ESc201 : Introduction to Electronics

Transient Analysis of Capacitive and Inductive Circuits

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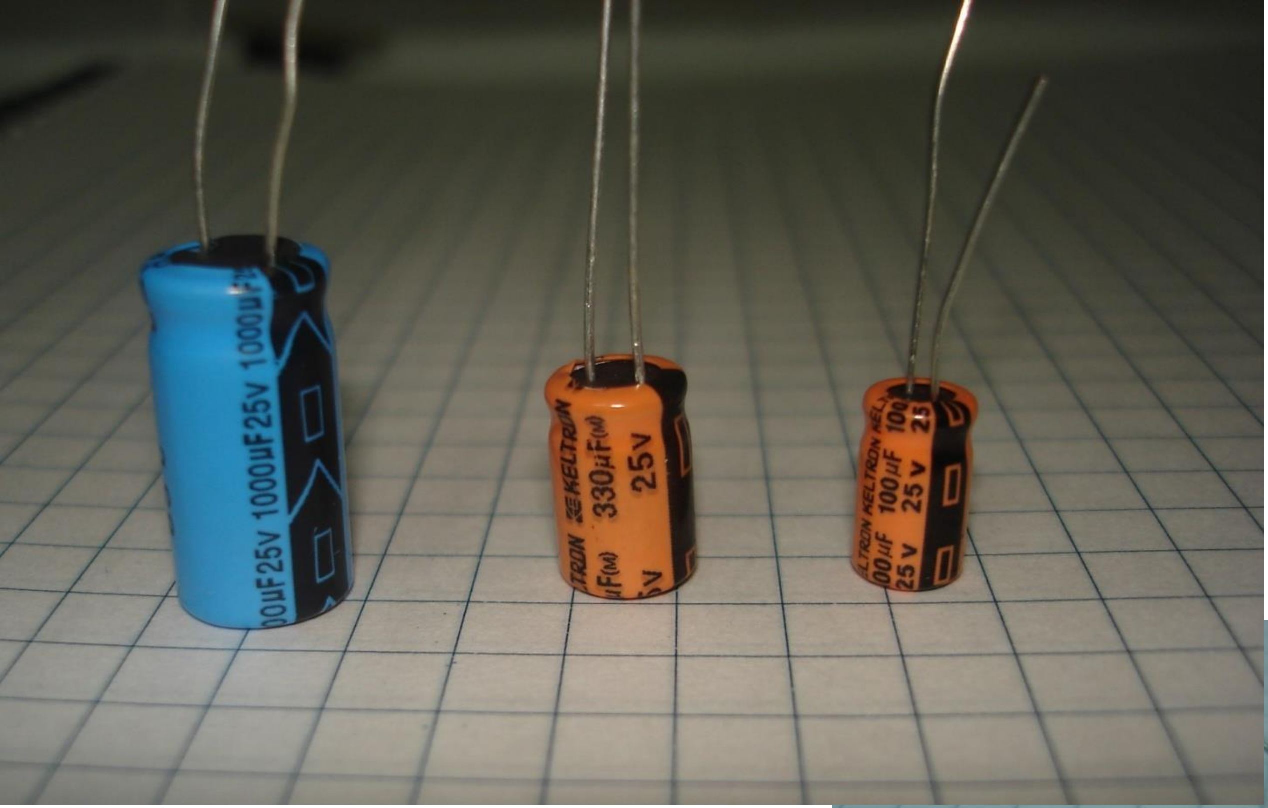
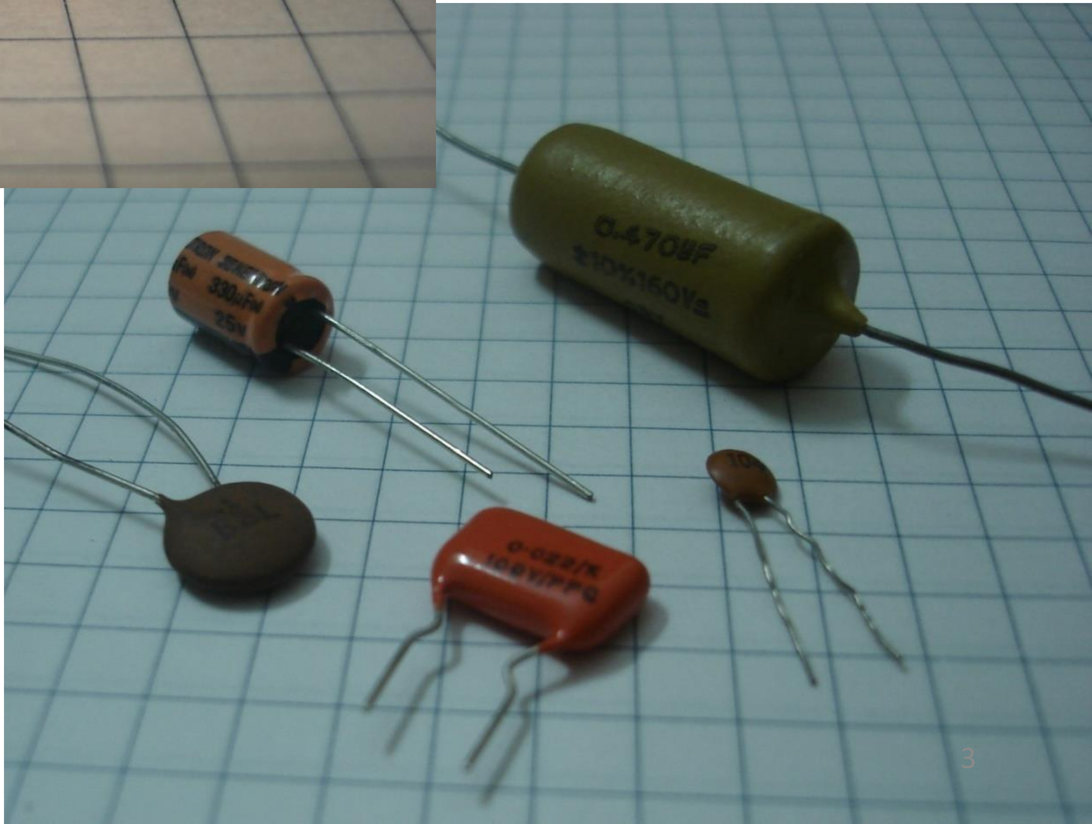
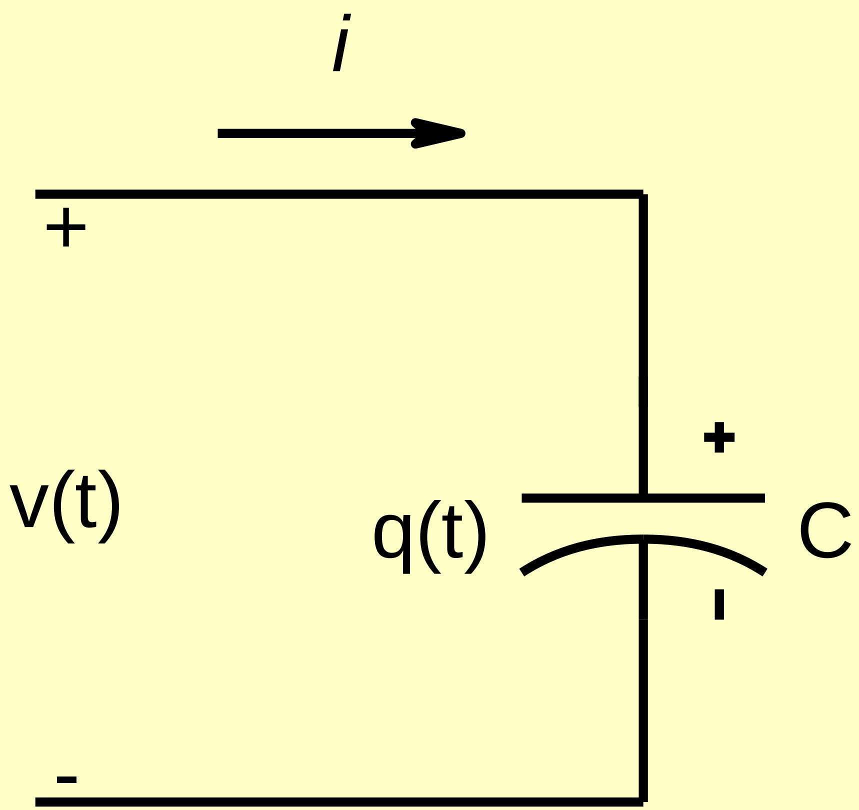
Capacitance

- Two sheets of conductors separated by a layer of insulating material
- Insulating material is called dielectric - could be air, polyester, …


1000µF25V
KELTRON
330µF(M)
25V
KELTRON
100µF
25 V


330µF
25V



$$q = C \times v$$

Coulombs

Farad

Volt

$$i = \frac{dq}{dt}$$

$$i = C\frac{dv}{dt}$$

$$v(t) = \frac{1}{C}\int_{t_o}^{t} i\,dt + v(t_o)$$

$$w_c(t) = \frac{1}{2} C \times v_c^2(t)$$

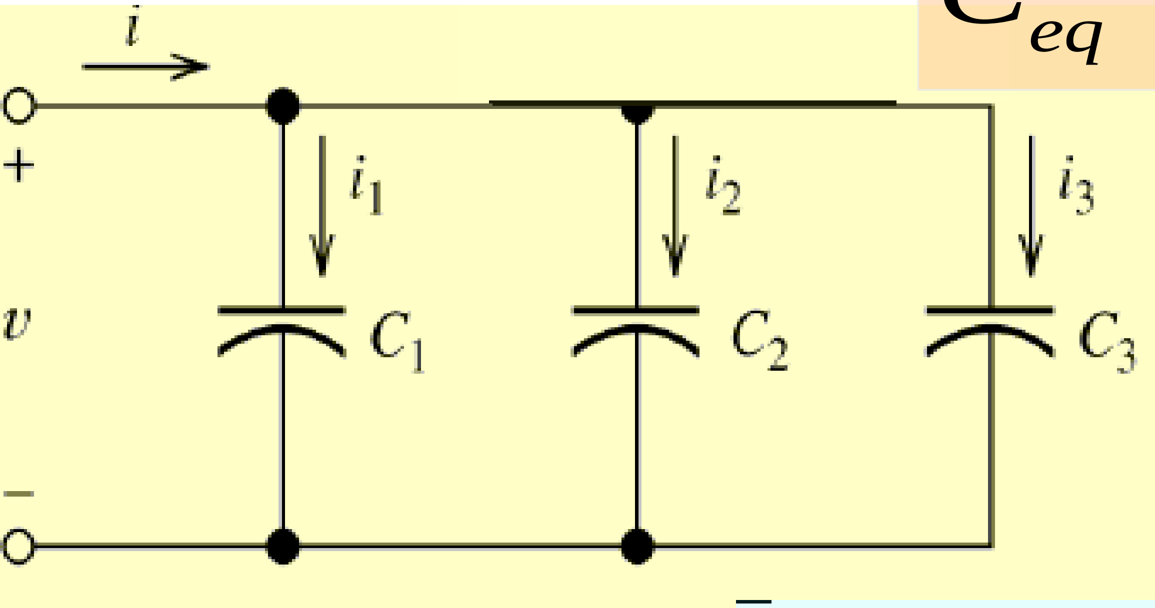
For dc or steady state when the voltage does not vary with time

A capacitor under dc or steady state acts like an

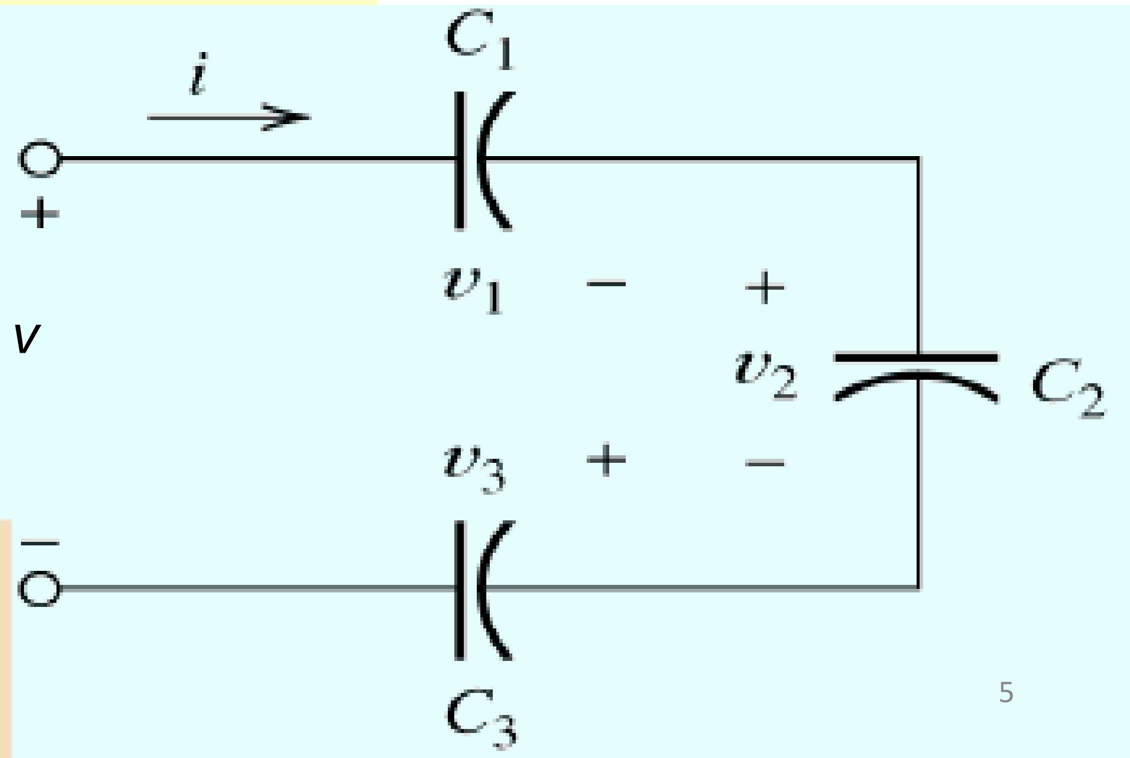
open circuit

$$i = 0$$

$$C_{eq} = C_1 + C_2 + C_3$$

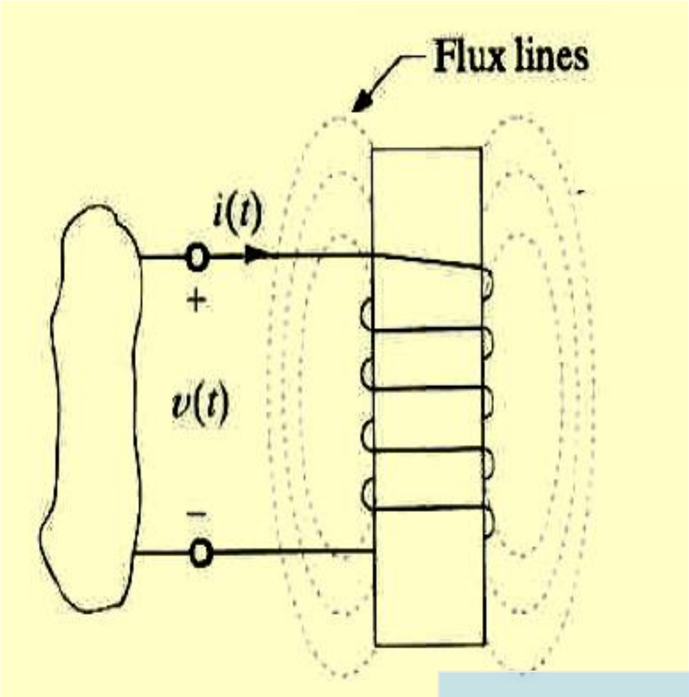


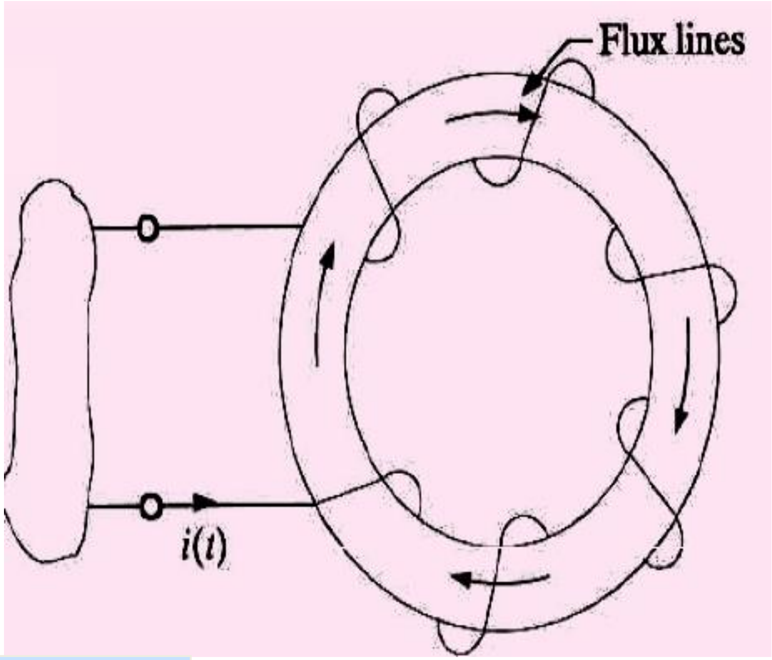
$$i = C\frac{dv}{dt}$$



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Inductance

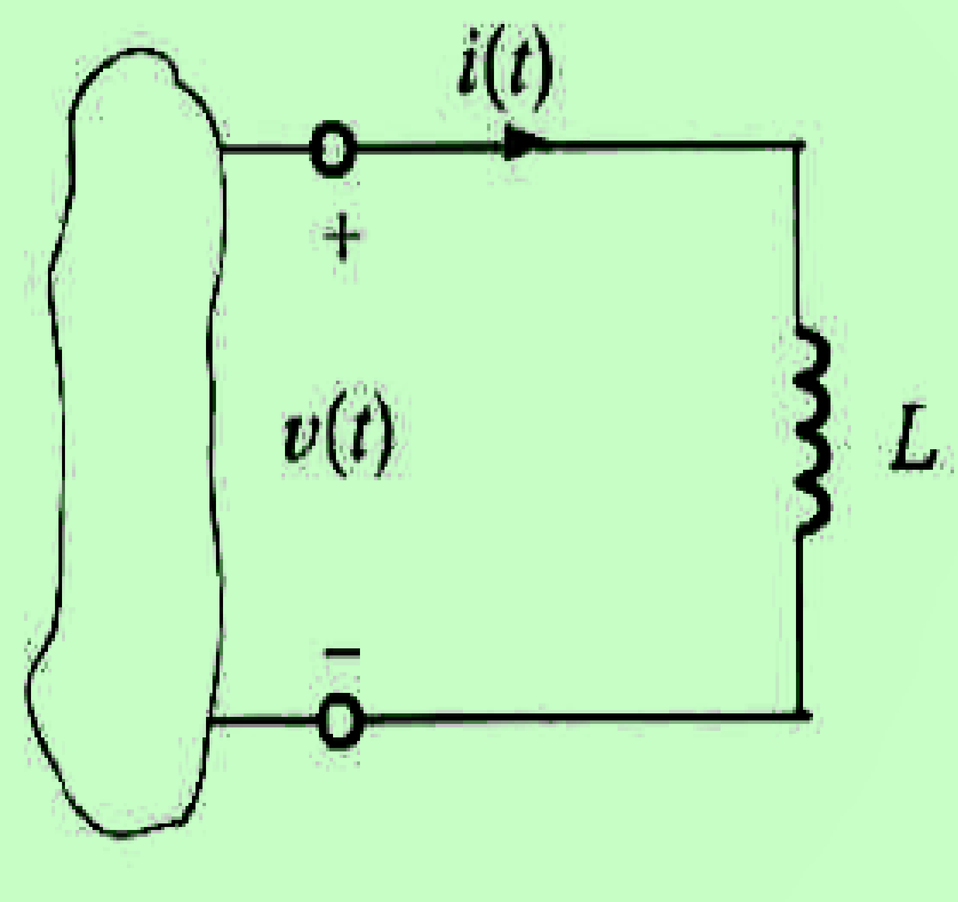




$$\phi = L \times i$$

A time varying flux causes voltage to appear across the device terminals

$$v = \frac{d\phi}{dt} = L \times \frac{di}{dt}$$



$$v = L \times \frac{di}{dt}$$

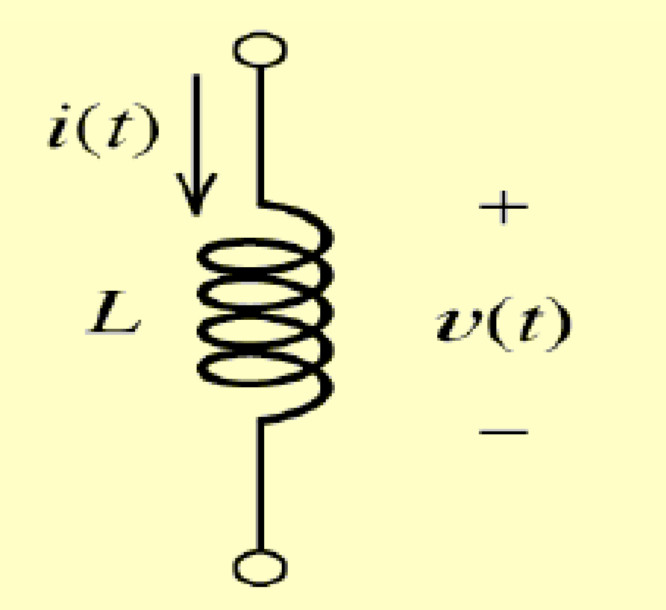
Henry

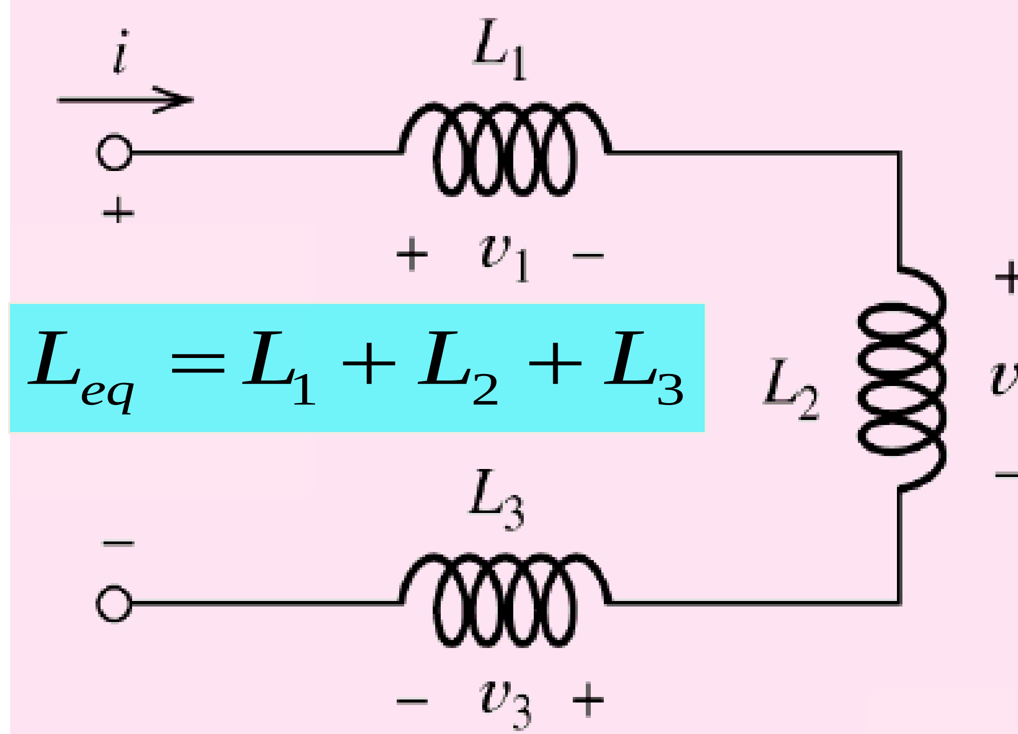
For dc or steady state when the current does not vary with time

$$v = 0$$

An inductor under dc or steady state acts like a short circuit

Typical Inductors

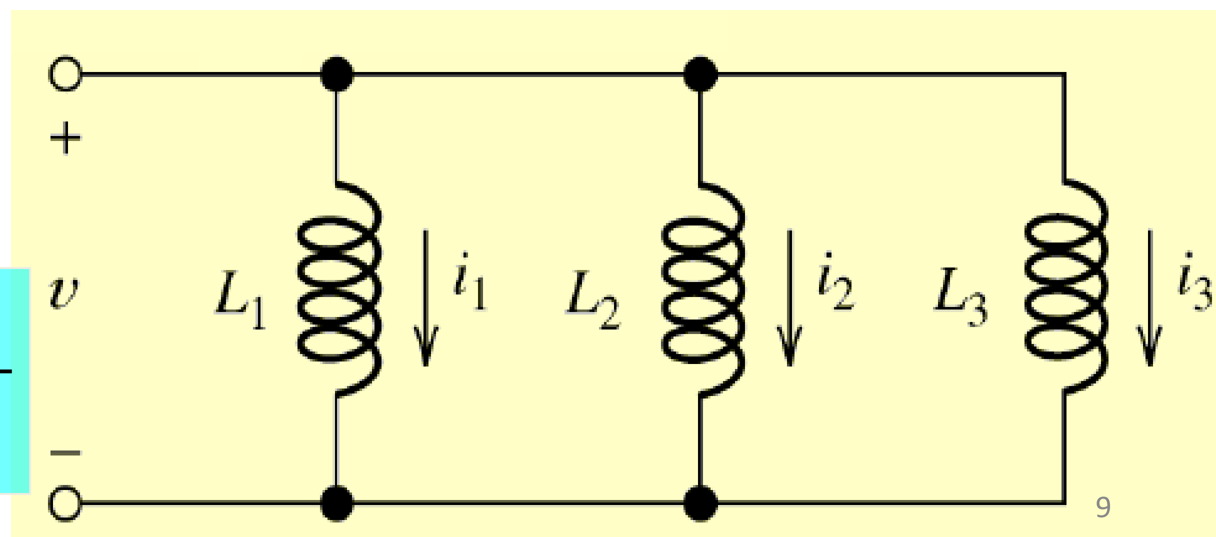




$$L_{eq} = L_1 + L_2 + L_3$$

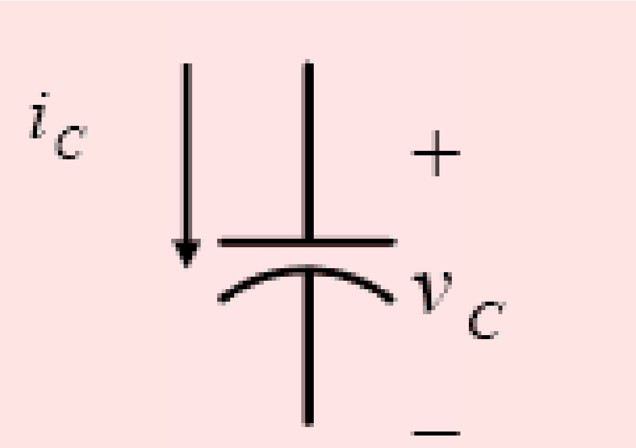
$$w_L(t) = \frac{1}{2} L \times i^2(t)$$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$



Two important concepts

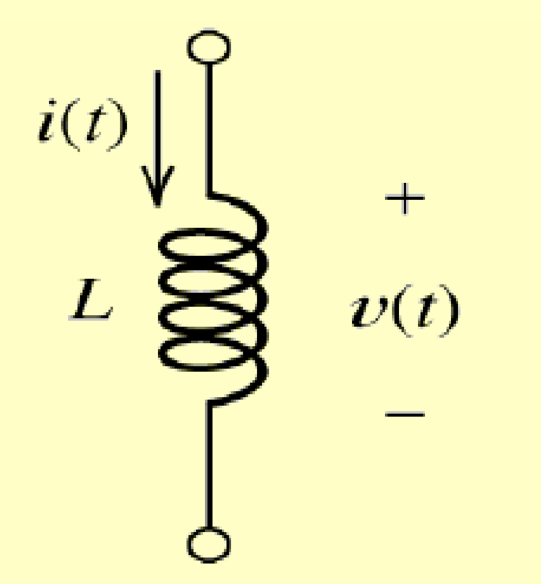
Voltage across a capacitor cannot change instantaneously



$$i_c = C\frac{dv_c}{dt}$$

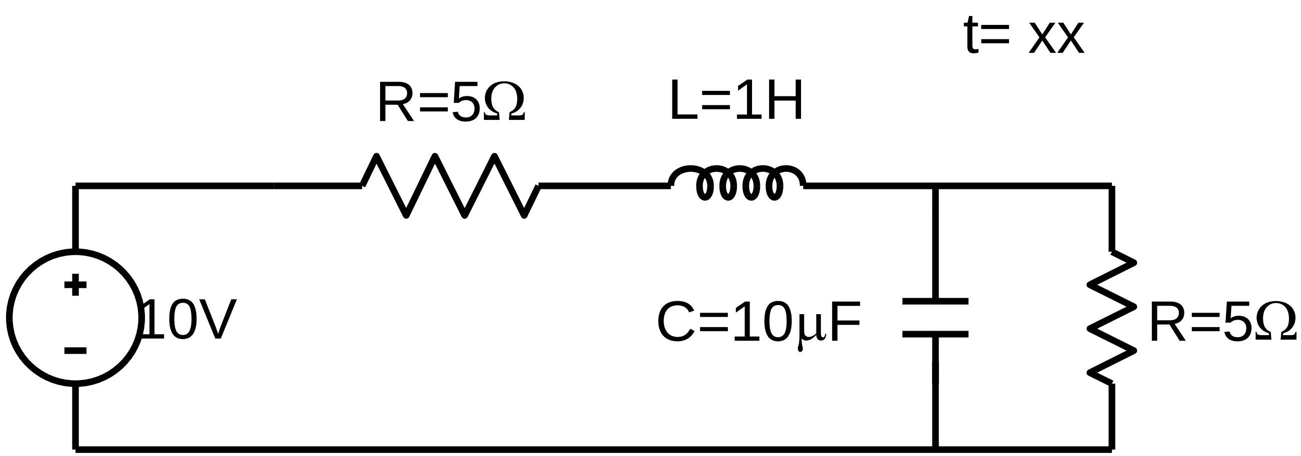
Instantaneous change in voltage implies infinite current!

Current through an inductor cannot change instantaneously



$$v = L\frac{di}{dt}$$

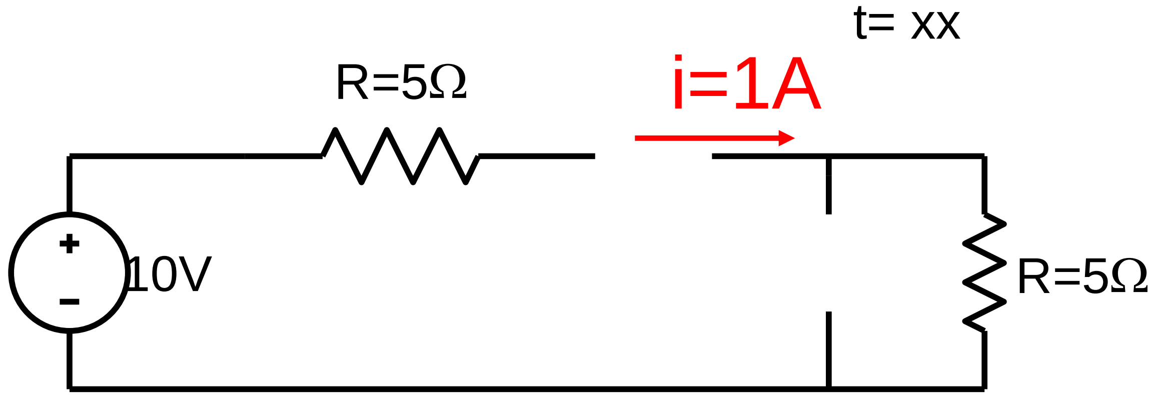
Instantaneous change in current implies infinite voltage!



What is current through the inductor or voltage across the capacitor ?

We can't give an answer unless we have some knowledge of the past state of the circuit

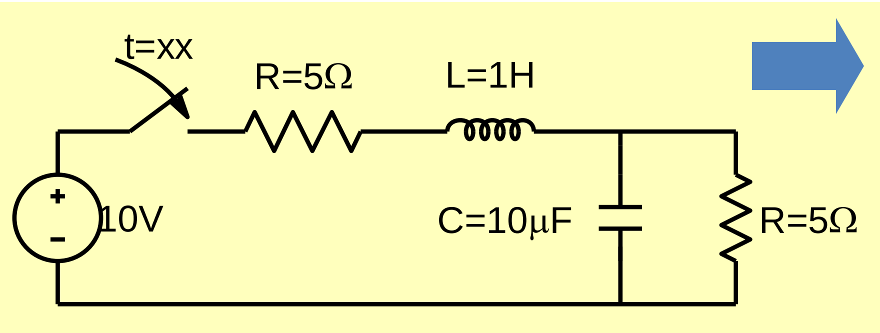
Suppose we are told that circuit has been in this state for a very very long time

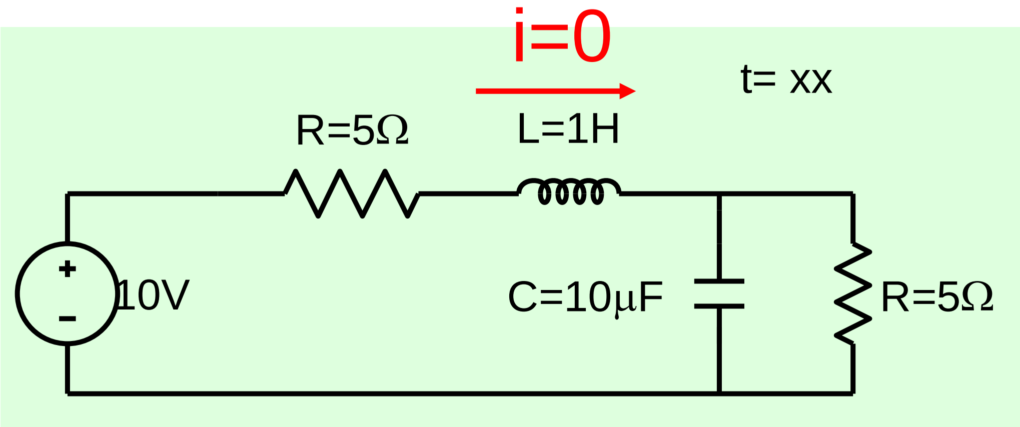


$$i_c = C\frac{dv_c}{dt}$$

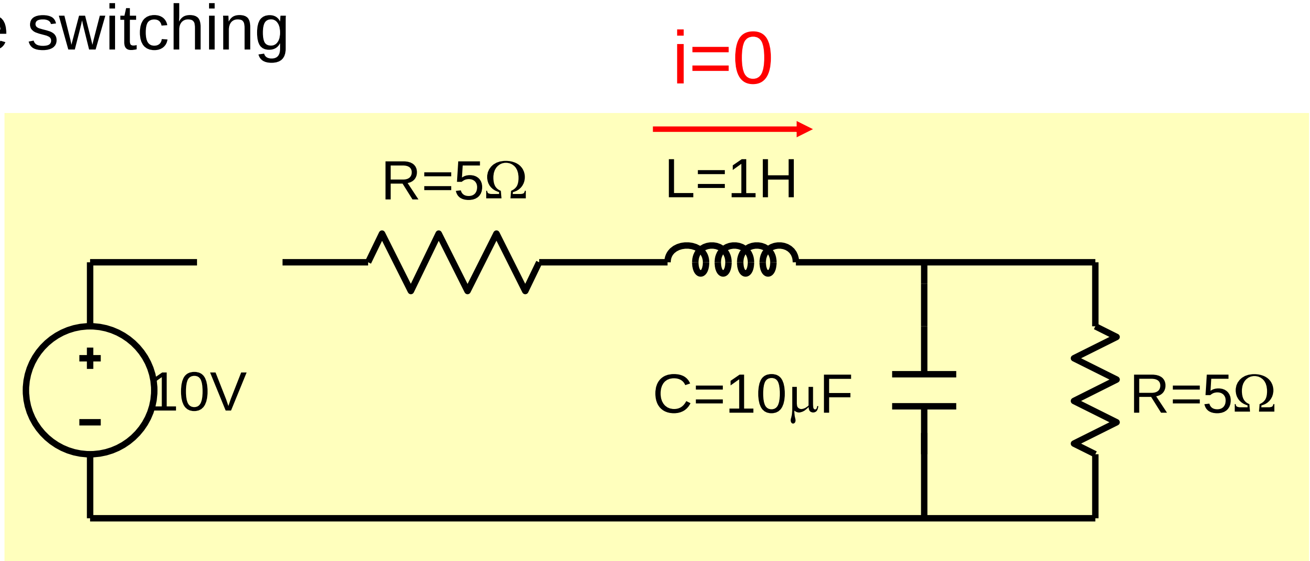
$$v = L\frac{di}{dt}$$

Or suppose we are told that circuit was switched at t= xx



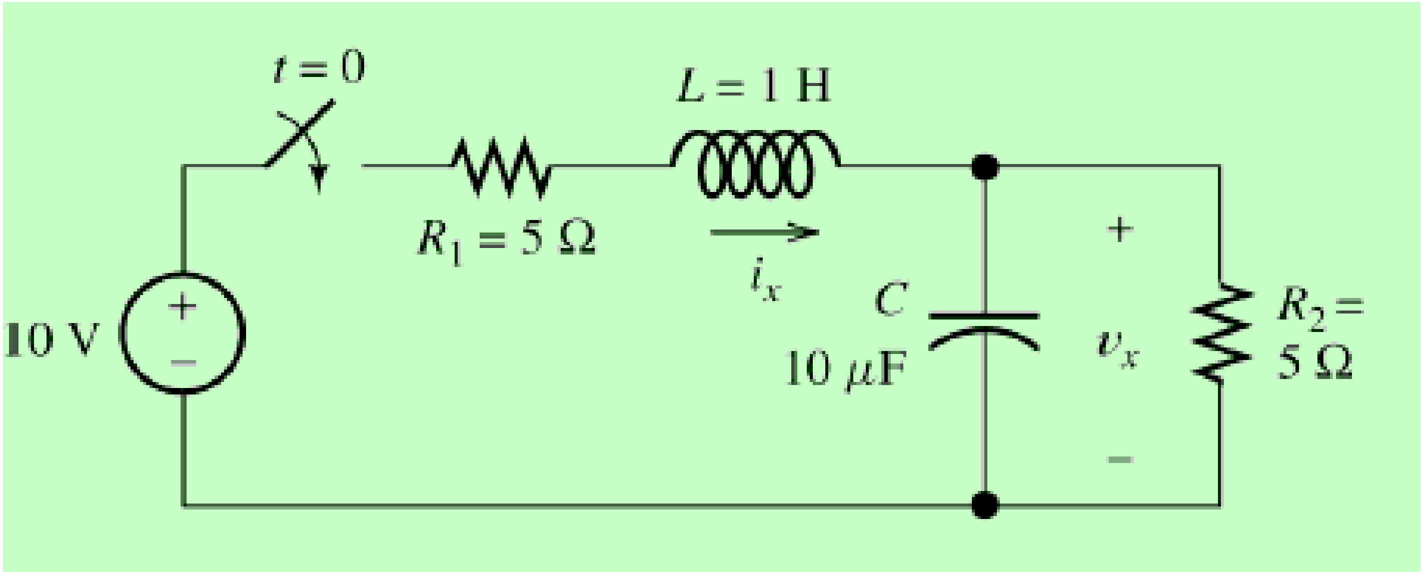


Circuit before switching



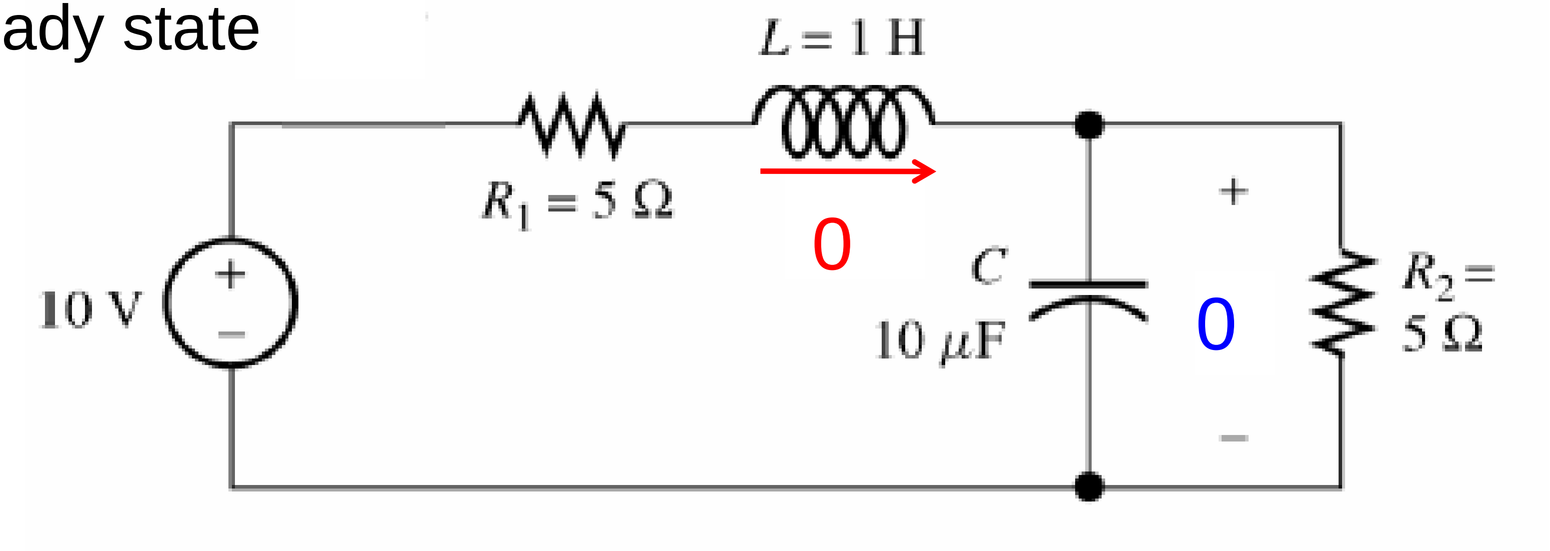
Circuits containing inductors or capacitors have a memory

Example



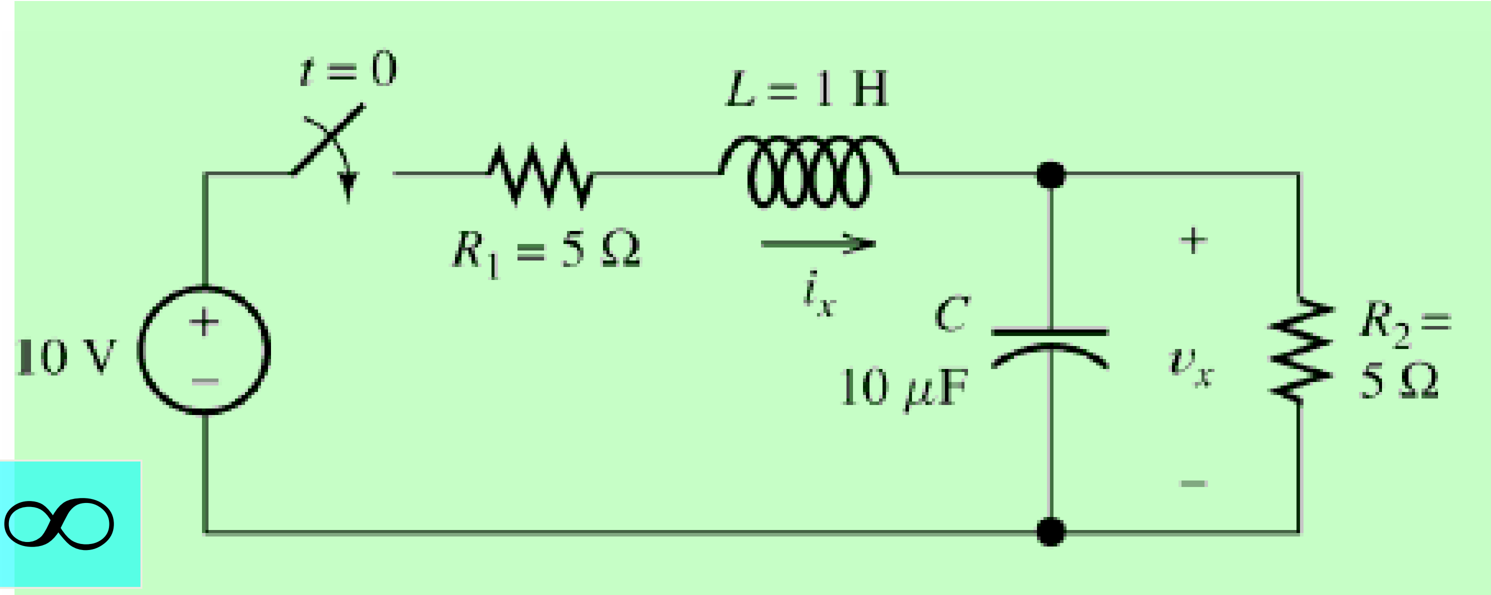
Find voltage and current immediately after closing the switch and in steady state

$t = 0^+$



Current through an inductor cannot change instantaneously

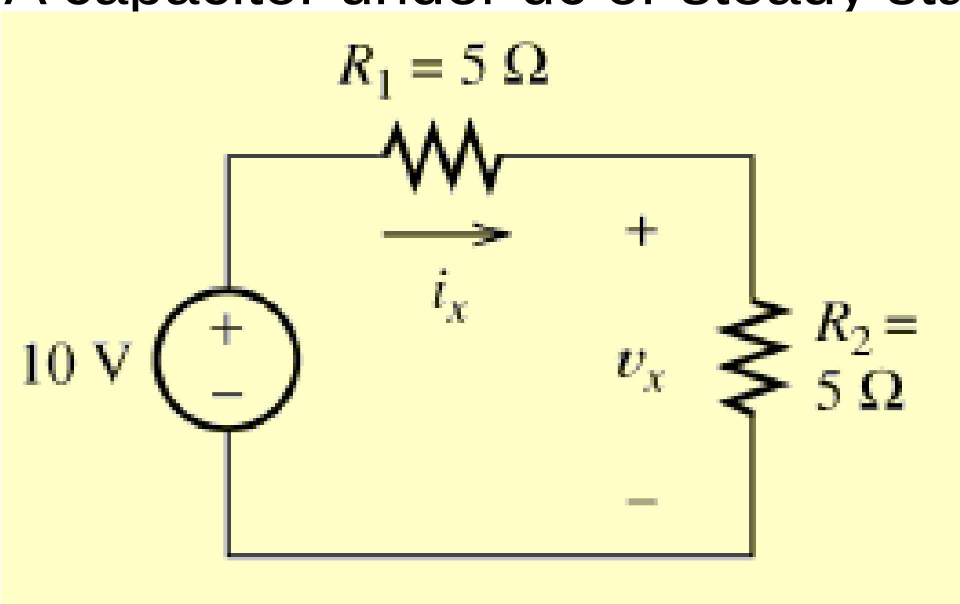
Voltage across a capacitor cannot change instantaneously



$t \to \infty$

An inductor under dc or steady state acts like a short circuit

A capacitor under dc or steady state acts like an open circuit



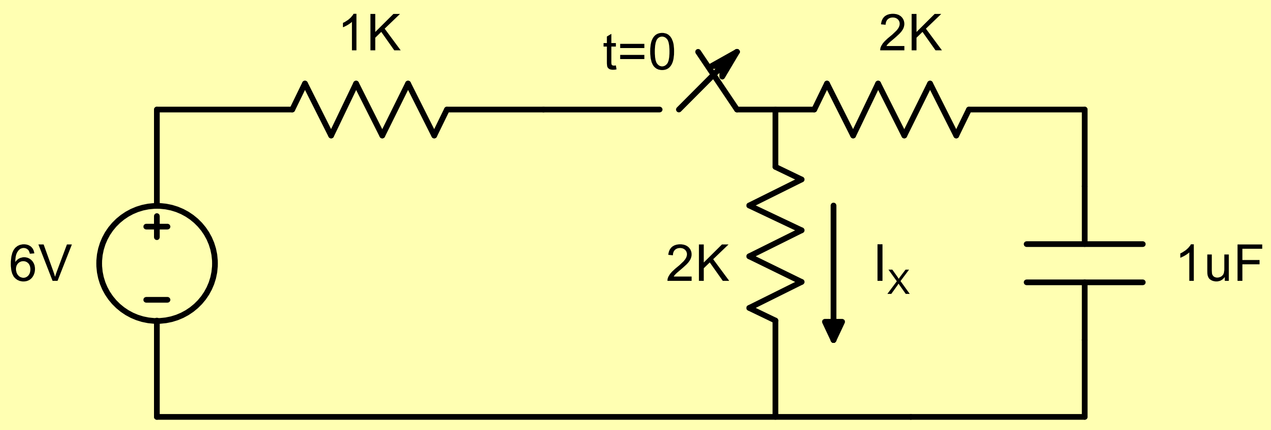
$$i_x = \frac{10}{R_1 + R_2} = 1 \text{ A}$$

$$v_x = R_2 i_x = 5 \text{ V}$$

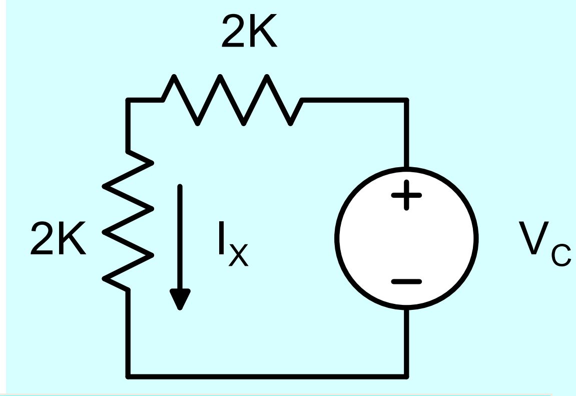
$$i_L(t \to \infty) = 1 \quad \text{A}$$

$$v_c(t \to \infty) = 5 \quad \text{V}$$

Determine the current I_X immediately after switch is opened.



Circuit for t>0

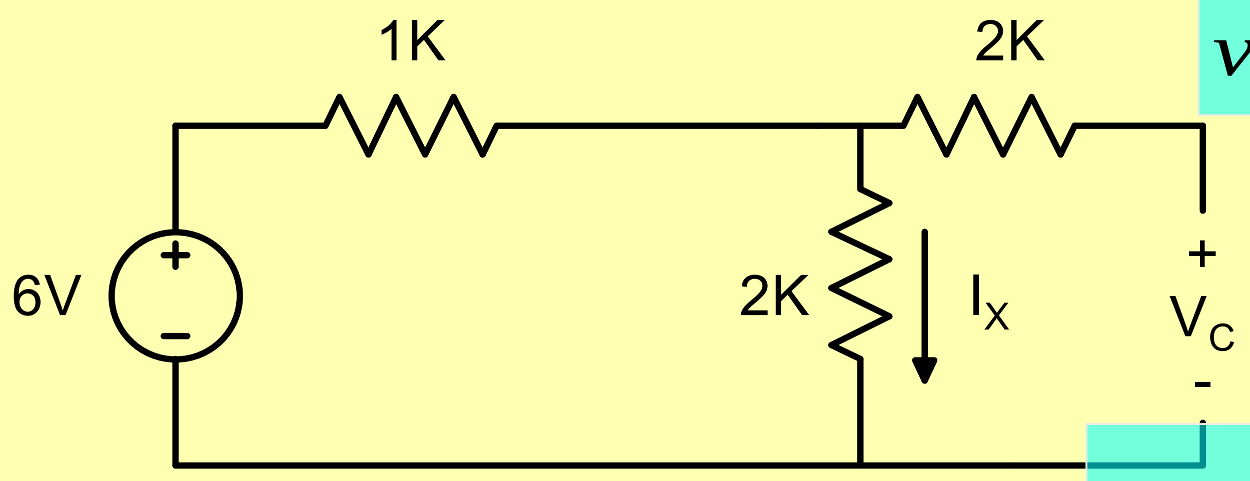


$$i_X(0^+) = \frac{v_C(0^+)}{4K} = 1mA$$

First find voltage $V_C(0^-)$

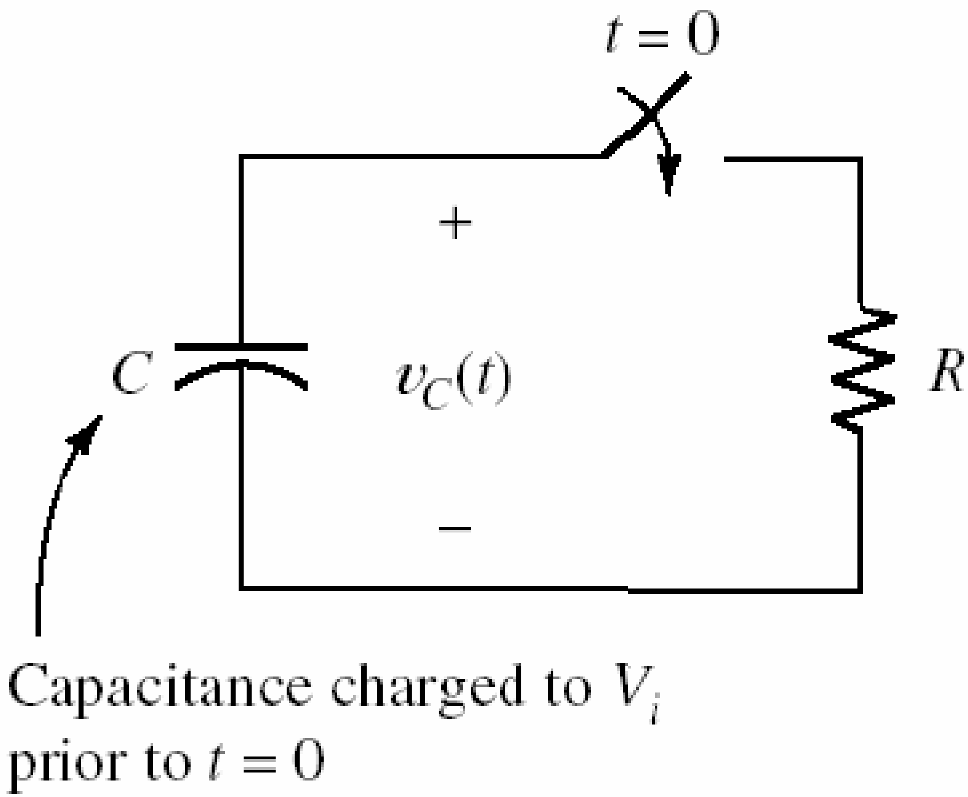
$$v_C(0^+) = v_C(0^-)$$

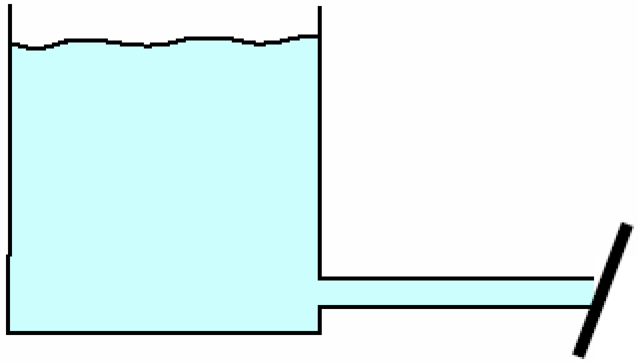
Circuit for t<0



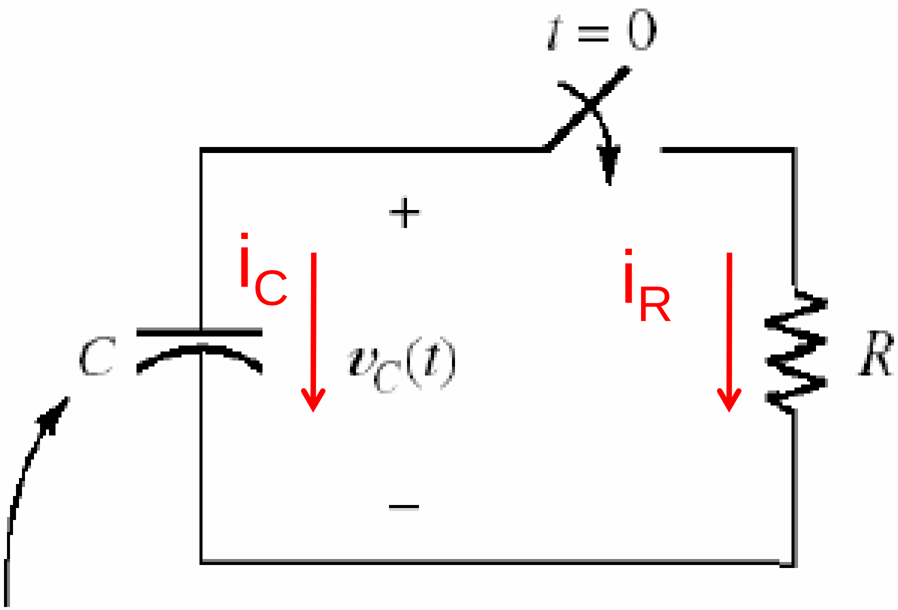
$$v_C(0^-) = \frac{2}{3} \times 6 = 4V$$

Discharge of a capacitor through a Resistor

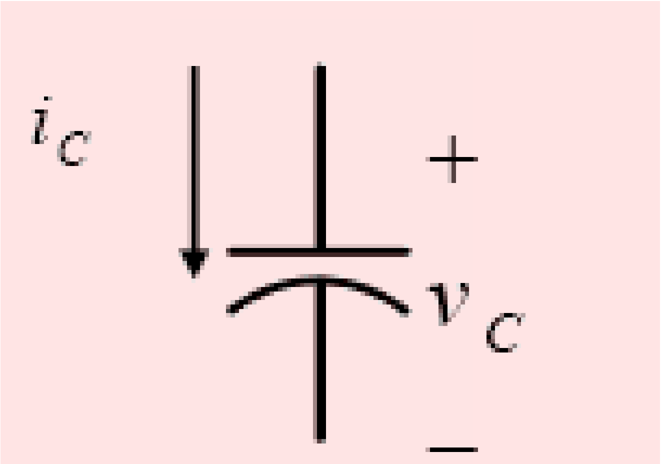




How long will it take for capacitor voltage to fall to half its initial value?



Capacitance charged to V_i prior to $t = 0$



$$i_c = C\frac{dv_c}{dt}$$

Write KCL at top node with switch closed:

$$i_c(t) + i_R(t) = 0$$

$$C\frac{dv_C(t)}{dt} + \frac{v_C(t)}{R} = 0$$

$$\frac{dv_C(t)}{dt} = -\frac{1}{RC}v_C(t)$$

First Order Differential Equation

$$\frac{dy}{dt} = -a\,y$$

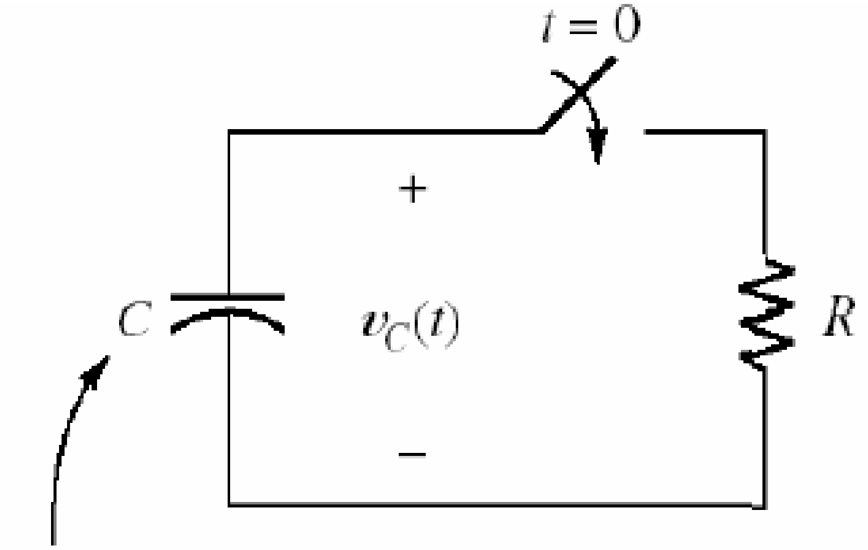
Solution:

$$y(t) = K\,e^{-at}$$

Constant K is often found from the initial condition

$$K = y(0)$$

$$y(t) = y(0)\;e^{-at}$$



Capacitance charged to V_i prior to $t = 0$

$$\frac{dv_C(t)}{dt} = -\frac{1}{RC} v_C(t)$$

$$v_C(t) = v_C(0)\ e^{-\frac{t}{RC}}$$

$$v_C(t) = v_C(0^+)\ e^{-\frac{t}{RC}}$$

$$\frac{dy}{dt} = -a\,y$$

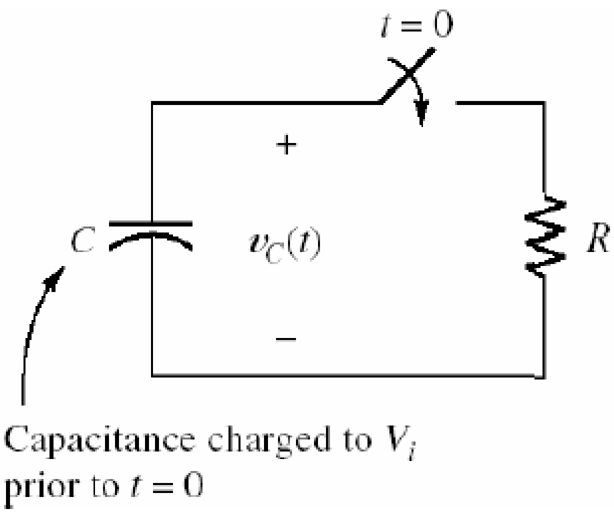
We know: $v_C(0^-) = V_i$

$$y(t) = y(0)\ e^{-at}$$

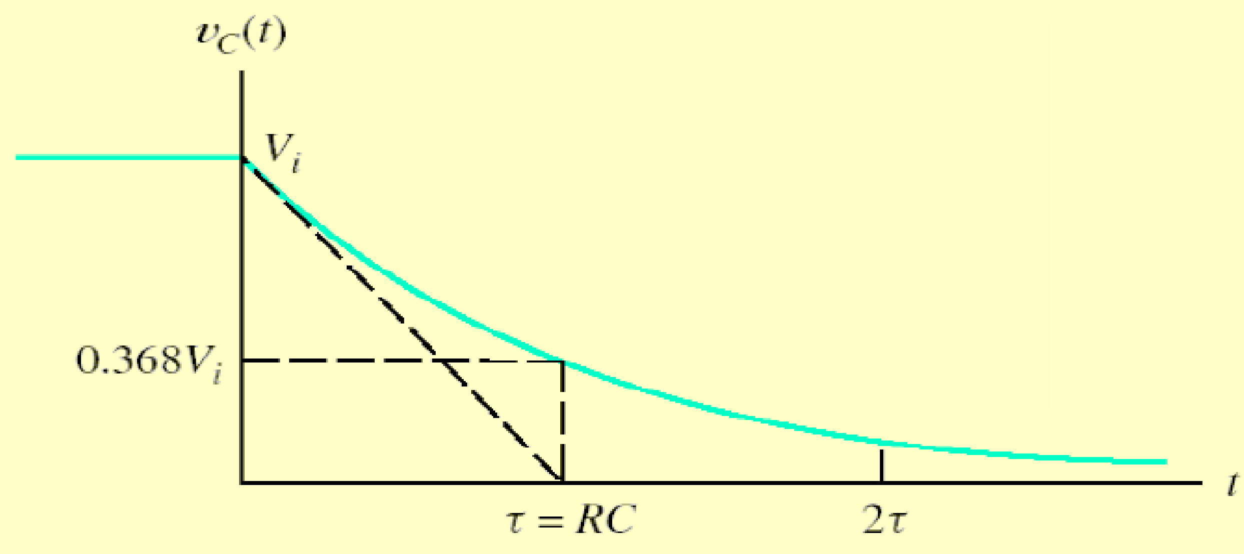
Voltage across a capacitor cannot change instantaneously

$$v_C(0^+) = v_C(0^-) = V_i$$

$$v_C(t) = V_i\ e^{-\frac{t}{RC}}$$



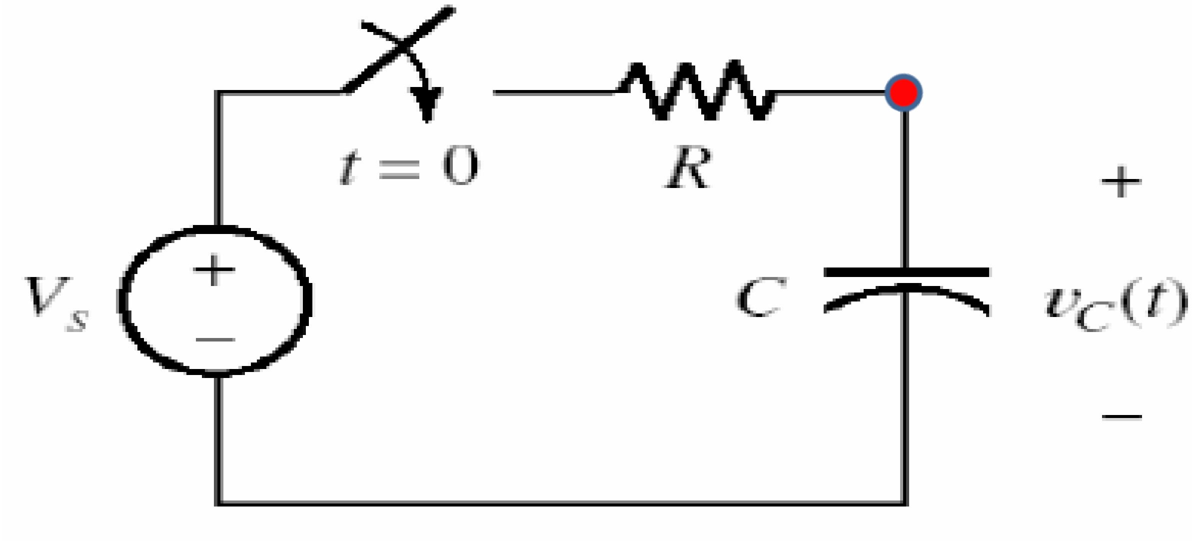
$$v_C(t) = V_i e^{-t/RC}$$



The time interval $\tau = RC$ is called the time constant of the circuit. After about **five time constants**, the voltage remaining on the capacitor will be negligible compared to the initial value

Time	τ	2τ	3τ	4τ	5τ
$V(t)/V_i$	0.368	0.135	.05	0.018	0.0067

Charging a capacitor



$$i_c = C\frac{dv_c}{dt}$$

Application of KCL at the indicated node gives

$$C\frac{dv_c(t)}{dt} + \frac{v_c(t) - V_s}{R} = 0$$

$$RC\frac{dv_c(t)}{dt} + v_c(t) = V_s$$

$$\frac{dx}{dt} = -a_1\, x + a_2$$

Solution: $$x(t) = K_1 + K_2\, e^{-a_1 t}$$

$$RC\frac{dv_c(t)}{dt} + v_c(t) = V_s$$

$$x(\infty) = K_1$$

$$x(t) = x(\infty) + K_2\, e^{-a_1 t}$$

Use initial condition: $$x(0) = x(\infty) + K_2$$

$$x(t) = x(\infty) + \{x(0) - x(\infty)\}\, e^{-a_1 t}$$

$$\frac{dx}{dt} = -a_1\, x + a_2$$

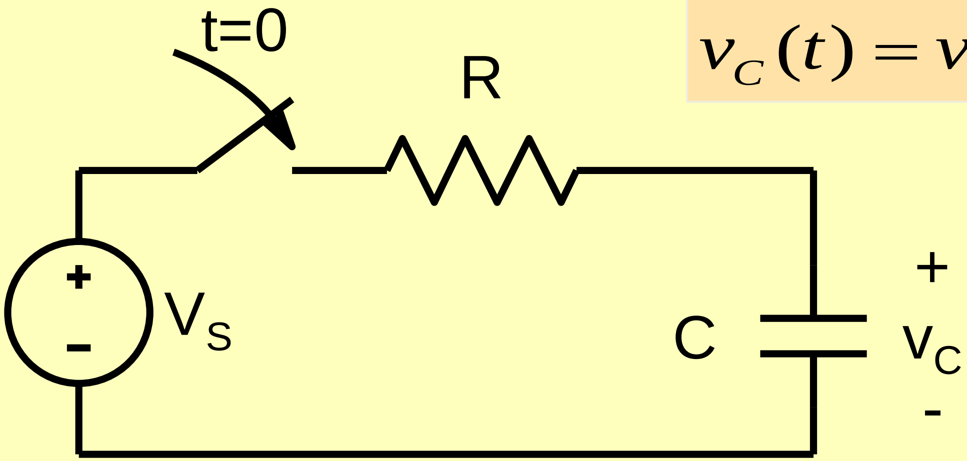
$$x(t) = x(\infty) + \{x(0) - x(\infty)\}\, e^{-a_1 t}$$

$$RC\,\frac{dv_c(t)}{dt} + v_c(t) = V_s$$

$$a_1 = \frac{1}{RC}$$

$$v_C(t) = v_C(\infty) + \{v_C(0^+) - v_C(\infty)\}\, e^{-\frac{t}{RC}}$$

$$v_C(t) = v_C(\infty) + \{v_C(0^+) - v_C(\infty)\}\, e^{-\frac{t}{RC}}$$

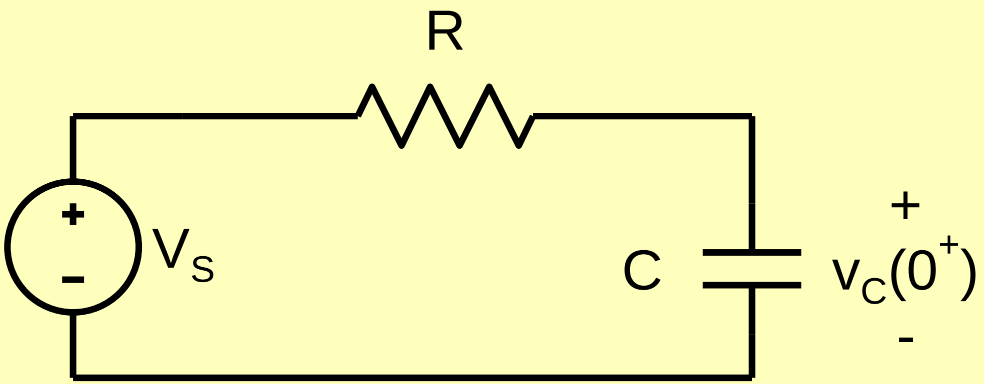


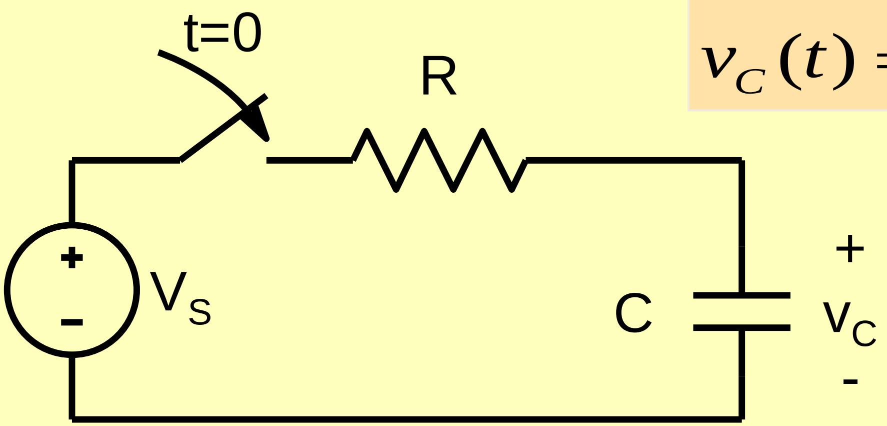
What is $v_C(\infty)$?

A capacitor under dc or steady state acts like an open circuit

$$v_C(\infty) = V_S$$

What is $v_C(0^+)$?





$$v_C(t) = v_C(\infty) + \{v_C(0^+) - v_C(\infty)\}\, e^{-\frac{t}{RC}}$$

$$v_C(0^+) = v_C(0^-)$$

We use the fact that voltage across a capacitor cannot change instantly

If the capacitor does not have any initial charge, then

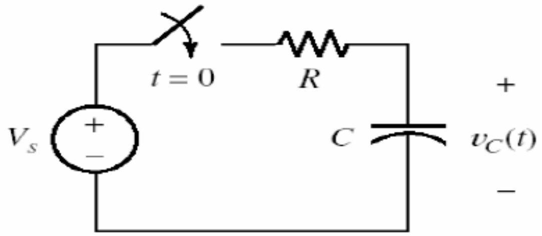
$$v_C(0^+) = v_C(0^-) = 0$$

$$v_C(\infty) = V_S$$

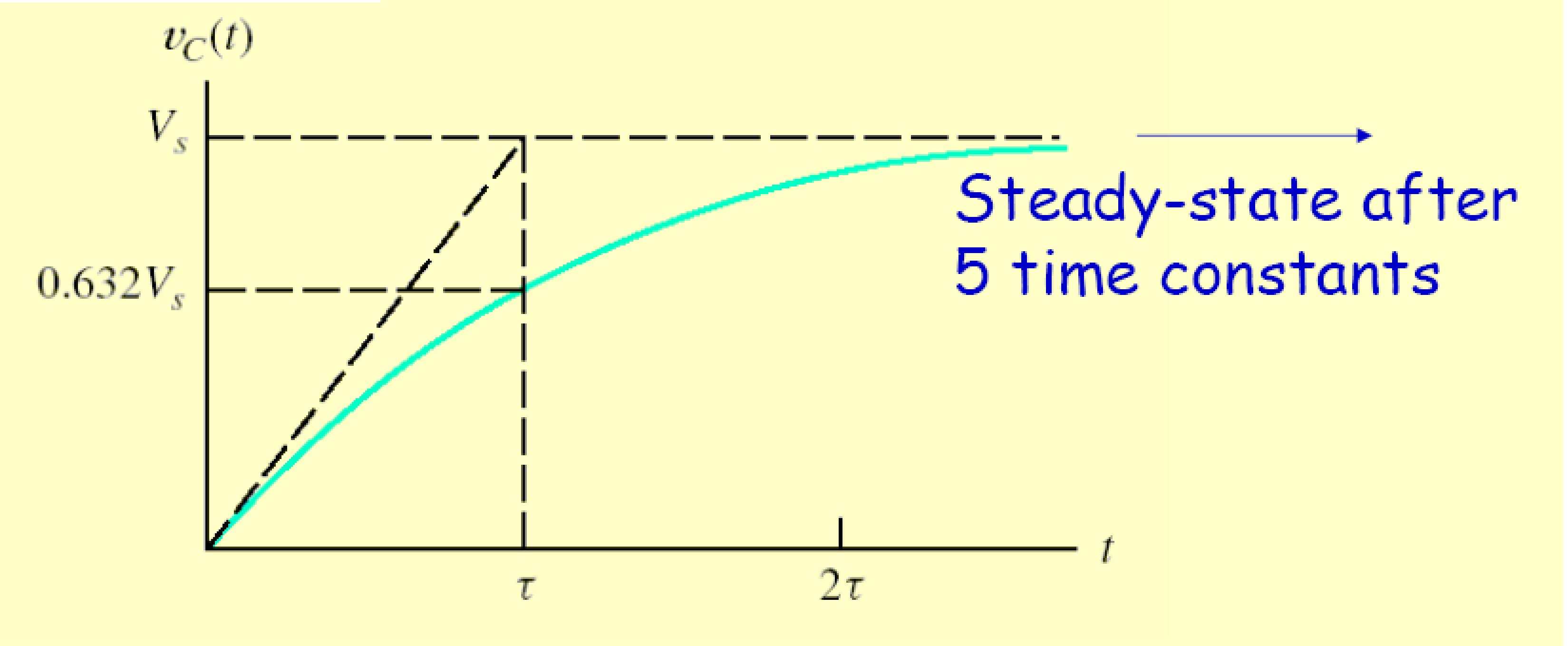
$$\tau = RC$$

$$v_C(t) = V_S(1 - e^{-\frac{t}{RC}})$$

Charging a Capacitor

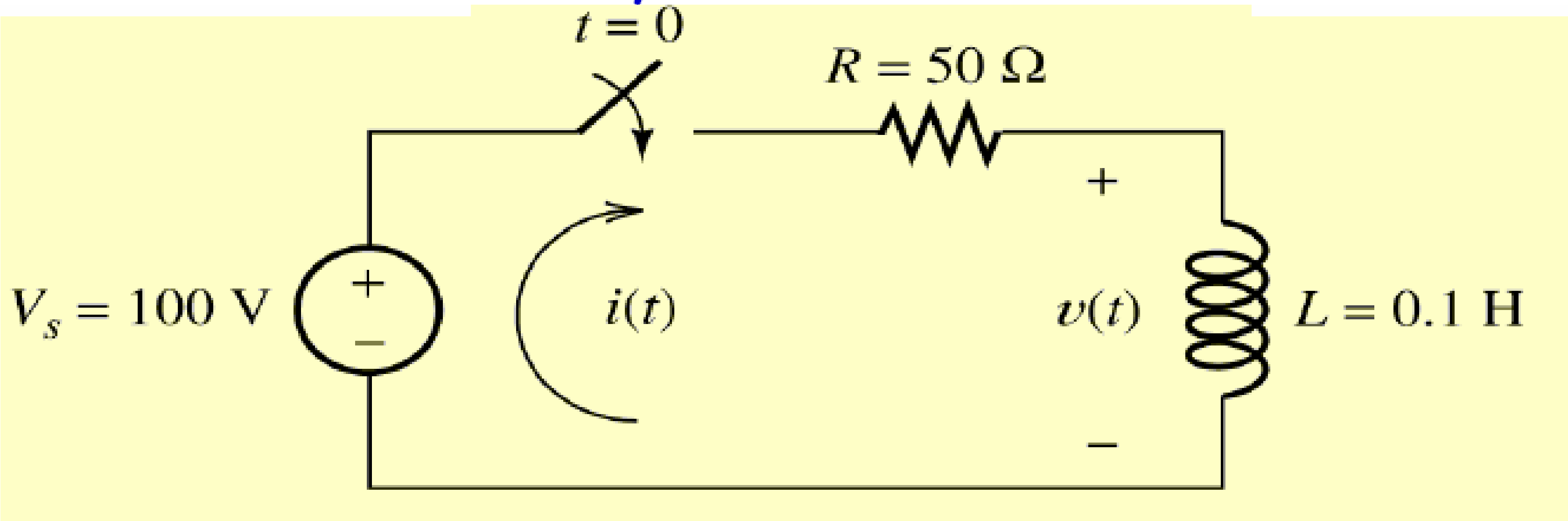


$$v_C(t) = V_s - V_s e^{-t/\tau}$$



Time	τ	2τ	3τ	4τ	5τ
$V(t)/V_i$	0.632	0.865	.95	0.982	0.993

RL Transient Analysis



Write KVL equation:

$$Ri(t) + L\frac{di}{dt} = V_s$$

$$v = L\frac{di}{dt}$$

$$\frac{dx}{dt} = -a_1\,x + a_2$$

$$x(t) = x(\infty) + \{x(0) - x(\infty)\}\,e^{-a_1 t}$$

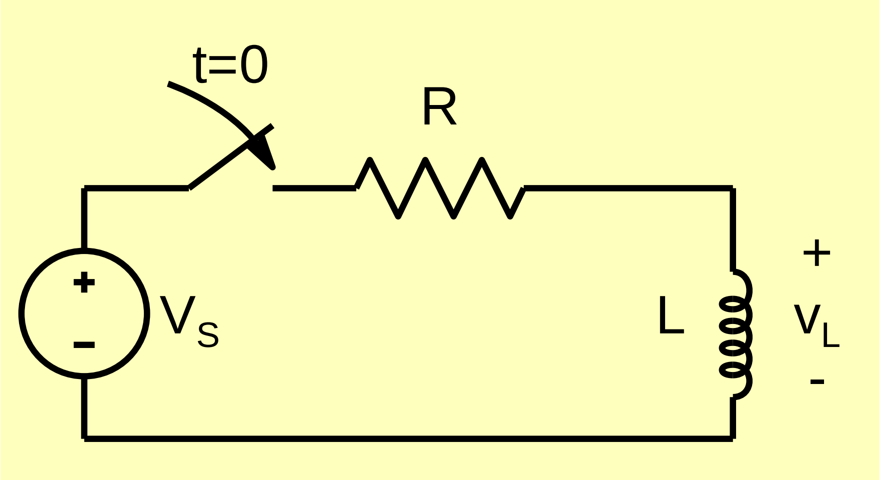
$$i(t) = i(\infty) + \{i(0) - i(\infty)\}\,e^{-\frac{R}{L}t}$$

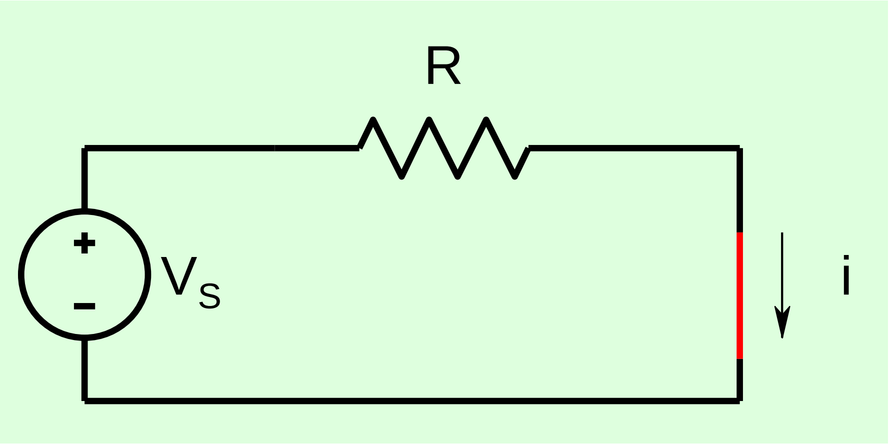
$$e^{-\frac{t}{\tau}}$$

$$\text{Time Constant}: \tau = \frac{L}{R}$$

What is $i(\infty)$?

Inductor in steady state is like a short circuit



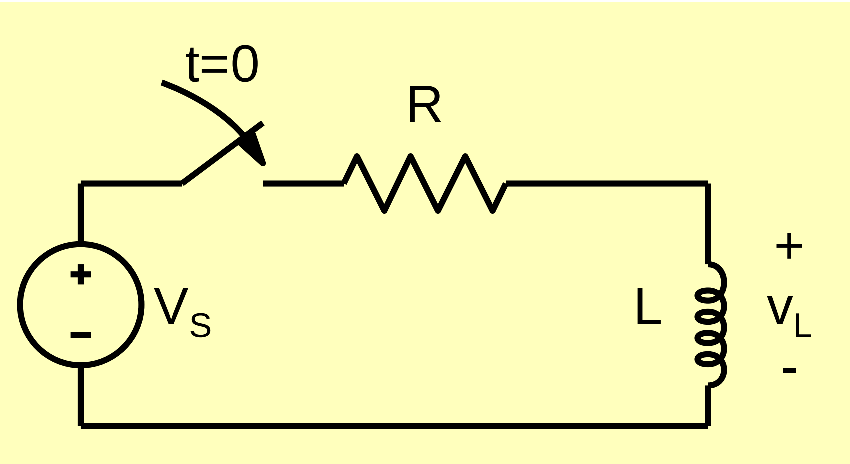


$$i(\infty) = \frac{V_S}{R}$$

$$i(t) = \frac{V_S}{R} + \{i(0) - \frac{V_S}{R}\}\, e^{-\frac{R}{L}t}$$

We also note that inductor current cannot change instantly

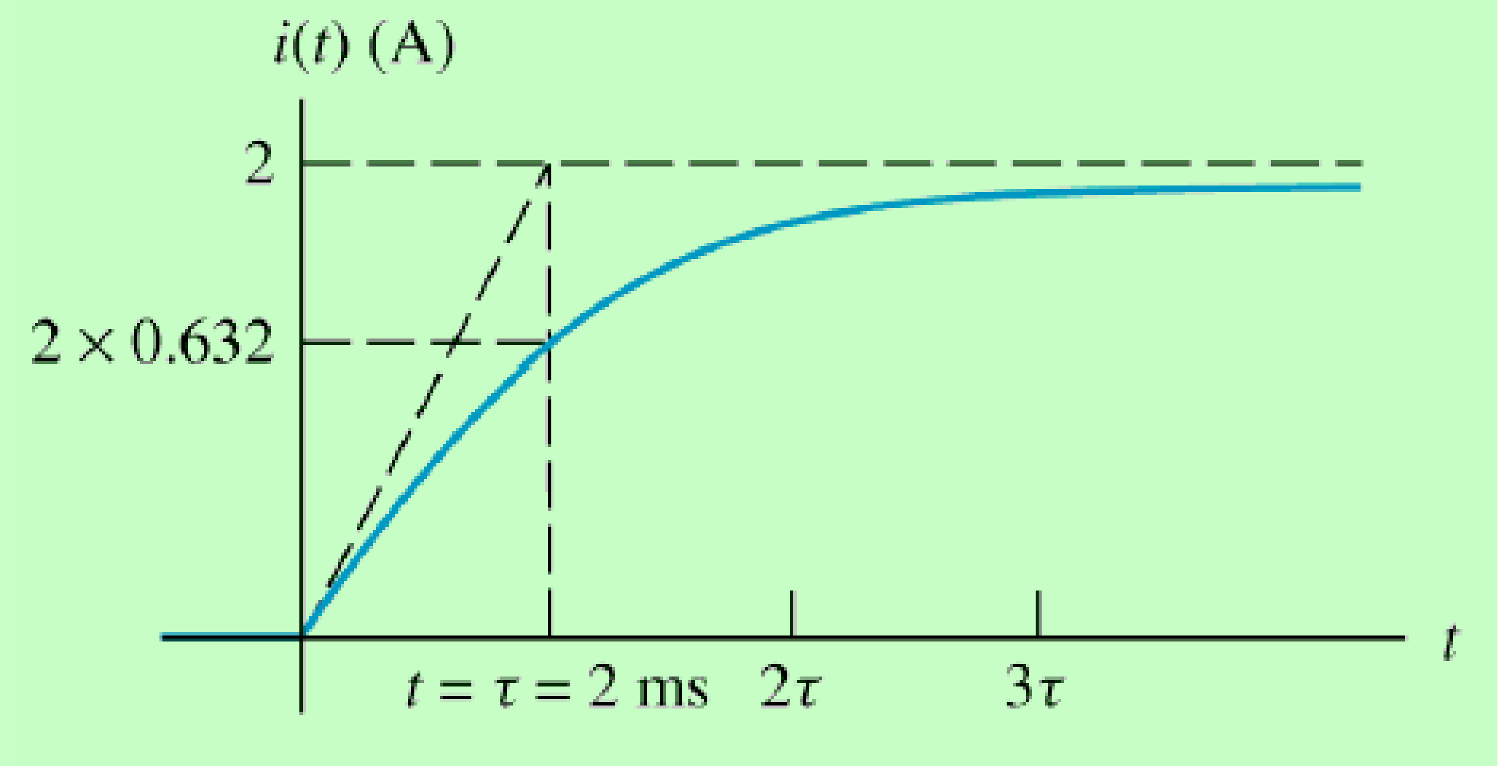
Current through an inductor cannot change instantaneously



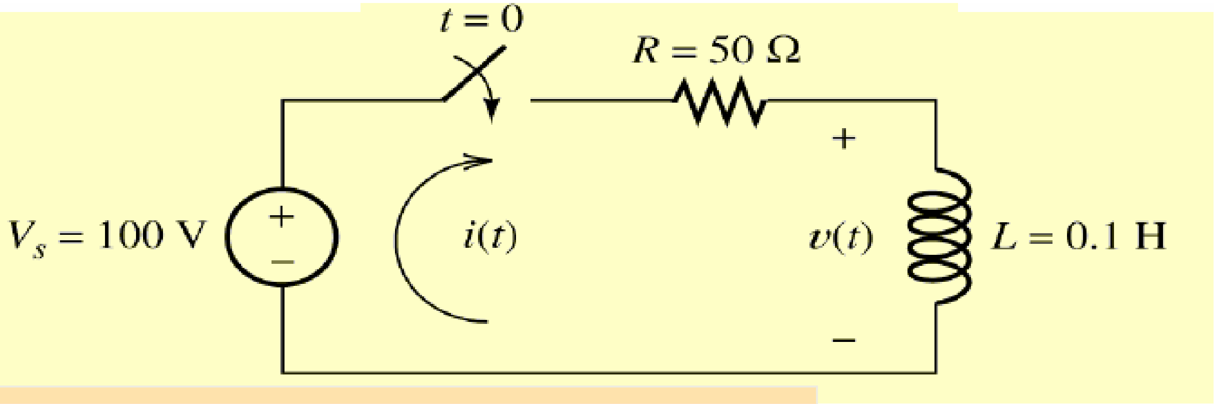
$$i(0^+) = i(0^-)$$

If $$i(0^+) = i(0^-) = 0$$

$$i(t) = \frac{V_S}{R} \times (1 - e^{-\frac{t}{\tau}})$$



What about voltage across the Inductor?



$$i(t) = \frac{V_S}{R} \times (1 - e^{-\frac{t}{\tau}})$$

$$v(t) = V_s e^{-\frac{t}{\tau}}$$