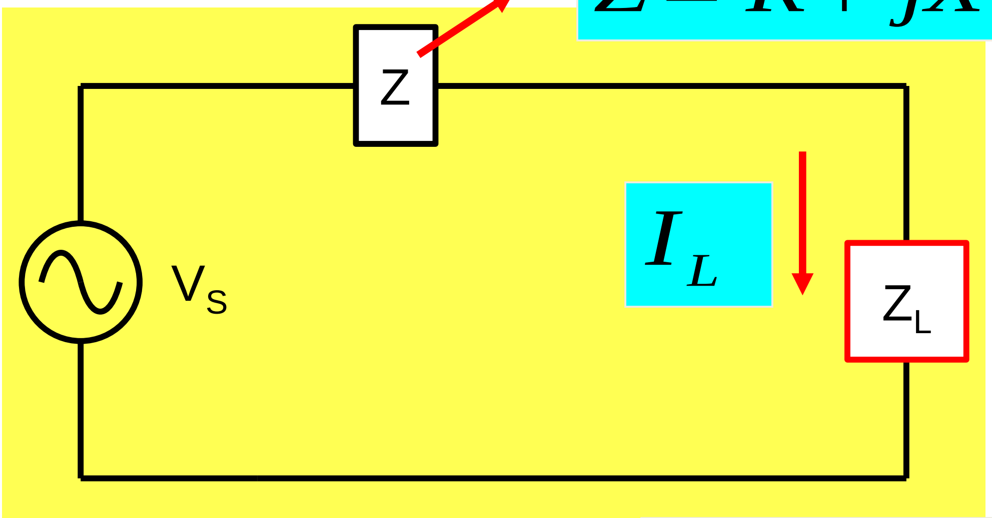
Be careful!

Voltage across R or voltage across X but not Z

$$Q = V_{rms} I_{rms} \sin\theta = I_{rms} \,|\, Z \,|\, I_{rms} \sin\theta = I_{rms}^2 \,|\, Z \,|\, \frac{X}{|\, Z \,|}$$

Maximum Power Transfer for sinusoidal input

$$Z = R + jX$$



$$I_L$$

$$Z_L = R_L + jX_L$$

$$P = I_{Rrms}^2 \times R = \frac{1}{2}\left|I_R\right|^2 R$$

$$I_L = \frac{V_S}{R + R_L + j(X + X_L)}$$

$$P_L = \frac{1}{2}\frac{V_S^2}{(R+R_L)^2 + (X + X_L)^2} R_L$$

For maximum load power : $X_L = -X$

$$P_L = \frac{V_S^2}{(R+R_L)^2} R_L$$

Choose $R_L = R$ to maximize load power

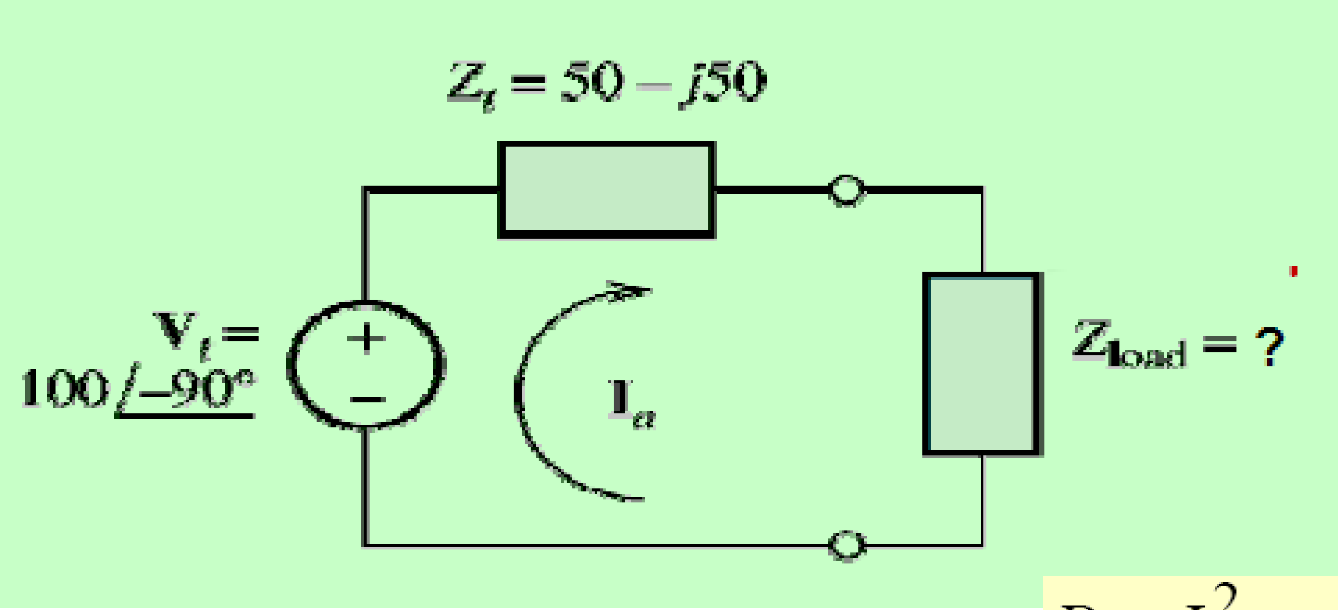
$$Z_L = \bar{Z}$$

Maximum power is transferred to the load when load is complex conjugate of source impedance

Maximum Power Transfer for sinusoidal input

Maximum power is transferred to the load when load is complex conjugate of source impedance

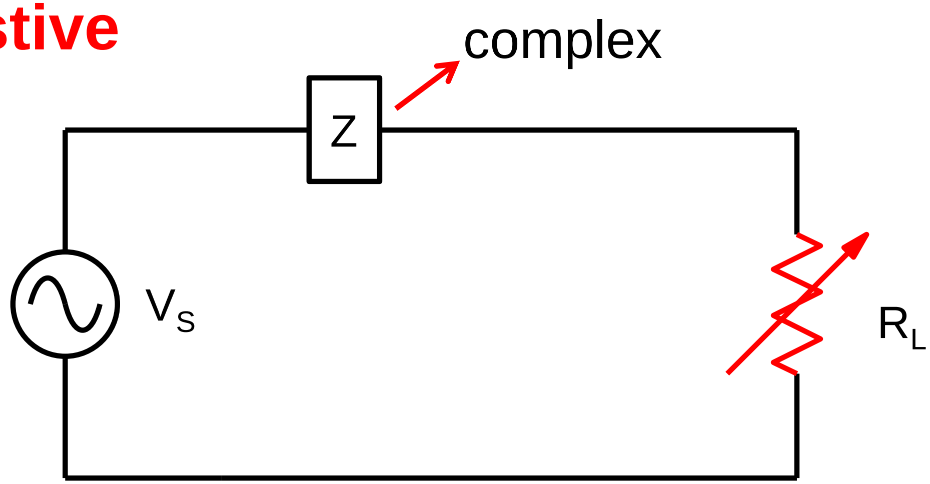
$$Z_L = \bar{Z}$$



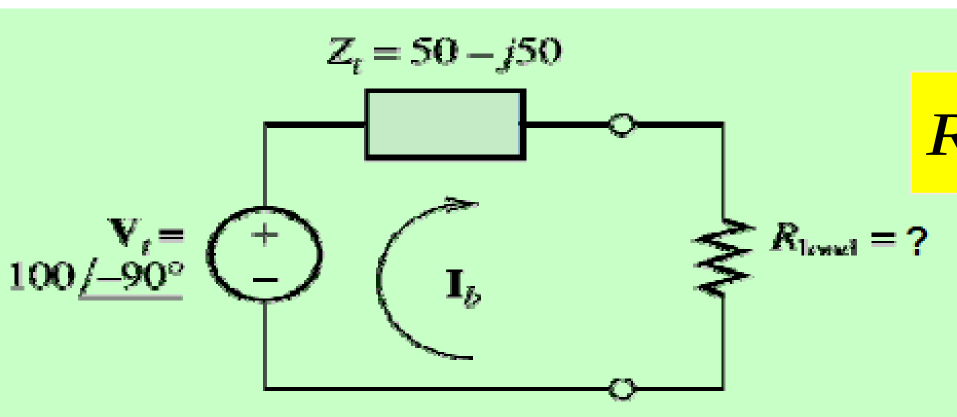
$$P = I_{arms}^2 R_{load}$$
$$= (\frac{1}{\sqrt{2}})^2 \times 50 = 25\mathbf{W}$$

$$Z_L = 50 + j50\,\Omega$$

Maximum Power Transfer for sinusoidal input when load is Resistive



Maximum power is transferred to the load when $\boxed{R_L = |Z|}$



$$R_L = |Z| = \sqrt{50^2 + 50^2} = 70.71\Omega$$

$$P = I_{brms}^2 R_{load}$$
$$= \left(\frac{0.7654}{\sqrt{2}}\right)^2 \times 70.71 = 20.7$$

Important Announcements

1. Major Quiz-1 on Thursday, Sept 1 during tutorial in the assigned tutorial rooms.

2. Syllabus: Lecture 1 to Lecture 12

3. Assignment-4 uploaded on mooKIT