ESc201 : Introduction to Electronics

Logic Gates

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Example (recap)

Adding or subtracting numbers with addition operation alone

To get a negative number, 2's complement of positive number is taken

$$\begin{array}{r} +5 \\ +2 \\ \hline +7 \end{array} \qquad \begin{array}{r} 0\,1\,0\,1 \\ +\,0\,0\,1\,0 \\ \hline 0\,1\,1\,1 \\ \hline \end{array}$$

$$\begin{array}{r} +5 \\ -2 \\ \hline +3 \\ \hline \end{array} \qquad \begin{array}{r} 0\,1\,0\,1 \\ +\,1\,1\,1\,0 \\ \hline 0\,0\,1\,1 \\ \hline \end{array}$$

2's comp. of +2

$$\begin{array}{r} -5 \\ +2 \\ \hline -3 \\ \hline \end{array} \qquad \begin{array}{r} 1\,0\,1\,1 \\ +\,0\,0\,1\,0 \\ \hline 1\,1\,0\,1 \\ \hline \end{array}$$

2's comp. of +5

2's complement is 0011 = 3

$$\begin{array}{r} -5 \\ -2 \\ \hline -7 \\ \hline \end{array} \qquad \begin{array}{r} 1\,0\,1\,1 \\ +\,1\,1\,1\,0 \\ \hline 1\,0\,0\,1 \\ \hline \end{array}$$

2's comp. of +5

2's comp. of +2

2's complement is 0111 = 7

Boolean Algebra (recap)

Algebra on Binary numbers

A variable x can take two values {0,1}

0 → False, No, Low voltage

1 → True, Yes, High voltage

Basic operations:

$$\text{AND:} \quad y = x_1 . x_2$$

y is 1 if and only if both x_1 and x_2 are 1, otherwise zero

Truth Table

x_1	x_2	y
0	0	0
0	1	0
1	0	0
1	1	1

Basic operations (recap)

$$\text{OR:} \quad y = x_1 + x_2$$

y is 1 if either x_1 or x_2 is 1. y= 0 if and only if both variables are zero

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	1

$$\text{NOT:} \quad y = \overline{x}$$

x	y
0	1
1	0

How do we get the chocolate?

Design Flow

System Description

↓

Truth Table

↓

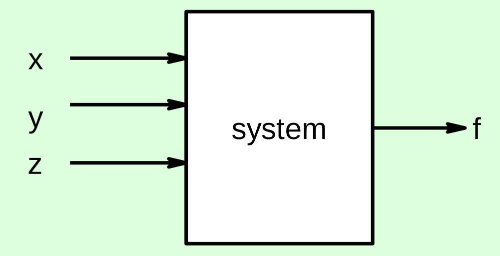
Boolean Expression

↓

Minimized Boolean Expression

↓

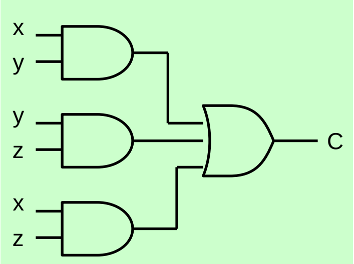
Gate Netlist



x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

$$f = \overline{x}.\overline{y}.z + \overline{x}.y.z + x.\overline{y}.z + x.y.z$$

$$\Rightarrow f = \overline{x}\,.\overline{z} + x.z$$



Proving Theorems

P1.a: $x + 0 = x$

P2.a: $x + y = y + x$

P3.a: $x.(y+z) = x.y+x.z$

P4.a: $x + \overline{x} = 1$

P1.b: $x . 1 = x$

P2.b: $x . y = y . x$

P3.b: $x+y.z = (x+y).(x+z)$

P4.b: $x . \overline{x} = 0$

Prove : $x + 1 = 1$

$x + 1 = x+(x+ \overline{x})$

$= (x+x)+ \overline{x}$

$= x + \overline{x}$

$= 1$

$x + x .y = x$

$= x . 1 + x. y$

$= x. (1+ y)$

$= x . 1$

$=x$

DeMorgan's Theorem

$$\overline{(x_1 + x_2 + x_3 + \ldots)} = \overline{x_1} . \overline{x_2} . \overline{x_3} .$$

$$\overline{(x_1 . x_2 . x_3 \ldots)} = (\overline{x_1} + \overline{x_2} + \overline{x_3} + \ldots)$$

Simplification of Boolean expressions

$$\overline{(x_1 + x_2 + x_3 + ...)} = \overline{x_1} \,.\, \overline{x_2} \,.\, \overline{x_3} \,.$$

$$\overline{(x_1 .\, x_2 .\, x_3 .....)} = (\overline{x_1} + \overline{x_2} + \overline{x_3} + ...)$$

$$\overline{(\overline{x_1} . x_2 + \overline{x_2} . x_3)} = ?$$

$$= \left(\overline{\overline{x_1} .\, x_2}\right) . \left(\overline{\overline{x_2} \,.\, x_3}\right)$$

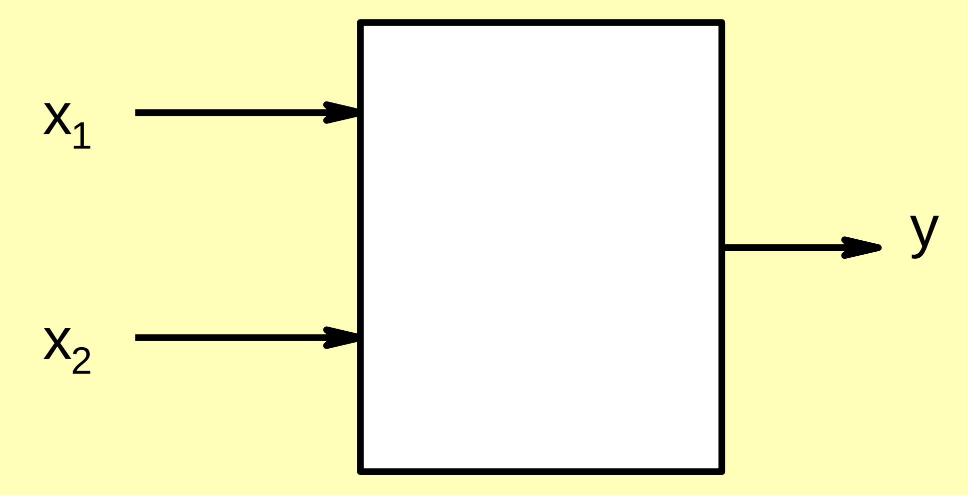
$$= (\overline{\overline{x_1}} + \overline{x_2}) \,.\, (\overline{\overline{x_2}} + \overline{x_3})$$

$$= (x_1 + \overline{x_2}) \,.\, (x_2 + \overline{x_3})$$

$$= x_1 .\, x_2 + \overline{x_2} . x_2 + x_1 \,.\, \overline{x_3} + \overline{x_2} .\, \overline{x_3}$$

$$= x_1 .\, x_2 + x_1 \,.\, \overline{x_3} + \overline{x_2} .\, \overline{x_3}$$

Function of Boolean variables



x_1	x_2	y
0	0	0
0	1	1
1	0	0
1	1	0

y = 1 when x_1 is 0 and x_2 is 1

$$y = \overline{x_1} \cdot x_2$$

Boolean expression

Obtaining Boolean expressions from truth Table

x_1	x_2	y
0	0	1
0	1	0
1	0	0
1	1	0

$$y = \overline{x_1} \,.\, \overline{x_2}$$

x_1	x_2	y
0	0	0
0	1	0
1	0	1
1	1	0

$$y = x_1 \,.\, \overline{x_2}$$

x_1	x_2	y
0	0	1
0	1	0
1	0	0
1	1	1

$$\overline{x_1} \,.\, \overline{x_2}$$

$$x_1 \,.\, x_2$$

$$y = \overline{x_1} \,.\, \overline{x_2} + x_1 . x_2$$

Obtaining Boolean expressions from truth Table

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0

$$y = \overline{x_1} . x_2 + x_1 . \overline{x_2}$$

Instead of writing expressions as sum of terms that make y equal to 1, we can also write expressions using terms that make y equal to 0

x_1	x_2	y
0	0	1
0	1	1
1	0	1
1	1	0

$$y = \overline{x_1} . \overline{x_2} + \overline{x_1} . x_2 + x_1 . \overline{x_2}$$

$$y = \overline{x_1} + \overline{x_2}$$

x_1	x_2	y
0	0	1
0	1	0
1	0	1
1	1	1

$$y = x_1 + \overline{x_2}$$

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	1

$$y = x_1 + x_2$$

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0

$$x_1 + x_2$$

$$y = (x_1 + x_2).(\overline{x_1} + \overline{x_2})$$

$$\overline{x_1} + \overline{x_2}$$

Obtaining Boolean expressions from truth Table

$$y = \overline{x_1}.\overline{x_2}.x_3 + \overline{x_1}.x_2.x_3 + x_1.\overline{x_2}.x_3 + x_1.x_2.x_3$$

x_1	x_2	x_3	y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

Sum of Products (SOP) form

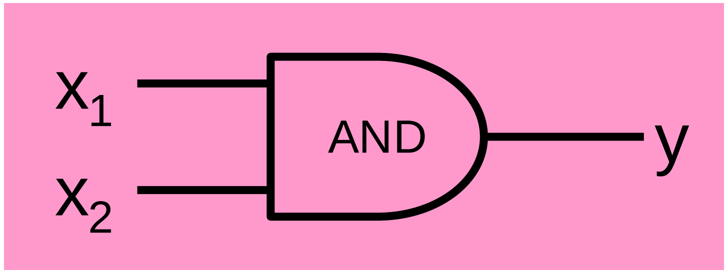
$$y = (x_1 + x_2 + x_3).(x_1 + \overline{x_2} + x_3).(\overline{x_1} + x_2 + x_3).(\overline{x_1} + \overline{x_2} + x_3)$$

Product of Sum (POS) form

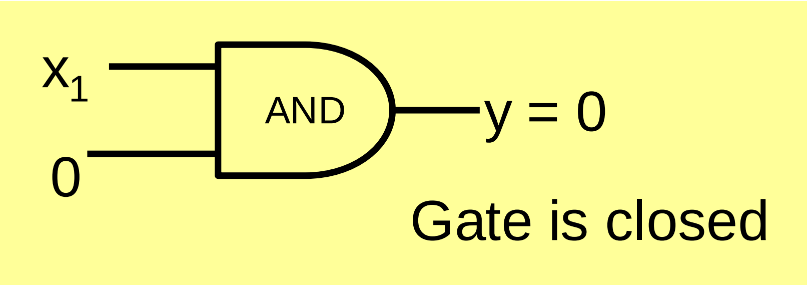
Implementing Boolean expressions

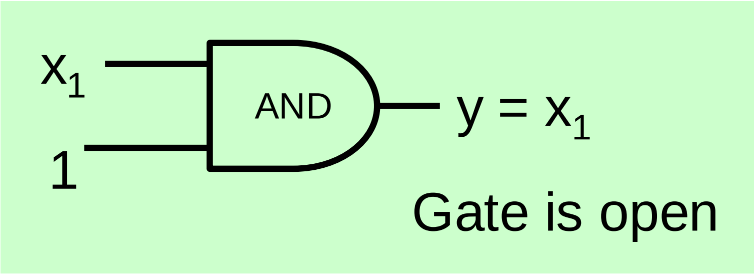
Elementary Gates

$$\text{AND:} \quad y = x_1 . x_2$$

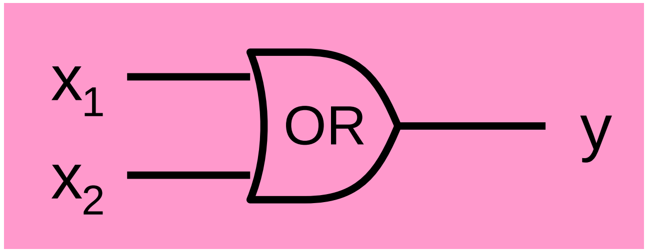


Why call it a gate?

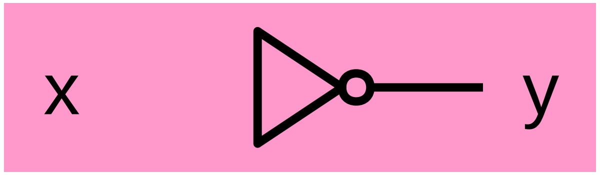




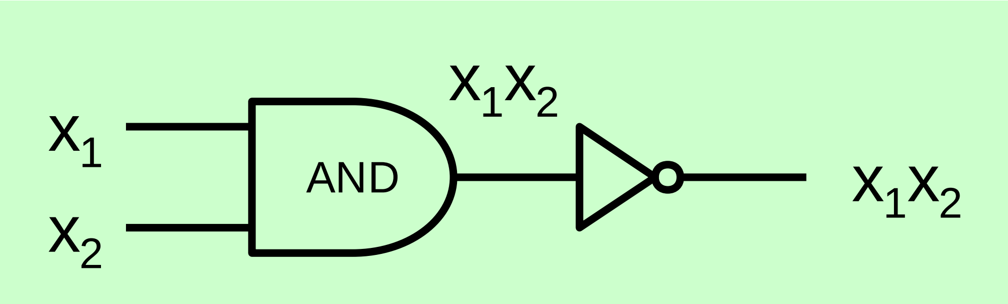
$$\text{OR:} \quad y = x_1 + x_2$$

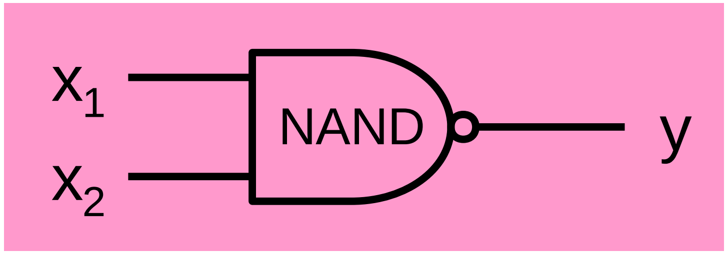


$$\text{NOT:} \quad y = \overline{x}$$

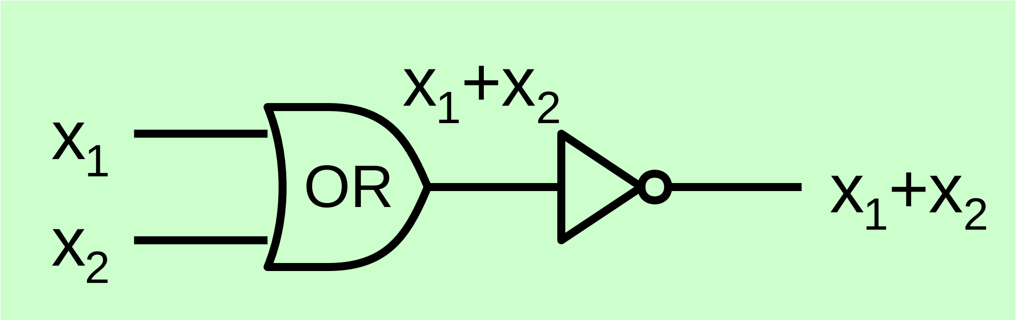


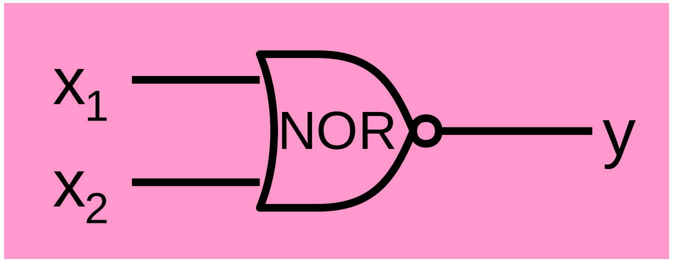
NAND: $y = \overline{x_1 . x_2}$





NOR: $y = \overline{x_1 + x_2}$

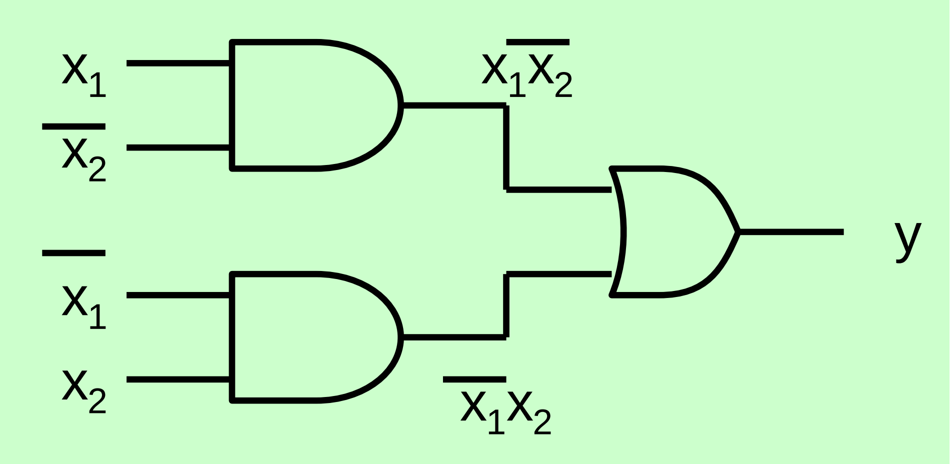


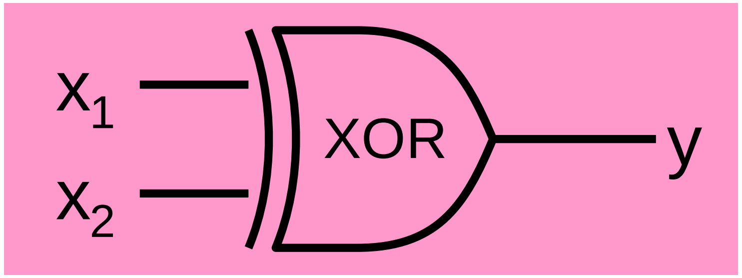


XOR: $y = x_1 \oplus x_2 = x_1 . \overline{x_2} + \overline{x_1} . x_2$

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0

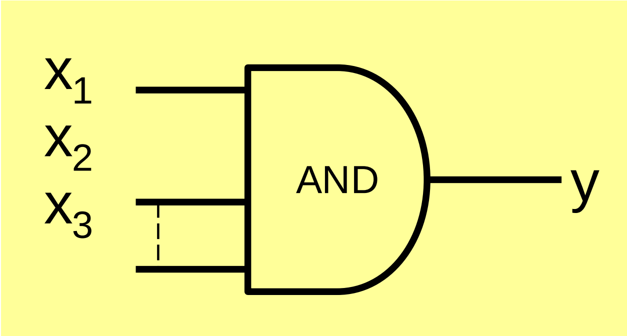
y is 1 if only one variable is 1 and the other is zero



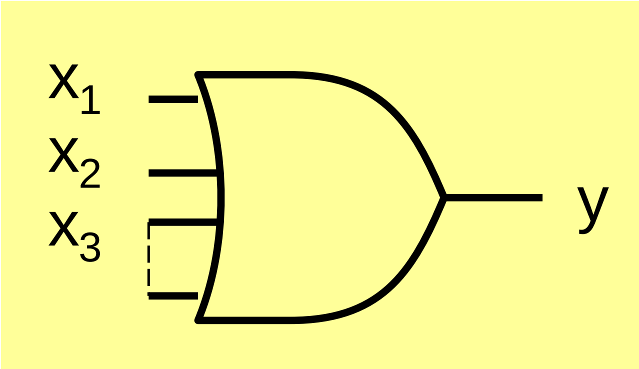


Gates with more than 2 inputs

$$\text{AND:} \quad y = x_1 . x_2 . x_3 \ldots$$



$$\text{OR:} \quad y = x_1 + x_2 + x_3 + \ldots.$$



$$\text{XOR:} \quad y = x_1 \oplus x_2 \oplus x_3 = x_1 . \overline{x_2}\,\overline{x_3} + \overline{x_1} . x_2 . \overline{x_3} + \overline{x_1} . \overline{x_2} . x_3 + x_1 . x_2 . x_3$$

y = 1 only if odd number of inputs is 1

How do we get the chocolate?

Design Flow

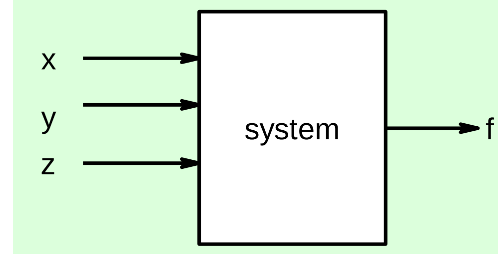
System Description

Truth Table

Boolean Expression

Minimized Boolean Expression

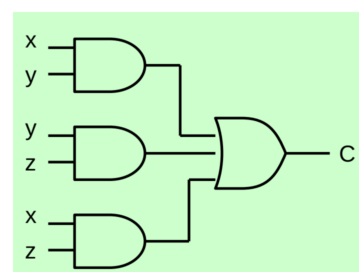
Gate Netlist



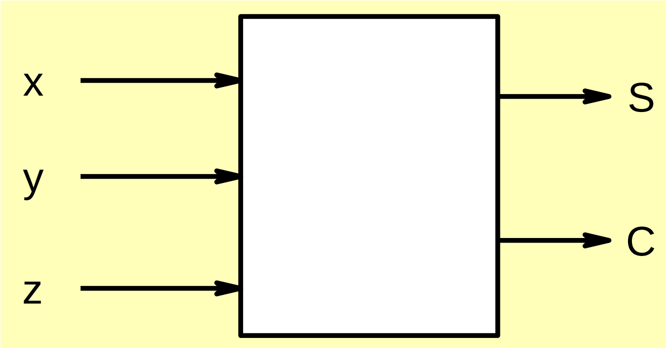
x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

$$f = \overline{x}.\overline{y}.z + \overline{x}.y.z + x.\overline{y}.z + x.y.z$$

$$\Rightarrow f = \overline{x}\,.\overline{z} + x.z$$



Implementing Boolean expressions using gates



$$S = \overline{x}.\overline{y}.z + \overline{x}.y.\overline{z} + x.\overline{y}.\overline{z} + x.y.z$$

$$C = x.y + x.z + y.z$$