ESc201 : Introduction to Electronics

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Recap

- Capacitors $$i = C\frac{dv}{dt}$$

A capacitor under dc or steady state acts like an open circuit

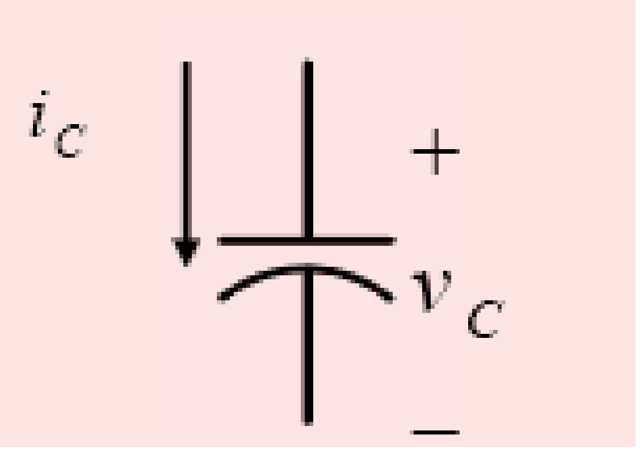
- Inductors $$v = \frac{d\phi}{dt} = L \times \frac{di}{dt}$$

An inductor under dc or steady state acts like a short circuit

Circuits containing inductors or capacitors have a memory

Recap: Two important concepts

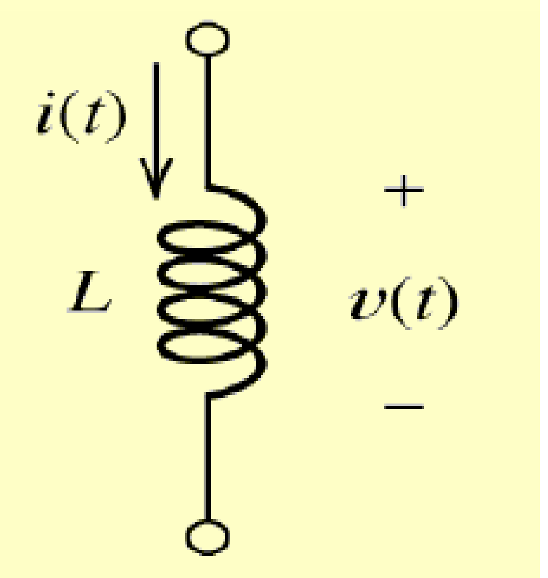
Voltage across a capacitor cannot change instantaneously



$$i_c = C\frac{dv_c}{dt}$$

Instantaneous change in voltage implies infinite current!

Current through an inductor cannot change instantaneously

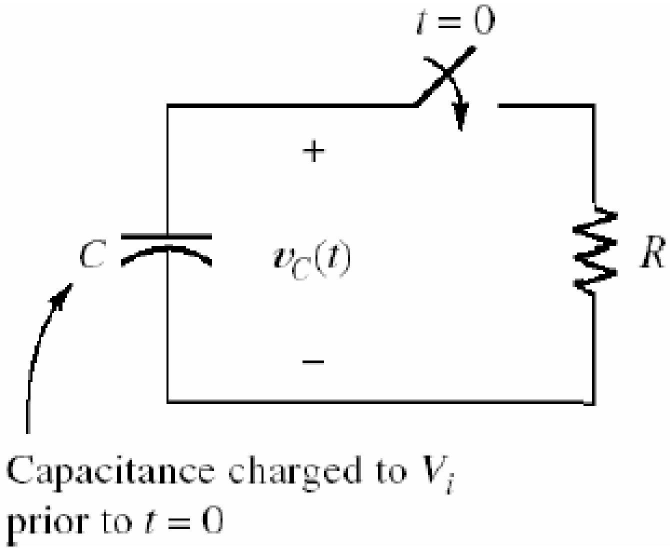


$$v = L\frac{di}{dt}$$

Instantaneous change in current implies infinite voltage!

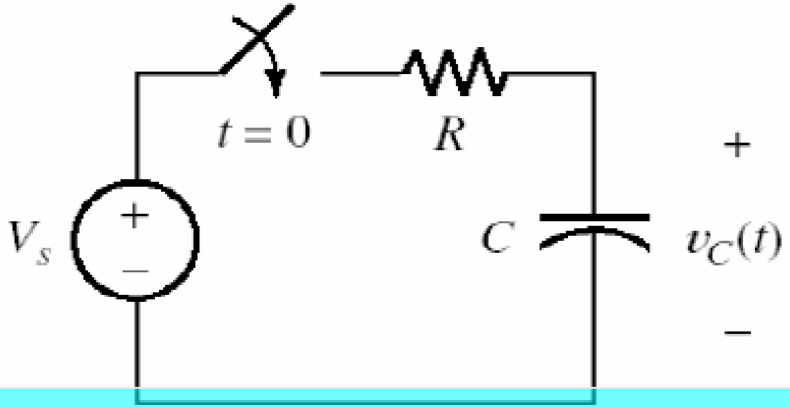
Recap

Discharging of capacitor



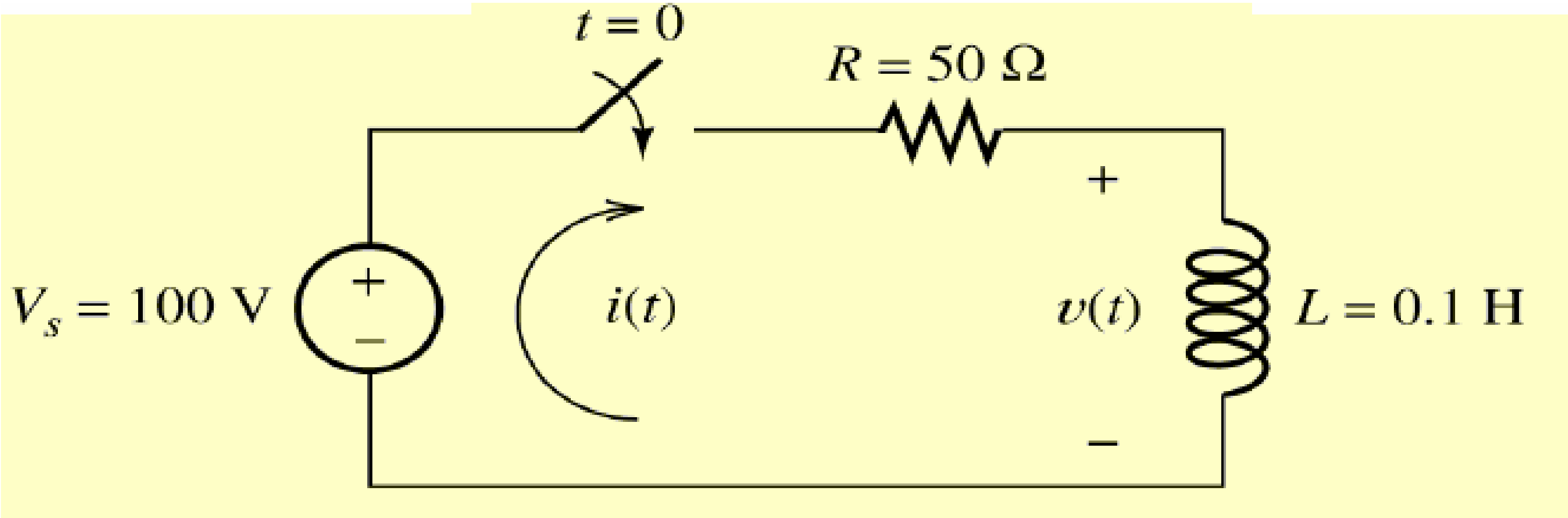
$$v_C(t) = V_i\, e^{-\frac{t}{RC}}$$

Charging a capacitor



$$v_C(t) = v_C(\infty) + \{v_C(0^+) - v_C(\infty)\}\, e^{-\frac{t}{RC}}$$

Recap: RL Transient Analysis

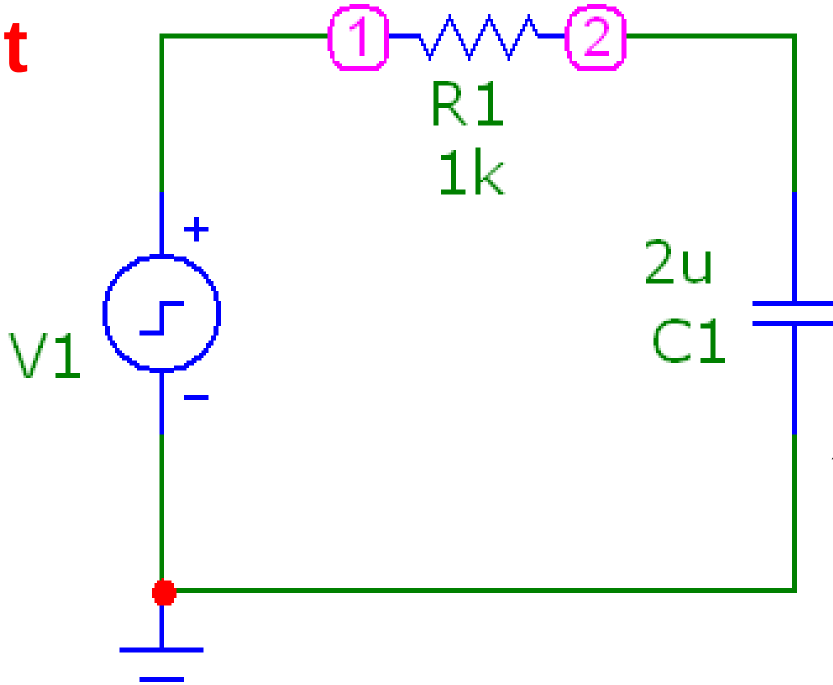


$$i(t) = i(\infty) + \{i(0) - i(\infty)\}\, e^{-\frac{R}{L}t}$$

$$e^{-\frac{t}{\tau}}$$

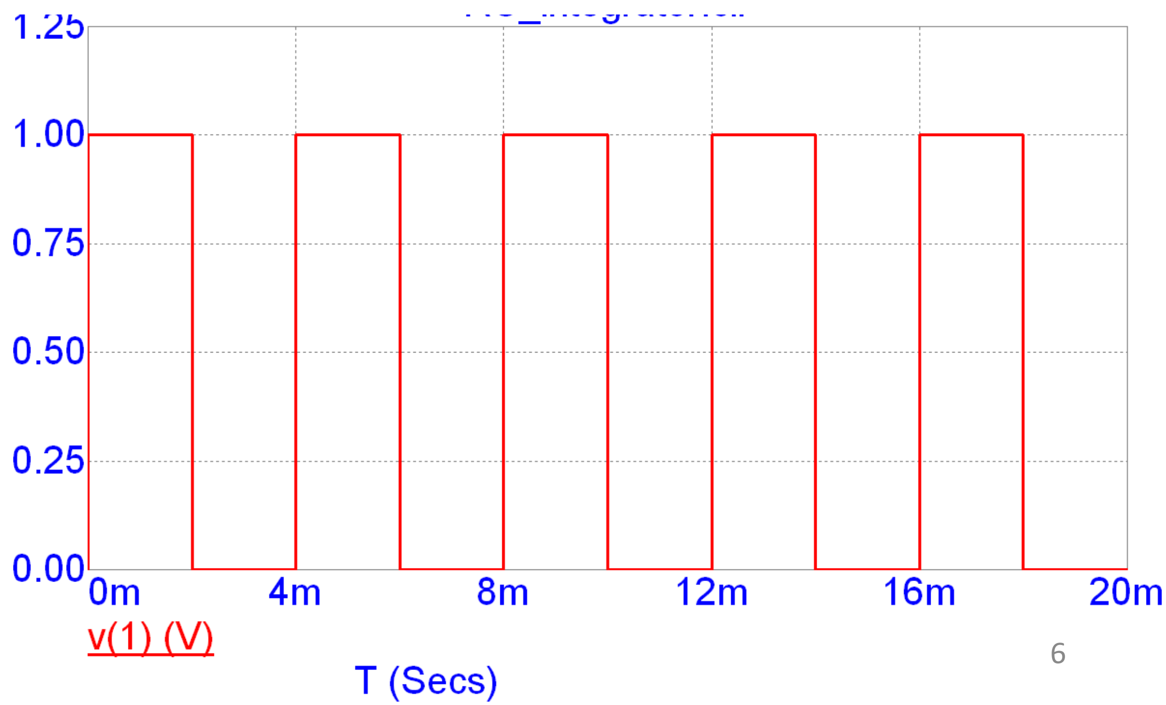
$$\text{Time Constant} : \tau = \frac{L}{R}$$

RC Circuit



$$\tau = RC = 2\,\mathrm{ms}$$

Input voltage



RC Circuit

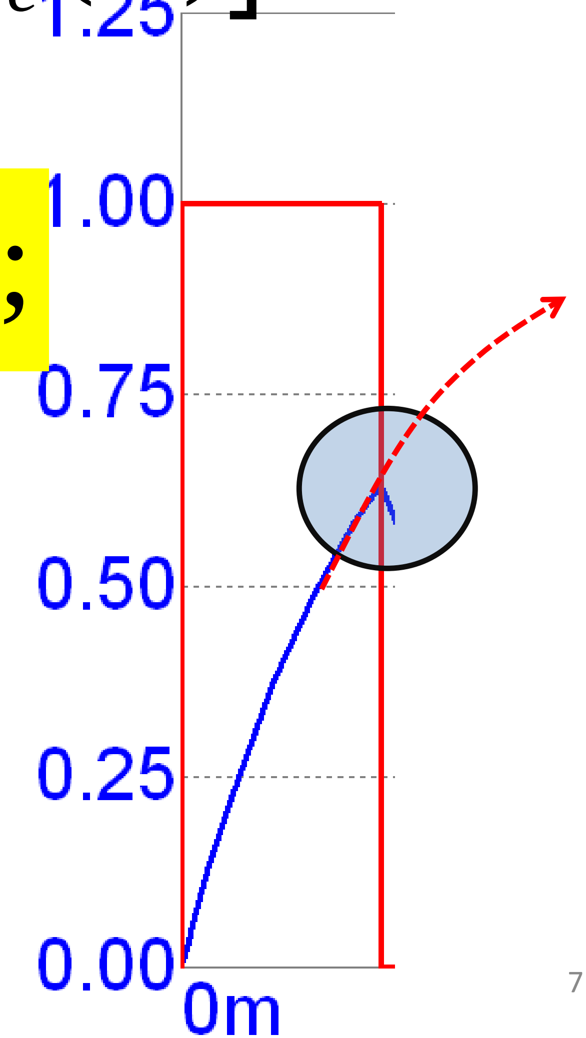
$$\tau = RC = 2\,\text{ms}$$

$$v_c(t) = v_c(\infty) + \left[v_c(0^+) - v_c(\infty)\right] e^{-\frac{t}{\tau}}$$

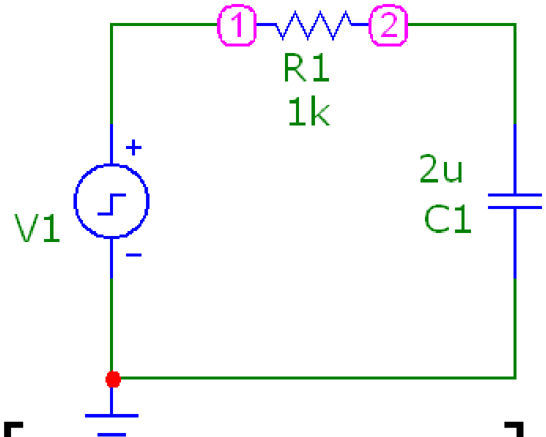
$$v_c(\infty) = 1V; \quad v_c(0^+) = 0;$$

$$v_c(t) = 1 - e^{-\frac{t}{2}}$$

$$v_c(2) = 1 - e^{-\frac{2}{2}} = 0.63V$$



RC Circuit



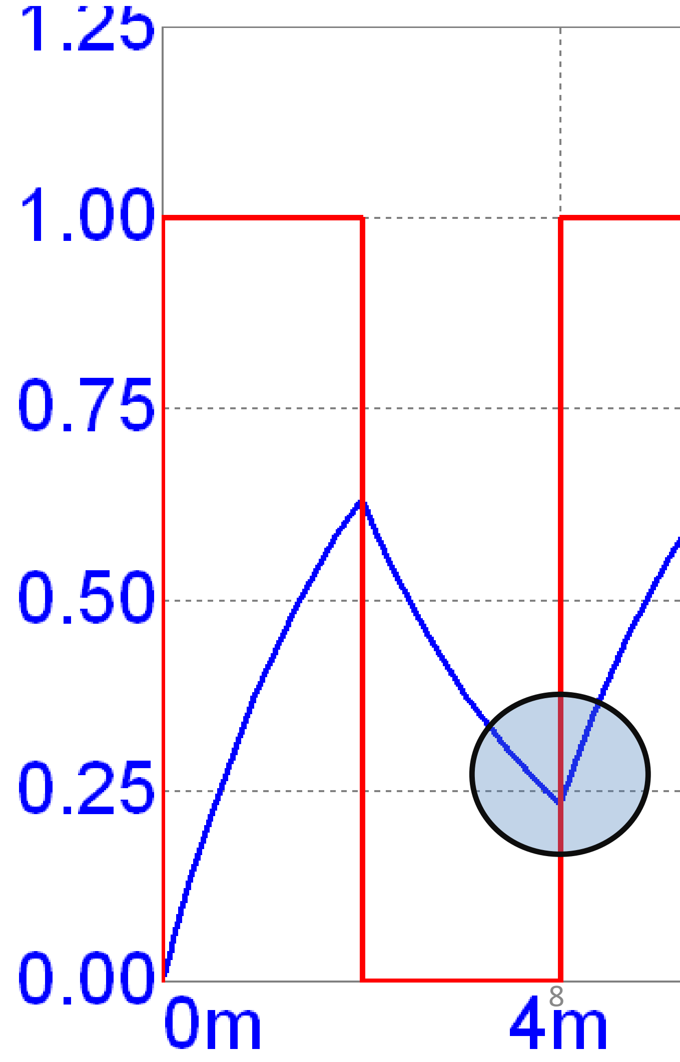
$$\tau = RC = 2\ \mathrm{ms}$$

$$v_c(t) = v_c(\infty) + \left[v_c(0^+) - v_c(\infty)\right]e^{-\frac{t}{\tau}}$$

$$v_c(\infty) = 0; v_c(0^+) = 0.63V;$$

$$v_c(t) = 0.63e^{-\frac{(t-2)}{2}}; 2 \le t \le 4$$

$$v_c(4) = 0.63\,e^{-\frac{2}{2}} = 0.23V$$

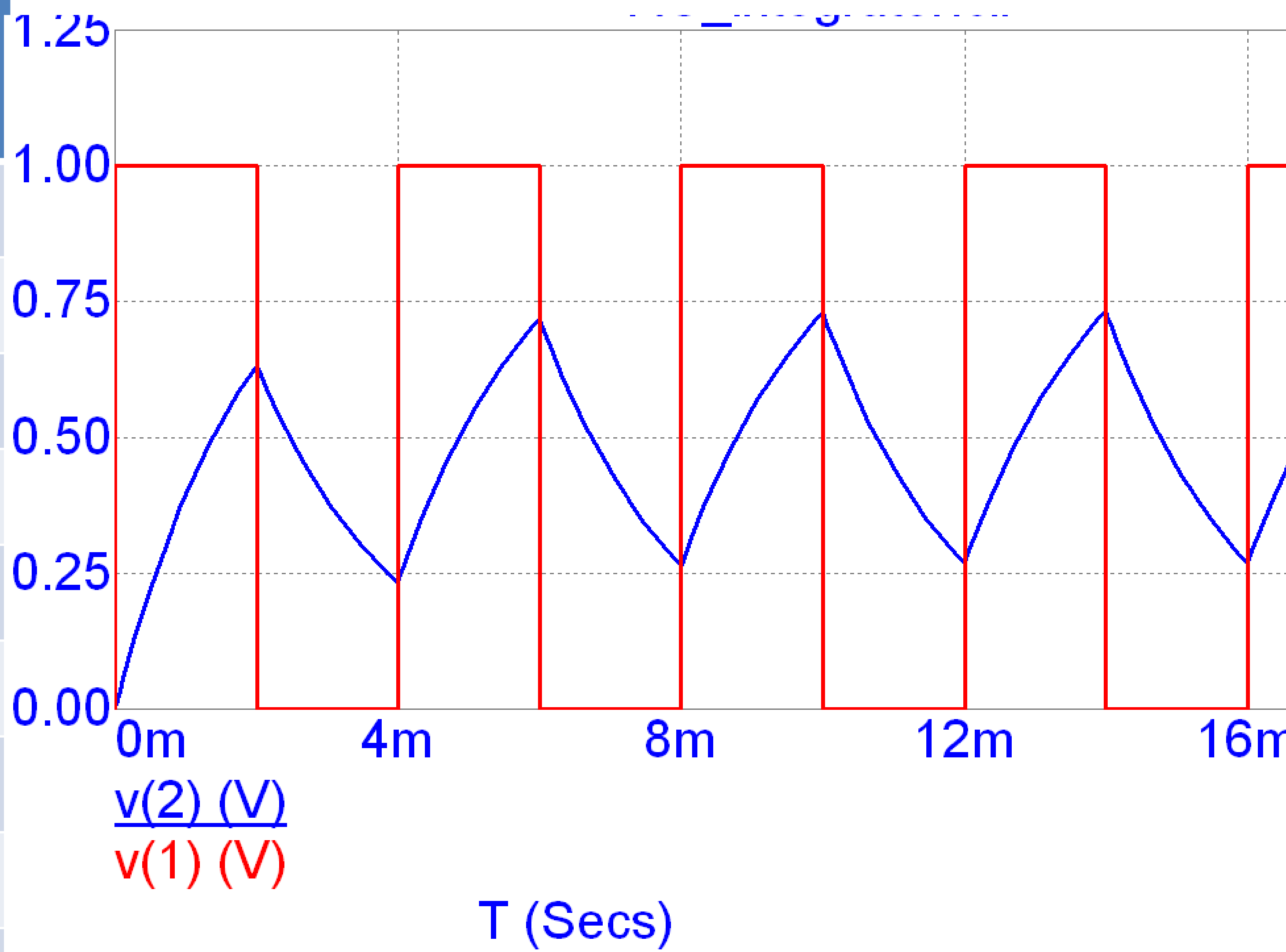


time *t* is in ms

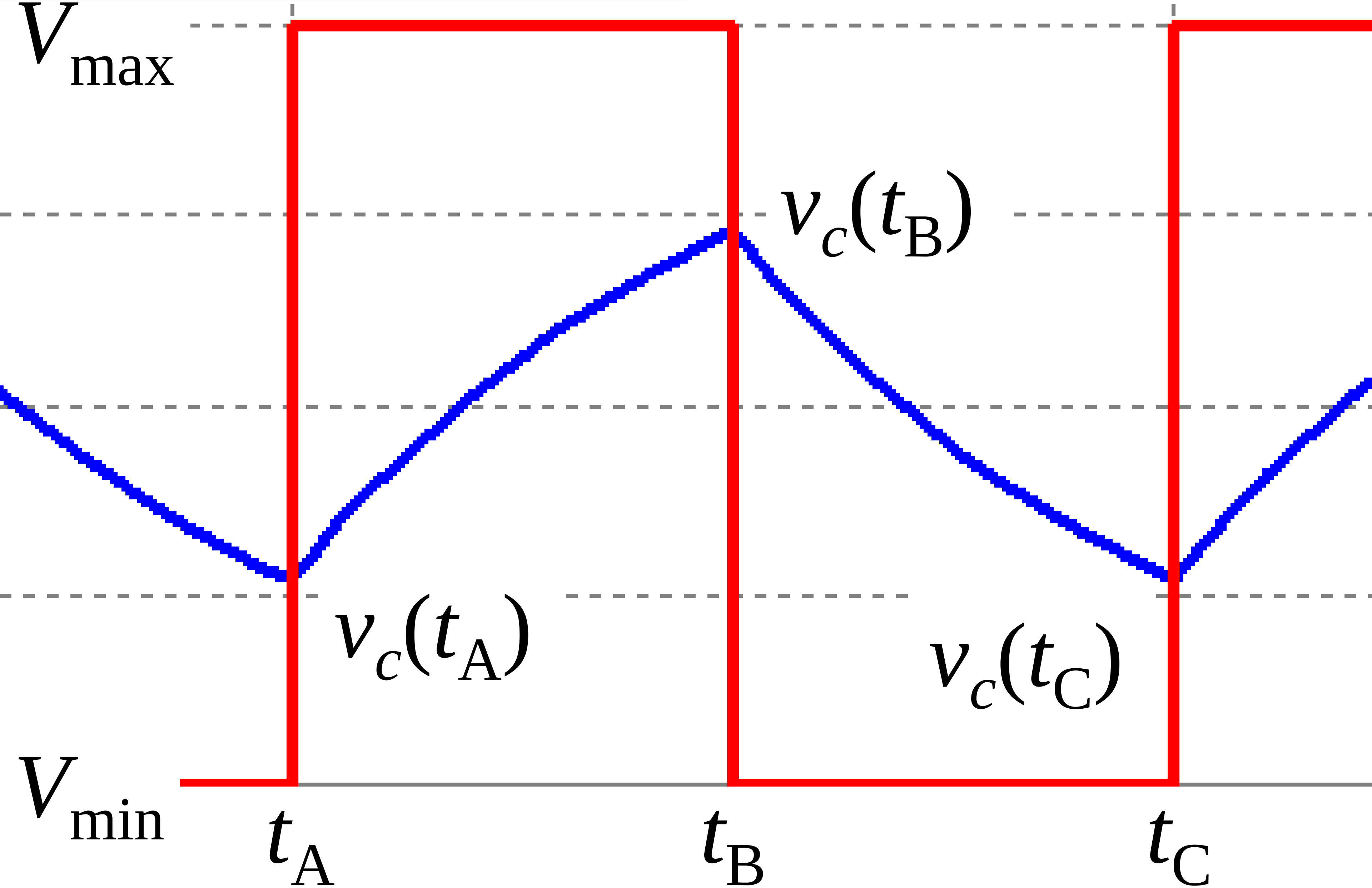
RC Circuit

$$\tau = RC = 2\ \text{ms}$$

t (ms)	$v_c(t)$ (V)
0	0
2	0.63
4	0.23
6	0.72
8	0.265
10	0.73
12	0.269
14	0.731
16	0.269



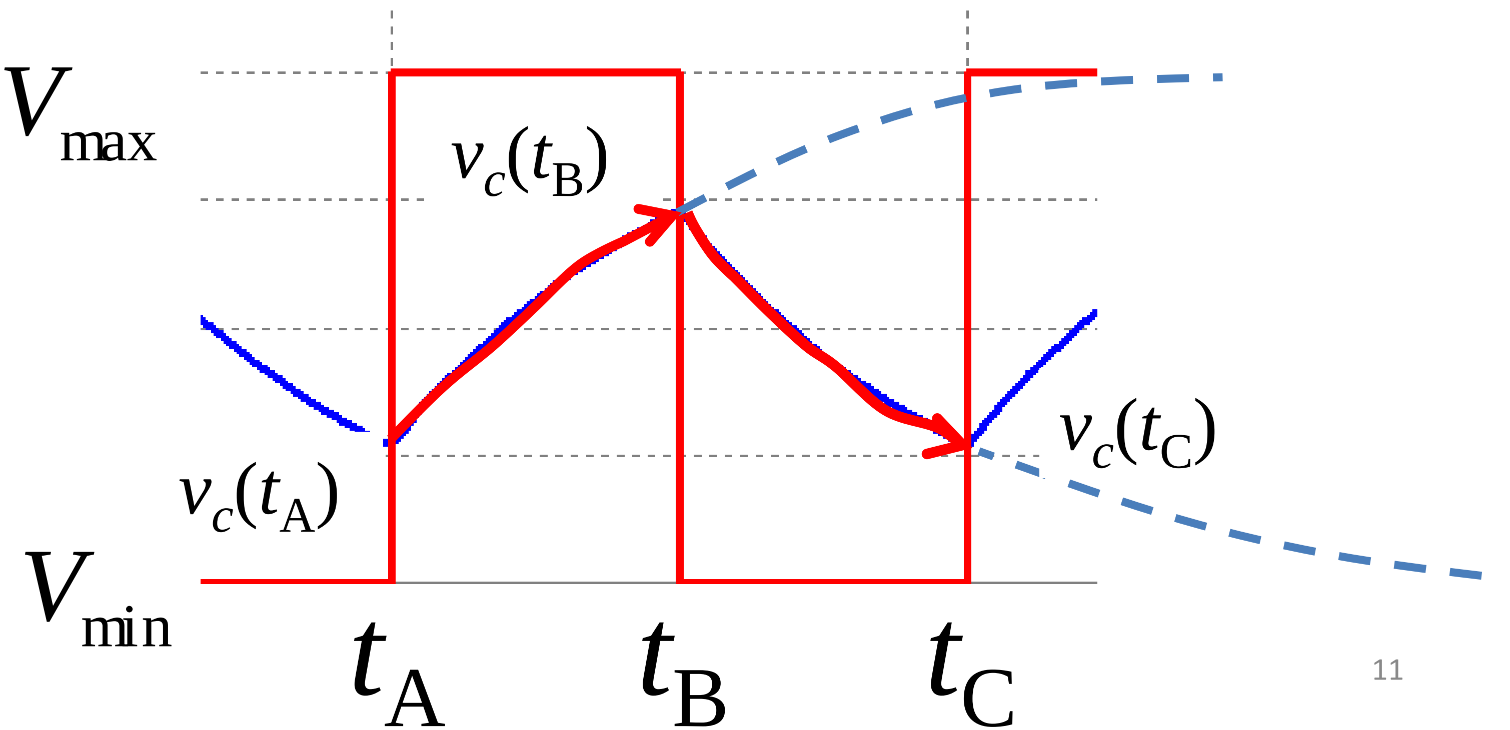
RC Circuit – Steady State



$$v_c(t) = v_c(\infty) + \left[v_c(0^+) - v_c(\infty)\right] e^{-\frac{t}{\tau}}$$

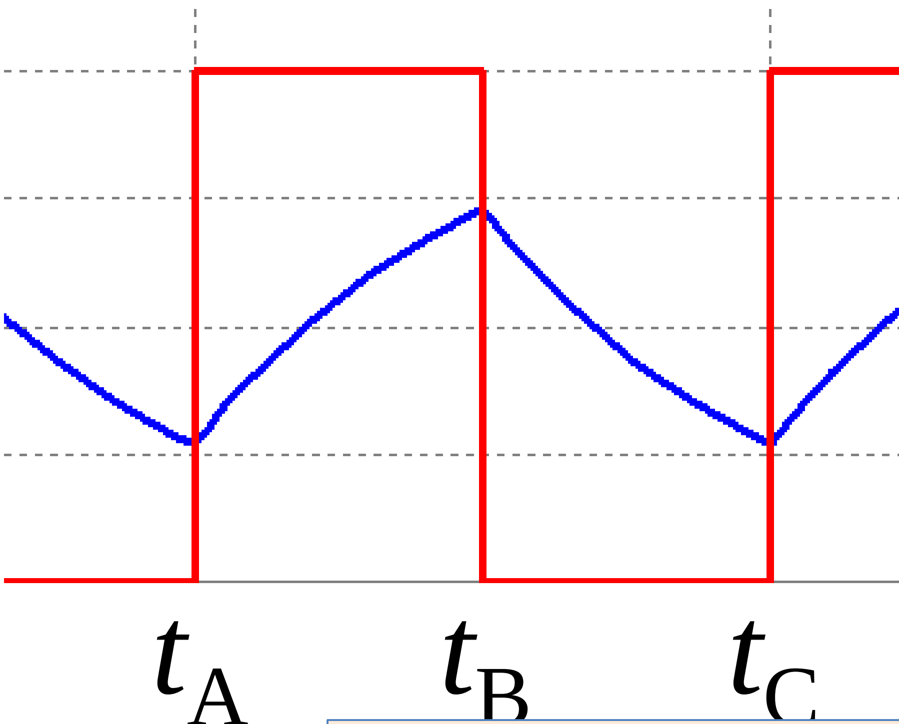
$$v_c(t) = V_{\max} + \left[v_c(t_A) - V_{\max}\right] e^{-\frac{(t-t_A)}{\tau}} \qquad t_A \le t \le t_B$$

$$v_c(t) = V_{\min} + \left[v_c(t_B) - V_{\min}\right] e^{-\frac{(t-t_B)}{\tau}} \qquad t_B \le t \le t_C$$



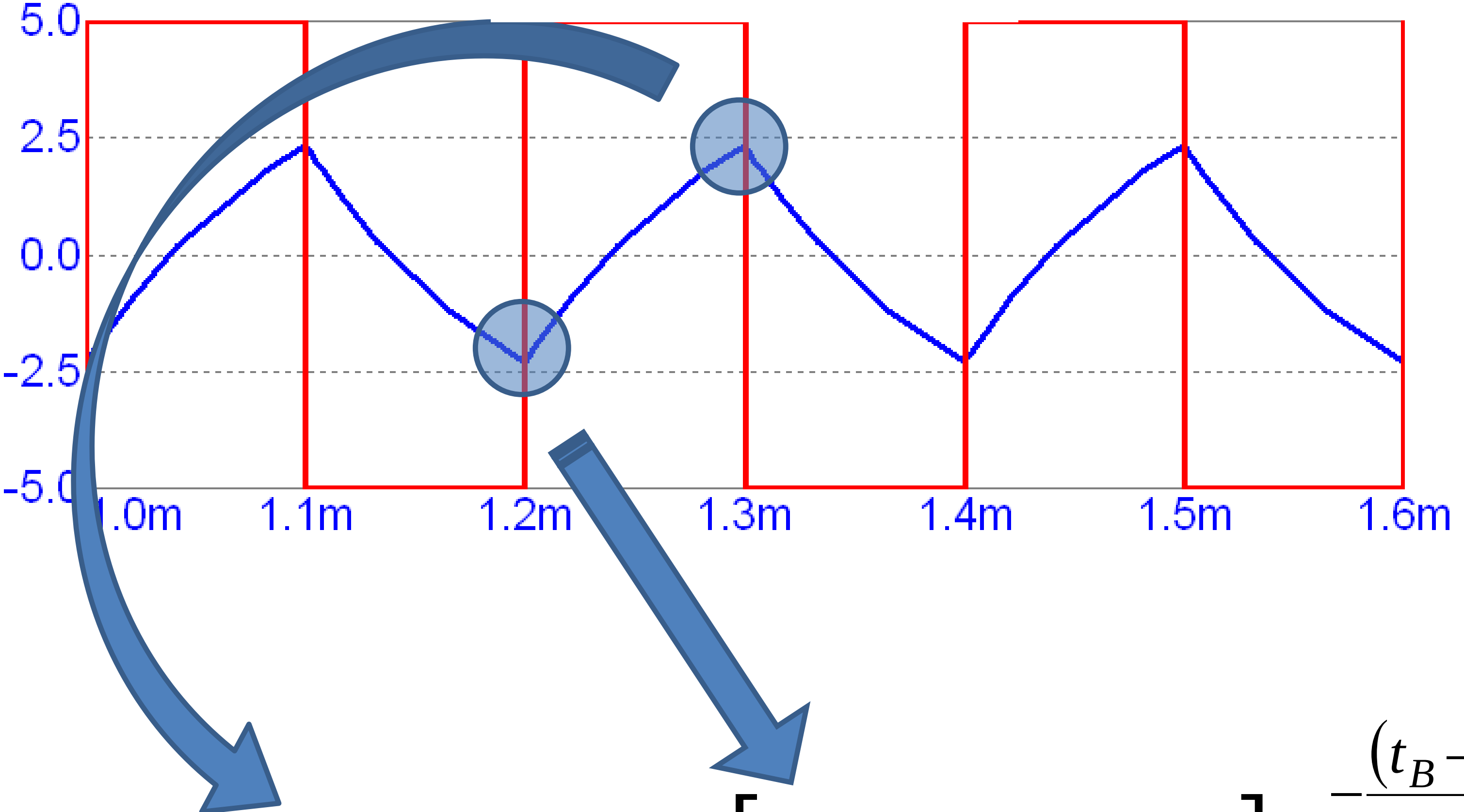
$$v_c(t_B) = V_{\max} + \left[v_c(t_A) - V_{\max}\right] e^{-\frac{(t_B - t_A)}{\tau}}$$

$$v_c(t_C) = V_{\min} + \left[v_c(t_B) - V_{\min}\right] e^{-\frac{(t_C - t_B)}{\tau}} = v_c(t_A)$$



$$\boxed{(t_B - t_A) = (t_C - t_B) = \frac{T}{2}}$$

Determine $v_c(t_A)$ and $v_c(t_B)$ in terms of $V_{\max}$ and $V_{\min}$

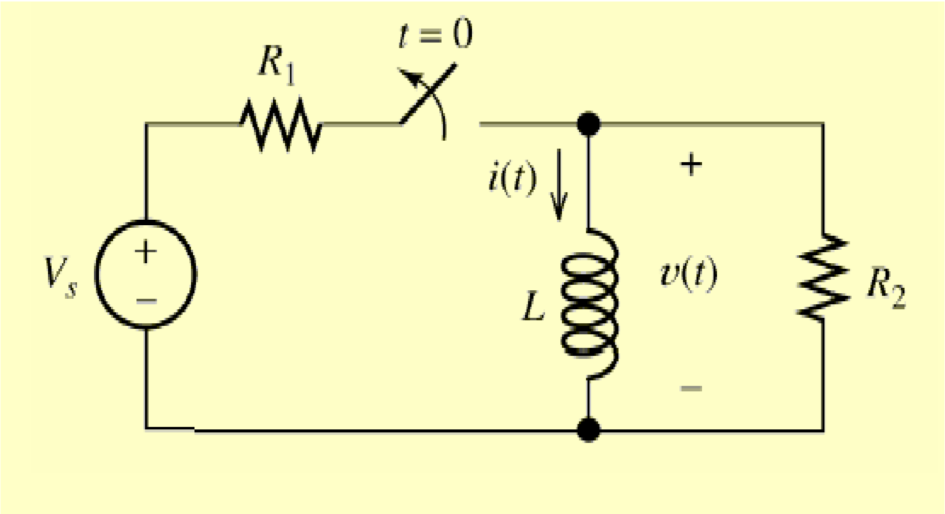


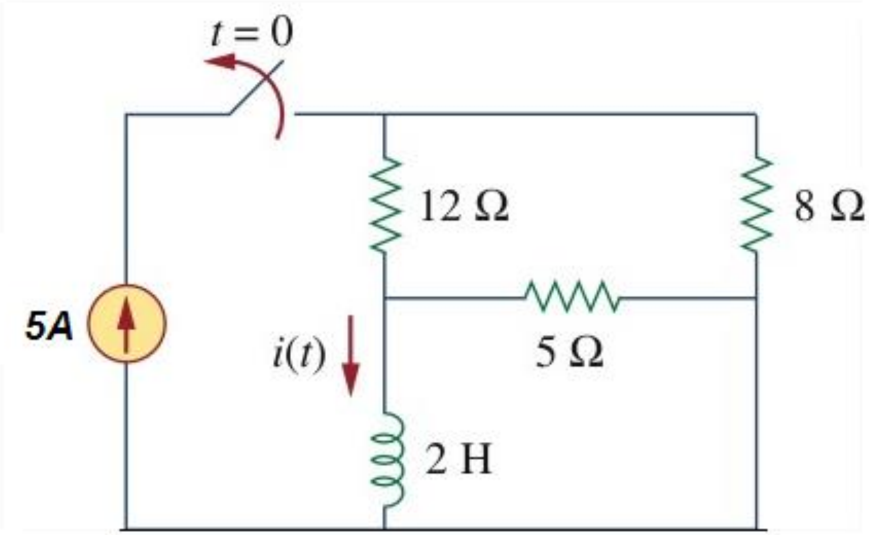
$$v_c(t_B) = V_{\max} + \left[v_c(t_A) - V_{\max}\right] e^{-\frac{(t_B - t_A)}{\tau}}$$

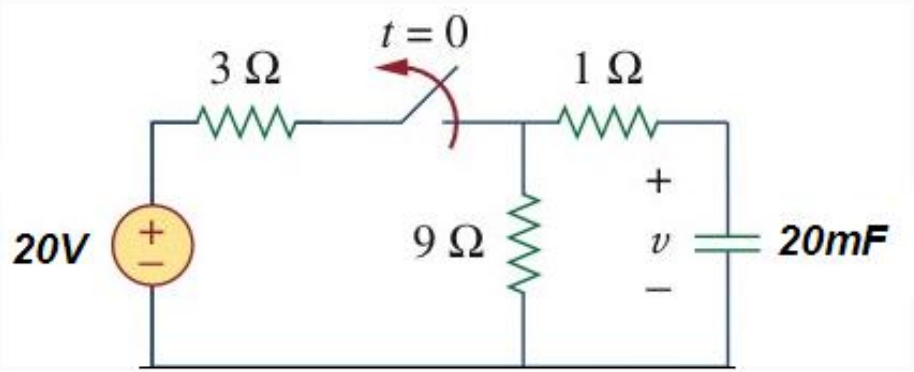
$$2.3 = 5 + \left[-2.3 - 5\right] e^{-\frac{0.1\,\mathrm{ms}}{\tau}}$$

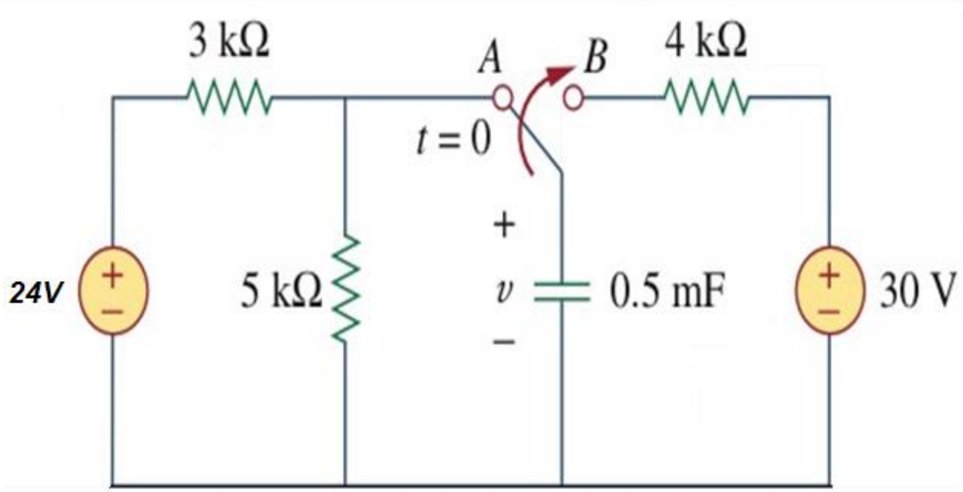
$$\tau = 0.1\,\mathrm{ms}$$

How do we solve more complex circuits containing a single inductor or a capacitor?



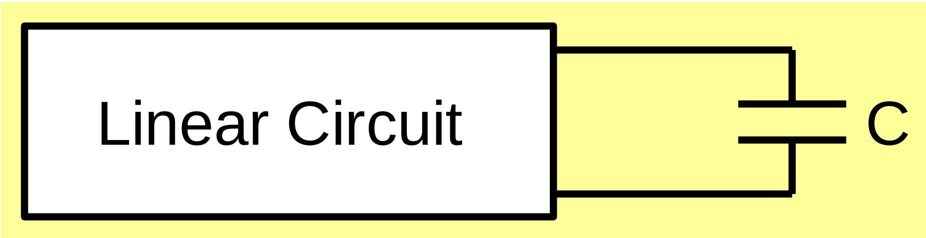


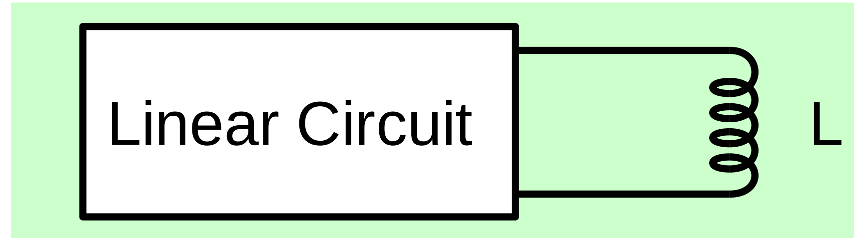


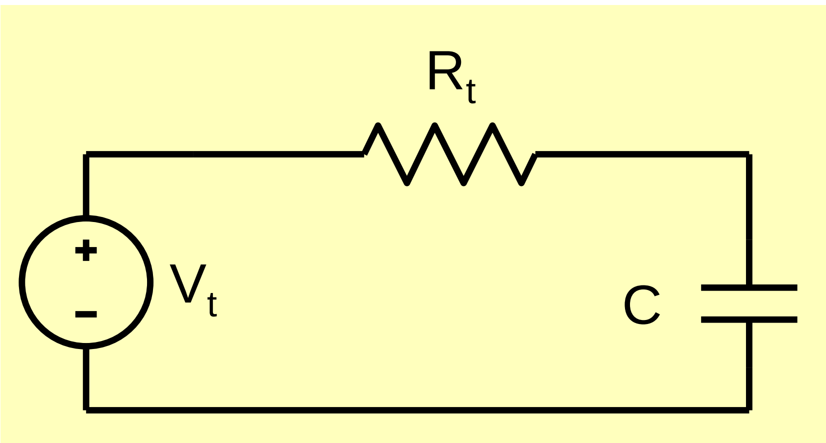


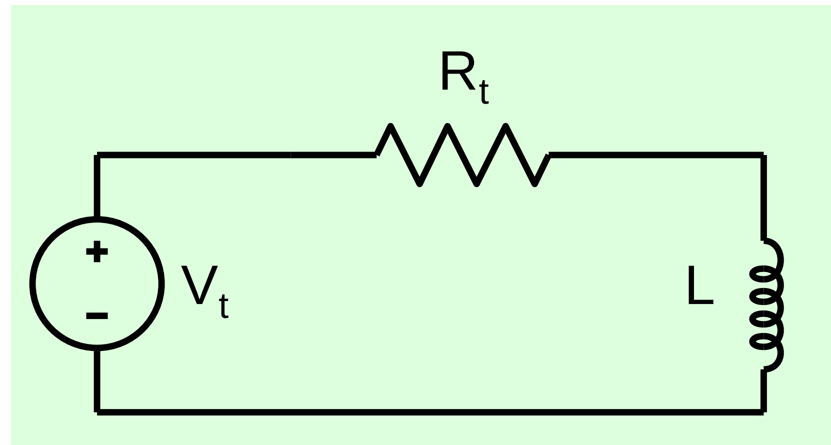
Method for circuits containing a single capacitor or inductor

Circuit for t > 0





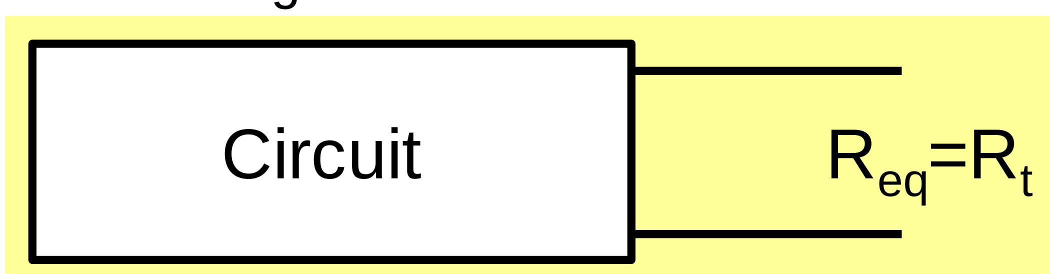




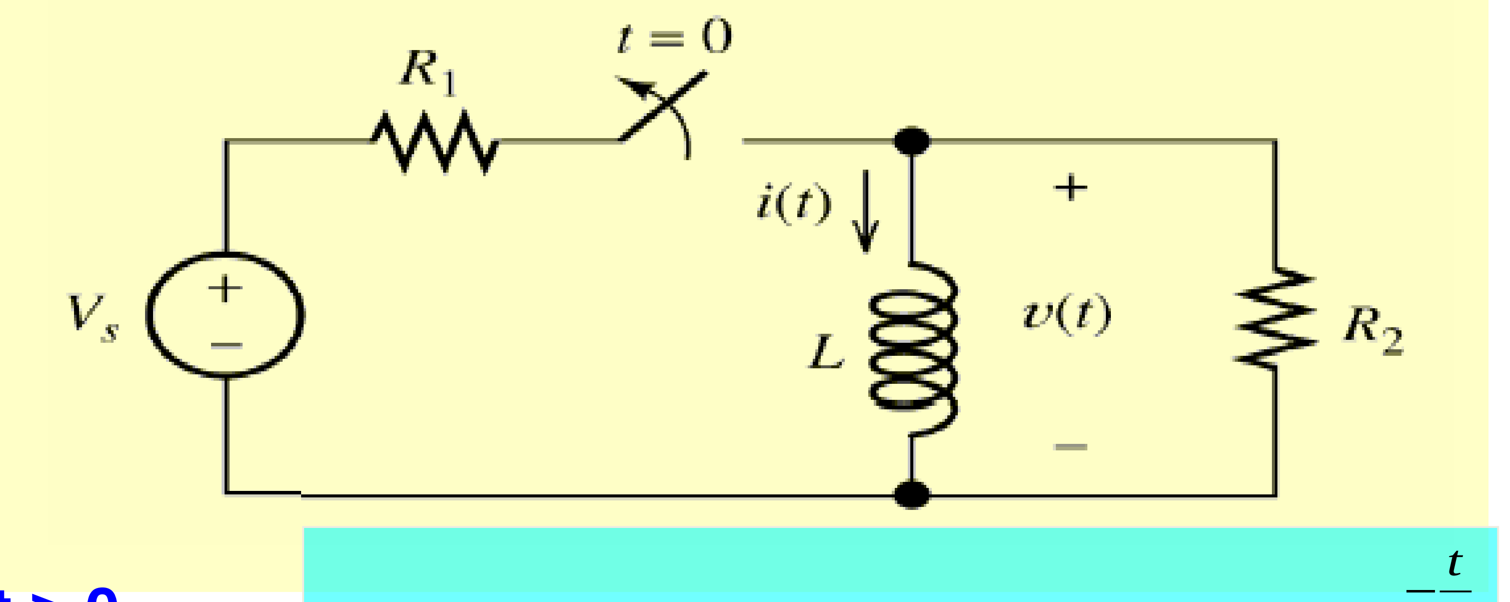
$$x(t) = x(\infty) + \{x(0^+) - x(\infty)\}e^{-\frac{t}{\tau}}$$

Where x is capacitor voltage or inductor current

$$\tau = \frac{L}{R_{eq}} \text{ or } R_{eq}C$$

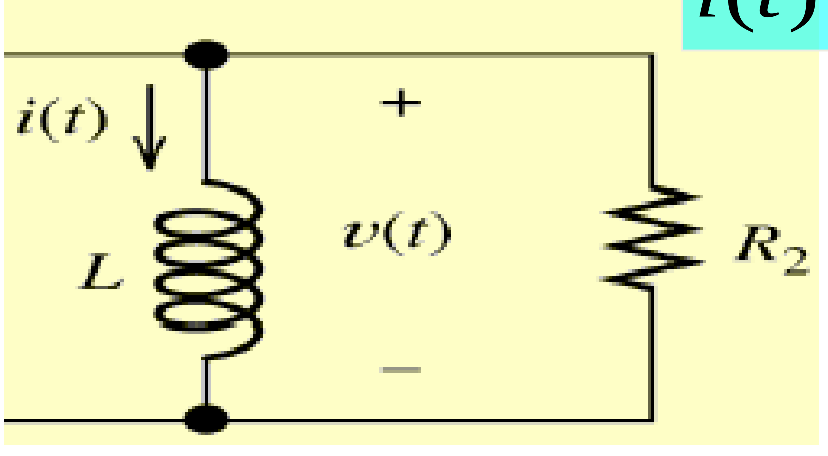


Example-1



Circuit for t > 0

$$i(t) = i(\infty) + \{i(0^+) - i(\infty)\} \times e^{-\frac{t}{\tau}}$$



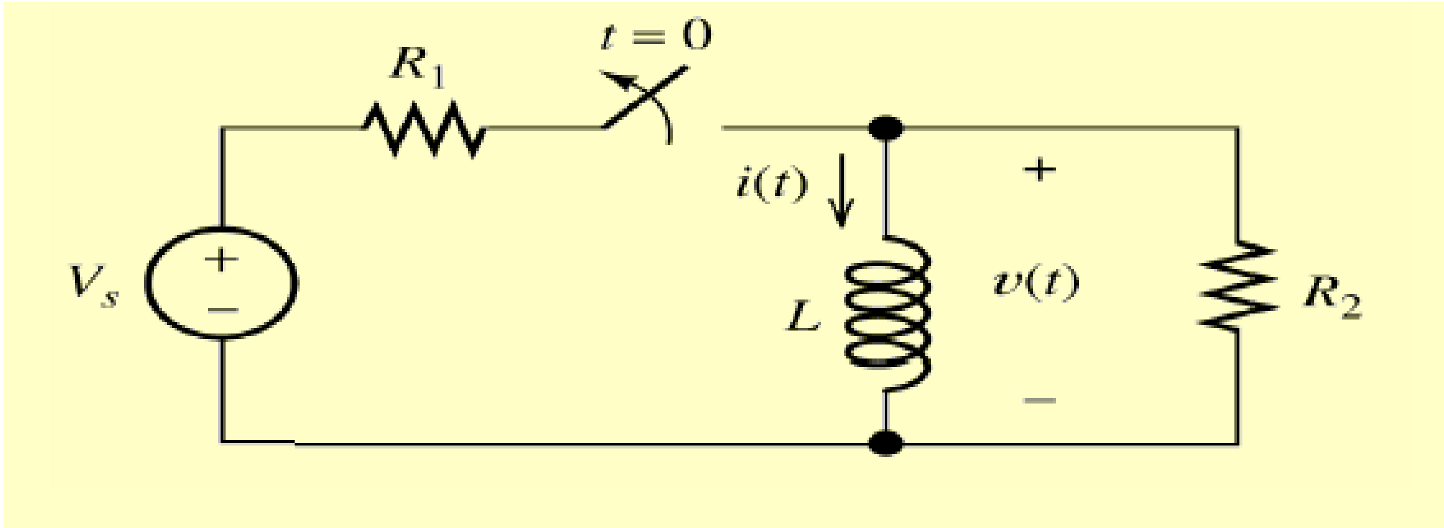
$$\tau = \frac{L}{R_2}$$

Steady state Solution:

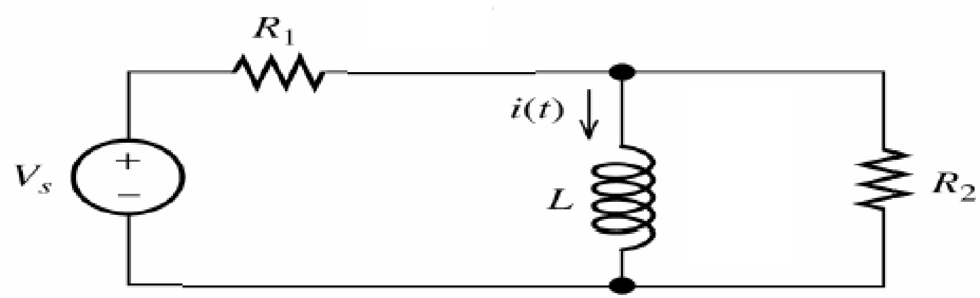
$$i(t \rightarrow \infty) = 0$$

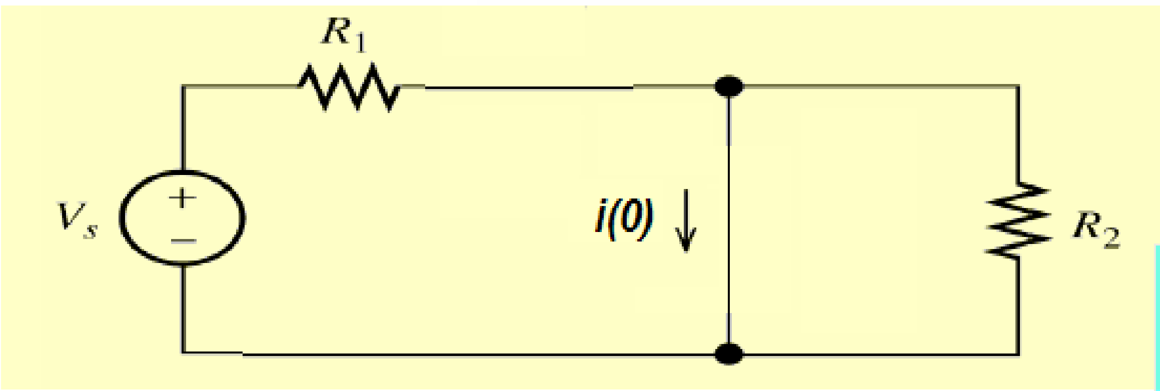
$$i(t) = i(0^+) \times e^{-\frac{t}{\tau}}$$

Initial condition



Circuit for t < 0

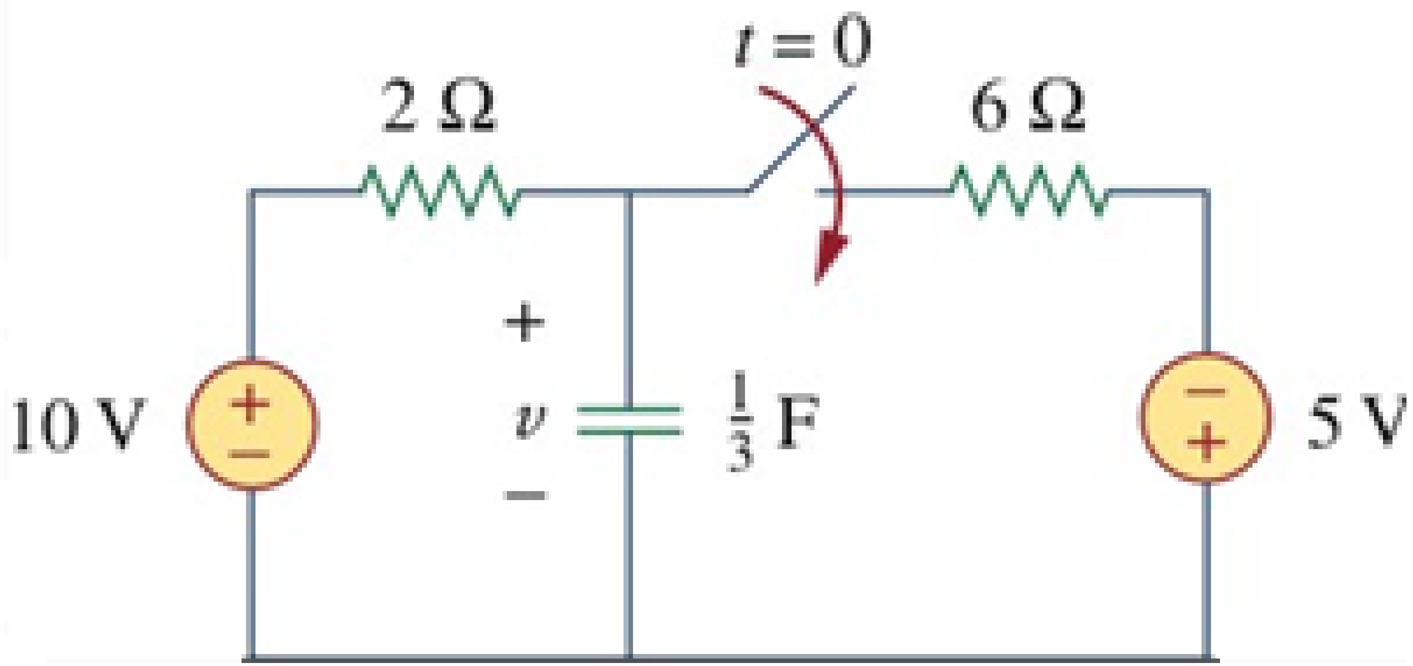




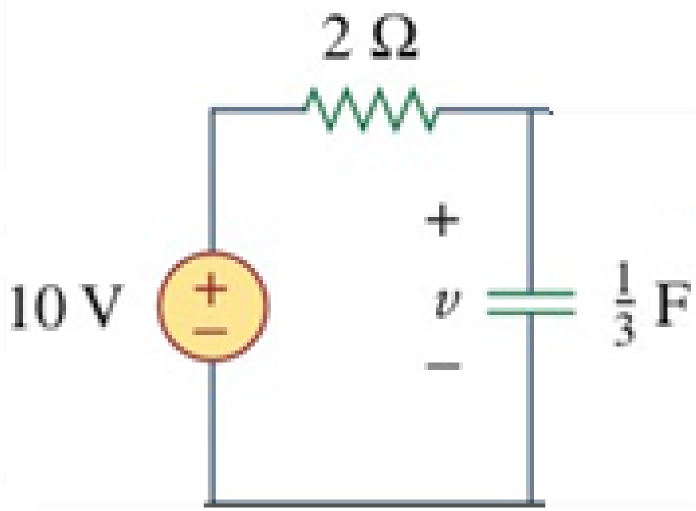
$$i(0^+) = i(0^-) = \frac{V_S}{R_1}$$

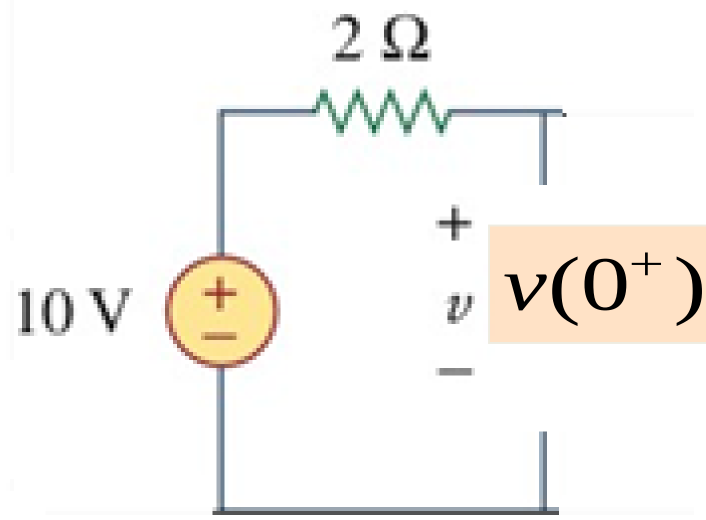
$$i(t) = \frac{V_S}{R_1} e^{-\frac{R_2}{L}t}$$

Determine the capacitor voltage as a function of time.



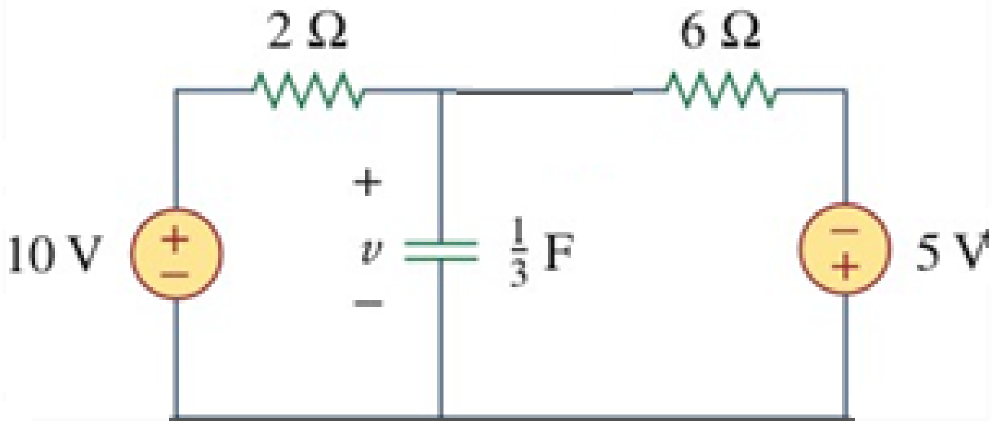
Circuit for t < 0





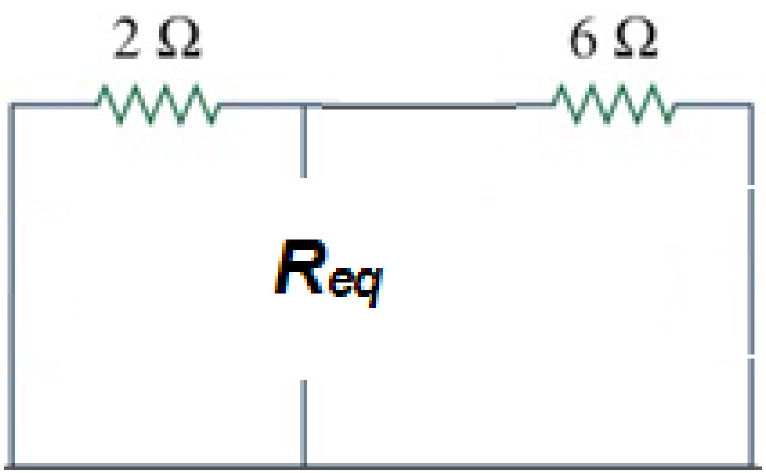
$$v(0^+) = 10 \quad \text{V}$$

Circuit for t > 0



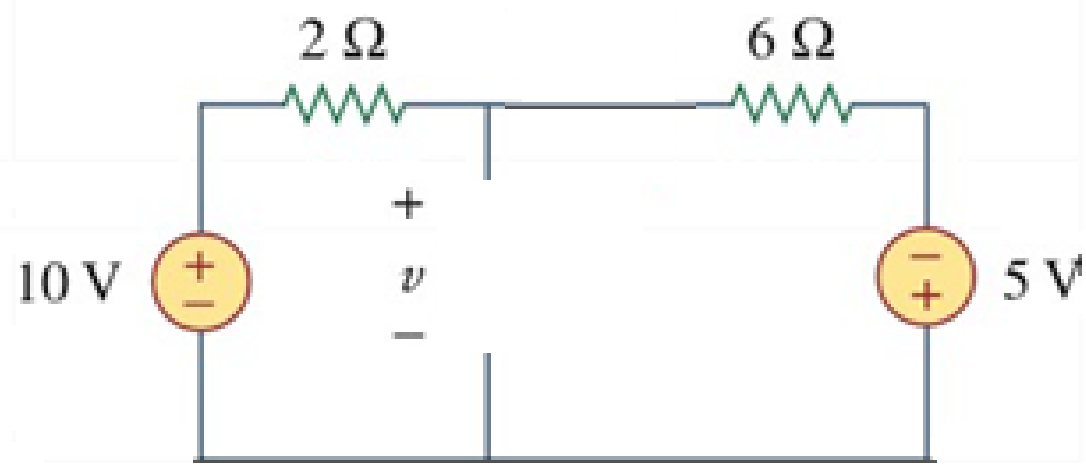
Determine the Thevenin equivalent, as seen by the capacitor:

Equivalent resistance



$$R_{eq} = 2 \| 6 = 1.5\Omega$$

We next find voltage long after closing the switch



$$v(\infty) = \frac{25}{4} \text{ V}$$

Final Solution:

$$v(t) = v(\infty) + \{v(0^+) - v(\infty)\}e^{-\frac{t}{\tau}}$$

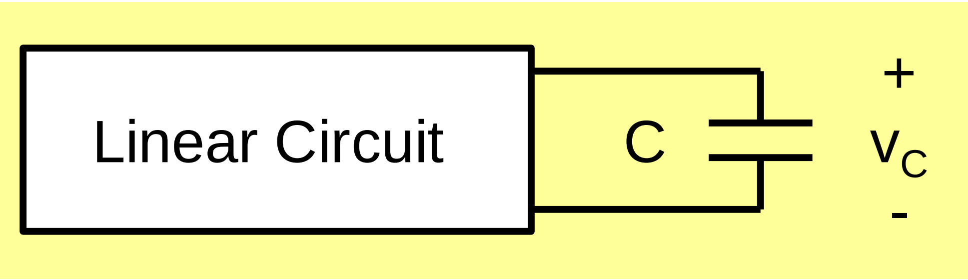
$$v(0^+) = 10 \text{ V}$$

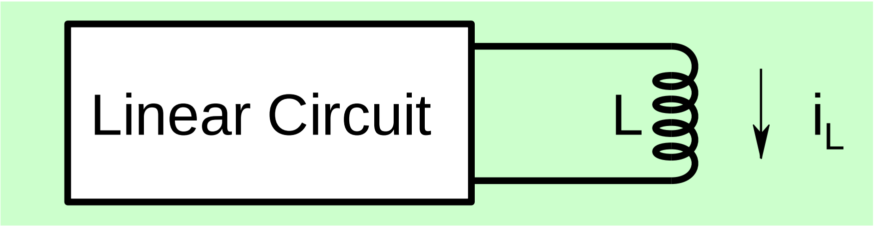
$$v(\infty) = \frac{25}{4} \text{ V}$$

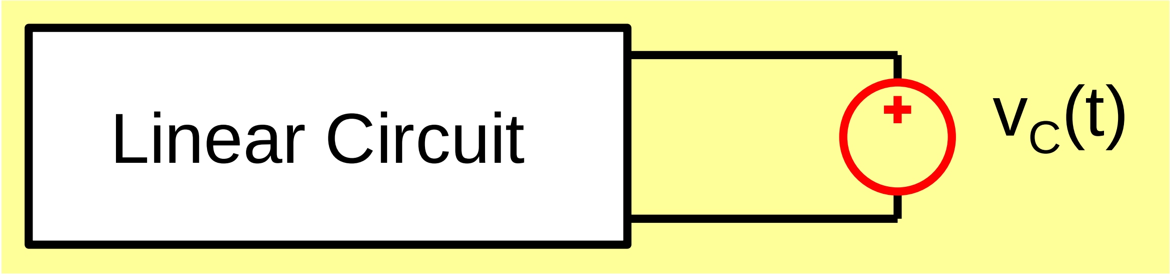
$$\tau = C \times R_{eq} = \frac{1}{3} \times 1.5 = 0.5s$$

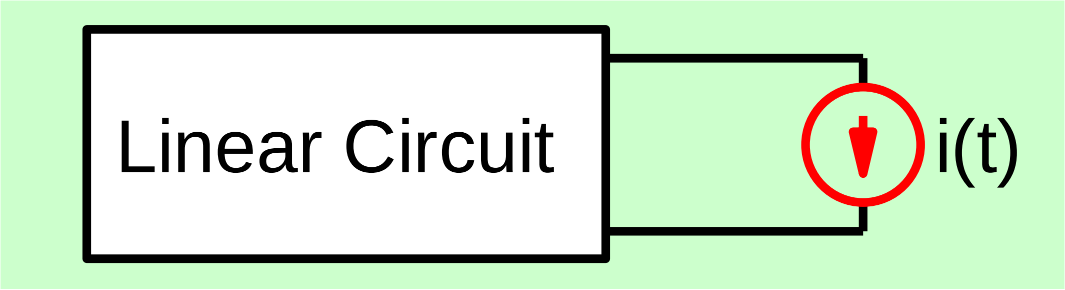
$$v(t) = \frac{25}{4} + \frac{15}{4}e^{-2t} \text{ V}$$

How do we find voltages and currents elsewhere in the circuit?

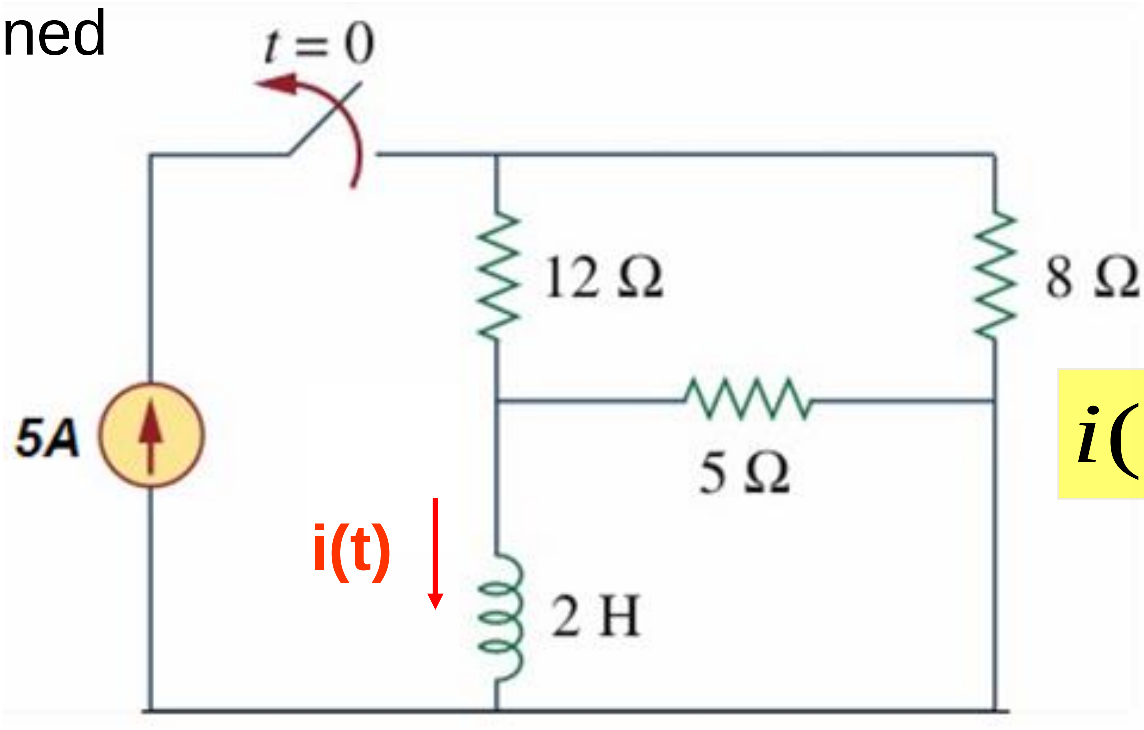








Find current in 8Ω resistor as a function of time after the switch is opened



$$i(t) = 2 \times e^{-2t} \text{ A}$$

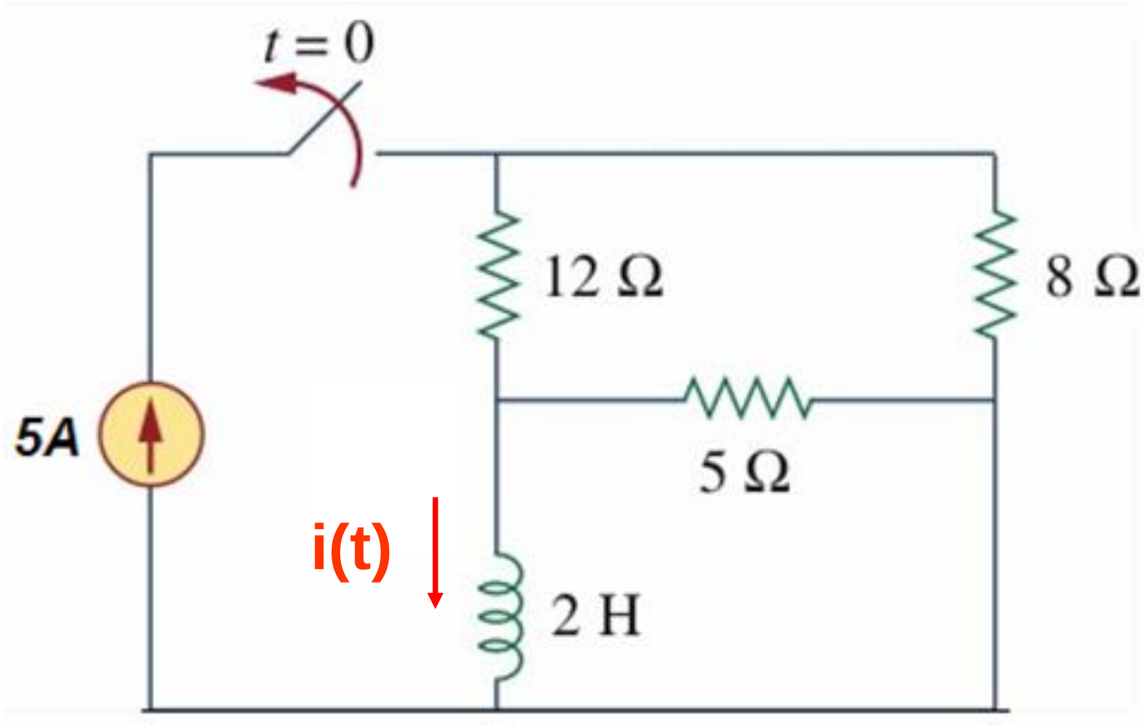
$$i(t=0^-) = 5A \times \left(\frac{8}{12+8} \right) = 2A$$

No current will flow in 5 Ω resistance for t < 0

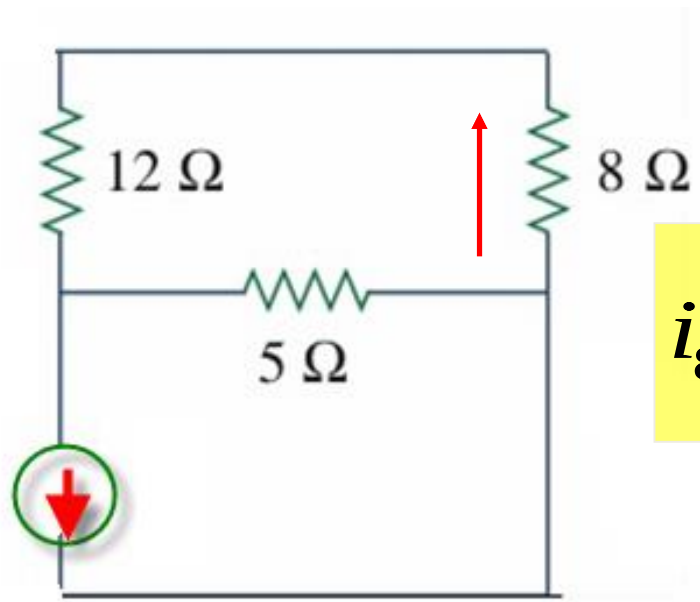
For t > 0

$$R_{eq} = (12+8) \,||\, 5 = 4 \quad \Omega$$

$$\tau = \frac{L}{R_{eq}} = 0.5 \sec$$

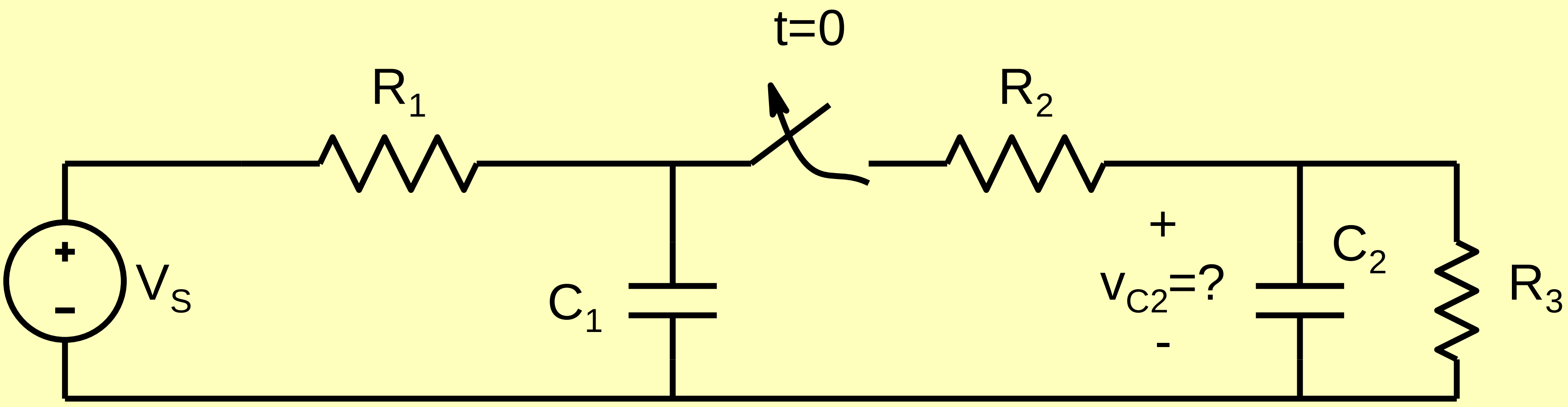


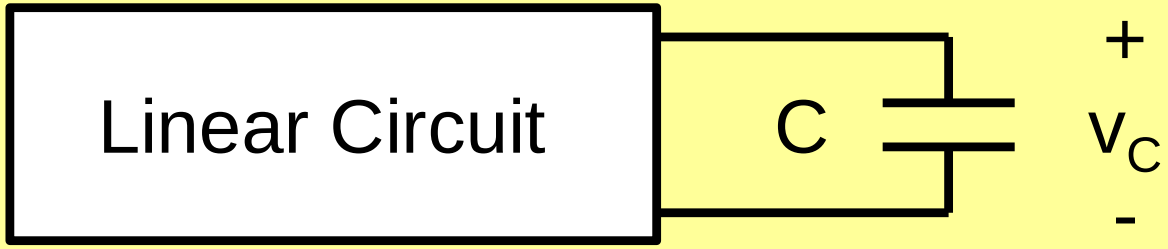
$$i(t) = 2 \times e^{-2t} \text{ A}$$



$$i_8 = i(t) \times \frac{5}{5+20} = 0.4 \times e^{-2t} \text{ A}$$

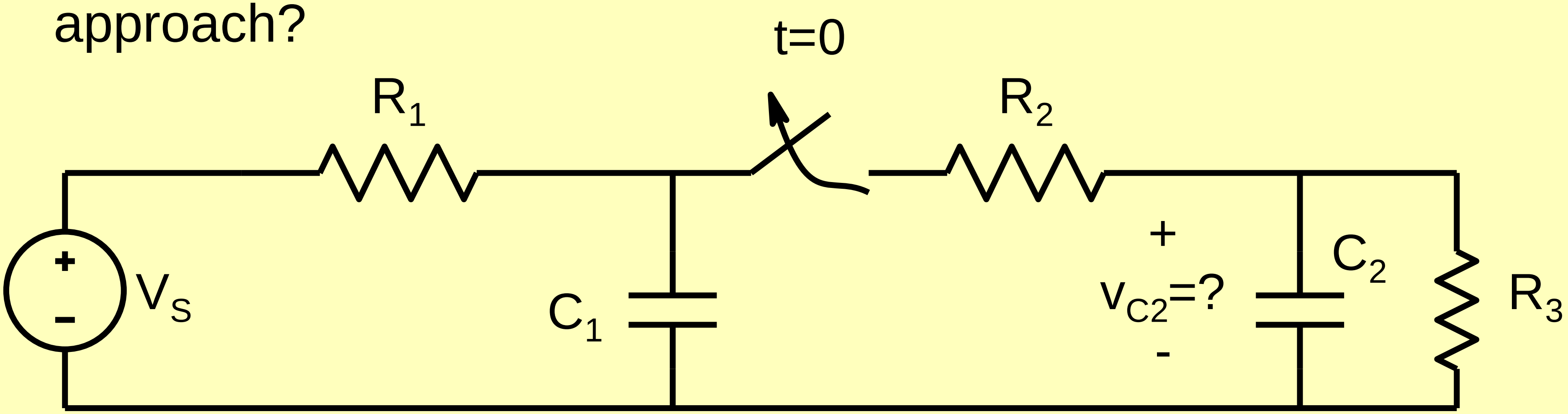
Can we solve this 2 capacitor problem using our present approach?

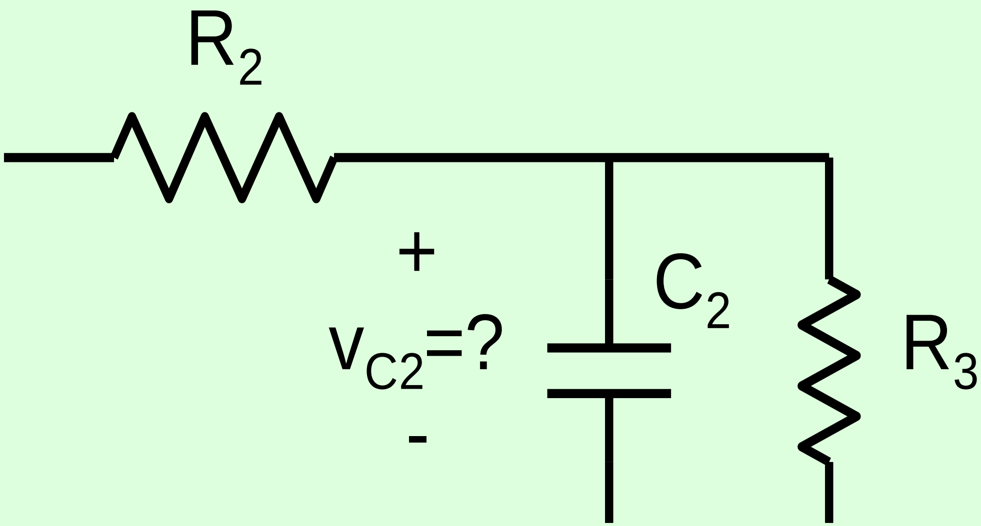




Circuit for t > 0

Can we solve this 2 capacitor problem using our present approach?



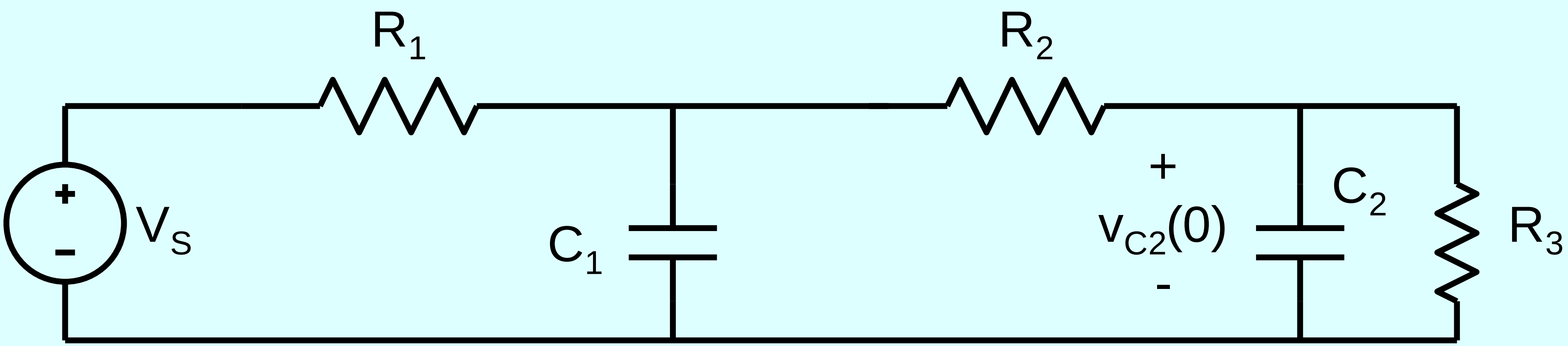


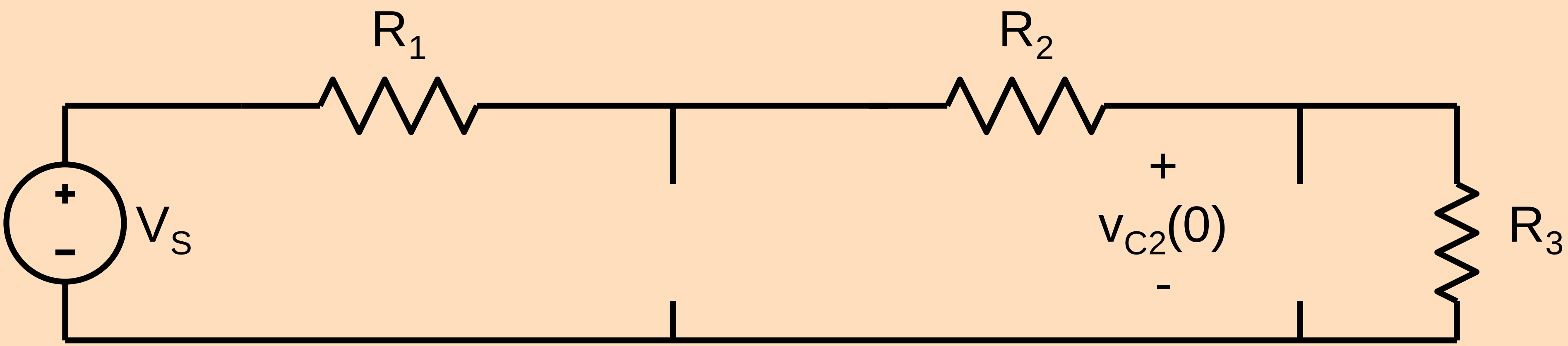
Circuit for t > 0

$$v_{c2}(0^+) = v_{c2}(0^-)$$

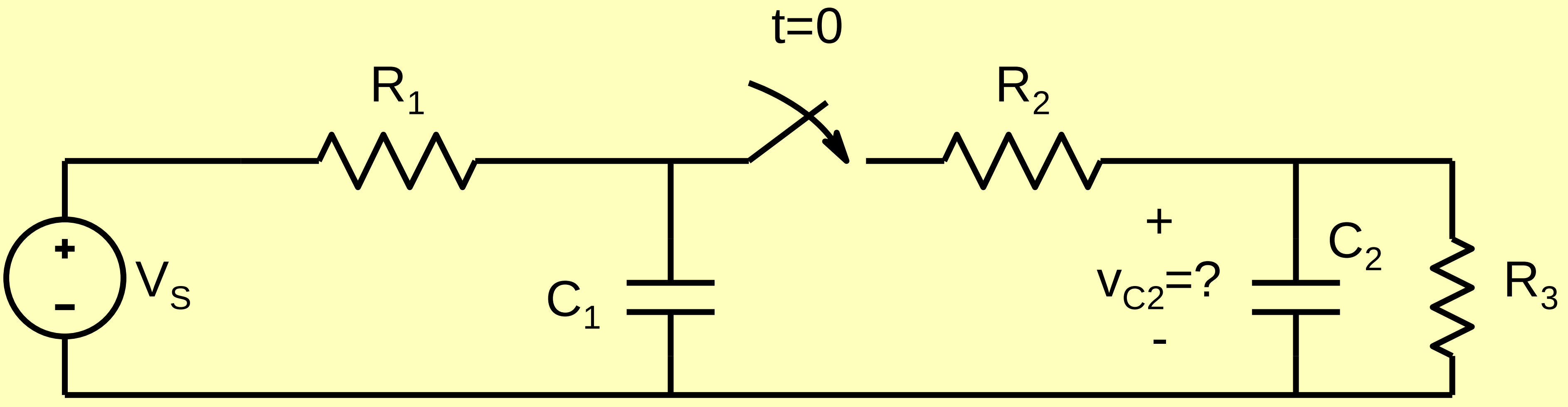
$$v_{c2}(t) = v_{c2}(\infty) + \{v_{c2}(0^+) - v_{c2}(\infty)\}e^{-\frac{t}{\tau}}$$

$$v_{c2}(0^+) = v_{c2}(0^-)$$





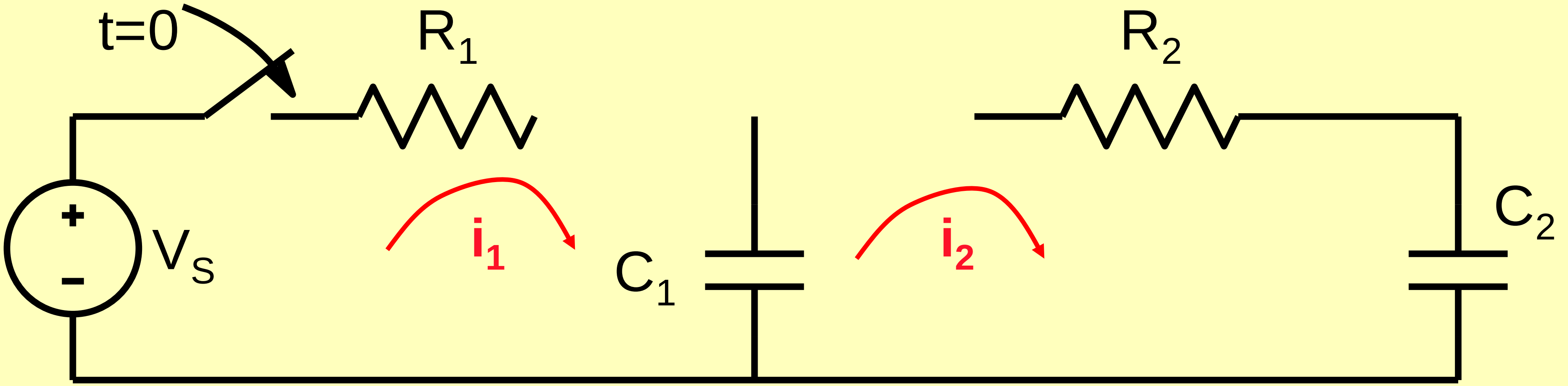
Will our approach work here?



No, because circuit for $t > 0$ has two capacitances

As long as the circuit has single capacitor or inductor for the time interval for which the analysis is being carried out, the stated approach will work fine.

What happens when there is more than one storage element?



$$V_S = i_1 R_1 + v_{C1} \quad (1)$$

$$v_{C1} = i_2 R_2 + v_{c2} \quad (2)$$

$$i_1 - i_2 = C_1 \frac{dv_1}{dt} \quad (3)$$

$$i_2 = C_2 \frac{dv_2}{dt} \quad (4)$$

$$R_1 R_2 C_1 C_2 \frac{d^2 v_{c2}}{dt^2} + (R_1 C_1 + R_1 C_2 + R_2 C_2) \frac{dv_{c2}}{dt} + v_{c2} = V_S$$