ESc201 : Introduction to Electronics

Frequency Domain Response

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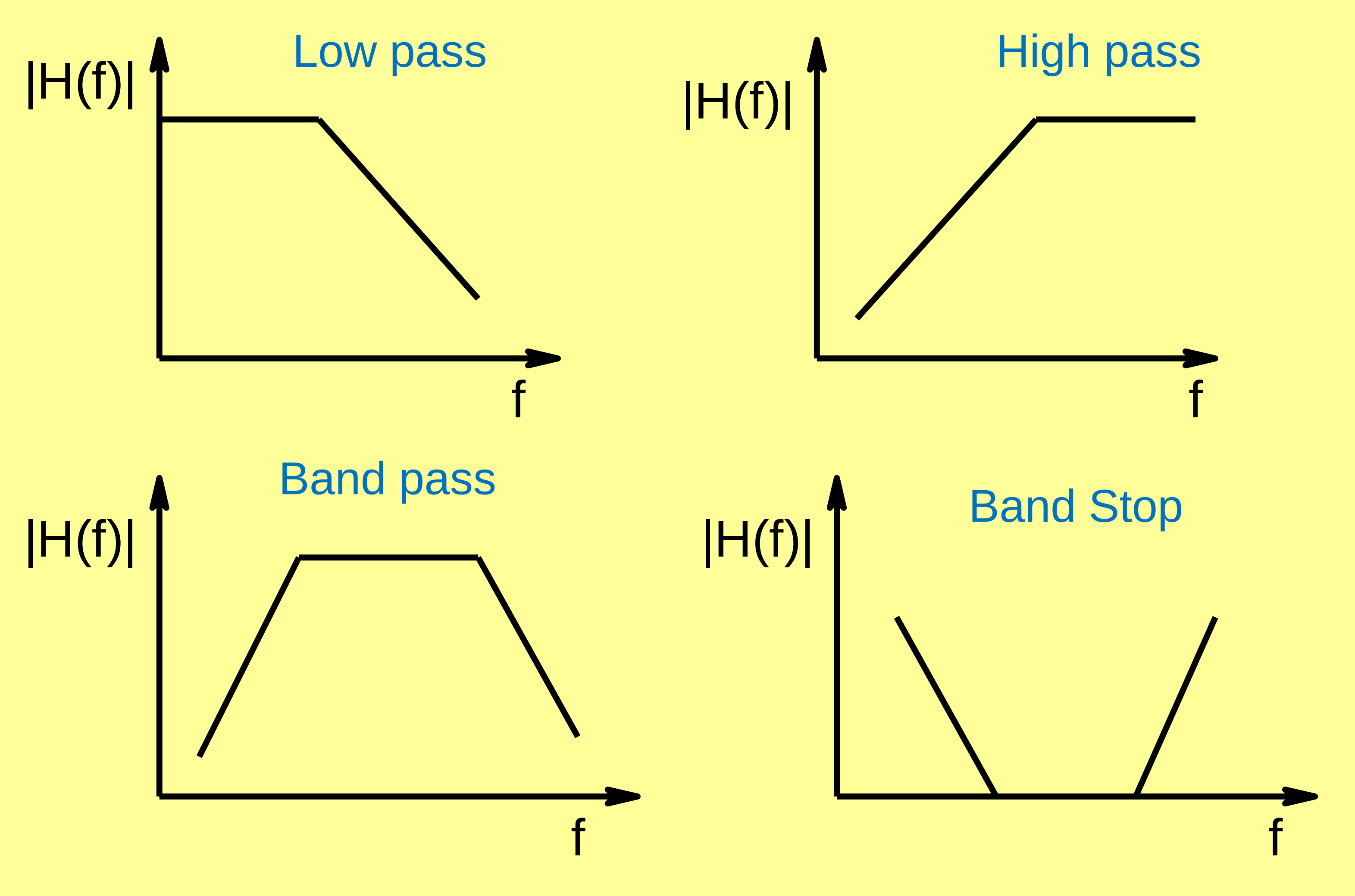
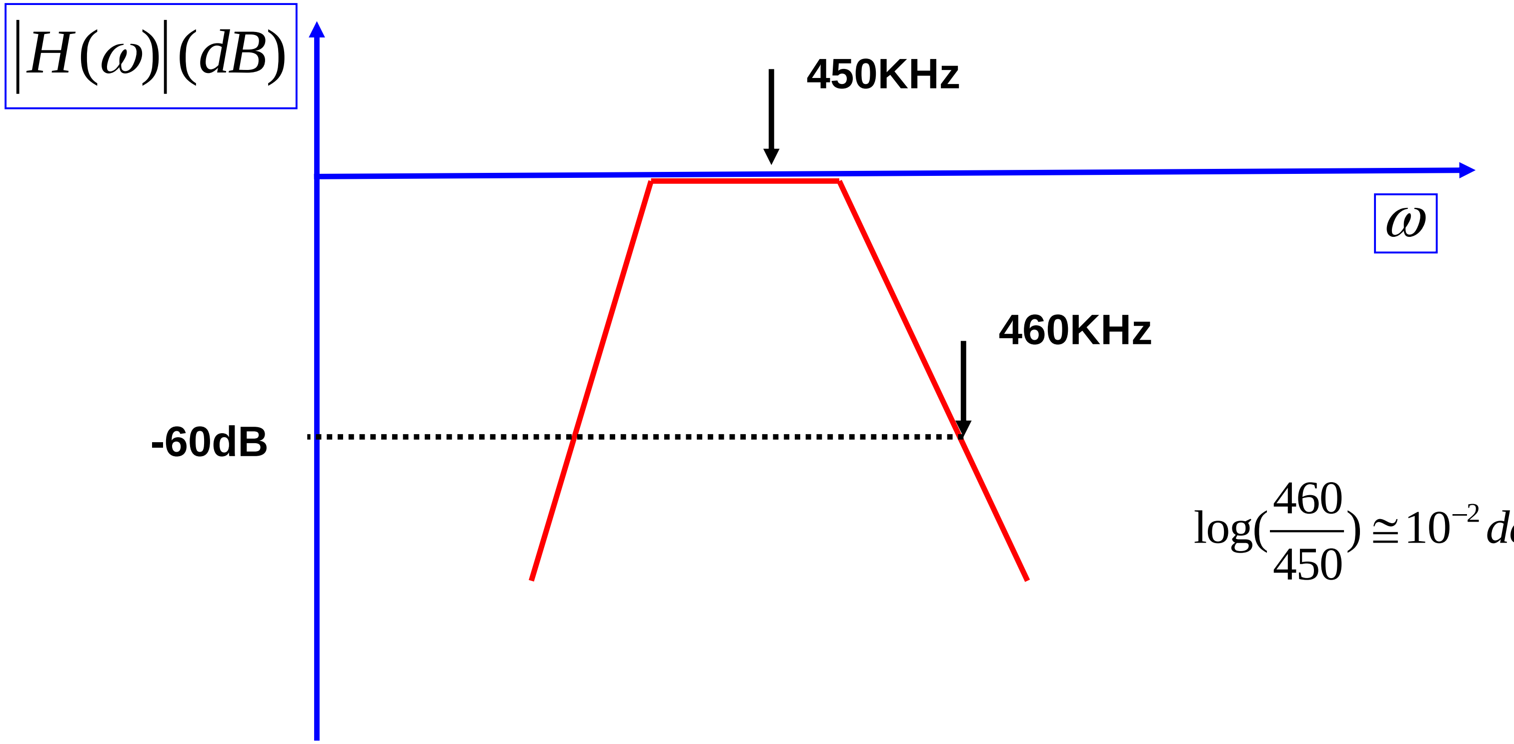
Filter -pass a band of frequency and reject the remaining



Amplitude Modulated (AM) Radio

Different radio channels are separated by very narrow frequency interval.

For example, one may want to receive a 450KHz signal but reject 460KHz or 440KHz

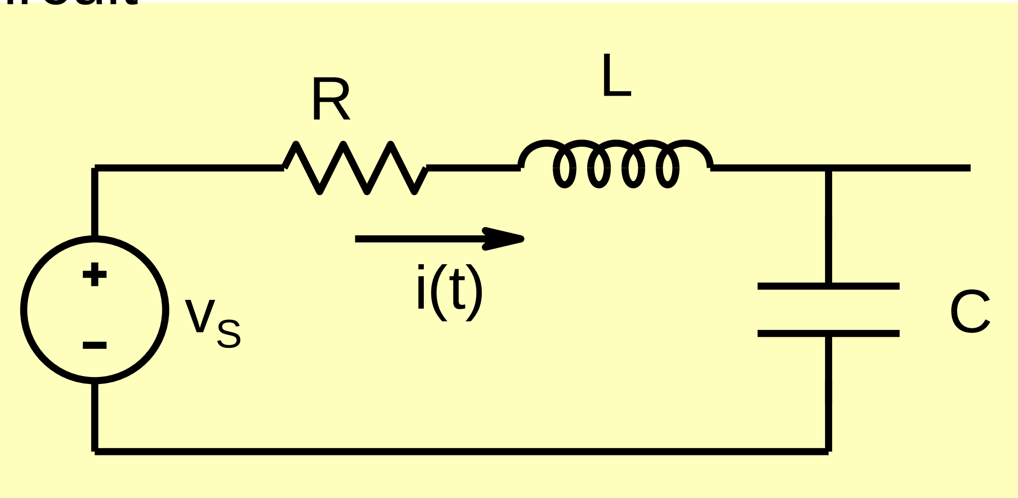


$$\log\left(\frac{460}{450}\right) \cong 10^{-2}\, decades$$

This implies an attenuation of -6000 dB/decade !!

Series Resonant Circuit

Resonance is a condition in which capacitive and inductive reactance cancel each other to give rise to a purely resistive circuit



$$Z_{eq} = R + j\omega L - j\frac{1}{\omega C}$$

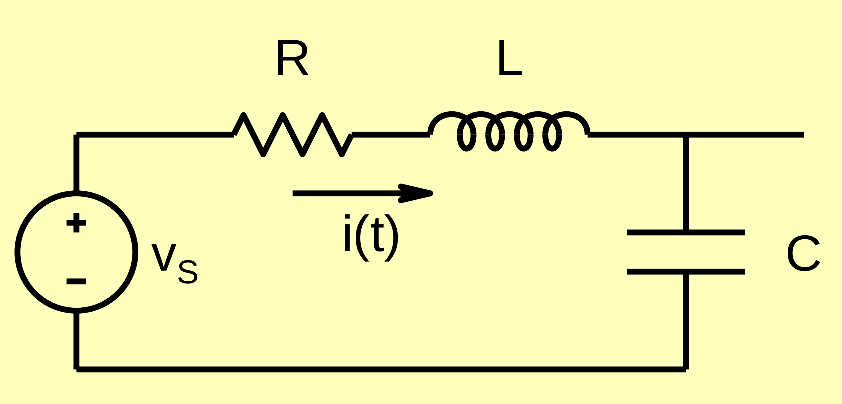
Resonant frequency:

$$j\omega_O L - j\frac{1}{\omega_O C} = 0 \Rightarrow \omega_O = \frac{1}{\sqrt{LC}}$$

$$f_O = \frac{1}{2\pi\sqrt{LC}}$$

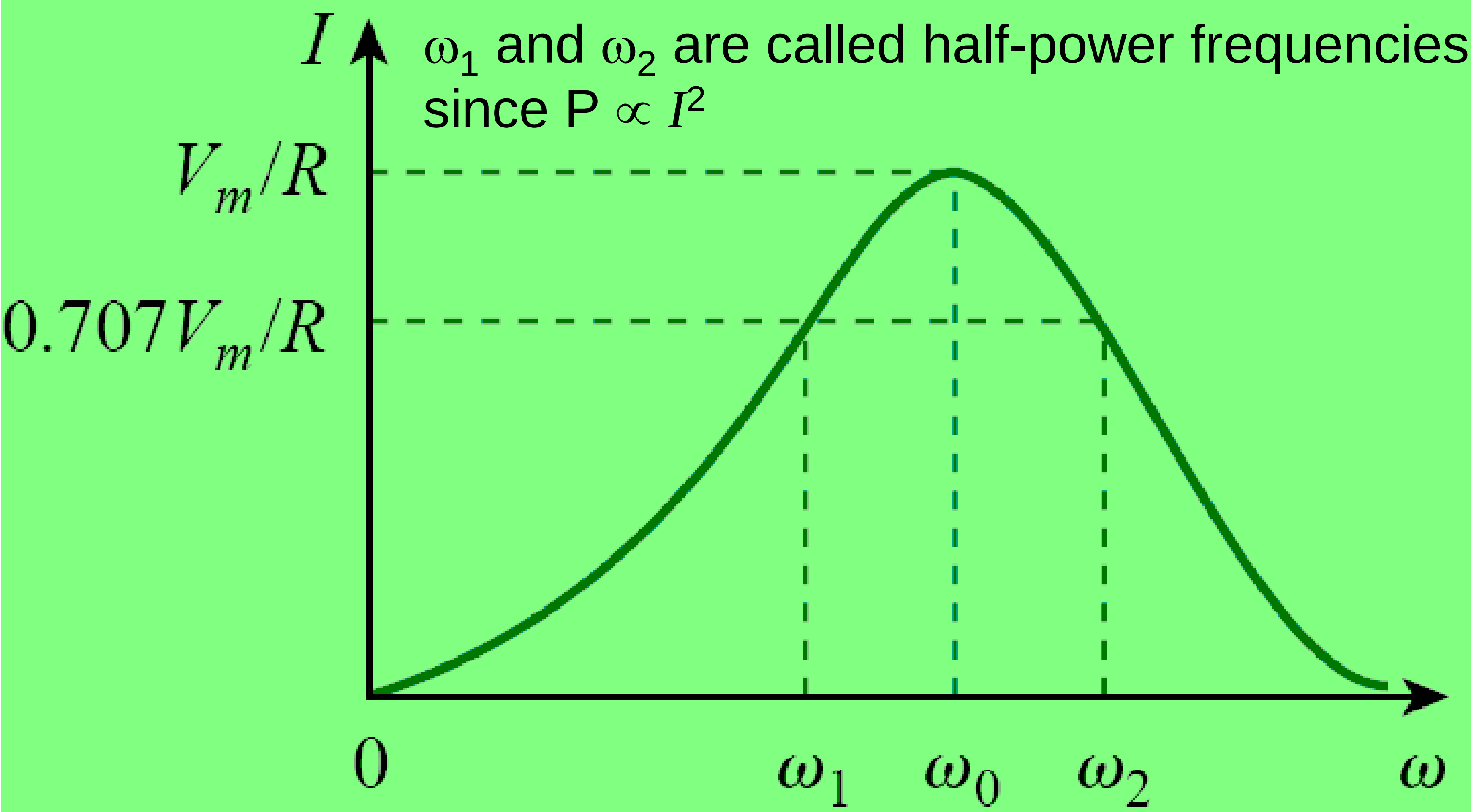
$$Z_{eq} = R$$

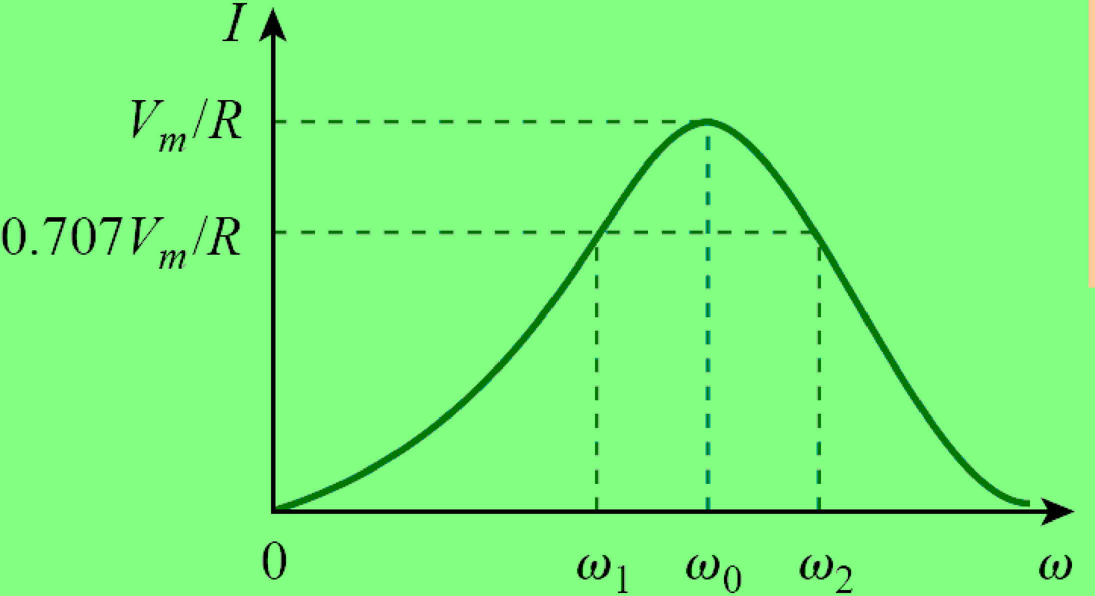
Current and voltage are in phase (power factor is unity) and current is maximum !



$$|I(\omega)| = \frac{V_m}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

ω_1 and ω_2 are called half-power frequencies since P $\propto$ I^2





$$|I(\omega)| = \frac{V_m}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

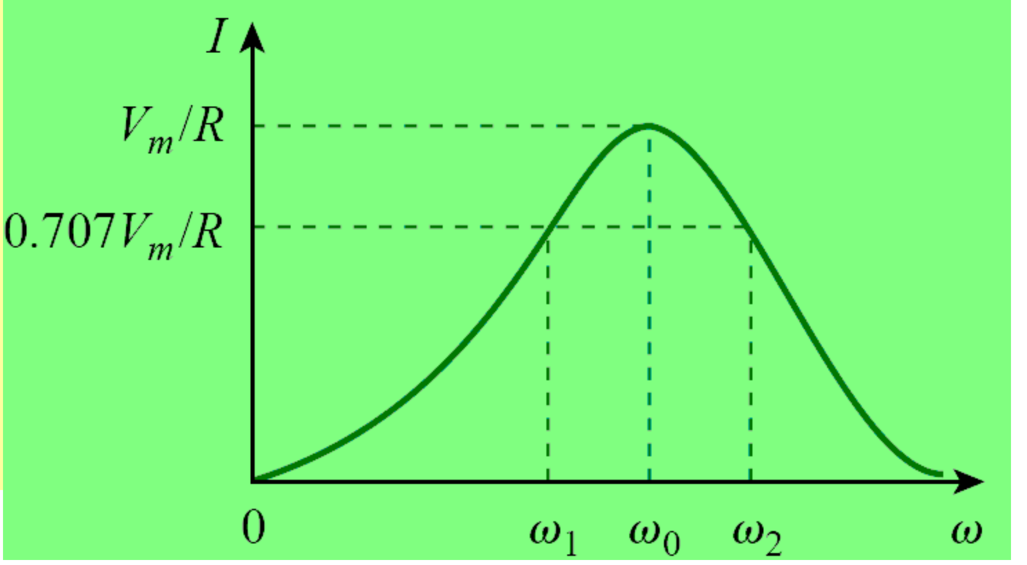
$$|I(\omega_1)| = \frac{V_m}{\sqrt{R^2 + (\omega_1 L - \frac{1}{\omega_1 C})^2}} = \frac{V_m}{\sqrt{2}R}$$

$$|I(\omega_2)| = \frac{V_m}{\sqrt{R^2 + (\omega_2 L - \frac{1}{\omega_2 C})^2}} = \frac{V_m}{\sqrt{2}R}$$

ω_1 and ω_2 are called half-power frequencies since $P \propto I^2$

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$



$$\omega_O = \sqrt{\omega_1 \omega_2}$$

$$B = \omega_2 - \omega_1 = \frac{R}{L}$$

Quality (Q) factor: Sharpness of resonance

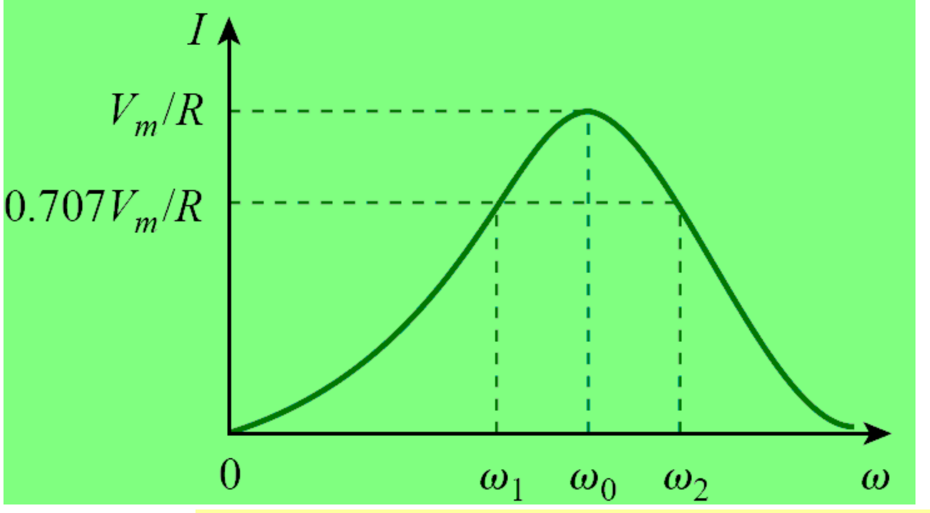
$$Q = \frac{\omega_O L}{R}$$

$$\omega_O = \frac{1}{\sqrt{LC}} \Rightarrow Q = \frac{1}{\omega_O CR}$$

$$B = \omega_2 - \omega_1 = \frac{R}{L}$$

$$Q = \frac{\omega_O}{B} = \frac{\omega_O}{\Delta\omega}$$

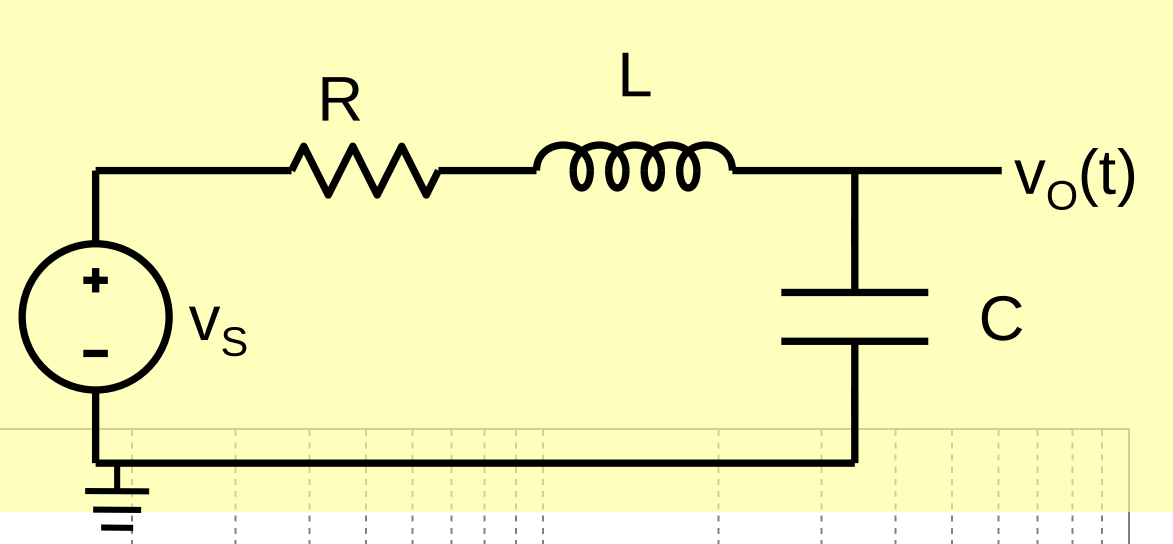
Hence Q represents sharpness of resonance

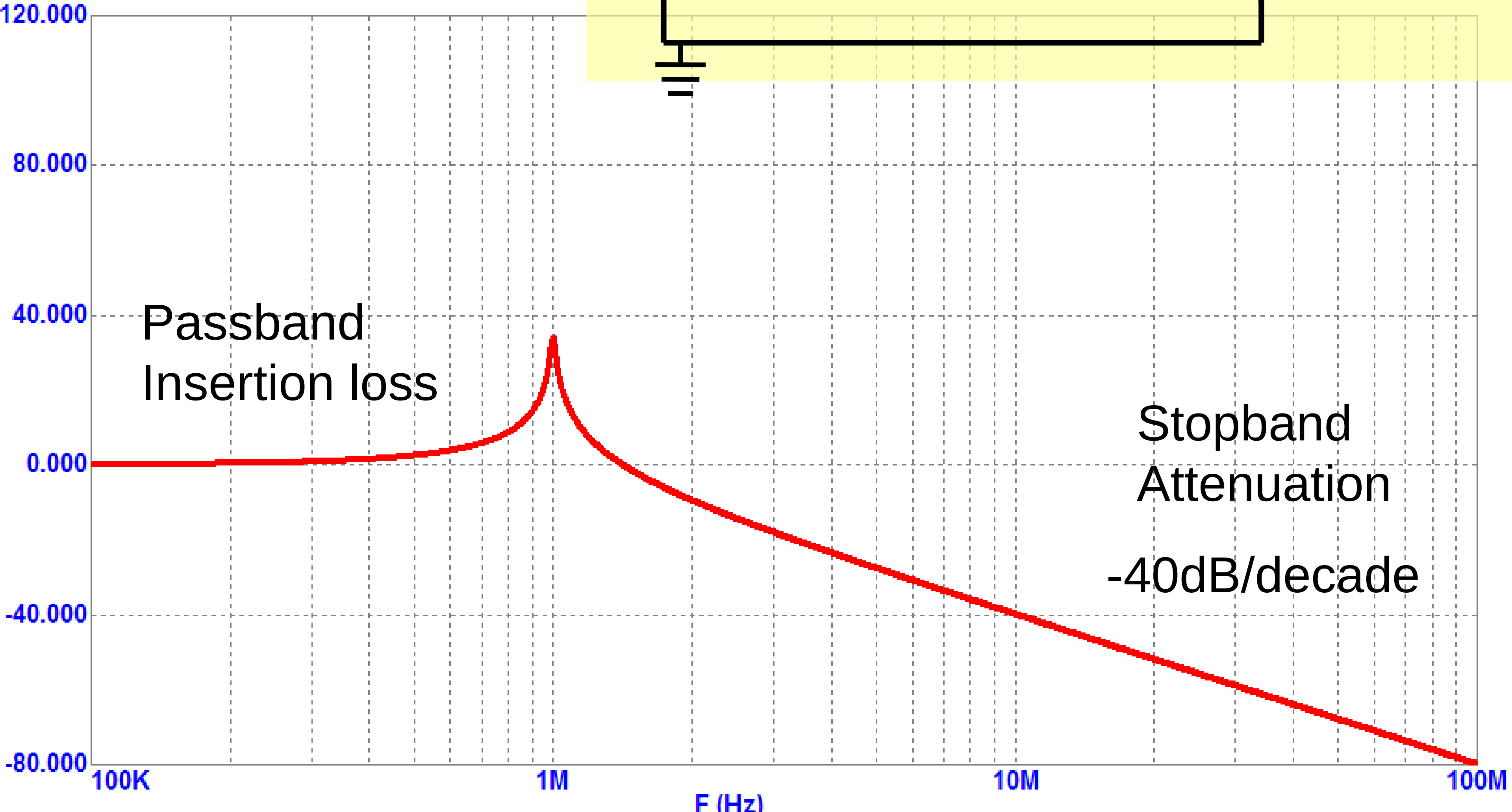


For high Q circuits:

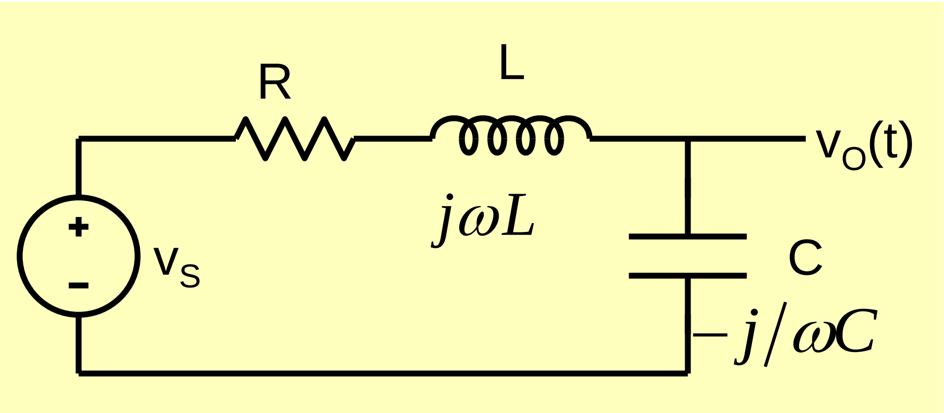
$$\omega_1 \simeq \omega_0 - \frac{B}{2}, \qquad \omega_2 \simeq \omega_0 + \frac{B}{2}$$

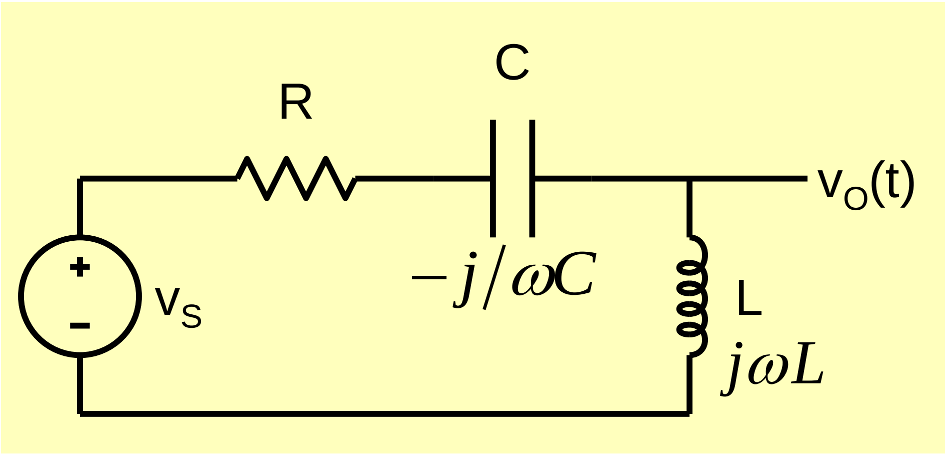
$$Q = \frac{\omega_O L}{R}$$

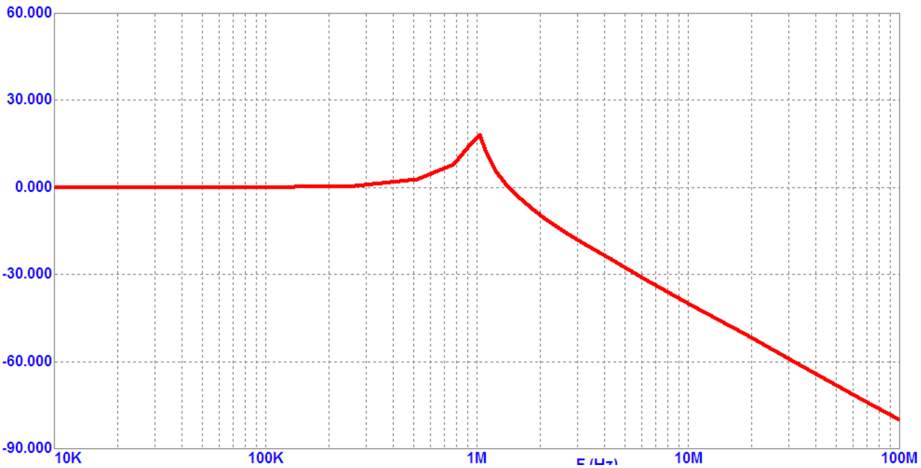


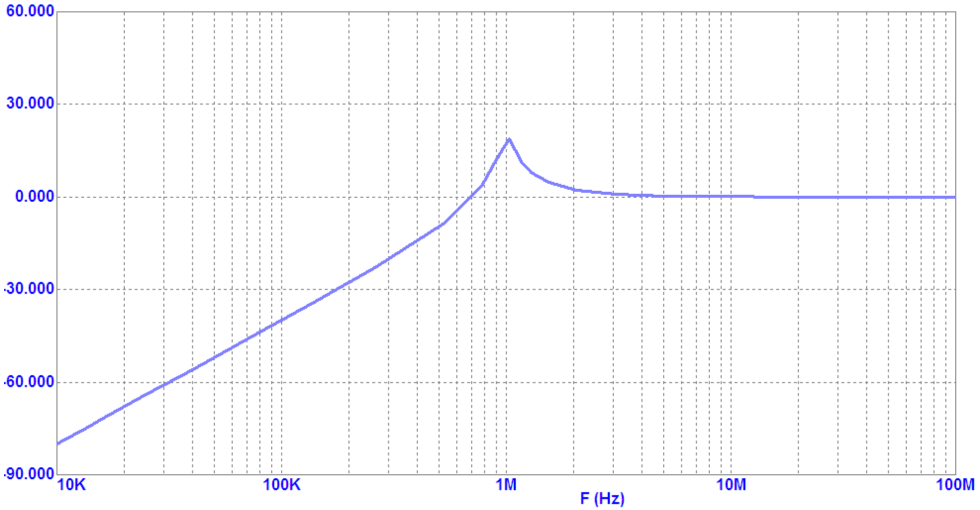


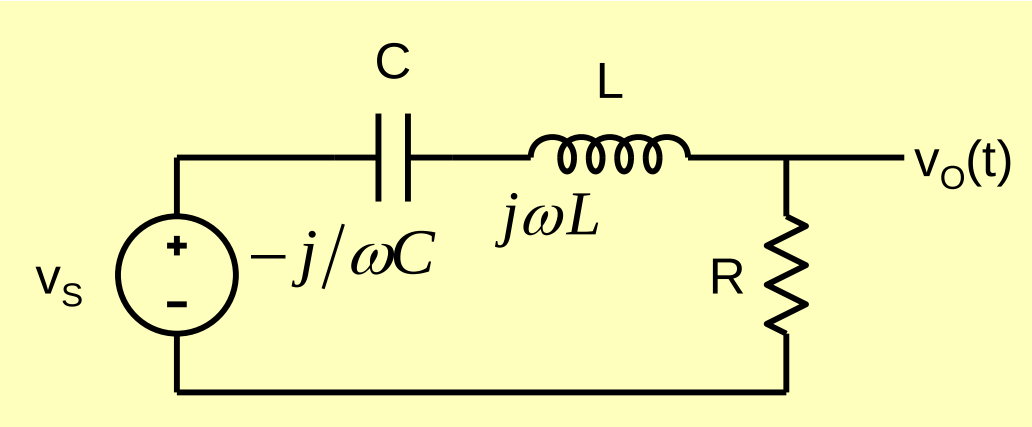
R-L-C filters

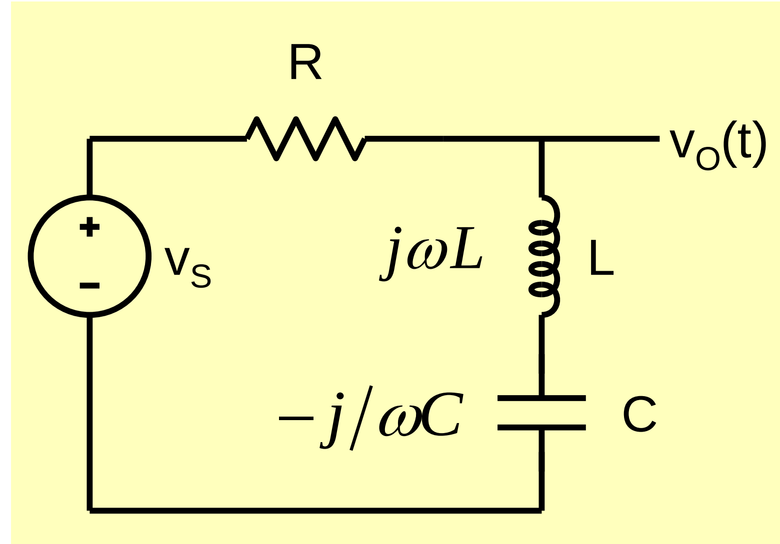

R
L
$v_O(t)$
$j\omega L$
v_S
C
$-j/\omega C$

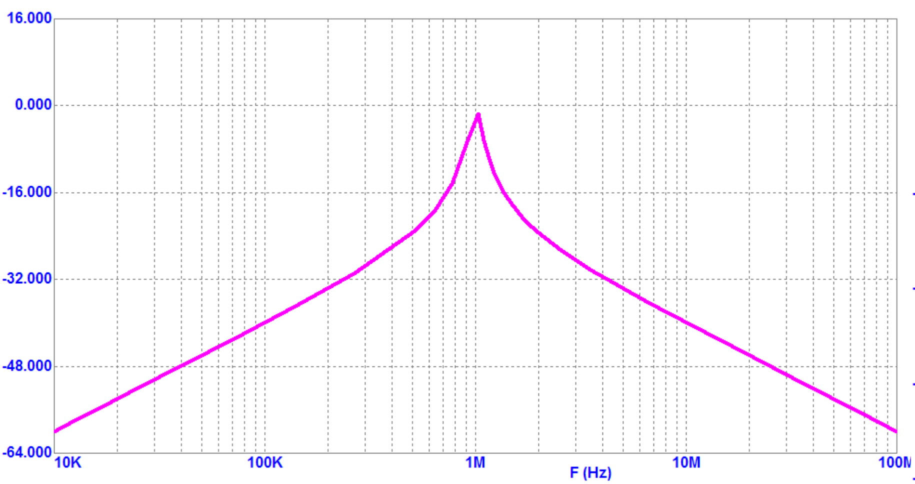

C
R
$v_O(t)$
v_S
$-j/\omega C$
L
$j\omega L$

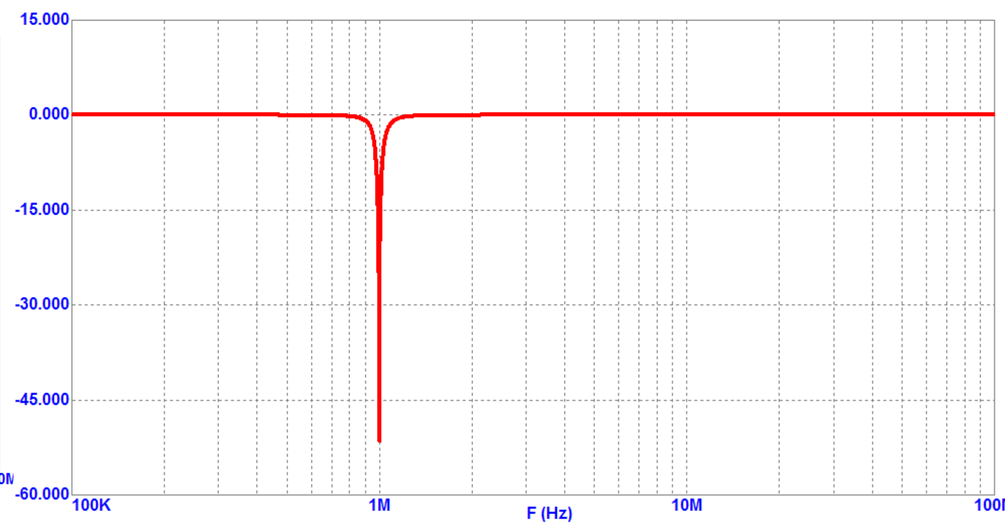

60.000
30.000
0.000
-30.000
-60.000
-90.000
10K
100K
1M
10M
100M

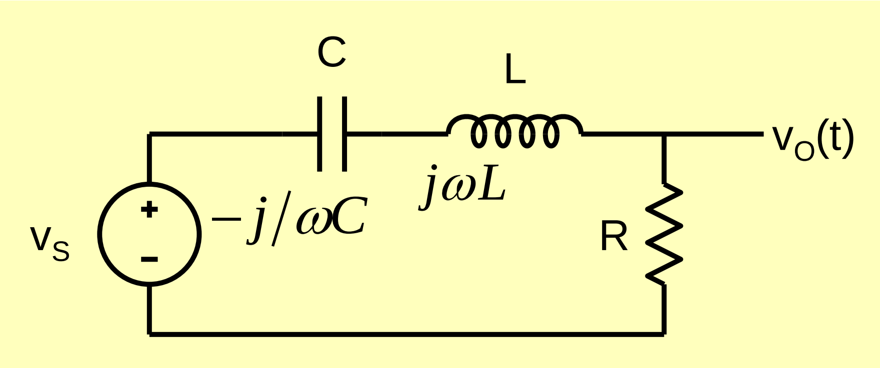

60.000
30.000
0.000
-30.000
-60.000
-90.000
10K
100K
1M
F (Hz)
10M
100M

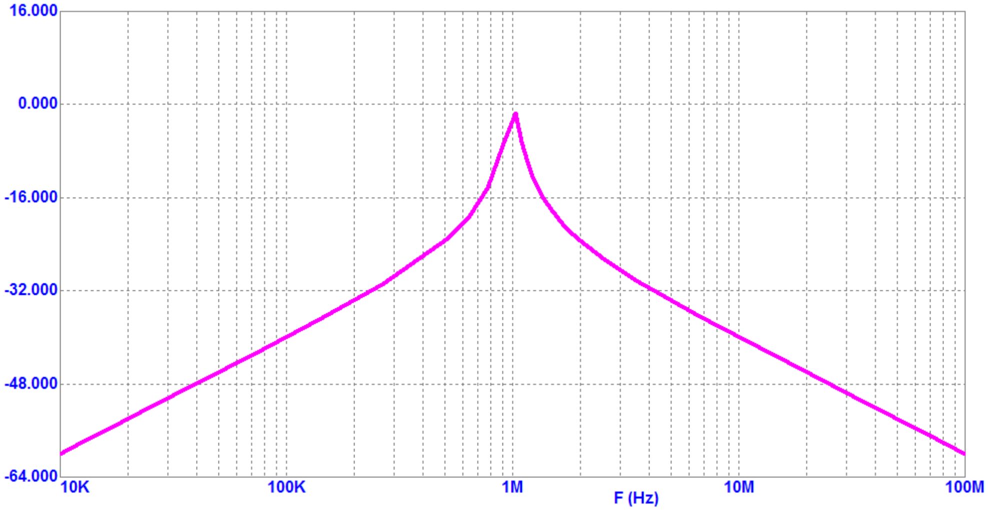

C
L
$v_O(t)$
v_S
$-j/\omega C$
$j\omega L$
R


R
$v_O(t)$
v_S
$j\omega L$
L
$-j/\omega C$
C


16.000
0.000
-16.000
-32.000
-48.000
-64.000
10K
100K
1M
F (Hz)
10M
100M


15.000
0.000
-15.000
-30.000
-45.000
-60.000
100K
1M
F (Hz)
10M
100M





How much Q do we need to pass 450KHz but reject 460KHz by 60dB?

$$|H(\omega)| = \left|\frac{V_O(\omega)}{V_{IN}(\omega)}\right| = \frac{R}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

Assuming V_{IN} = 1V and noting that $Q = \omega_O L/R$

$$|V_O(\omega)| = \frac{1}{\sqrt{1 + Q^2(\frac{\omega^2}{\omega_O^2} - 1)^2}}$$

For $\omega=\omega_O$, V_O = 1 so the signal simply passes through !

$$\omega_O = 2 \times \pi \times 450 \times 10^3 = 2.8 \times 10^6\, rad/s$$

$$|V_O(\omega)| = \frac{1}{\sqrt{1 + Q^2 (1 - \frac{\omega_O^2}{\omega^2})^2}}$$

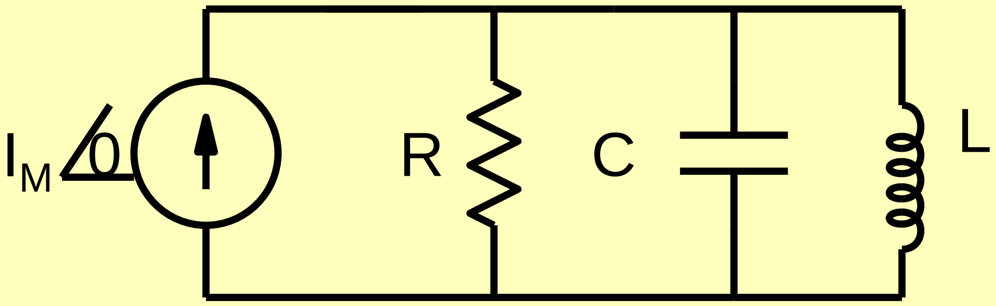
$$\omega_0 = 2\pi \times 450 \times 10^3 = 2.827 \times 10^6 \, rad/s$$

$$\omega = 2\pi \times 460 \times 10^3 = 2.89 \times 10^6 \, rad/s$$

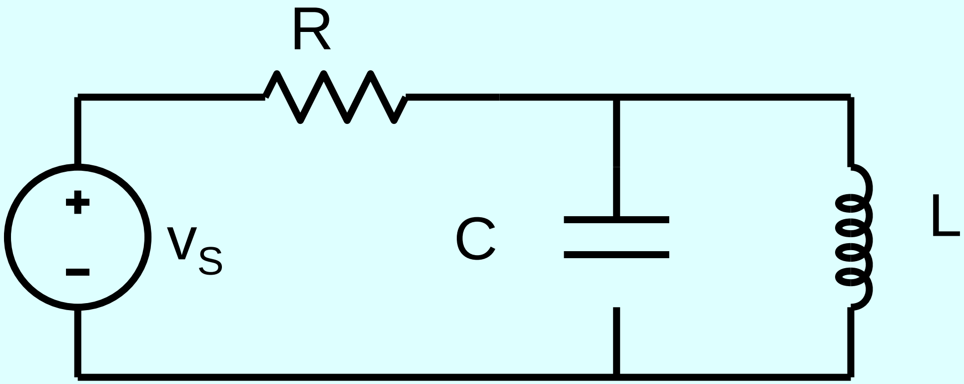
For an attenuation of -60dB or 10^{-3} at ω: **Q=23,000**

This is a large value of Q!

Parallel Resonance



$$Y_{eq} = \frac{1}{R} + j\omega C - j\frac{1}{\omega L}$$

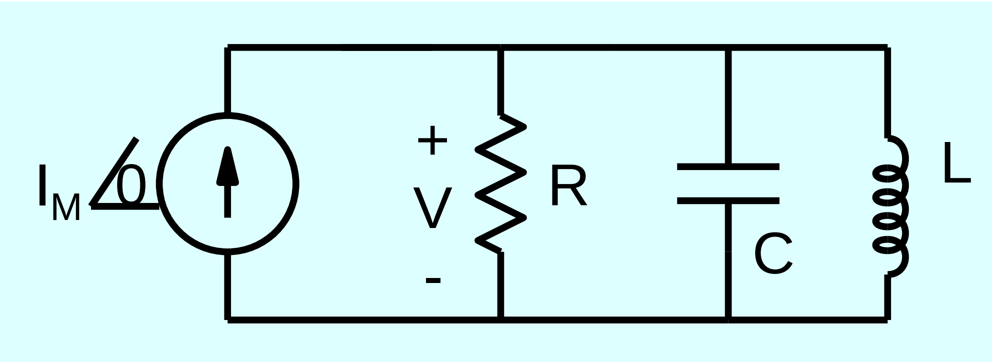


Resonant frequency:

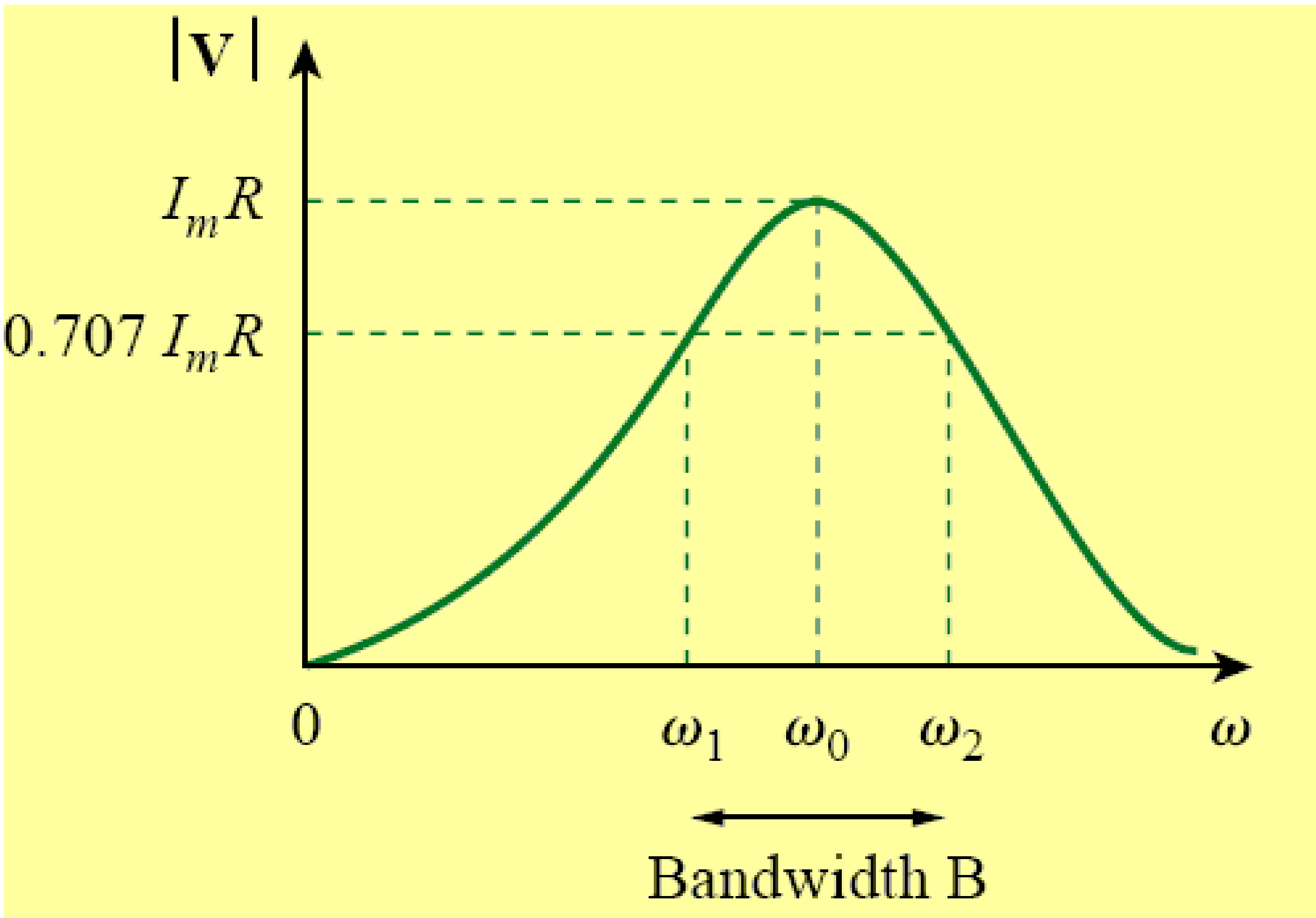
$$j\omega_O C - j\frac{1}{\omega_O L} = 0 \Rightarrow \omega_O = \frac{1}{\sqrt{LC}}$$

$$f_O = \frac{1}{2\pi\sqrt{LC}}$$

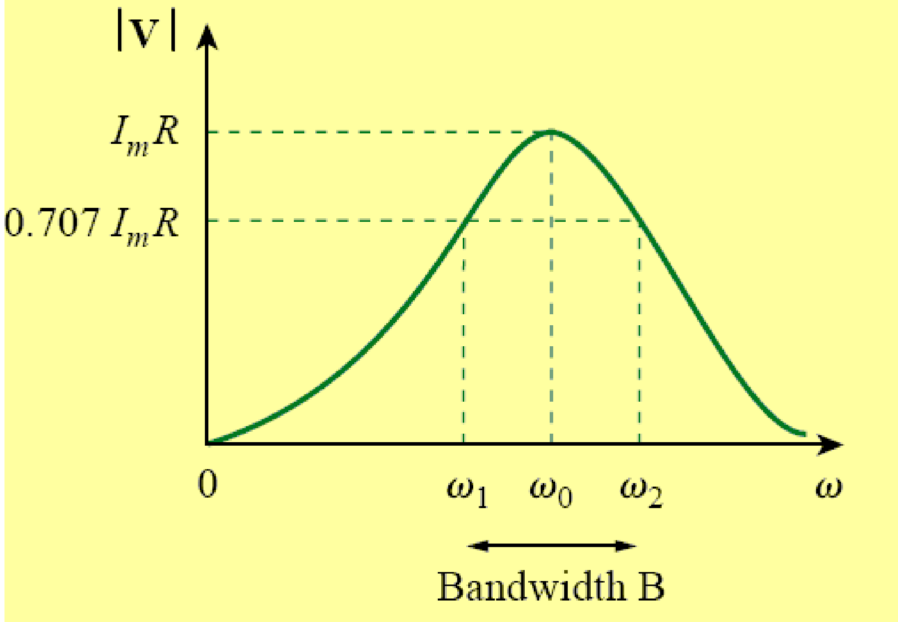
$$Z_{eq} = R$$



$$|V(\omega)| = \frac{I_m R}{\sqrt{1 + \frac{R^2 C^2}{L^2}\left(\omega L - \frac{1}{\omega C}\right)^2}}$$



$$|V(\omega)| = \frac{I_m R}{\sqrt{1 + \frac{R^2C^2}{L^2}(\omega L - \frac{1}{\omega C})^2}}$$



$$\omega_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$B = \omega_2 - \omega_1 = \frac{1}{RC}$$

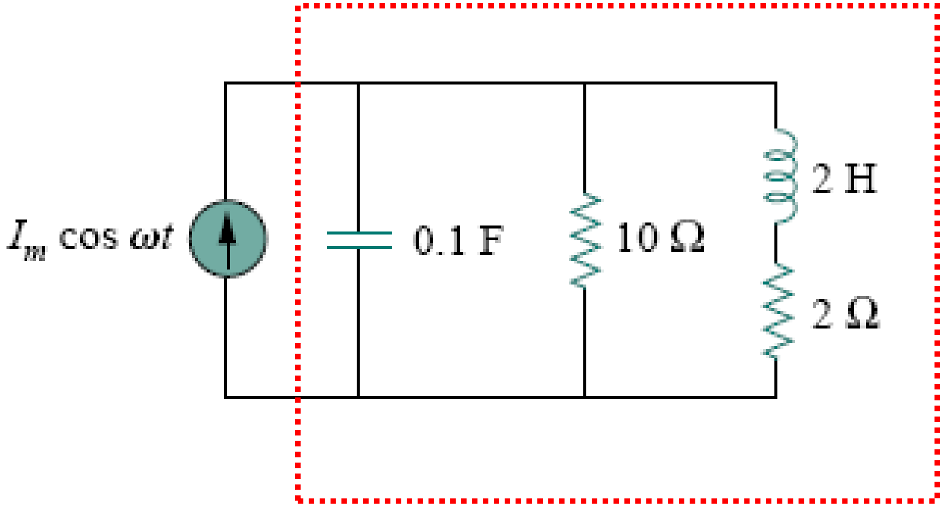
$$Q = \frac{\omega_0}{B} = \omega_0 RC = \frac{R}{\omega_0 L}$$

$$\omega_1 = \omega_0\sqrt{1 + \left(\frac{1}{2Q}\right)^2} - \frac{\omega_0}{2Q}, \qquad \omega_2 = \omega_0\sqrt{1 + \left(\frac{1}{2Q}\right)^2} + \frac{\omega_0}{2Q}$$

For high Q:

$$\omega_1 \simeq \omega_0 - \frac{B}{2}, \qquad \omega_2 \simeq \omega_0 + \frac{B}{2}$$

What is the resonant frequency ?



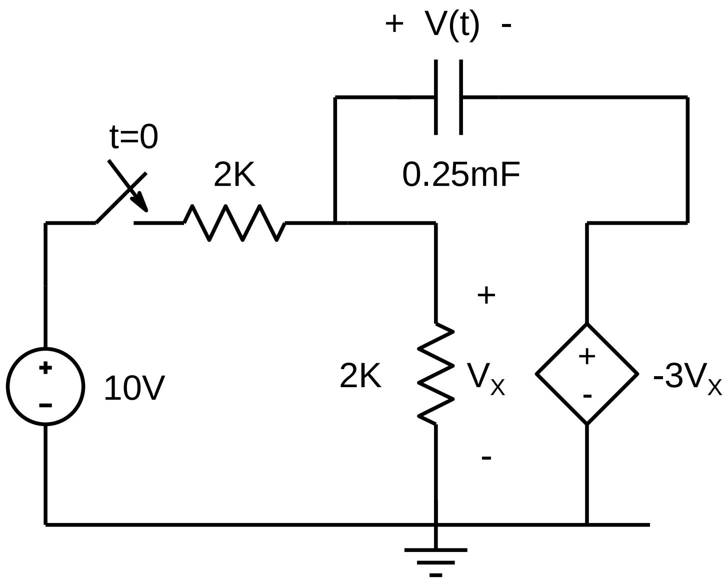
$$\mathbf{Y} = j\omega 0.1 + \frac{1}{10} + \frac{1}{2 + j\omega 2} = 0.1 + j\omega 0.1 + \frac{2 - j\omega 2}{4 + 4\omega^2}$$

At resonance, $\mathrm{Im}(\mathbf{Y}) = 0$

$$\omega_0 0.1 - \frac{2\omega_0}{4 + 4\omega_0^2} = 0 \qquad \Longrightarrow \qquad \omega_0 = 2 \text{ rad/s}$$

$$f_o = \frac{\omega_o}{2\pi}$$

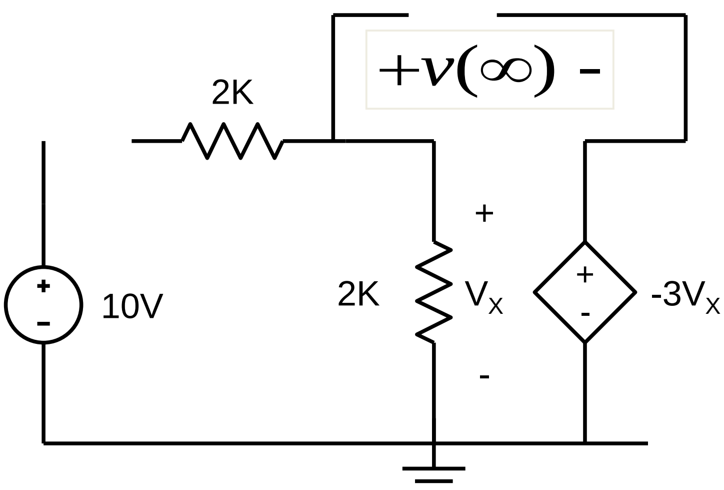
Q.1 Assuming that the capacitor does not have any initial charge, determine the voltage across the capacitor V(t) as a function of time after the switch is closed at t = 0. Also find the energy stored in the capacitor at t = 1 s.



$$v(t) = v(\infty) + \{v(0^+) - v(\infty)\}e^{-\frac{t}{\tau}}$$

$$v(0^-) = v(0^+) = 0$$

$v(\infty)$ can be found from the circuit

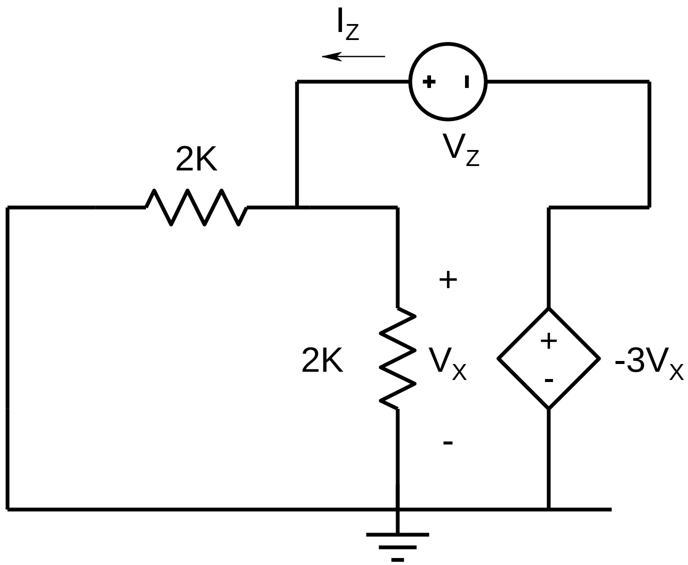


$$v_X = \frac{2K}{2K+2K} \times 10 = 5V$$

$$v(\infty) = V_X - (-3V_X) = 4V_x = 20V$$

$$\tau = CR_{eq}$$

R_{eq} can be found from the circuit:



$$R_{eq} = \frac{v_Z}{i_Z}$$

$$v_Z = v_X - -3v_X = 4v_X$$

$$i_Z = \frac{v_X}{1K}$$

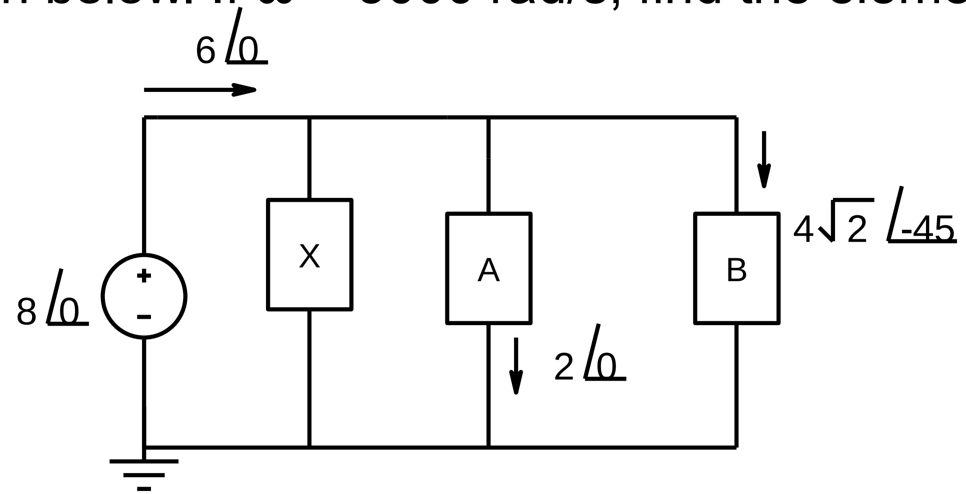
$$R_{eq} = \frac{v_Z}{i_Z} = 4K$$

$$\tau = CR_{eq} = 1s$$

$$v(t) = 20\{1 - e^{-t}\}$$

$$Energy\ Stored = \frac{1}{2} * C * v(t = 1)^2 = 20\ mJ$$

Q.2 Determine the impedance of element X for the given currents and voltages in the circuit shown below. If ω = 5000 rad/s, find the element X value ?



$$6\angle 0 = i_X + 2\angle 0 + 4\sqrt{2}\angle -45$$
$$= i_X + 2 + 4 - 4j$$

$$\Rightarrow i_X = 4j$$

$$\Rightarrow Z_X = \frac{8}{4j} = -2j$$

Element X is a capacitor

$$-\frac{j}{\omega C} = -2j \rightarrow C = \frac{1}{2\omega} \rightarrow C = 100\ \mu F$$