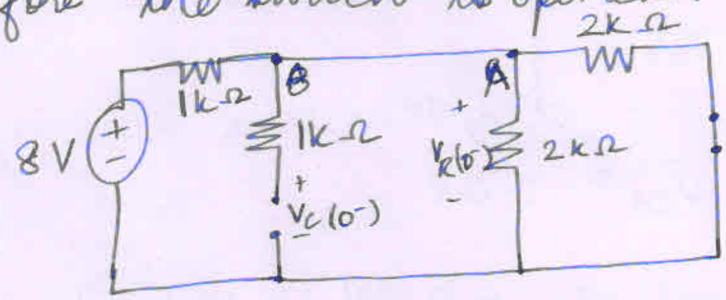
ESC201

Assignment 3 Solutions

Ans1. Before the switch is opened:



Applying nodal at node B,

$$\frac{V_c(0^-) - 8}{1} + \frac{V_c(0^-)}{2} + \frac{V_c(0^-)}{2} = 0$$

$$\Rightarrow V_c(0^-) = 4V = V_c(0^+) = V_R(0^-)$$

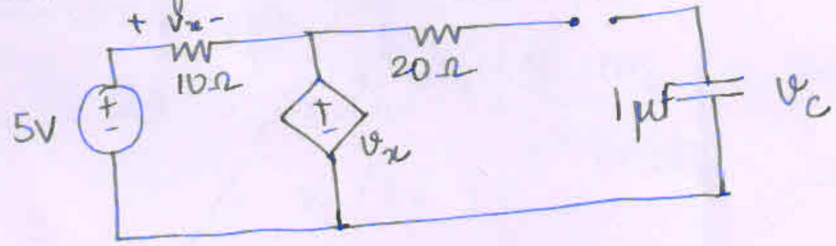
$$i_L(0^-) = \frac{4}{2} mA = 2mA = i_L(0^+)$$

At $t = 0^+$: It is a source free circuit.

$$\therefore \tau = \frac{L}{R} = \frac{1H}{4k} = 0.25ms$$

$$\therefore v_R(t)\Big|_{t\geq 0} = -i_L(t\geq 0)\cdot 2k \ V$$
$$= -2k \times i_L(0^+) e^{-t/\tau} \ V$$
$$= -2 \times 2 \ e^{-4t} \ V$$
$$= -4e^{-4t} \ V$$

Ans2. Before the switch is closed:

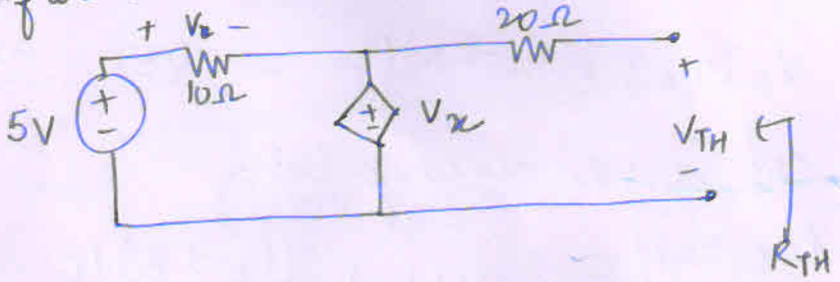


Applying KVL in the first loop,

$$5 - v_x - v_x = 0$$

or $v_x = 2.5V$

After the switch is closed:



Thevenin voltage,

$V_{TH} = 2.5V$

Short circuit current $= I_{sc} = \frac{2.5}{20} A = 125 mA$

$\therefore R_{TH} = \frac{2.5V}{125mA} = 20\Omega$

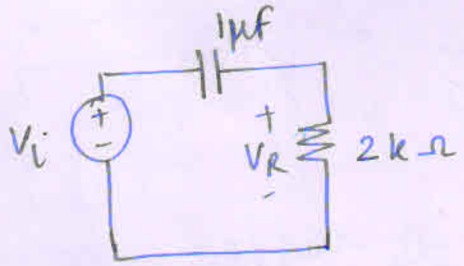
$\therefore \tau = 20 \times 10^{-6} s = 20\mu s$

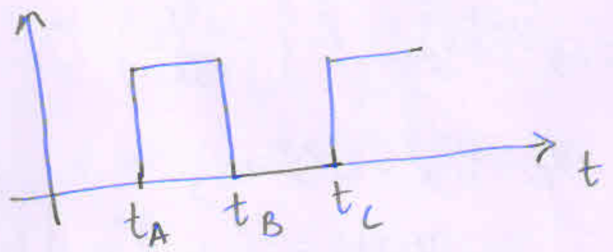
$\therefore v_c(t) = v_c(\infty) + [v_c(0) - v_c(\infty)] e^{-t/\tau}$

$= 2.5 + (0 - 2.5) e^{-t/2\times10^{-5}} V$

$= 2.5(1 - e^{-t/2\times10^{-5}}) V$

Ans 3.





$\tau = RC = 10^{-6} \times 2 \times 10^3 s = 2ms$

$V_{max} = 1V$, $V_{min} = 0V$

Charging cycle: $t_A < t < t_B$

$v_c(t) = V_{max} + [v_c(t_A^-) - V_{max}] e^{-(t-t_A)/\tau}$

$= 1 + [v_c(t_A^-) - 1] e^{-(t-t_A)/\tau}$ —— (1)

Discharging cycle: $t_B < t < t_C$

$v_c(t) = V_{min} + [v_c(t_B^-) - V_{min}] e^{-(t-t_B)/\tau}$

$= v_c(t_B^-) e^{-(t-t_B)/\tau}$ —— (2)

In steady state,

$v_c(t_A) = v_c(t_C)$

Inserting this condition in equations (1) and (2)

$$\therefore v_c(t_B) = v_c(t_A)\, e^{(t_C - t_B)/\tau} \quad \text{— (3)}$$

Put $t = t_B$ in (1) and (2) and using (3),

$$1 + [v_C(t_A) - 1]\, e^{-(t_B - t_A)/\tau} = v_C(t_A)\, e^{(t_C - t_B)/\tau}$$

$$\therefore v_c(t_A) = \frac{1 - e^{-(t_B - t_A)/\tau}}{e^{(t_C - t_B)/\tau} - e^{-(t_B - t_A)/\tau}}$$

$t_B - t_A = 2ms$ and $t_C - t_B = 2ms$

$$\therefore v_c(t_A) = \frac{e-1}{e^2-1} = 0.269V$$

$$v_c(t_B) = 0.731V$$

$$\therefore v_R(t) = R\left(C\frac{dv_c}{dt}\right) = \tau \frac{dv_c}{dt}$$

v_c $t_A < t < t_B$:

$$v_c(t) = 1 + [v_c(t_A) - 1]\, e^{-(t - t_A)/\tau}$$

$t_B < t < t_C$

$$v_c(t) = v_c(t_A)\, e^{-(t - t_C)/\tau}$$

$\therefore$ $t_A < t < t_B$:

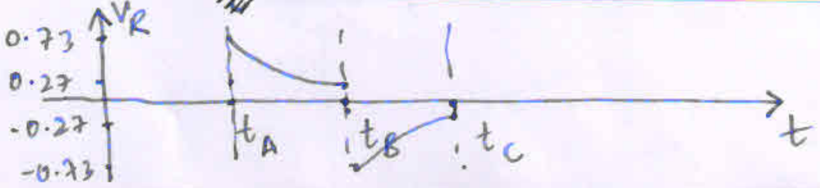
$$v_R(t) = -\tau\, [v_C(t_A) - 1]\, e^{-(t - t_A)/\tau} = 0.731\, e^{-(t - t_A)/\tau}$$

$[\tau = 2ms]$

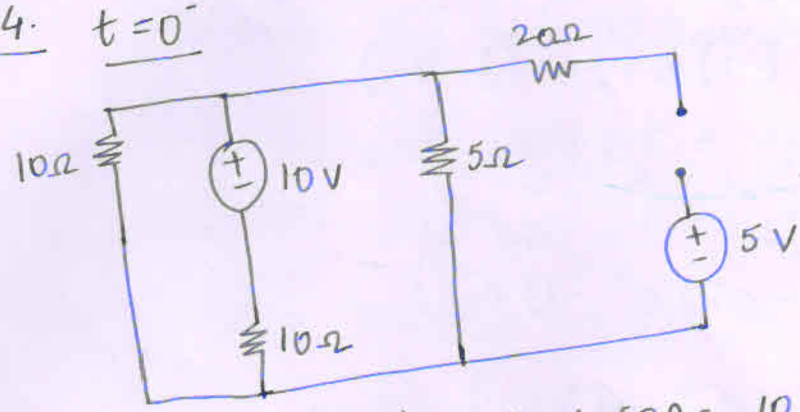
$t_B < t < t_C$:

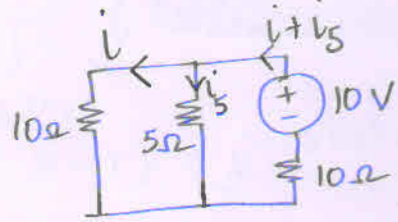
$$v_R(t) = -\tau\, v_c(t_A)\, e^{-(t - t_B - \tau)/\tau} = \cancel{0.27}\,{-0.731}\, e^{-(t - t_B)/\tau}$$

$$= -e\, v_c(t_A)\, e^{-(t - t_B)/\tau}$$



Ans4. $t = 0^-$



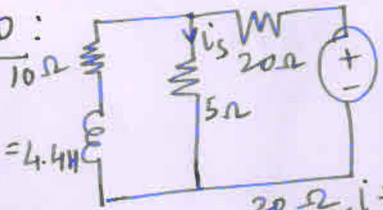


$$R_{eq} = 10 + \left(\frac{10 \times 5}{15}\right)\Omega = 10 + \frac{50}{15}\Omega = 10 + \frac{10}{3}\ \Omega$$

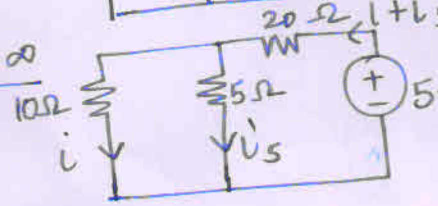
$$\therefore i_{5\Omega} = \left(\frac{10}{10+\frac{10}{3}}\right) \times \frac{10}{15} A = 0.5 A$$

$$10 - 10i - 10i - 10i_5 = 0$$
$$1 - i_5 = 2i$$
$$10 - 5i_5 - 10i - 10i_5 = 0$$
$$\Rightarrow i = \frac{1}{4} A$$
$$i(0^-) = 1/4 A = i(0^+)$$

$t > 0$:



$t \to \infty$



$$5 - 20i - 20i_5 - 5i_5 = 0$$
$$5 - 20i - 20i_5 - 10i = 0$$
$$\Rightarrow i = \frac{5}{70} A, \quad i_5 = \frac{10}{70} A$$
$$R_{TH} = 20 \| 5\ \Omega + 10\ \Omega = 14\ \Omega$$
$$\therefore \tau = \frac{4.4}{14} = 0.314 s$$

$t > 0$ $i(t) = i(\infty) + [i(0^+) - i(\infty)] e^{-t/\tau}$

$$\therefore i(t) = \frac{1}{14} + \left(\frac{1}{4} - \frac{1}{14}\right) e^{-3.2t} A$$

$$V_{5\Omega} = 10i + L\frac{di}{dt} = \frac{5}{7} + \left(\frac{5}{2} - \frac{5}{7}\right) e^{-3.2t} + 4.4\left(\frac{1}{4} - \frac{1}{14}\right) e^{-3.2t}(-3.2) V$$

$$\therefore V_{5\Omega} = \frac{10}{14}\left(1 - e^{-3.2t}\right) V$$

$$\therefore i_{5\Omega} = \frac{1}{7}\left(1 - e^{-3.2t}\right) A$$

Ans5. i) $v(t) = -110 \cos(\omega t + 30°) V$

$\Rightarrow v(t) = -110 \cos(180° + (\omega t - 150°)) V$

$\Rightarrow v(t) = \cancel{+\cos}\ 110 \cos(\omega t - 150°) V$

ii) $v(t) = 220 \sin(\omega t + 220°) V$

$\Rightarrow v(t) = 220 \sin(90° + (\omega t + 130°)) V$

$\Rightarrow v(t) = 220 \cos(\omega t + 130°) V$

iii) $v(t) = 10 \sin(\omega t + 110°) + 4 \cos(\omega t + 110°) V$

$\Rightarrow v(t) = \sqrt{100+16}\left[\frac{10}{\sqrt{116}} \sin(\omega t + 110°) + \frac{4}{\sqrt{116}} \cos(\omega t + 110°)\right]$

$$\Rightarrow v(t) = \sqrt{116}\left[\sin(68^\circ)\sin(\omega t+110^\circ) + \cos(68^\circ)\cos(\omega t+110^\circ)\right]$$

$$\Rightarrow v(t) = \sqrt{116}\cos(\omega t + 110^\circ - 68^\circ)$$

$$\Rightarrow v(t) = \sqrt{116}\cos(\omega t + 42^\circ)$$

iv) $v(t) = 10\cos(\omega t + 370^\circ)4\sin(\omega t+10^\circ)$ V

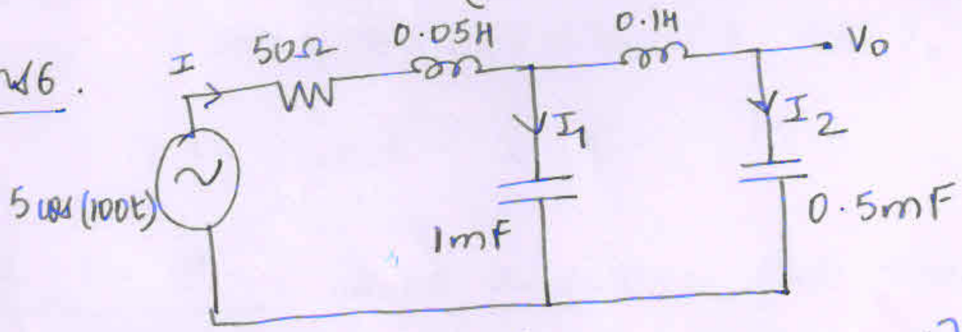
$$= 40\left[\sin(2\omega t + 380^\circ) - \sin(360^\circ)\right]$$

$$= 40\left[\sin(2\omega t + 380^\circ)\right]$$

$$= \cancel{40\cos(2\omega t + 2890}$$

$$= 40\cos(2\omega t + 290^\circ)\text{ V} = 40\cos(2\omega t - 70^\circ)\text{ V}$$

Ans6.



$$Z_{eq} = 50 + j5 + \left[-j10 \,||\, (j10 - j20)\right]$$

$$= 50 + j5 + (-j10 \,||\, -j10)$$

$$= 50 + j5 - j5$$

$$= 50\,\Omega$$

$$\therefore I = \frac{5}{50}\cos(100t)\text{A} = 0.1\cos(100t)\text{A}$$

$$\therefore I_2 = \left(\frac{-j10}{-j10 + j10 - j20}\right) I = \frac{-10}{-20} \times 0.1\cos(100t) = 0.05\cos(100t)\text{ A}$$

$$\therefore V_0 = I_2(-j20) = \cos(100t - 90^\circ)\text{ V}$$

$$= \sin(100t)\text{ V}$$