ESc201: Introduction to Electronics

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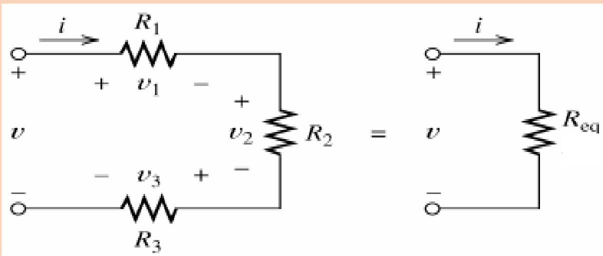
Recap: Techniques of Circuit Analysis

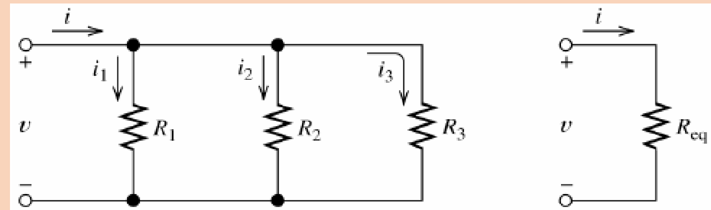
Concept of equivalent circuits

Two circuits are equivalent if they have the same current-voltage behavior

Series combination



Parallel combination



Voltage Division

A voltage applied to resistors connected in series is divided among them

Current Division

The total current flowing into a parallel combination of resistors is divided among them

Nodal Analysis

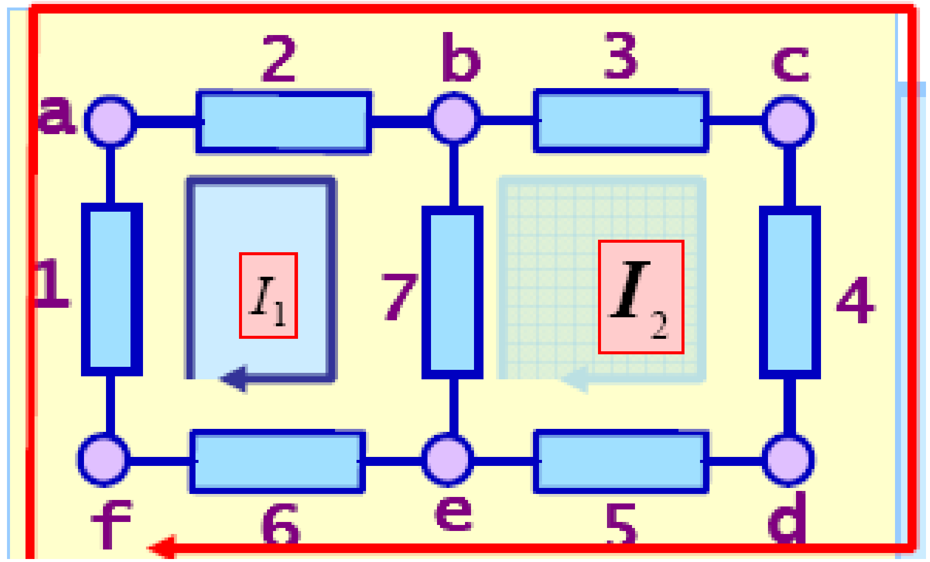
1. Identify and number the nodes
2. Pick Ground node/Reference node wisely, if it is not already specified
3. Writing KCL Equations in Terms of the Node Voltages

Mesh Analysis

1. Mesh analysis provides another general procedure for analyzing circuits using mesh currents as the circuit variables.

2. Mesh analysis applies KVL to find unknown currents.

3. A mesh is a loop which does not contain any other loop within it.



A loop is a closed path that does not go twice over any node. This circuit has three loops

A mesh is a loop that does not enclose any other loop. fabef, ebcde are meshes

fabef

ebcde

~~fabcdef~~

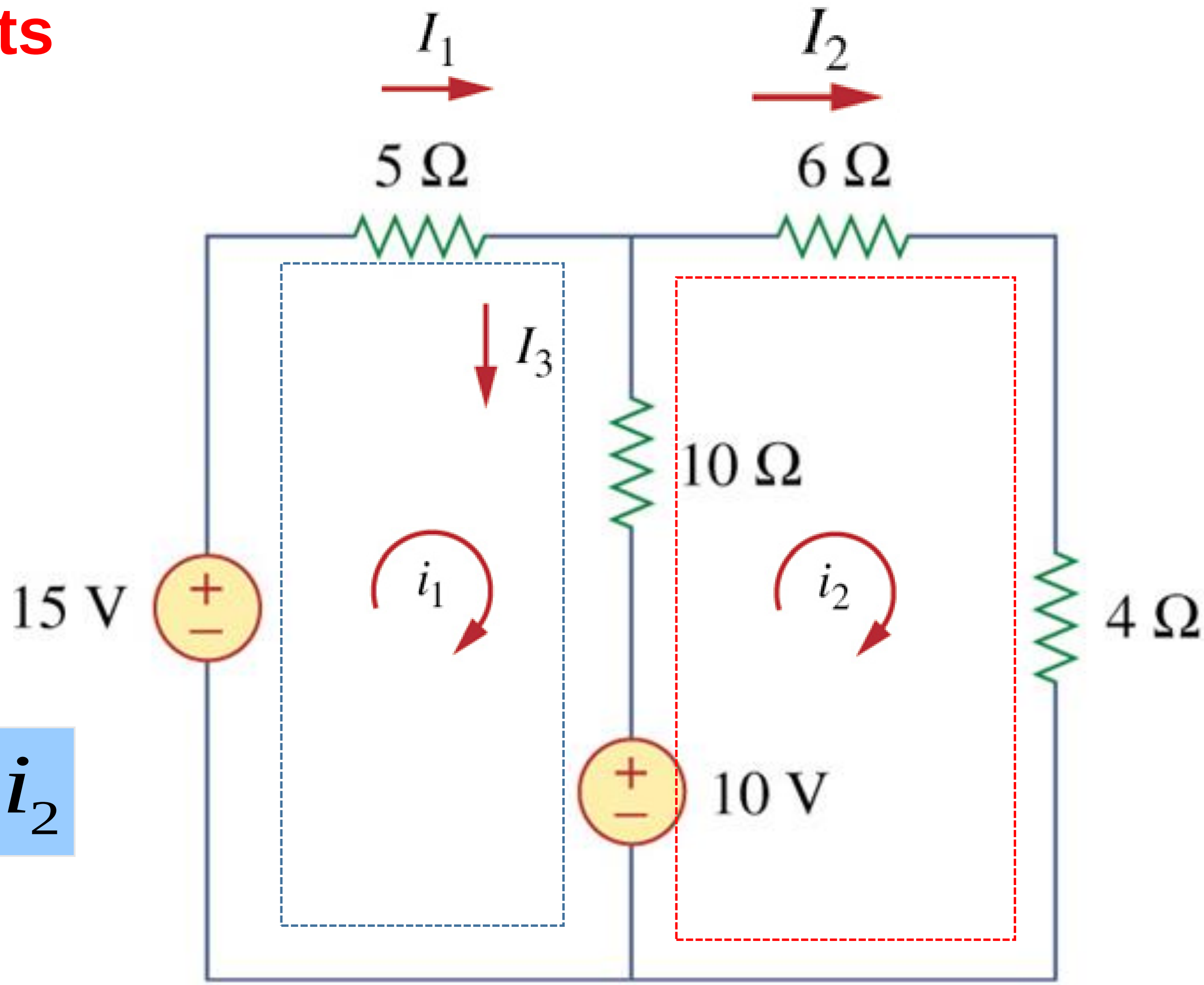
Mesh Currents

How many meshes?

$$I_1 = i_1$$

$$I_2 = i_2$$

$$I_3 = i_1 - i_2$$

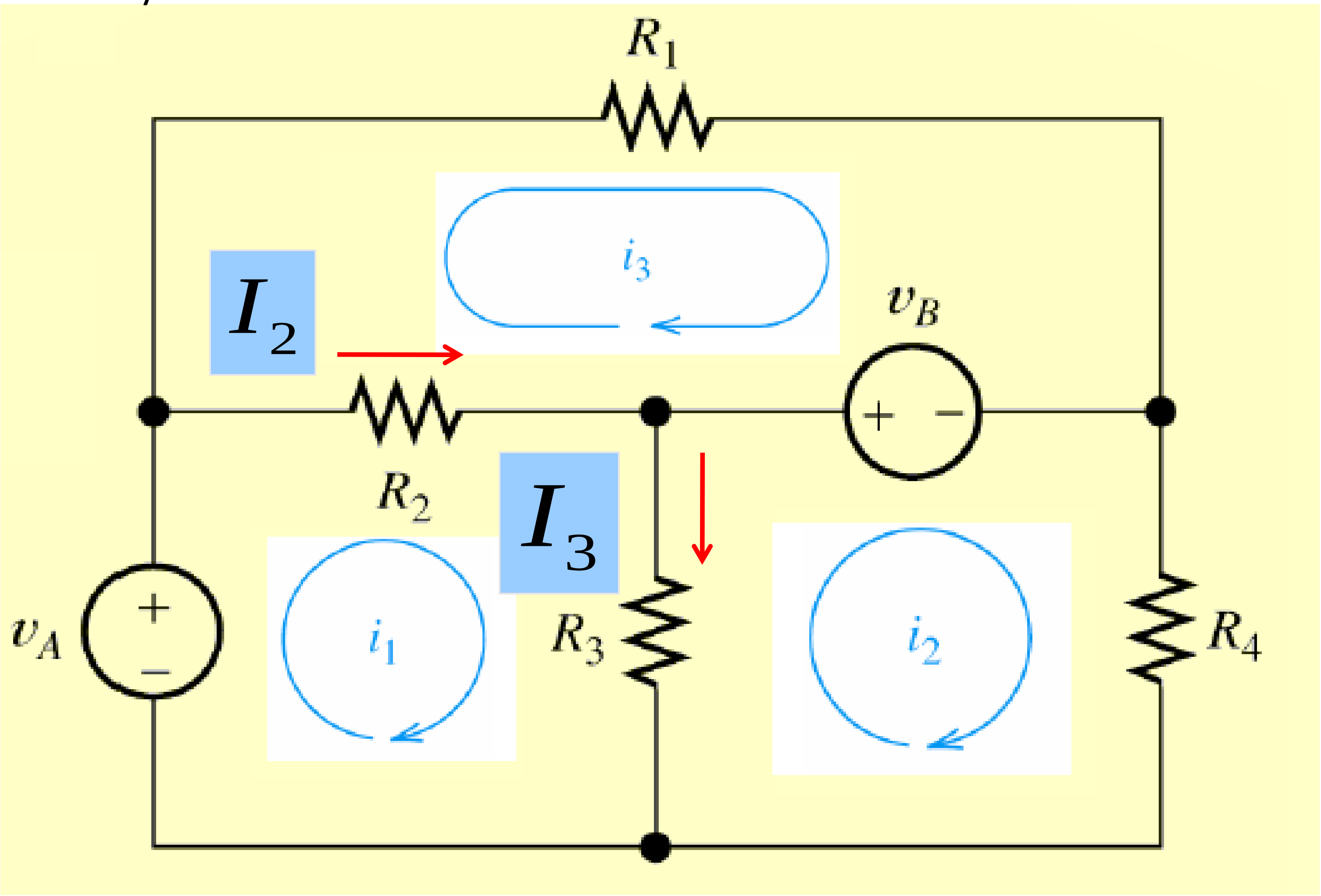


i_1 and i_2 are mesh current (imaginary, not measurable directly)

I_1, I_2 and I_3 are branch current (real, measurable directly)

Mesh Currents

How many meshes?

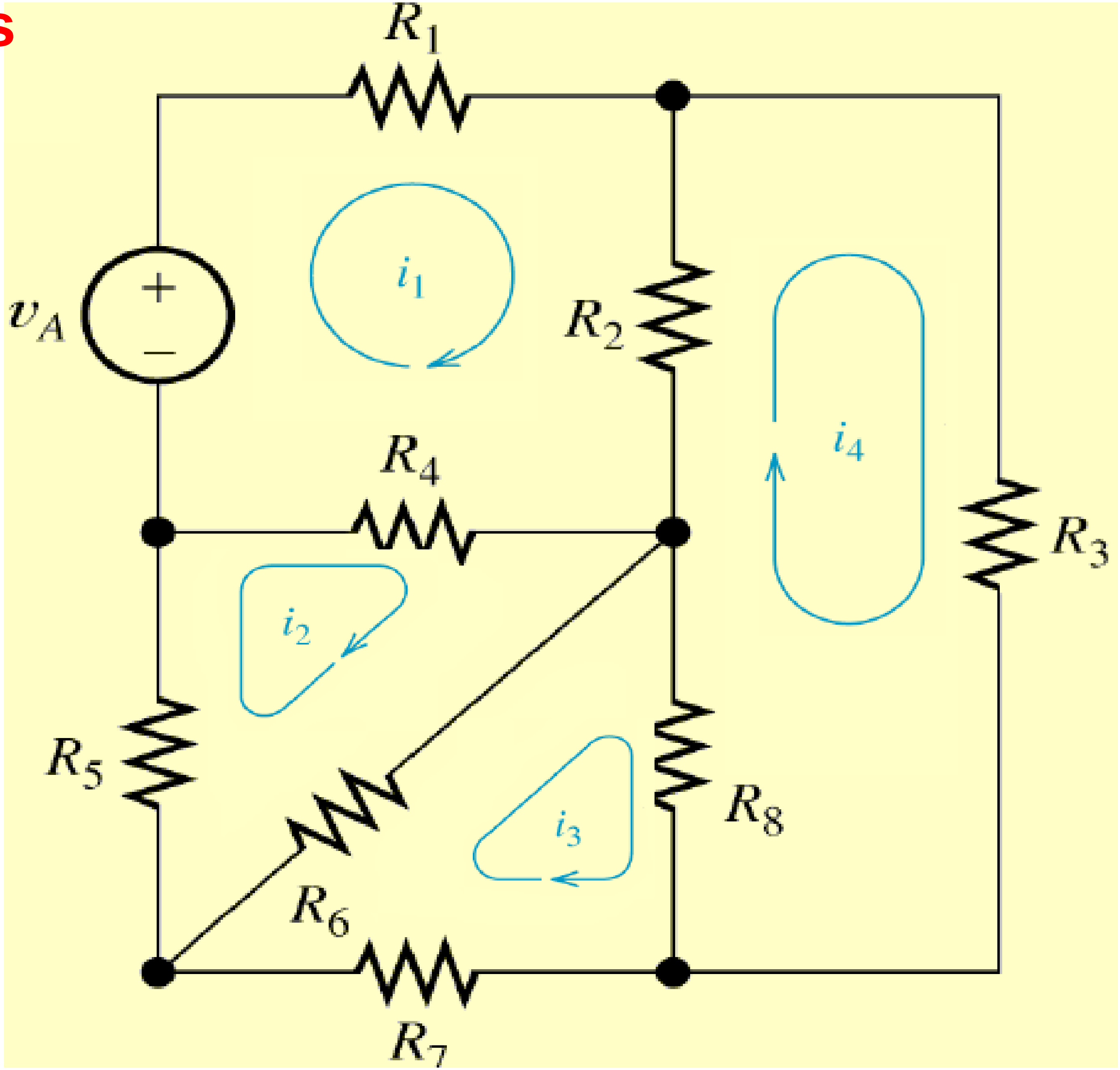


$$I_2 = i_1 - i_3$$

$$I_3 = i_1 - i_2$$

Mesh Currents

How many meshes?



Mesh Analysis

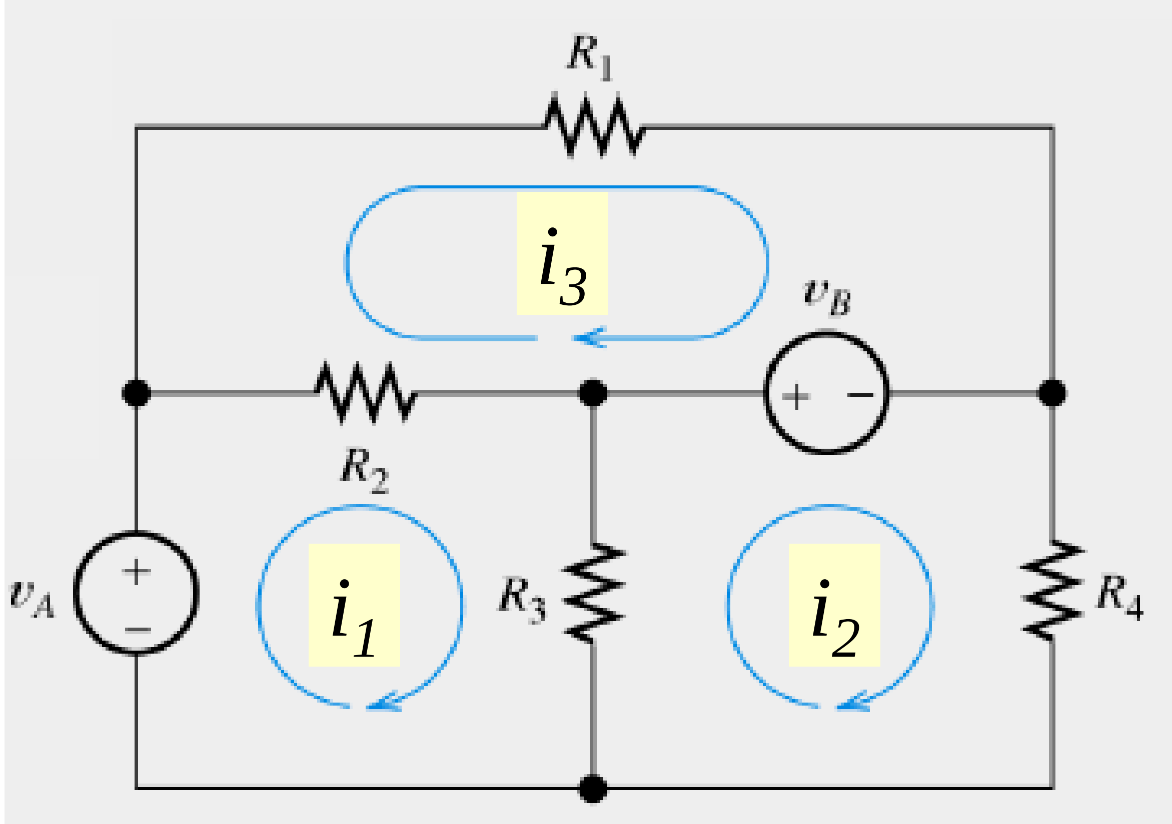
Steps to determine the mesh currents:

1. Assign mesh currents i_1, i_2, …, in to the n meshes.

2. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.

3. Solve the resulting n simultaneous equations to get the mesh currents.

Example

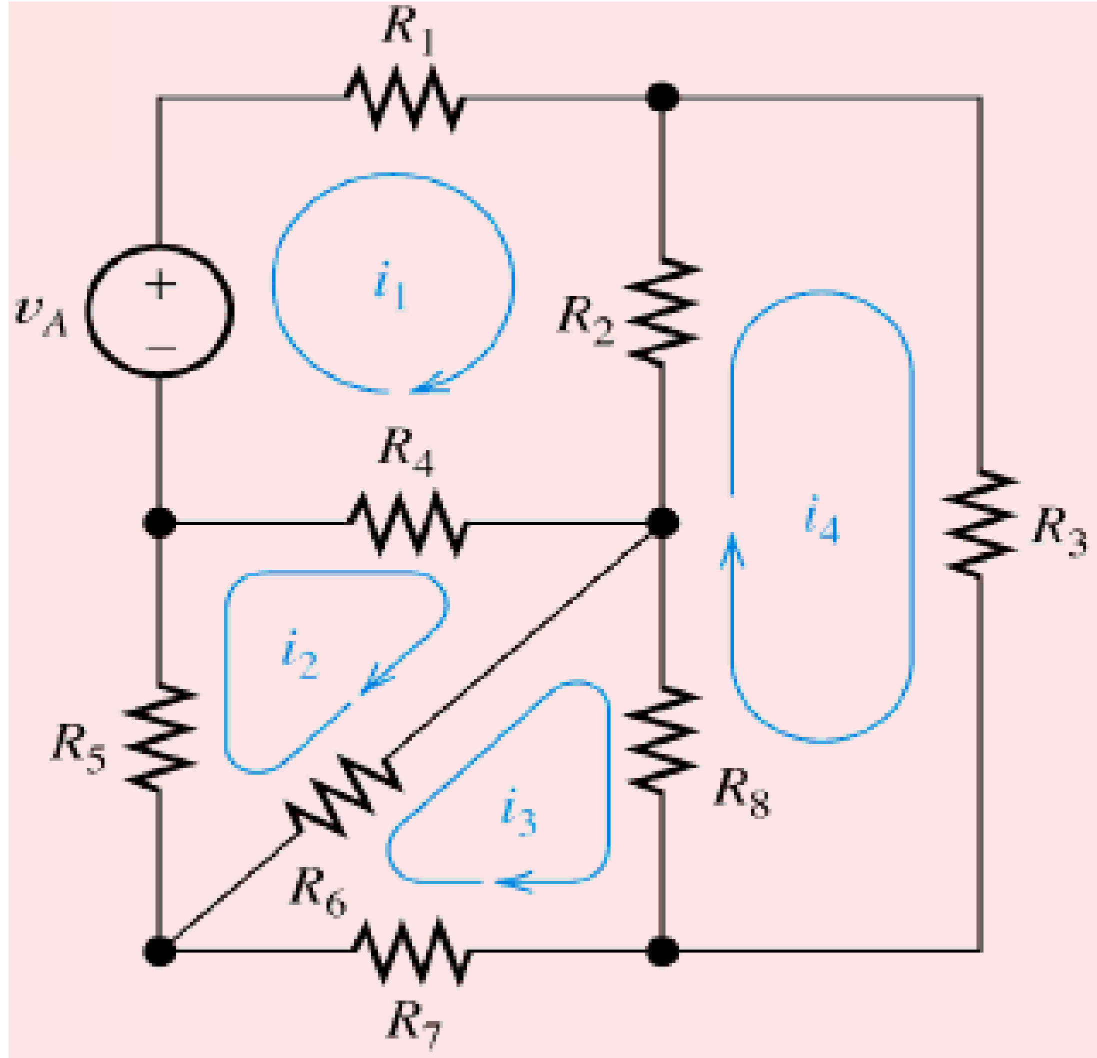


Mesh-1 $$R_2(i_1 - i_3) + R_3(i_1 - i_2) - v_A = 0$$

Mesh-2 $$R_3(i_2 - i_1) + v_B + R_4 i_2 = 0$$

Mesh-3 $$R_2(i_3 - i_1) + R_1 i_3 - v_B = 0$$

Example



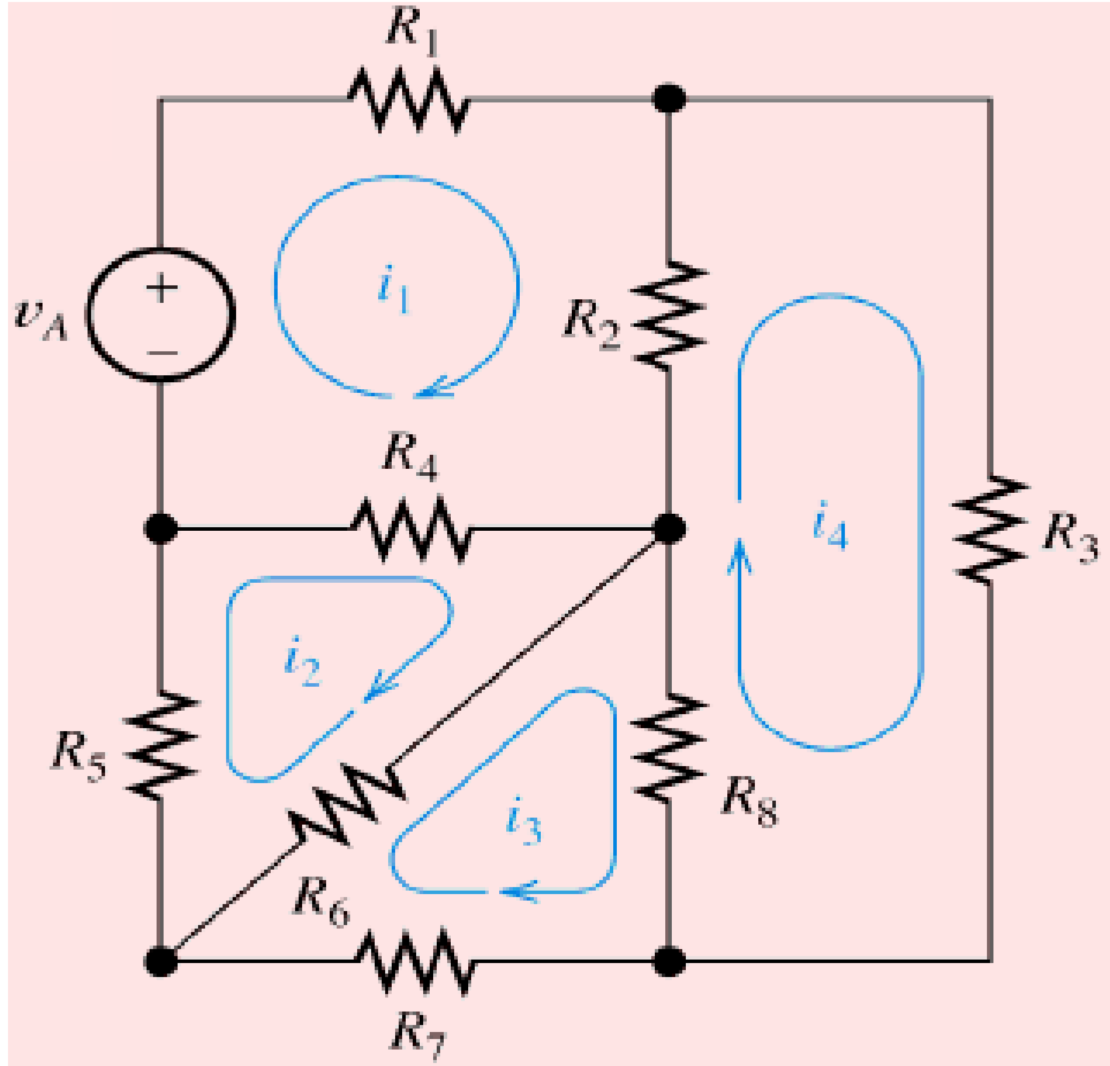
Mesh-1

$$R_1 i_1 + R_2 (i_1 - i_4) + R_4 (i_1 - i_2) - v_A = 0$$

Mesh-2

$$R_5 i_2 + R_4 (i_2 - i_1) + R_6 (i_2 - i_3) = 0$$

Example

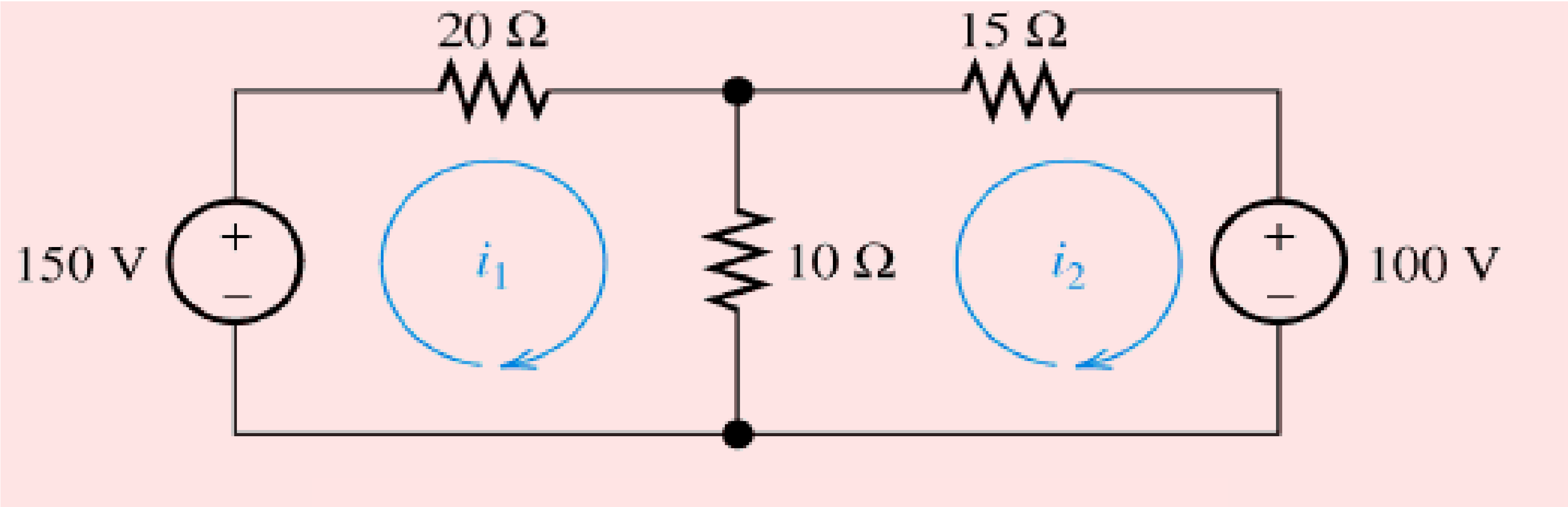


Mesh-3 $$R_7 i_3 + R_6\left(i_3 - i_2\right) + R_8\left(i_3 - i_4\right) = 0$$

Mesh-4 $$R_3 i_4 + R_2\left(i_4 - i_1\right) + R_8\left(i_4 - i_3\right) = 0$$

Example

Solve for the currents in each element in the circuit



$$\text{mesh 1}: 20i_1 + 10(i_1 - i_2) - 150 = 0$$

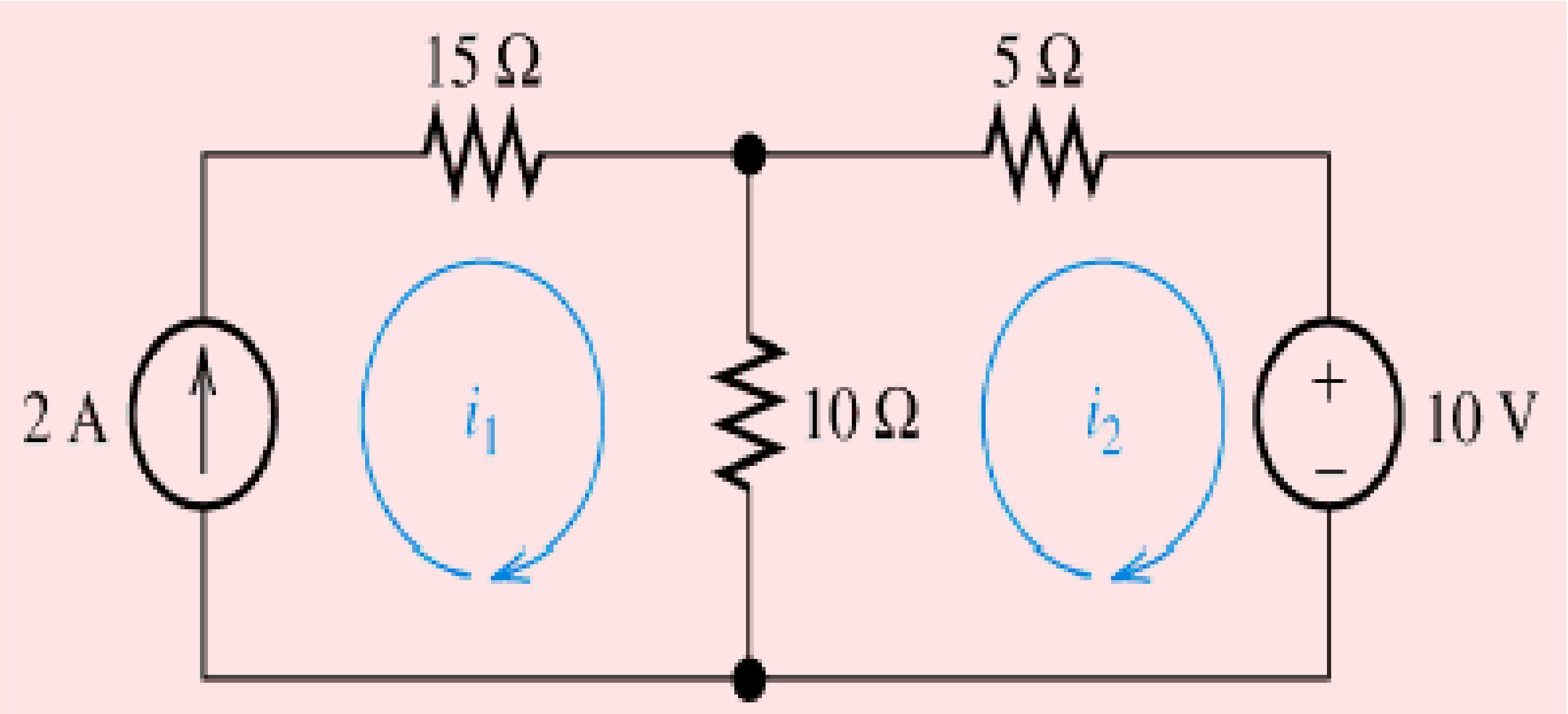
$$\text{mesh 2}: 10(i_2 - i_1) + 15i_2 + 100 = 0$$

$$30\ i_1 - 10\ i_2 = 150 \qquad i_1 = 4.231\ A$$

$$-10\ i_1 + 25\ i_2 = -100 \qquad i_2 = -2.308\ A$$

The current in the 10 - Ω is i_1-i_2 = 6.539 A

Mesh Currents in Circuits Containing Current Sources

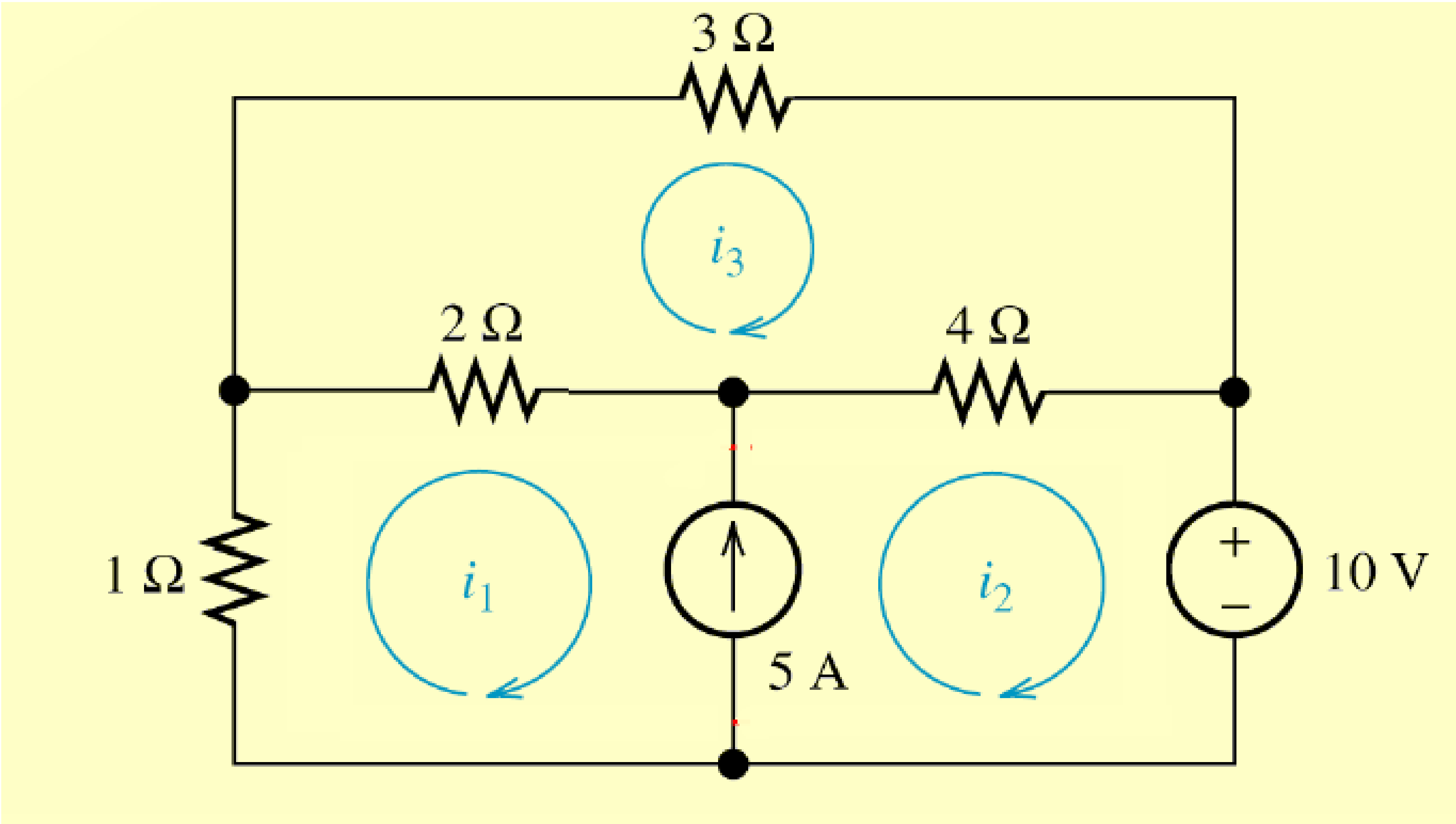


$$15i_1 + 10(i_1 - i_2) + ? = 0$$

$$i_1 = 2\text{A}$$

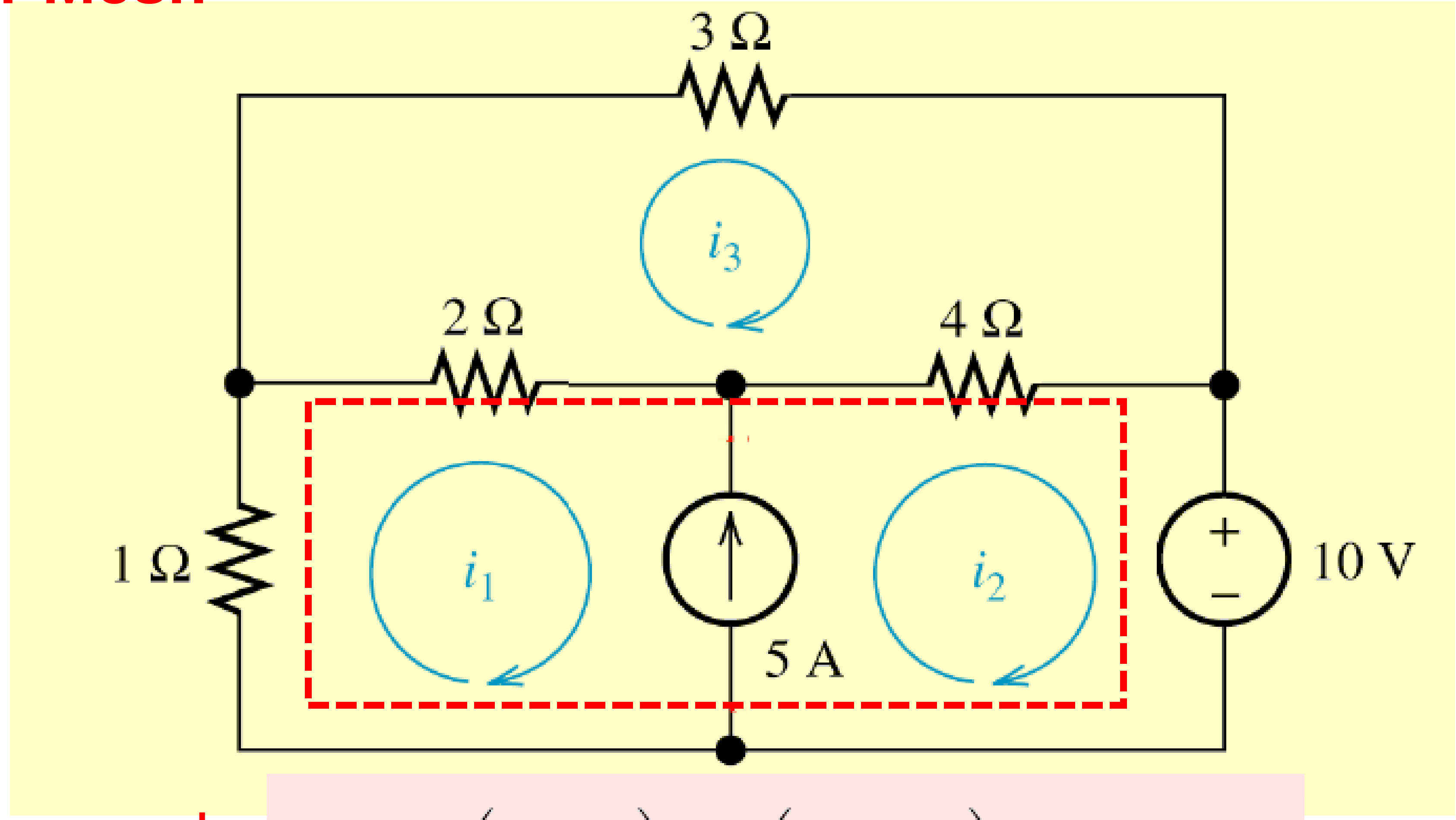
$$10(i_2 - i_1) + 5i_2 + 10 = 0$$

Current source common to 2 mesh



$$i_1 + 2(i_1 - i_3) + ? = 0$$

Super Mesh

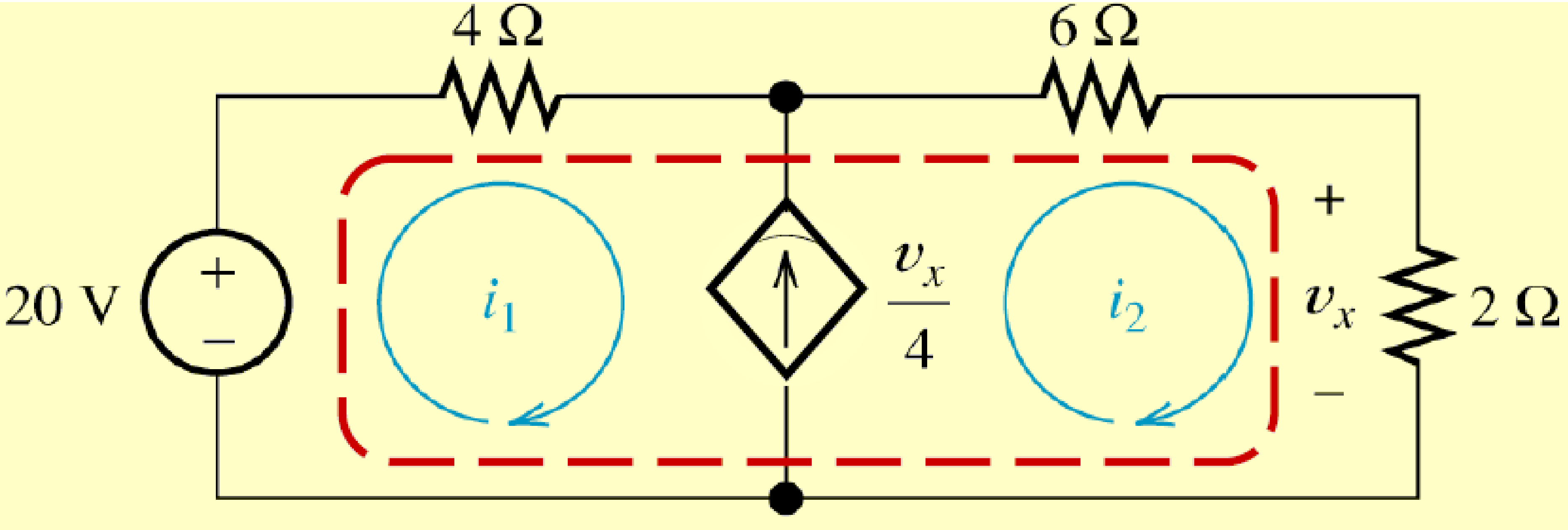


Super mesh $i_1 + 2(i_1 - i_3) + 4(i_2 - i_3) + 10 = 0$

Mesh-3 $3i_3 + 4(i_3 - i_2) + 2(i_3 - i_1) = 0$

$$i_2 - i_1 = 5$$

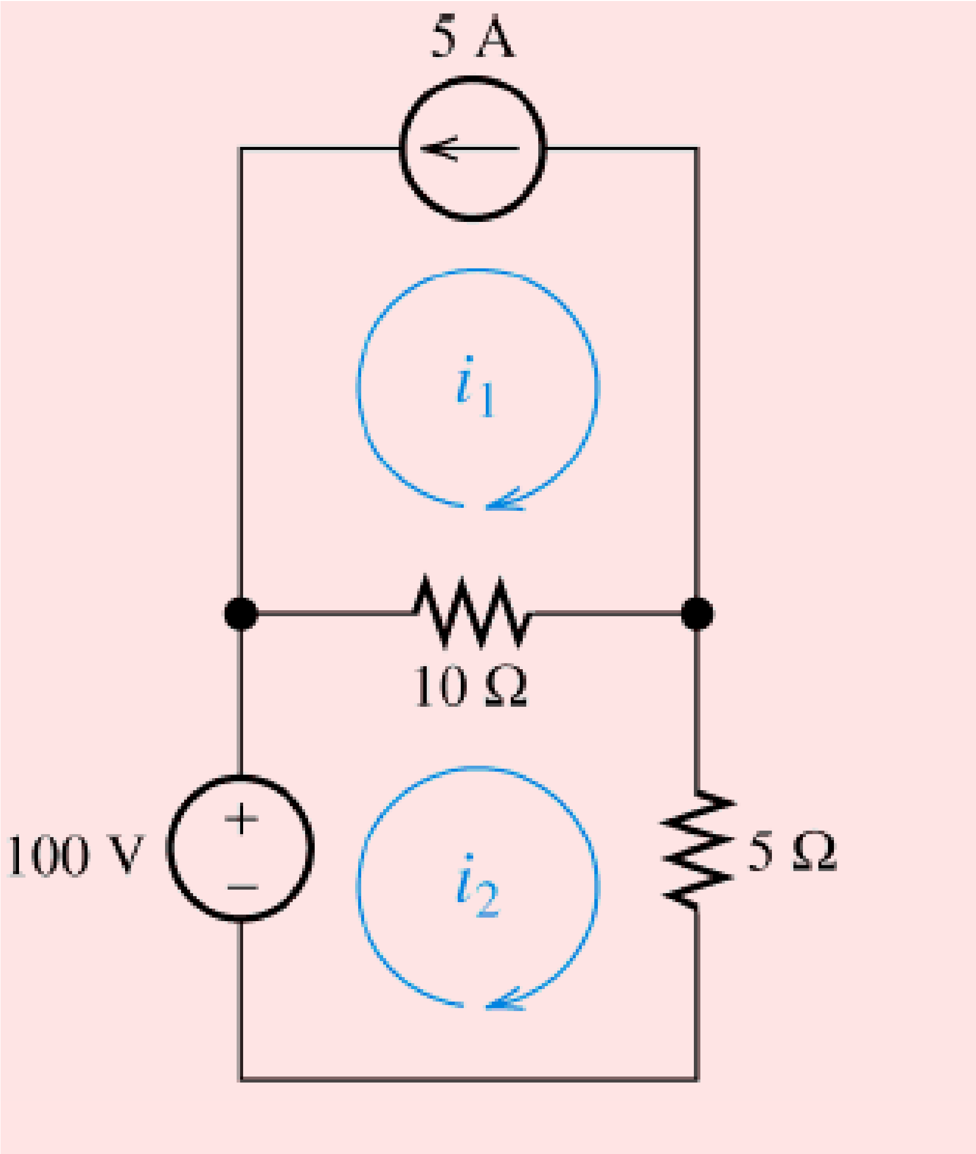
Example



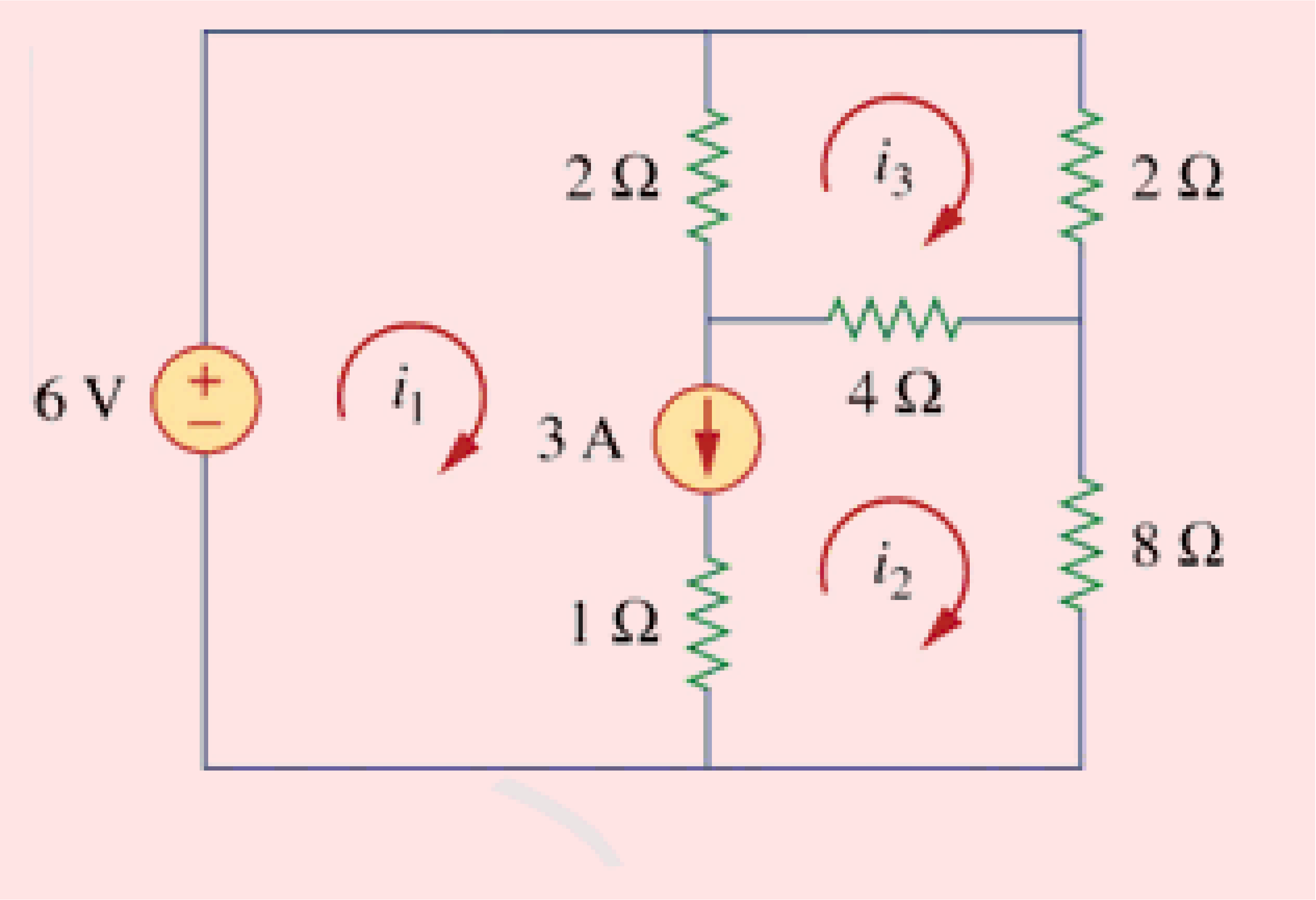
$$-20 + 4i_1 + 6i_2 + 2i_2 = 0$$

$$\frac{v_x}{4} = i_2 - i_1 \qquad v_x = 2i_2$$

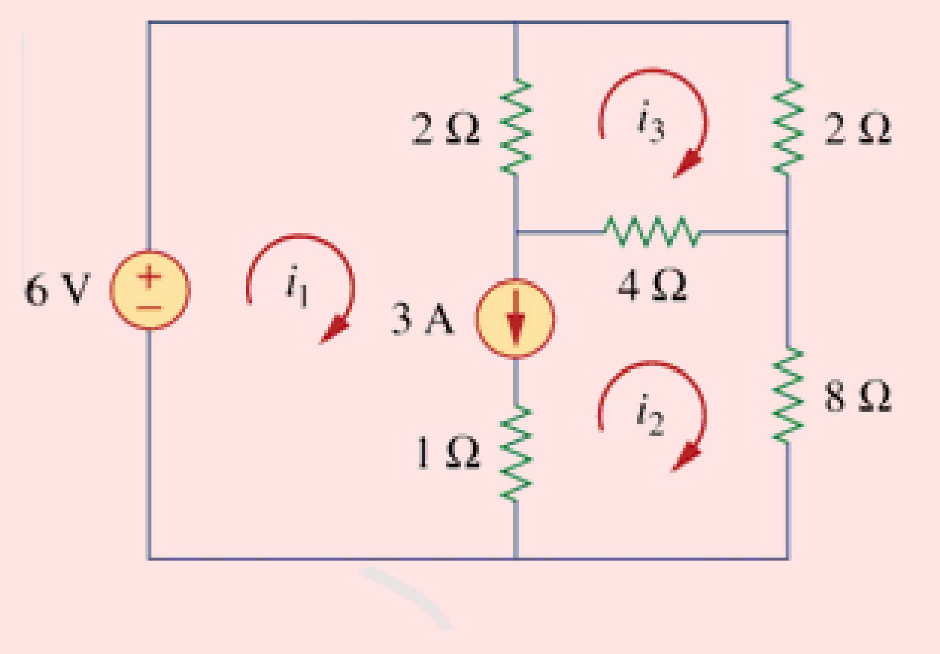
Exercise!



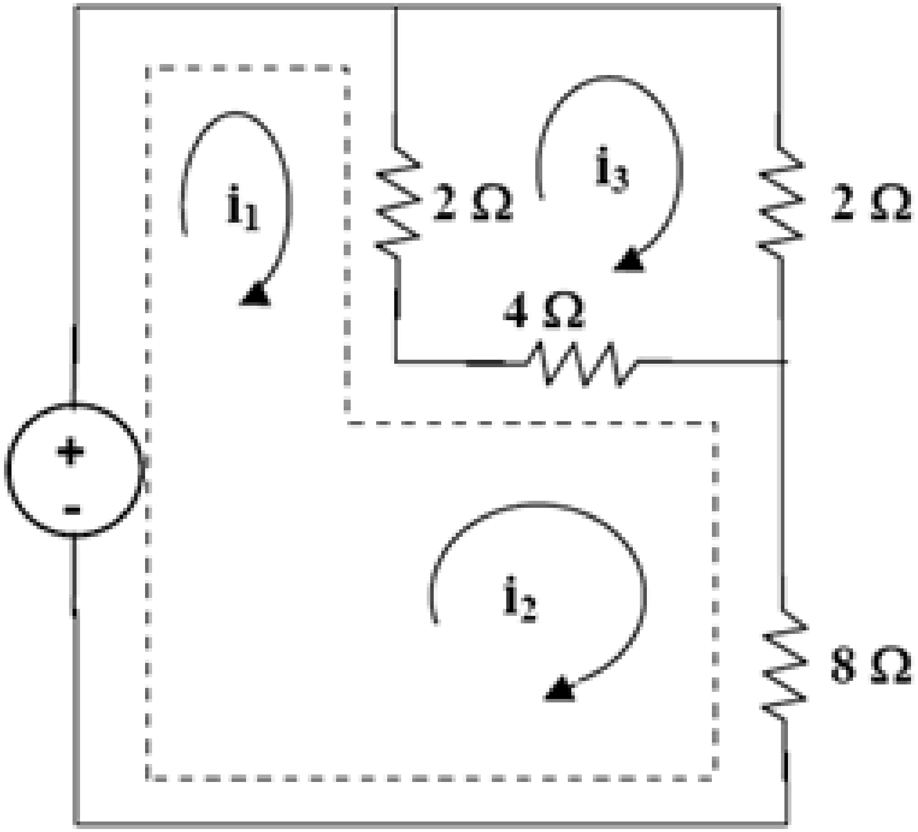
Example: Identify the super mesh



Example contd..:



Identify the super mesh



$$-6 + 2(i_1 - i_3) + 4(i_2 - i_3) + 8i_2 = 0$$

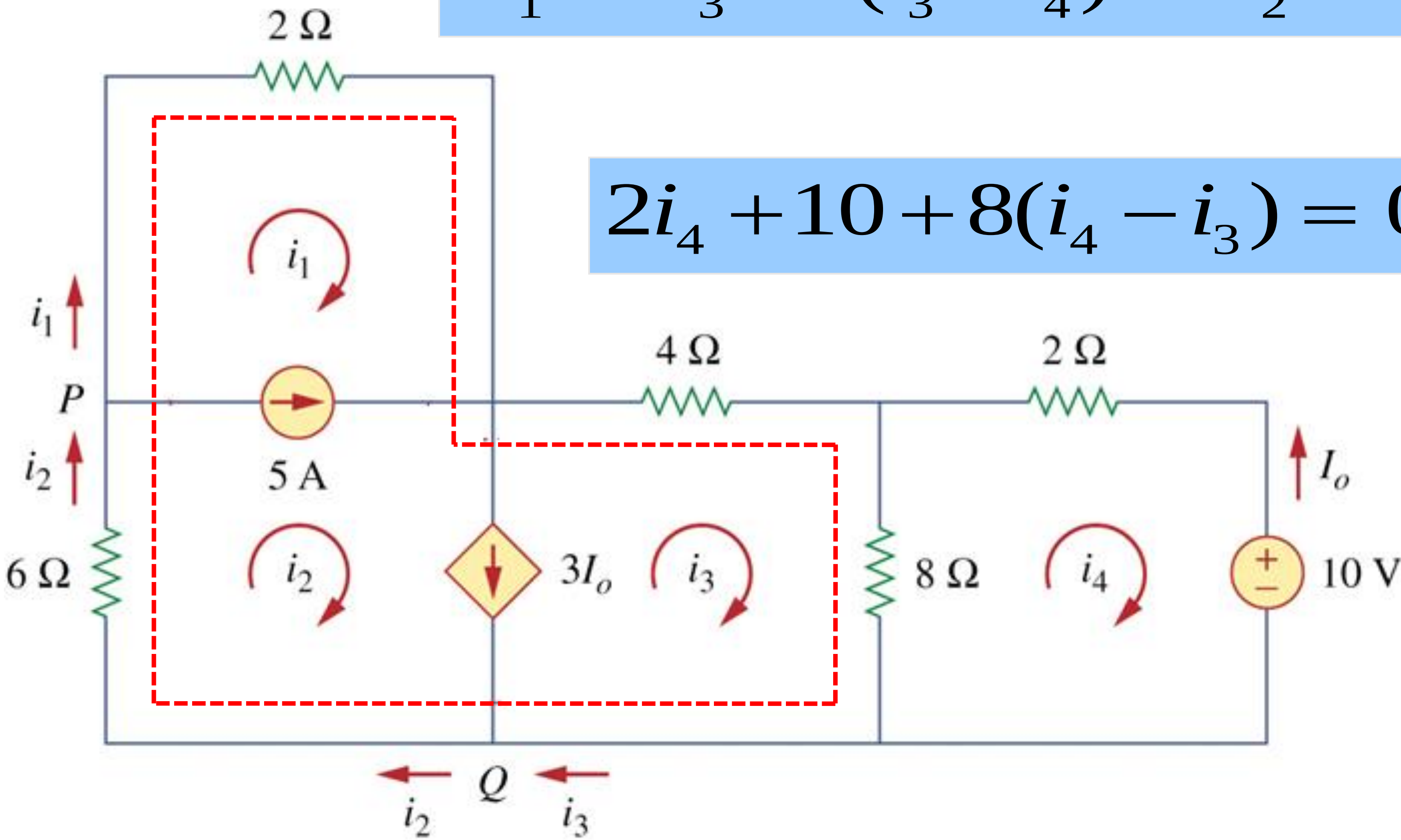
$$2i_3 + 4(i_3 - i_2) + 2(i_3 - i_1) = 0$$

$$i_1 - i_2 = 3$$

Example:

$$2i_1 + 4i_3 + 8(i_3 - i_4) + 6i_2 = 0$$

$$2i_4 + 10 + 8(i_4 - i_3) = 0$$



$$i_2 - i_1 = 5$$

$$i_2 - i_3 = 3I_o$$

$$I_o = -i_4$$

Nodal vs. Mesh Analysis

To select the method that results in the smaller number of equations. For example:

1. Choose nodal analysis for circuit with fewer nodes than meshes.

   *Choose mesh analysis for circuit with fewer meshes than nodes.

   *Networks that contain many series connected elements, voltage sources, or supermeshes are more suitable for mesh analysis.

   *Networks with parallel-connected elements, current sources, or supernodes are more suitable for nodal analysis.

2. If node voltages are required, it may be expedient to apply nodal analysis. If branch or mesh currents are required, it may be better to use mesh analysis.

Some more circuit analysis techniques

- Superposition Method
- Thevenin Method
- Norton Method

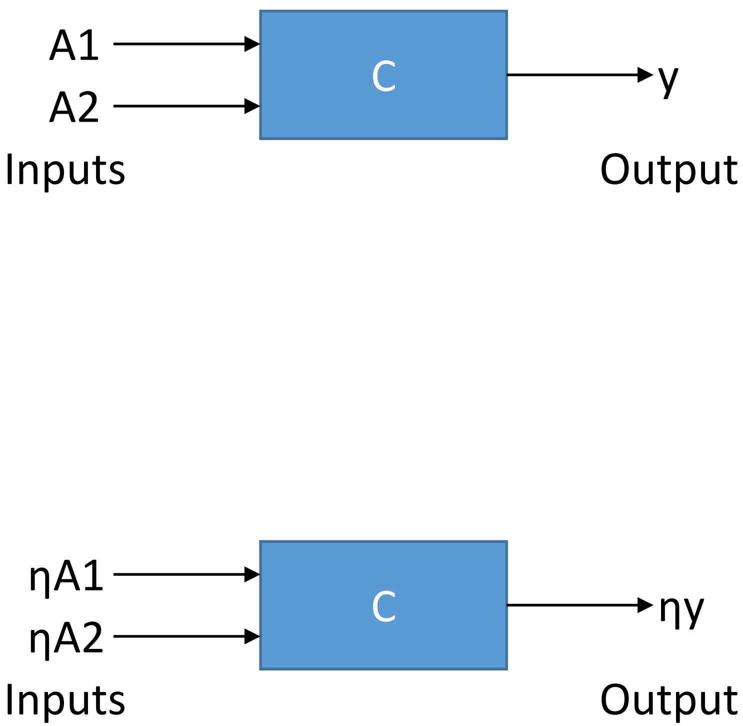
Applicable to only **linear** circuits

Homogeneity

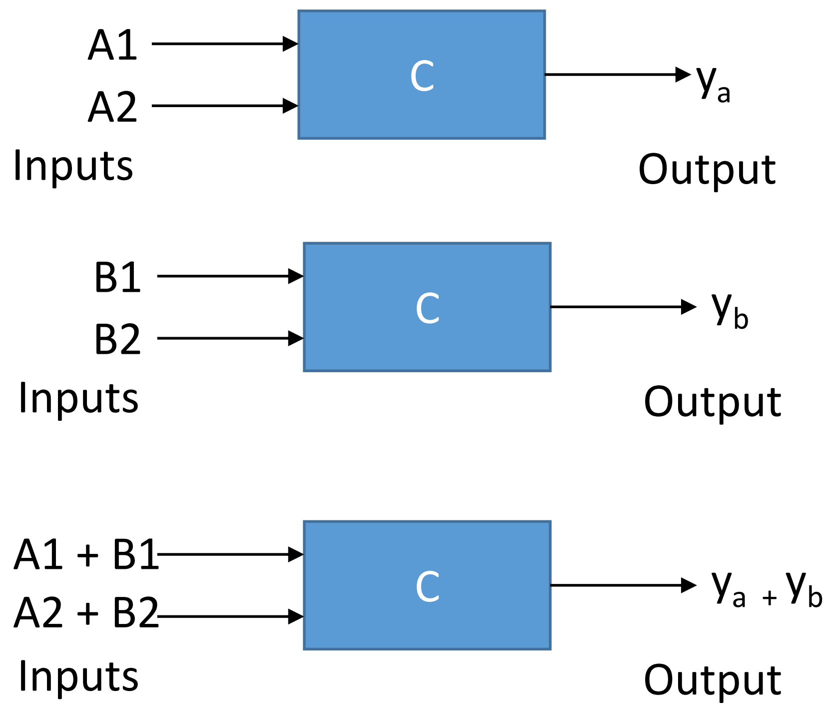
Superposition

Linear circuits

Homogeneity



Superposition



Reference: MIT 6.002 Circuits and Electronics, Spring 2007 (Anant Agarwal)

Example: Our favorite element 'Resistor'

$$V = IR$$

Increasing the current by a constant k

$$kIR = kV$$

Homogeneity

Response to two excitations:

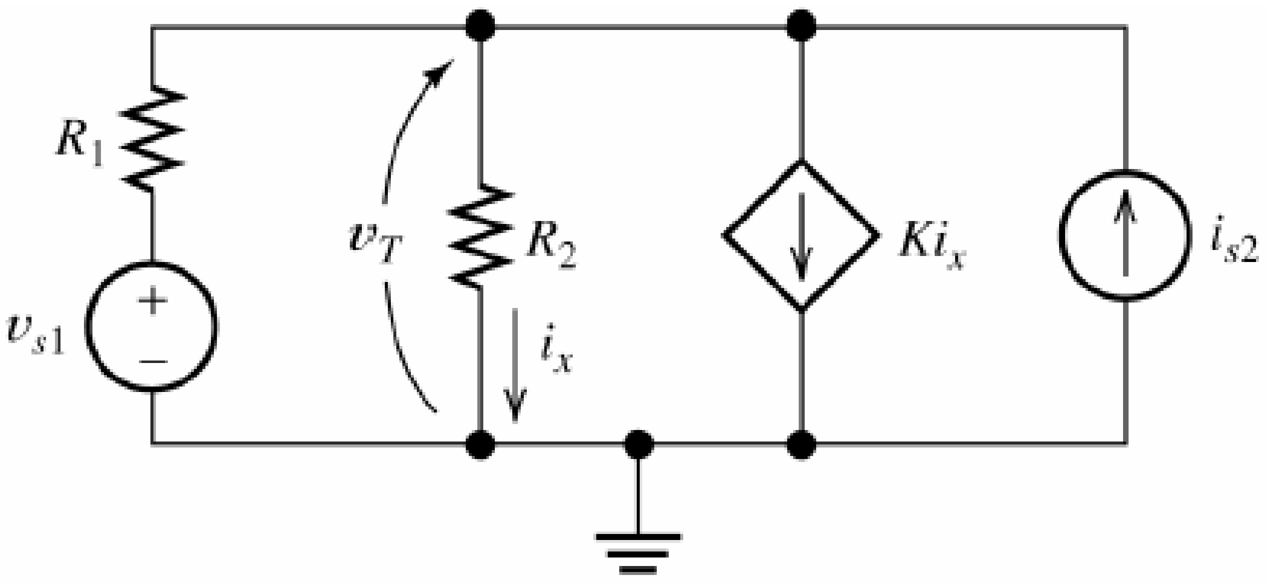
$$V_1 = I_1 R \qquad V_2 = I_2 R$$

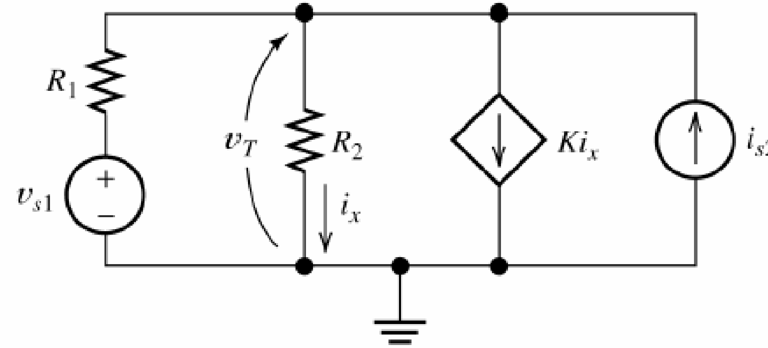
$$V = \left(I_1 + I_2\right)R = I_1 R + I_2 R = V_1 + V_2$$

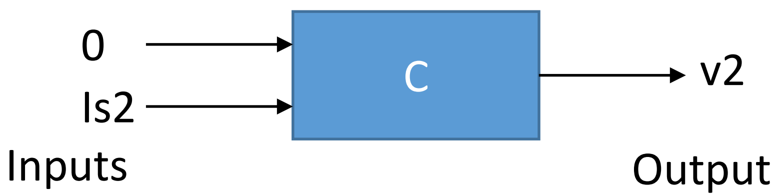
Superposition

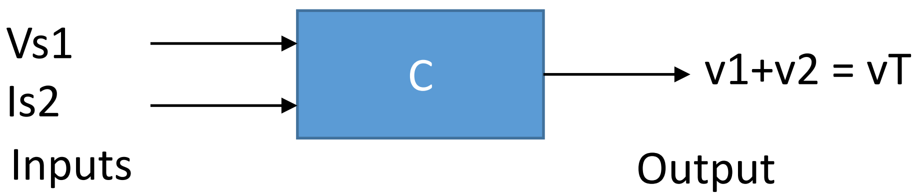
Superposition Principle

The superposition principle states that the total response is the sum of the responses to each of the independent sources acting individually.



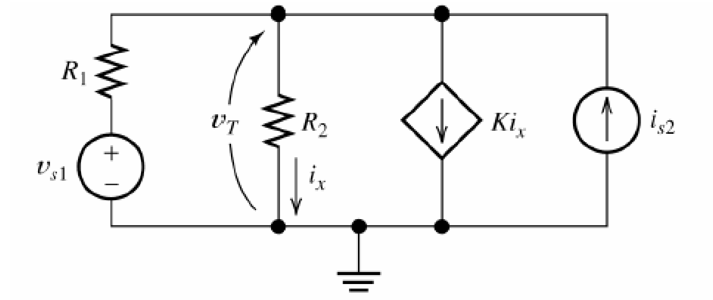


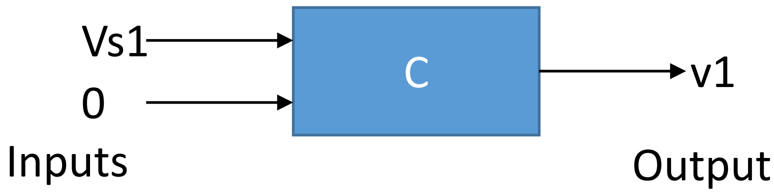


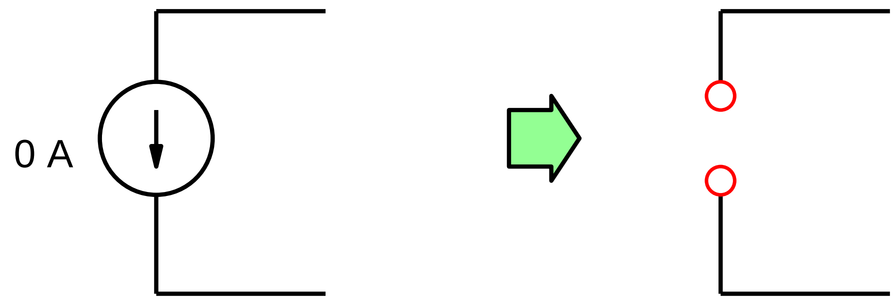


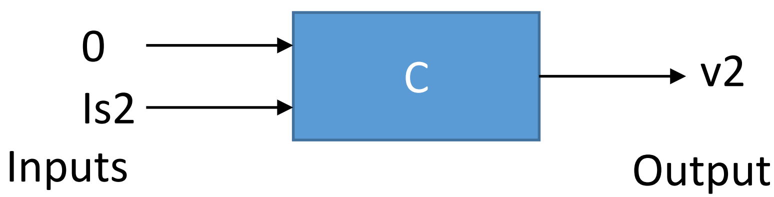
1. Find circuit response to each source acting alone
2. Sum up the individual/partial responses to get the total response

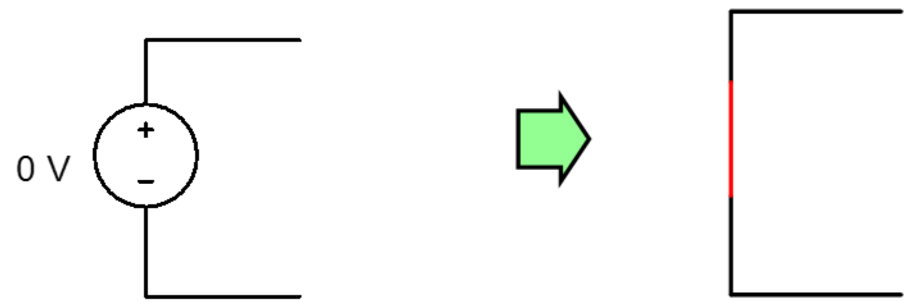
1. Find circuit response to each source acting alone



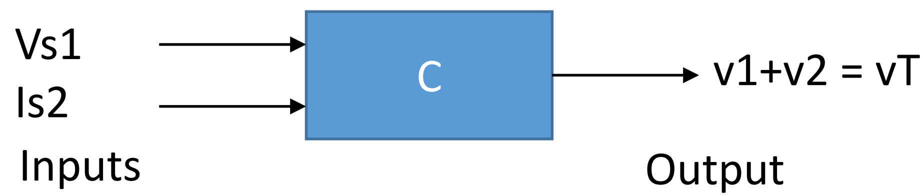




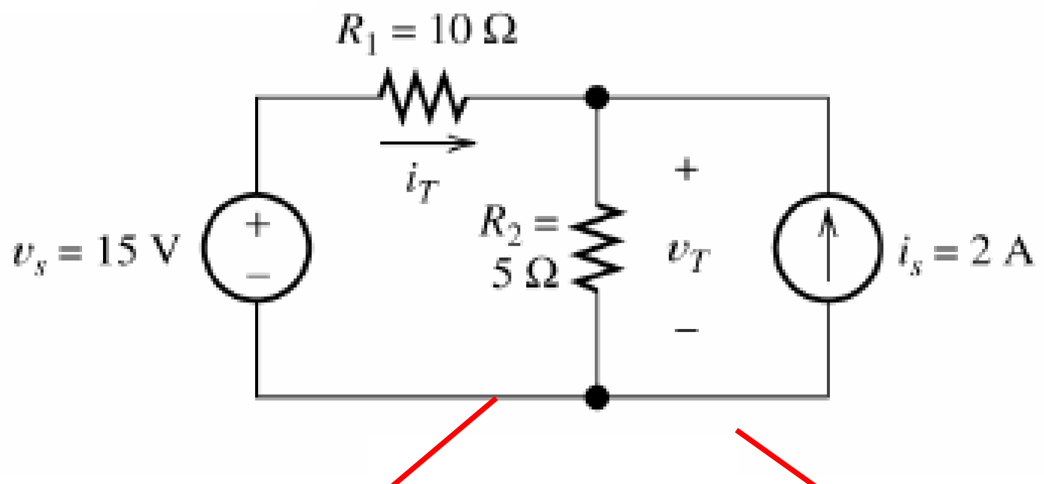


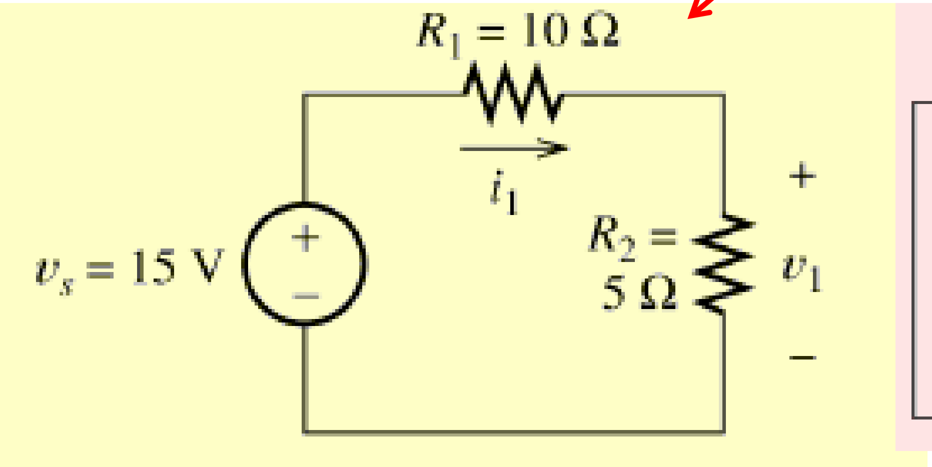


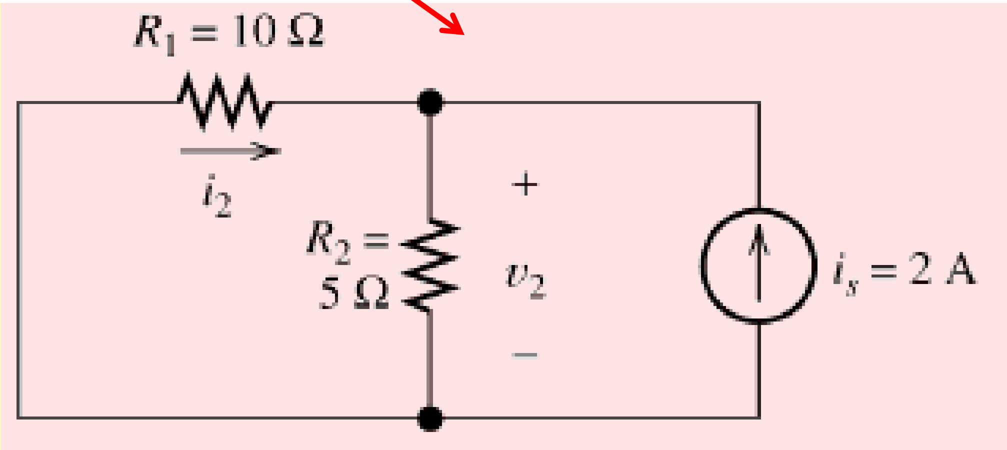
2. Sum up the individual/partial responses to get the total response



Example-1





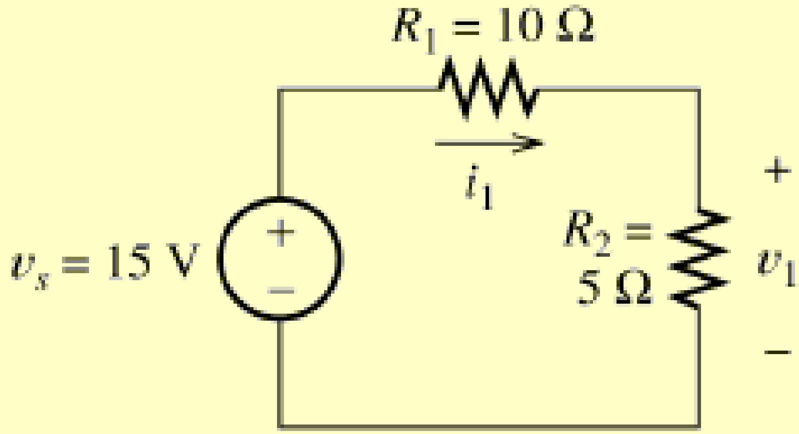


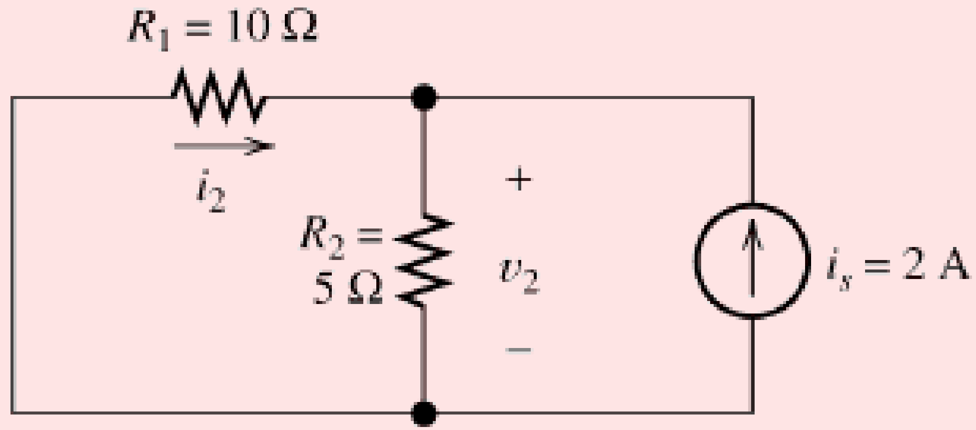
Circuit with only voltage source active. Current source is open circuited.

Circuit with only current source active. Voltage source is short circuited.

$$i_T = i_1 + i_2$$

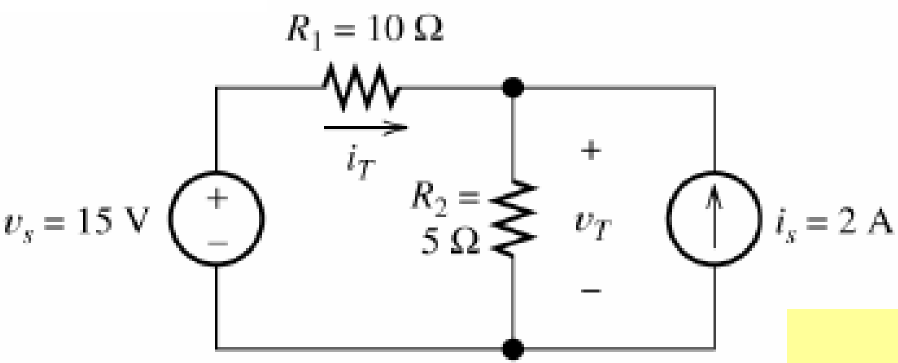
$$v_T = v_1 + v_2$$





$$v_1 = 15\frac{5}{15} = 5V$$

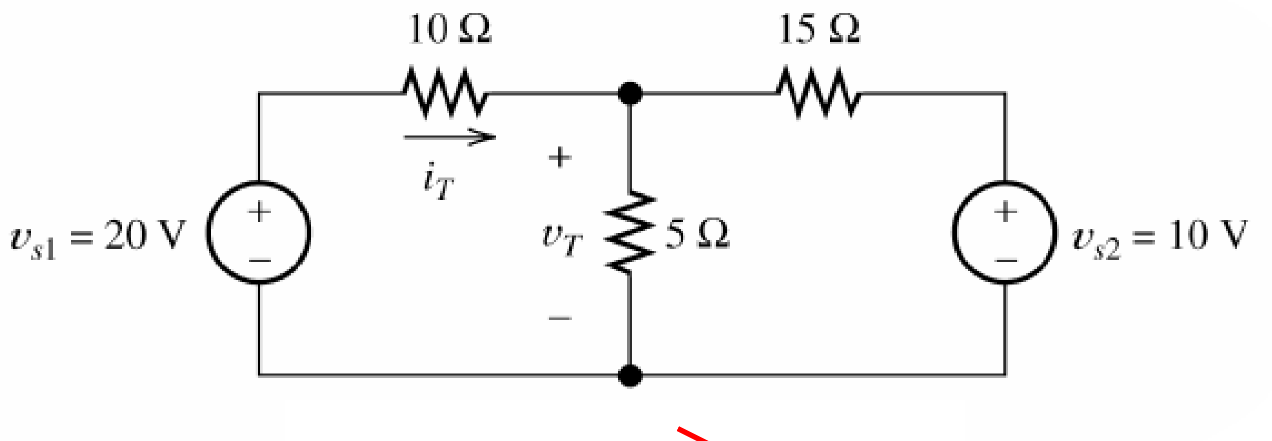
$$v_2 = 5 \times \left(2 \times \frac{10}{15}\right) = \frac{20}{3}V$$

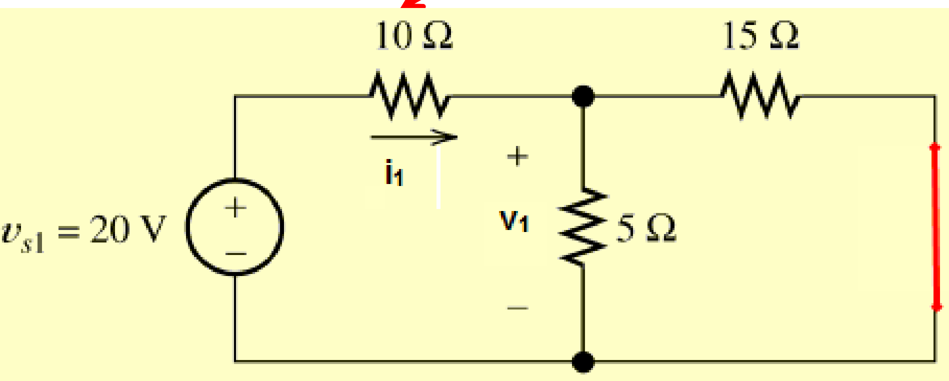


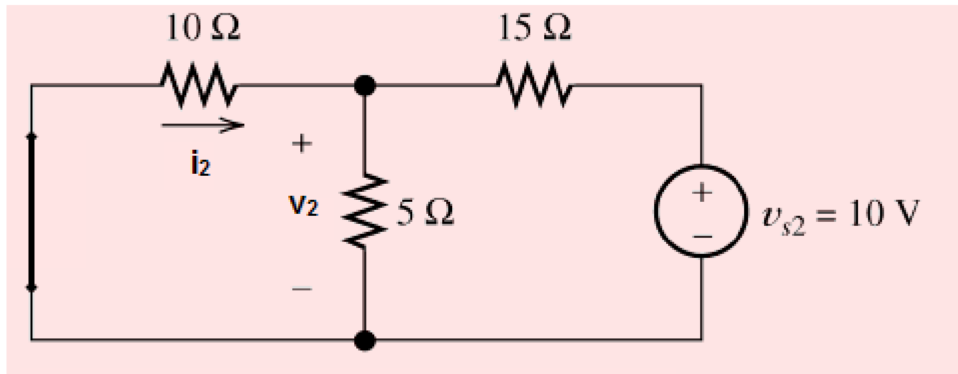
$$v_T = v_1 + v_2$$
$$i_T = i_1 + i_2$$

$$v_T = v_1 + v_2 = 5 + \frac{20}{3} = \frac{35}{3}V$$

Example-2



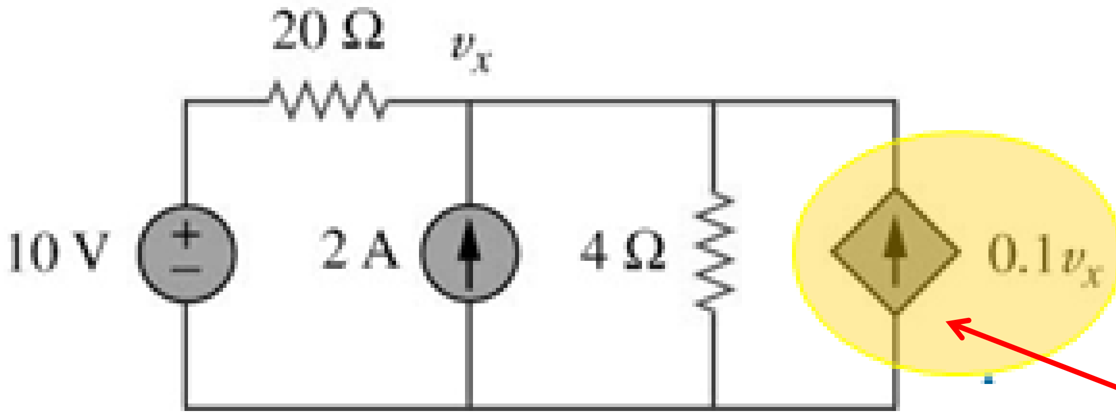




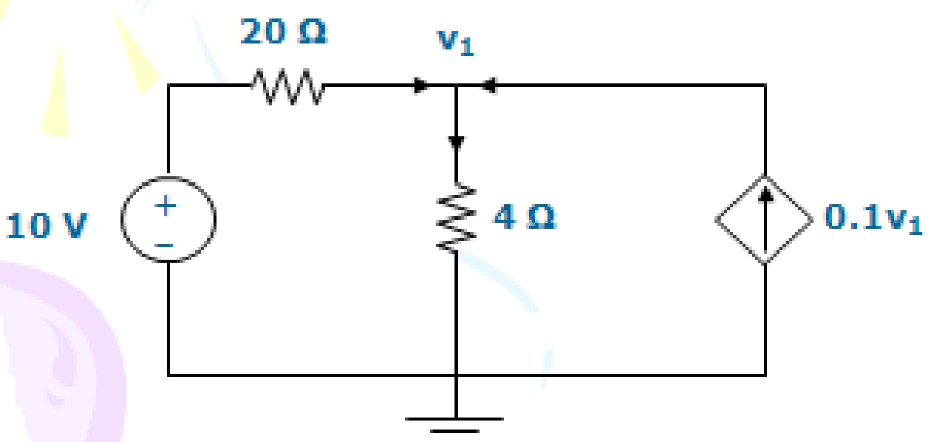
$$i_T = i_1 + i_2$$

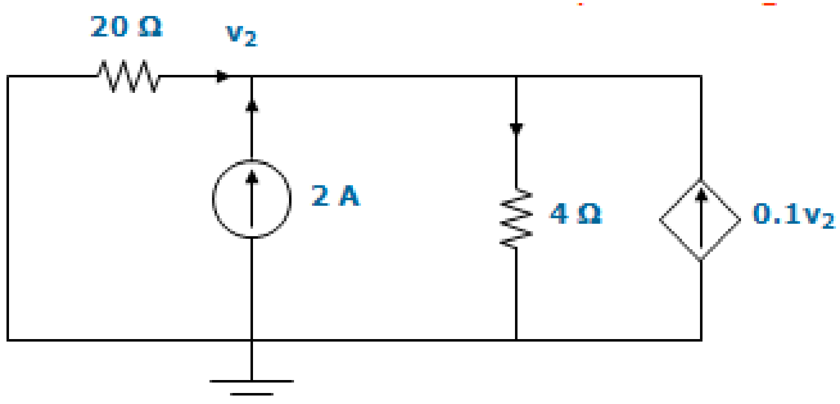
$$v_T = v_1 + v_2$$

Example-3



Dependant source keep unchanged





10V is discarded by short circuit

2A is discarded by open circuit