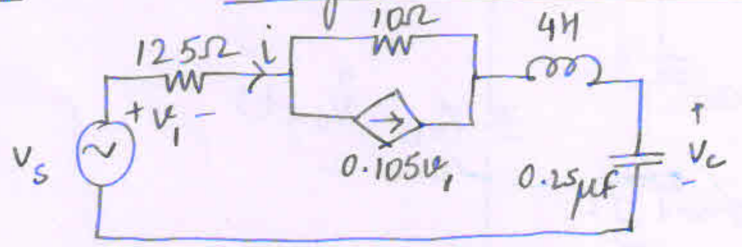
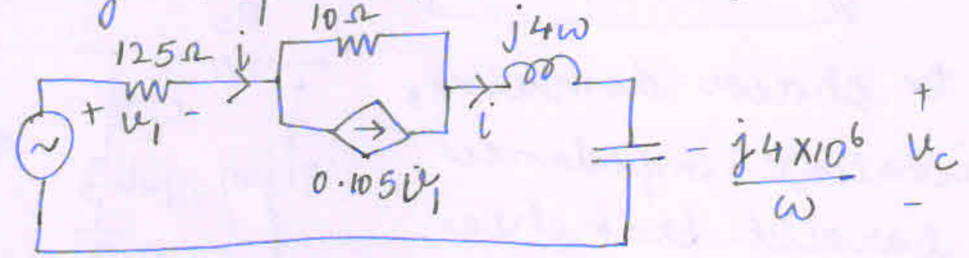
ESC 201 Assignment 5 Solutions

Ans 1.



Let the current flowing in the circuit be i.

Converting to phasor domain,



$$v_1 = 125i$$

Applying KVL,

$$v_S - 125i - 10(i - 0.105v_1) - j4\omega i + \frac{j4\times10^6}{\omega}i = 0$$

$$\Rightarrow v_S - 125i - 10i + 1.05(125i) - j4\omega i + \frac{j4\times10^6}{\omega}i = 0$$

$$\Rightarrow v_S = 3.75i + j4\omega i - \frac{j4\times10^6}{\omega}i$$

$$\Rightarrow \frac{v_S}{i} = 3.75 + j4\left(\omega - \frac{10^6}{\omega}\right) \quad \text{— (1)}$$

$$v_c = -\frac{j4\times10^6}{\omega}i$$

$$\text{or } \frac{v_c}{i} = -\frac{j4\times10^6}{\omega} \quad \text{— (2)}$$

from (1) and (2),

$$\frac{V_c}{V_S} = \frac{-j4\times10^6/\omega}{3.75 + j4(\omega - 10^6/\omega)} = \frac{4\times10^6/\omega}{-4(\omega - 10^6/\omega) + j3.75}$$

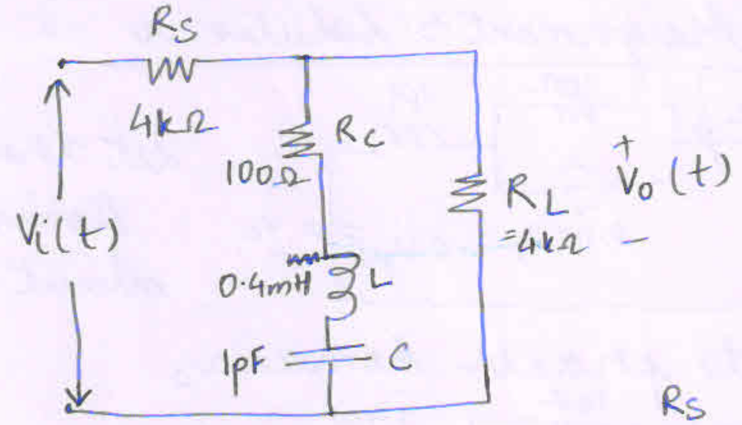
$$\frac{V_c}{V_S} = \frac{4\times10^6\omega}{-4\left(\omega - \frac{10^6}{\omega}\right) + j3.75} \times \frac{-4\left(\omega - \frac{10^6}{\omega}\right) - j3.75}{-4\left(\omega - \frac{10^6}{\omega}\right) - j3.75} \qquad \mathrm{Im}\left[\frac{V_c}{V_S}\right] = 0$$

$$\Rightarrow \omega^2 = 10^6 \text{ or } \omega = 10^3 \text{ rad/sec}$$

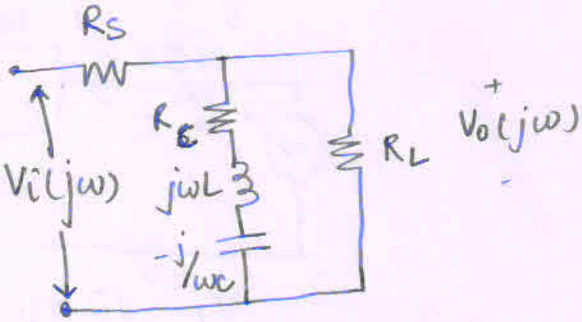
$$|H(j\omega)|_{\omega=1000} = \frac{4\times10^6/10^3}{3.75} = 1.066\times10^3$$

$$\therefore 20\log|H(j\omega)|_{\omega=10^3} = 60.56 \text{ dB}$$

Ans 2.



Converting to phasor domain, let the equivalent impedance of the two parallel branches be Z_1.



$$\therefore Z_1 = R_L \,||\, \left[R_C + j\left(\omega L - \frac{1}{\omega C}\right)\right]$$

$$\Rightarrow Z_1 = \frac{R_L\left[R_C + j\left(\omega L - \frac{1}{\omega C}\right)\right]}{R_L + R_C + j\left(\omega L - \frac{1}{\omega C}\right)}$$

Applying voltage division,

$$V_o(j\omega) = \frac{Z_1 \times V_i(j\omega)}{Z_1 + R_S}$$

$$\Rightarrow \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{Z_1}{Z_1 + R_S}$$

$$\Rightarrow \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{\dfrac{R_L\left[R_C + j\left(\omega L - \frac{1}{\omega C}\right)\right]}{R_L + R_C + j\left(\omega L - \frac{1}{\omega C}\right)}}{\dfrac{R_L\left[R_C + j\left(\omega L - \frac{1}{\omega C}\right)\right]}{R_L + R_C + j\left(\omega L - \frac{1}{\omega C}\right)} + R_S}$$

$$\Rightarrow \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{R_L R_C + jR_L\left(\omega L - \frac{1}{\omega C}\right)}{R_L R_C + jR_L\left(\omega L - \frac{1}{\omega C}\right) + R_S R_L + R_S R_C + jR_S\left(\omega L - \frac{1}{\omega C}\right)}$$

$$\Rightarrow \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{R_L R_C + jR_L\left(\omega L - \frac{1}{\omega C}\right)}{R_L R_C + R_S R_L + R_S R_C + j\left(\omega L - \frac{1}{\omega C}\right)(R_S + R_L)}$$

converting to the desired form,

$$\Rightarrow \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{R_L R_C\left[1 + \frac{j}{R_C}\left(\omega L - \frac{1}{\omega C}\right)\right]}{R_L R_C + R_S R_L + R_S R_C\left[1 + \frac{j(R_S + R_L)}{(R_L R_C + R_S R_L + R_S R_C)}\left(\omega L - \frac{1}{\omega C}\right)\right]}$$

$$\therefore K = \frac{R_L R_C}{R_L R_C + R_S R_L + R_S R_C}$$

Substituting the values of R_L, R_S, R_C, $K = \frac{1}{42}$

$$a = \frac{1}{RC}\left(\omega L - \frac{1}{\omega C}\right), \quad b = \frac{(R_S + R_L)\left(\omega L - \frac{1}{\omega C}\right)}{(R_L R_C + R_S R_L + R_S R_C)}$$

For very low frequencies, $\omega \to 0$, $X_L \to 0$ and $X_C \to \infty$.
Therefore, the R-L-C branch is open circuited.

$$|H(j\omega)|_{\omega \to 0} = \frac{R_L}{R_L + R_S} = \frac{4k}{4k + 4k} = 0.5$$

For very high frequencies, $\omega \to \infty$, $X_L \to \infty$, $X_C \to 0$.
~~Agm~~ Again, the R-L-C branch is open circuited.

$$\therefore |H(j\omega)|_{\omega \to \infty} = \frac{R_L}{R_S + R_L} = 0.5$$

For resonance, $\text{Im}[H(j\omega)] = 0$

$$\therefore K\left[\frac{1 + ja}{1 + jb} \times \frac{1 - jb}{1 - jb}\right] = K\left[\frac{1 - jb + ja + ab}{1 + b^2}\right] = K\left[\frac{1 + j(a - b) + ab}{1 + b^2}\right]$$

$$\Rightarrow a - b = 0$$

$$\Rightarrow \omega_0 = \frac{1}{\sqrt{LC}} \quad \Rightarrow f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$f_r = \frac{1}{2\pi\sqrt{0.4\times10^{-3}\times10^{-12}}}\ Hz$$

$$f_r = 7.96\,MHz$$

$$|H(j\omega)|_{\omega=2\pi f_r} = k\left|\frac{1+ab}{1+b^2}\right|$$

Substituting values,

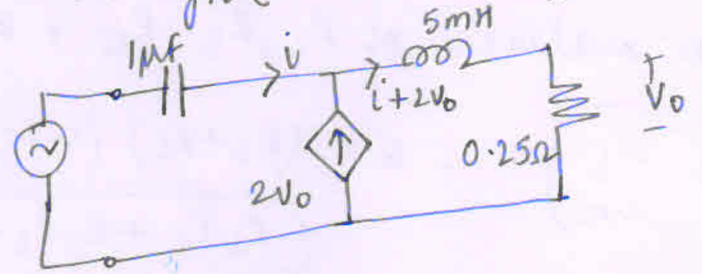
$$\begin{bmatrix} (a)_{f_r} = -0.08961 \\ (b)_{f_r} = -0.00189 \end{bmatrix}$$

$$= \frac{1}{42}\left[\frac{1+(-0.08961)(-0.00189)}{1+(-0.00189)^2}\right]$$

$$= 23.809\times10^{-3}$$

$$\text{Rejection} = 20\log_{10}(23.809\times10^{-3})\,dB = -32.465\,dB.$$

Ans 3.



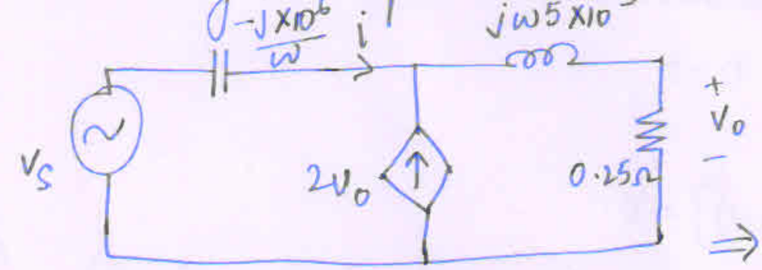
$$V_0 = (i+2V_0)0.25$$

$$\therefore V_0 = 0.25i + 0.5V_0$$

$$\Rightarrow 0.5V_0 = 0.25i$$

$$\text{or } \frac{V_0}{i} = \frac{0.25}{0.5} = \frac{1}{2}$$

Converting to phasor domain,



$$V_s + j\frac{\times10^6}{\omega}i - j\omega5\times10^{-3}(i+2V_0) - V_0 = 0$$

$$\Rightarrow V_s + j\frac{10^6}{\omega}i - j\omega5\times10^{-3}i - j\omega V_0 10^{-2} - V_0 = 0$$

$$\Rightarrow V_s = \left(j\omega5\times10^{-3} - j\frac{10^6}{\omega}\right)(2V_0) + j\omega V_0 10^{-2} + V_0$$

$$\Rightarrow \frac{V_0}{V_s} = \frac{1}{1 + j\left(\omega\times2\times10^{-2} - \frac{2\times10^6}{\omega}\right)}$$

At resonance, $X_L = X_C$

$$\therefore \omega_r = 10^4\ rad/sec$$

Ans4. (a) $P_{av} = V_{rms} \cdot I_{rms} \cdot \cos\phi$

$\Rightarrow 400 = 230 \times I_{rms} \times 0.8$

$\Rightarrow I_{rms} = 2.17 A$

$\cos\phi = \frac{V_R}{V} \Rightarrow V_R = V\cos\phi$

$= 230 \times 0.8$

$= I_{rms} \times R$

$\therefore R = \frac{230 \times 0.8}{2.17}\,\Omega = \underline{84.79\,\Omega}$

$\cos\phi = 0.8 \Rightarrow \phi = \cos^{-1}(0.8) = 36.86^\circ$

$\therefore X_L = R\tan\phi = 84.79 \times 0.75\,\Omega$

$\omega L = 84.79 \times 0.75$

$\Rightarrow L = \frac{84.79 \times 0.75}{2 \times 3.14 \times 50} H = \underline{0.2 H}$

(b) Equivalent impedance, $Z_{eq} = X_C || (X_L + R)$

$Z_{eq} = \frac{-j}{\omega C} || (R + j\omega L)$

$$\Rightarrow Z_{eq} = \frac{\frac{-j}{\omega C}(R + j\omega L)}{\frac{-j}{\omega L} + R + j\omega L}$$

$$= \frac{\frac{-j}{\omega C}(R + j\omega L)}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$= \frac{\frac{-j}{\omega C}(R + j\omega L)}{R + j\left(\omega L - \frac{1}{\omega C}\right)} \times \frac{R - j\left(\omega L - \frac{1}{\omega C}\right)}{R - j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$\Rightarrow Z_{eq} = \frac{\left[-\frac{j}{\omega C}(R+j\omega L)\right]}{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}\left[R-j\left(\omega L-\frac{1}{\omega C}\right)\right]$$

$$\Rightarrow Z_{eq} = \frac{\left[\frac{-jR}{\omega C}+\frac{L}{C}\right]\left[R-j\omega L+\frac{j}{\omega C}\right]}{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}$$

$$\Rightarrow Z_{eq} = \frac{\left[-\frac{jR^2}{\omega C}-\cancel{\frac{RL}{C}}+\frac{R}{\omega^2C^2}+\cancel{\frac{RL}{C}}-j\frac{\omega L^2}{C}+\frac{jL}{\omega C^2}\right]}{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}$$

$$\Rightarrow Z_{eq} = \frac{\left[\frac{R}{\omega^2C^2}+j\left(\frac{L}{\omega C^2}-\frac{\omega L^2}{C}-\frac{R^2}{\omega C}\right)\right]}{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}$$

$$\text{Imag}[Z_{eq}] = \frac{\frac{L}{\omega C^2}-\frac{\omega L^2}{C}-\frac{R^2}{\omega C}}{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2} = 0$$

$$\Rightarrow \frac{L}{\omega C^2}-\frac{\omega L^2}{C}-\frac{R^2}{\omega C}=0$$

$$\Rightarrow \frac{L-\omega^2L^2C-R^2C}{\omega C^2}=0$$

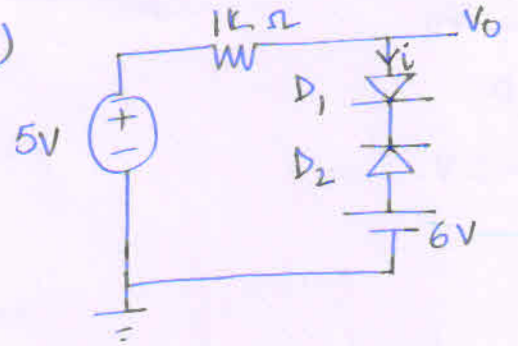
$$\Rightarrow L = (\omega^2L^2+R^2)C$$

$$\text{or} \quad C = \frac{L}{\omega^2L^2+R^2} = \frac{0.2}{(2\times3.14\times50\times0.2)^2+(84.79)^2}\ F$$

$$\Rightarrow C = \frac{0.2}{3943.84+7189.3441}\ F$$

$$\Rightarrow C = 17.96\ \mu F$$

Ans 5. (a)



Case i) If D_1 is on and D_2 is off,

$i = 0$

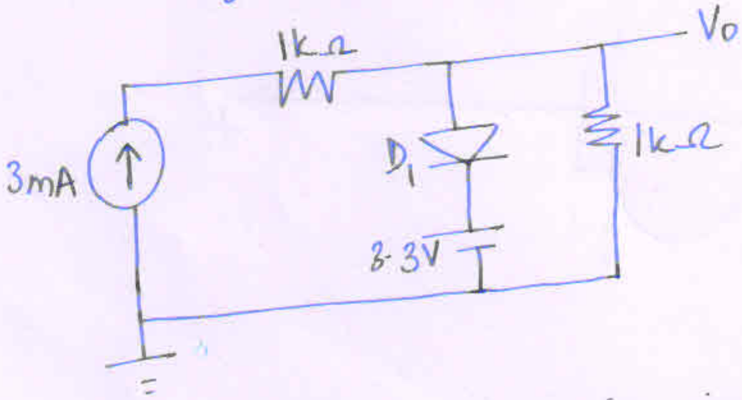
$\Rightarrow V_0 = 5V$

Case ii) If D_1 is off and D_2 is on,

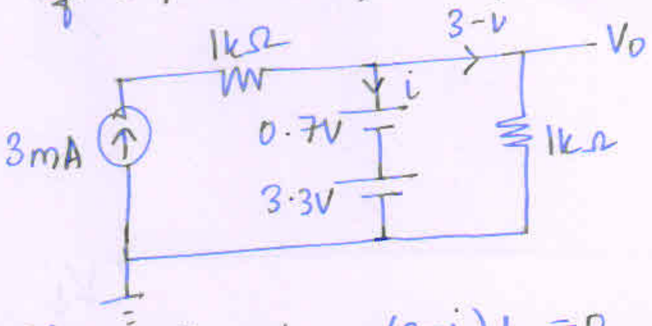
$i = 0$

$\Rightarrow V_0 = 5V$

(b)



Case i) If D_1 is on, the circuit becomes:



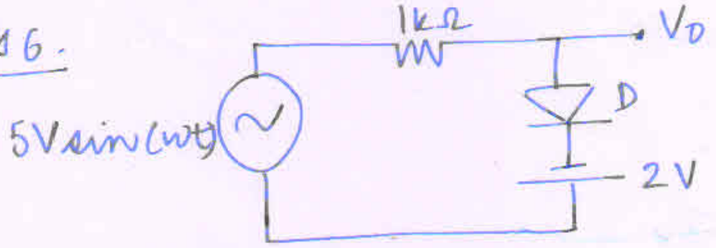
apply KVL, $4 - (3-i)1 = 0$

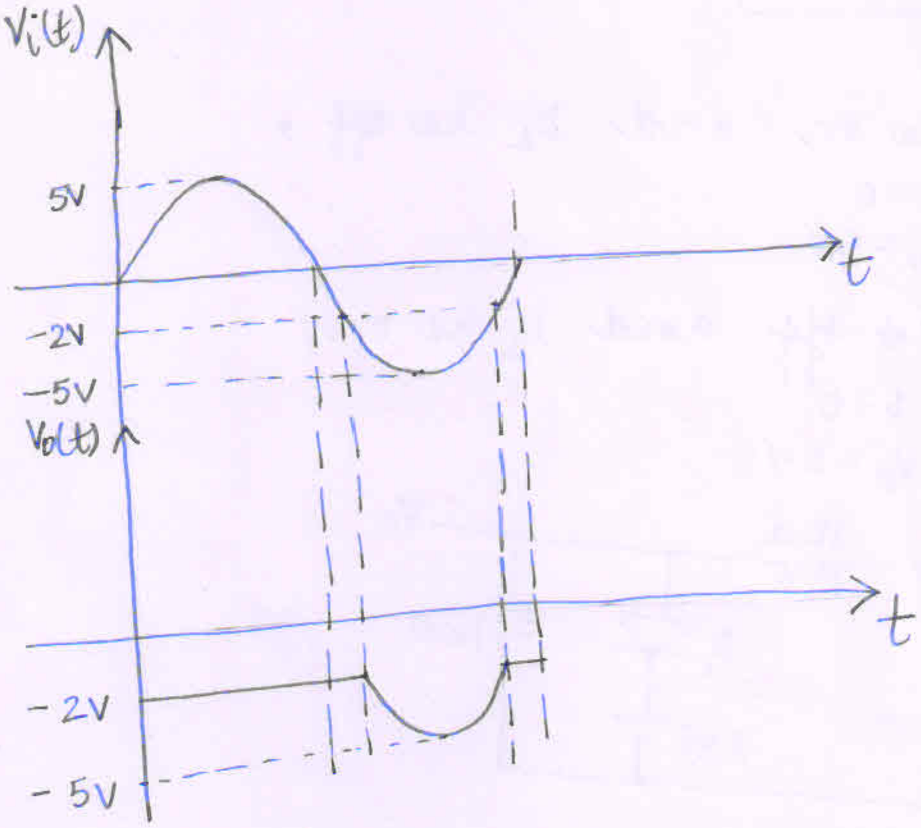
or $i = \cancel{1A}$ $-1mA$ which is not possible.

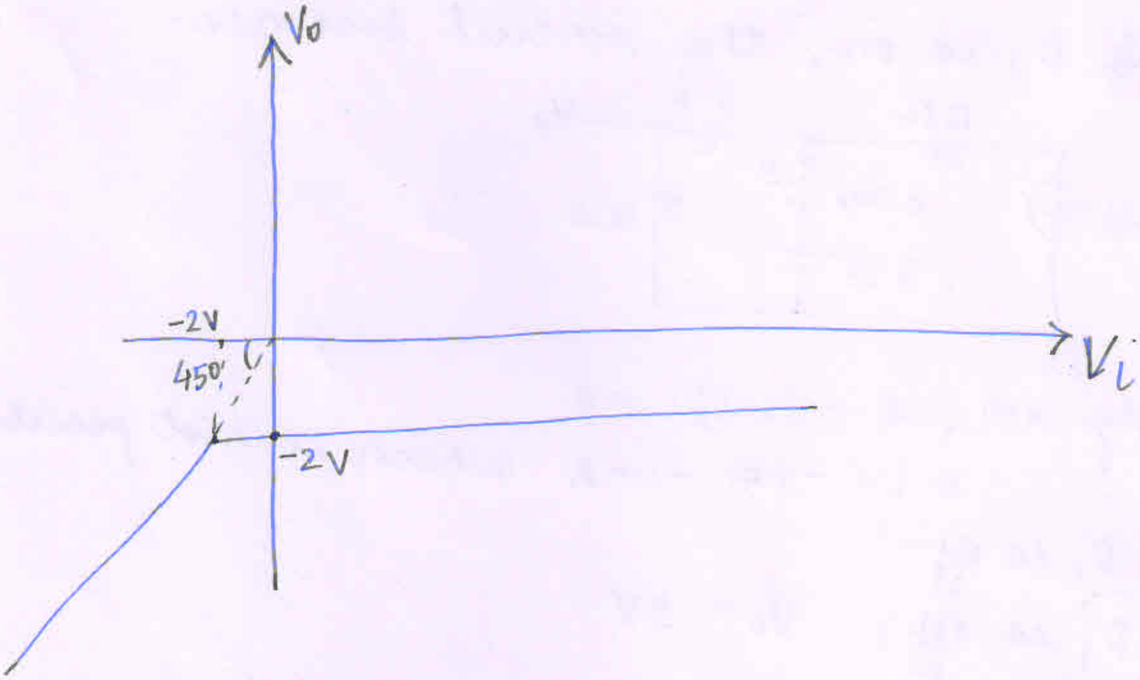
$\therefore D_1$ is off.

Case ii) D_1 is off, $V_0 = 3V$.

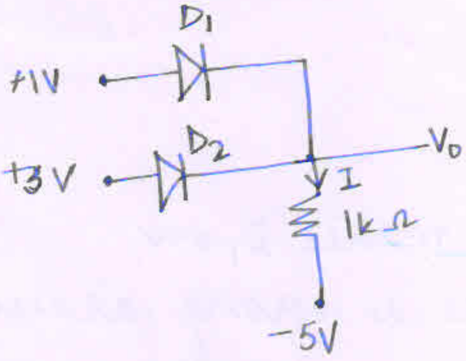
Ans 6.



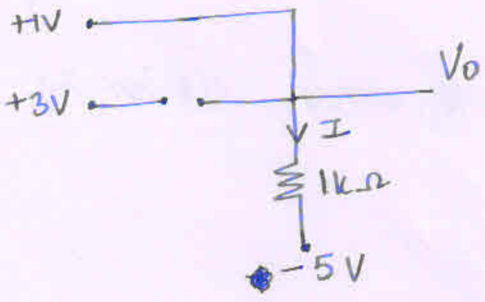




Ans 7. (a)

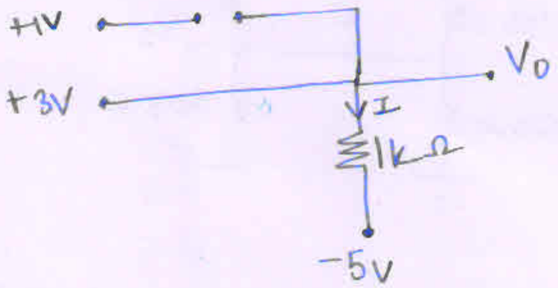


Case i) D_1 is on, D_2 is off



$V_0 = 1V$ which is not possible
Therefore, this is a wrong assumption.

Case ii) D_2 is on, D_1 is off.



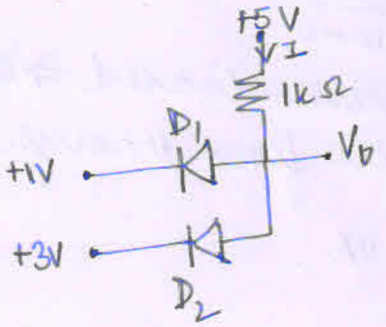
$\therefore V_0 = 3V$

$I = \dfrac{3-(-5)}{1k} A = 8mA$

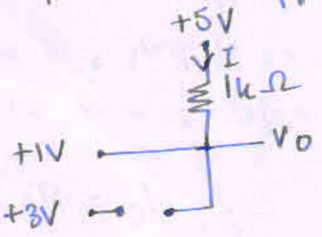
Case iii) Both diodes are on. This is not possible as V_0 cannot be 1V and 3V at the same time.

Case iv) Both diodes are off. $V_0 = -5V$. This is a wrong assumption as then diodes become forward biased and is contradictory.

(b)



Case i) D_1 on, D_2 off.

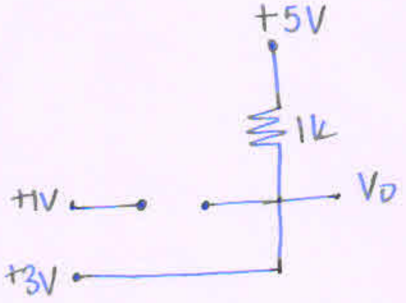


$V_0 = 1V$

$I = \dfrac{5-1}{1k} A$

$= 4mA$

Case ii) D_1 is off, D_2 is on



$V_O = 3V$

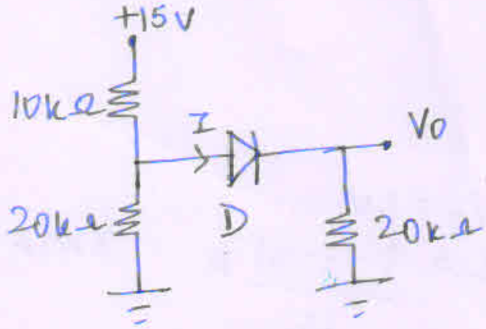
But this makes D_1 on.

So, this is a wrong assumption.

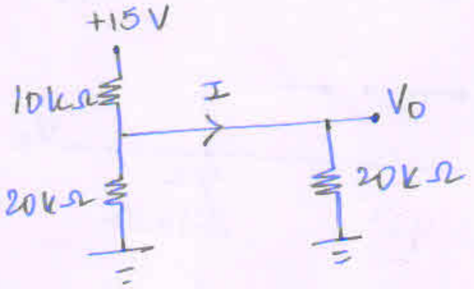
Case iii) Both diodes are on. This is not possible.

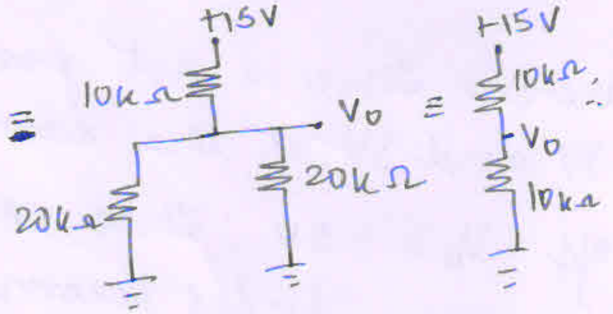
Case iv) Both diodes are off. $V_O = 5V$. This is not possible.

(C)



Case i) D is on



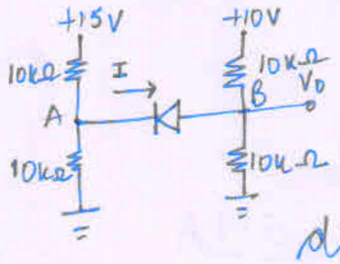


$\therefore V_O = \frac{15 \times 10}{20} V = 7.5V$

$I = \frac{7.5}{20k} A = 0.375mA$

Case ii) D off is not possible.

(d) Assume that the diode is OFF.



$V_A = \frac{15 \times 10}{10+10} V = 7.5$, $V_B = \frac{10 \times 10}{10+10} V = 5V$

Since $V_A > V_B$, diode is reverse biased ⇒ Diode is OFF.

Therefore, no current will flow through the diode. $\therefore V_O = V_B = 5V$, $I = 0A$.