ESC201A Assignment 3

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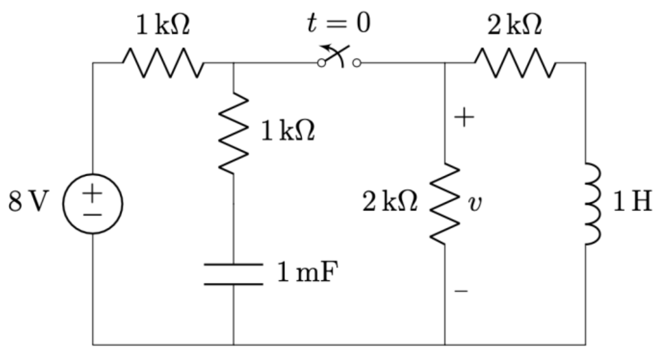
2023-2024 Semester I

Topics

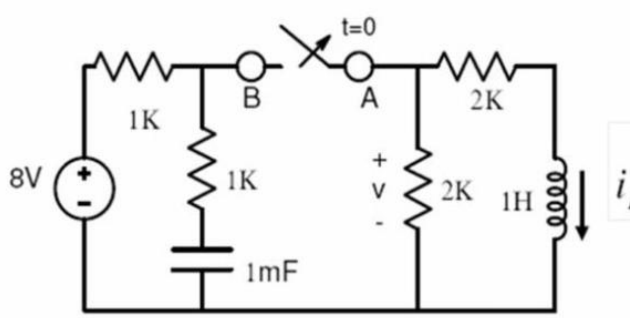
Transient analysis, Steady state, Sinusoidal sources, Phasors

Questions

1. For the circuit shown below, determine the voltage across the 2K resistor (vertical) as a function of time after the switch is opened at $t = 0$.



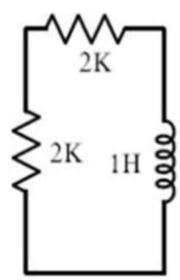
Solution:



First find the inductor current

$$i_L(t) = i_L(\infty) + \{i_L(0^+) - i_L(\infty)\} \times e^{-\frac{t}{\tau}}$$

Circuit after opening the switch (t > 0)



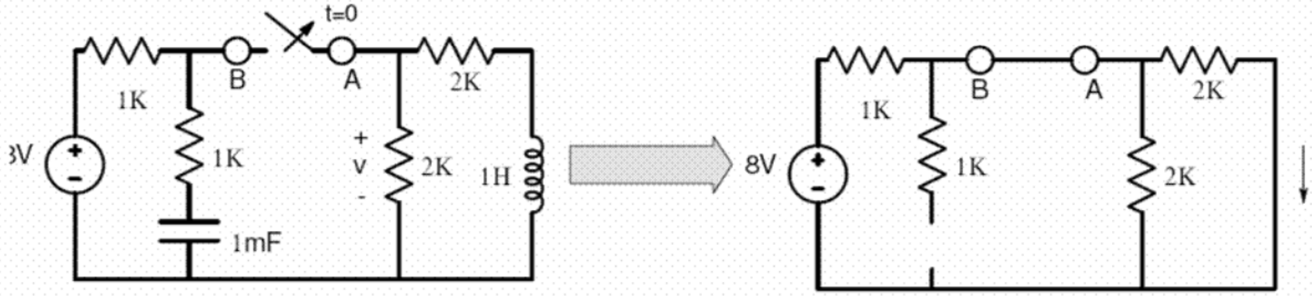
$$R_{eq} = 2K + 2K = 4K$$

$$\tau = \frac{L}{R_{eq}} = 0.25ms$$

One can also see that : $i_L(\infty) = 0$

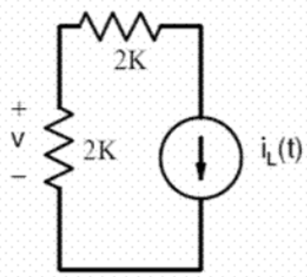
$i_L(0^+) = i_L(0^-)$

Circuit before opening the switch (t < 0) and assuming steady state condition:



$$i_L(0^+) = i_L(0^-) = \frac{8}{(2K \| 2K) + 1K} \times 0.5 = 2mA \quad \Rightarrow i_L(t) = 2 \times e^{-4000t} \, mA$$

Voltage across the 2K resistor:



$$v(t) = -2 \times 10^3 \times i_L(t) = -4 \times e^{-4000t} V$$

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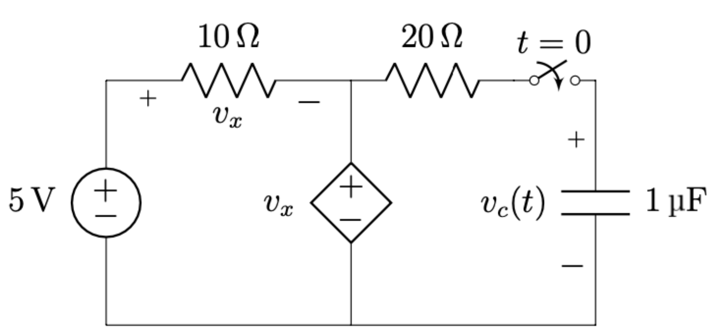
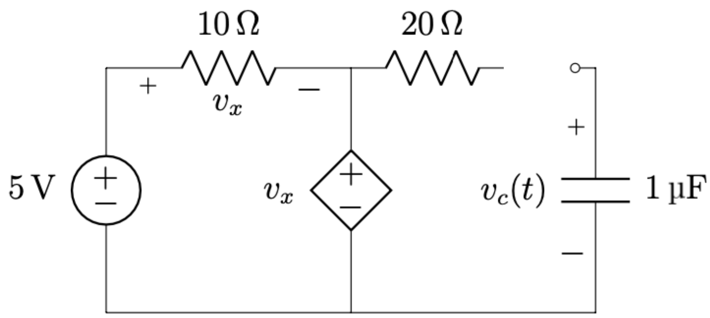
2. Find $v_c(t)$ for $t > 0$ in the following circuit if the capacitor voltage is zero for $t < 0$.



Solution:

Before the circuit is closed:



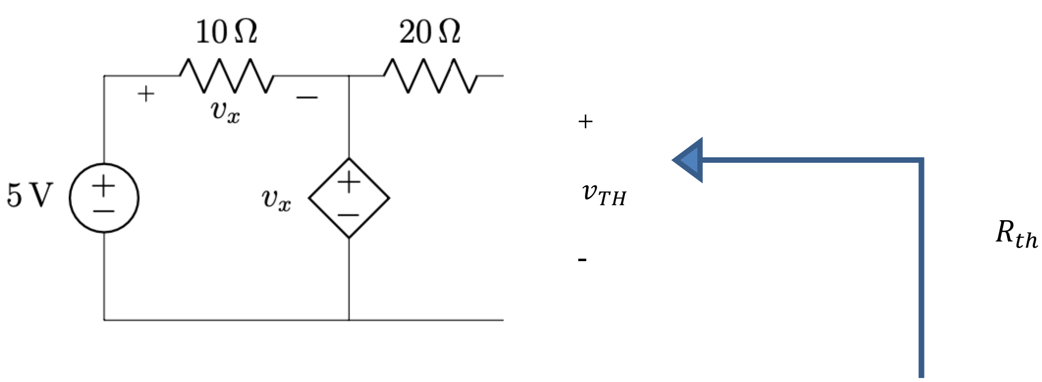
Applying KVL in the first loop

$$5 - V_x - V_x = 0$$

or

$$V_x = 2.5V$$

After the switch is closed



Thevenin voltage
Vth =2.5V

Short circuit current=
$Isc \ = \frac{2.5A}{20} = 125\ mA$

$R_{th} = \dfrac{2.5v}{125mA} = 20\Omega$

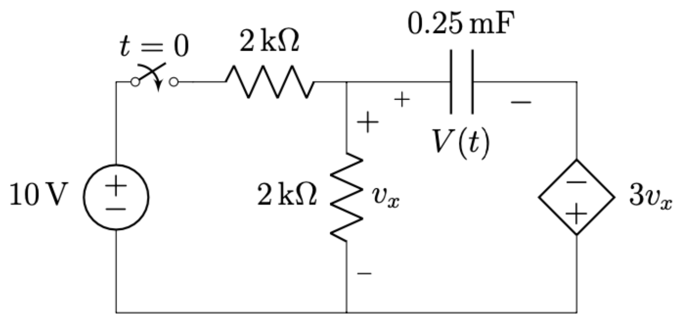
Hence, the time constant is

$$\tau = 20 \times 10^{-6} = 20\mu s$$

The voltage is given as

$$V_c(t) = V_c(\infty) + [V_c(0) - V_c(\infty)]e^{-\frac{t}{\tau}}$$
$$= 2.5 + (0 - 2.5)e^{-\frac{t}{2\times10^{-5}}}\ V$$
$$= 2.5\left(1 - e^{-\frac{t}{2\times10^{-5}}}\right)\ V$$

3. Assuming that the capacitor does not have any initial charge, determine the voltage across the capacitor $V(t)$ as a function of time after the switch is closed at $t = 0$.



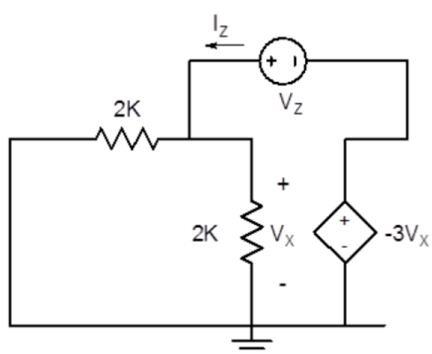
Solution:

$$v(t) = v(\infty) + \{v(0^+) - v(\infty)\}e^{-t/\tau}$$
$$v(0^+) = 0 \ [\mathbf{1}$$

At $t \to \infty$, the capacitor is open circuit. Therefore,

$$v_X = \frac{2K}{2K + 2K} * 10 = 5V$$
$$v(\infty) = V_X - (-3V_X) = 4V_X = 20V \ [\mathbf{1\ m}$$
$$\tau = CR_{eq}$$

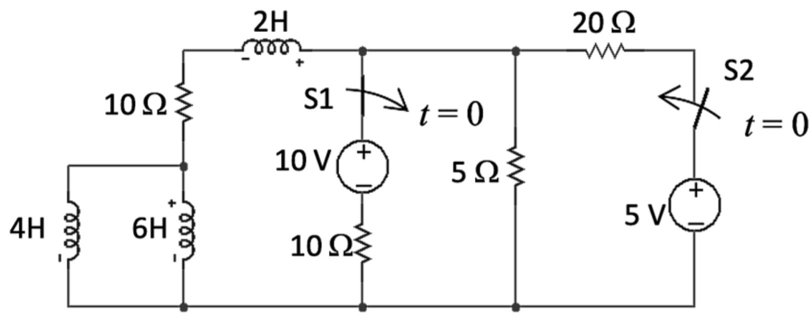
R_{eq} can be found from the circuit:



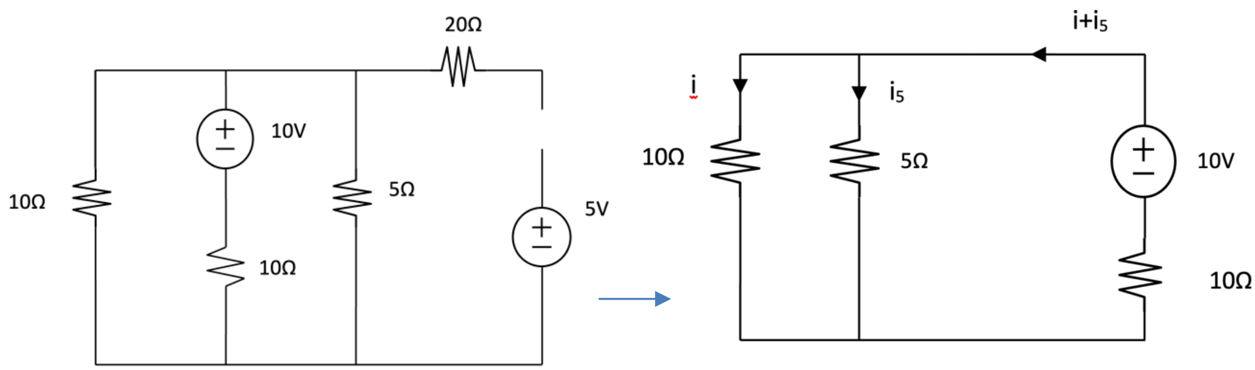
$$R_{eq} = \frac{v_Z}{i_Z}$$
$$v_Z = v_X - -3v_X = 4v_X$$
$$i_Z = \frac{v_X}{1K}$$
$$R_{eq} = \frac{v_Z}{i_Z} = 4K \ [\mathbf{1}$$

$$\tau = CR_{eq} = 1s \ [\mathbf{1}$$
$$v(t) = 20\{1 - e^{-t}\} \ [\mathbf{1}$$

4. In the following circuit the switch S1 is closed and S2 is left open for a long time. At t=0, S1 is opened and S2 is closed. Determine the current, i_5, through the 5Ω resistor for all time



Solution:
For t = 0^-



$$R_{eq} = 10 + \left(10\, x \frac{5}{15}\right) = \left(10 + \frac{50}{15}\right)\Omega = \left(10 + \frac{10}{3}\right)\Omega$$

$$i_{5\Omega} = \left(\frac{10}{10+\frac{10}{3}}\right) \times \frac{10}{15} A = 0.5\, A$$

$$10 - 10i - 10i - 10i_5 = 0$$

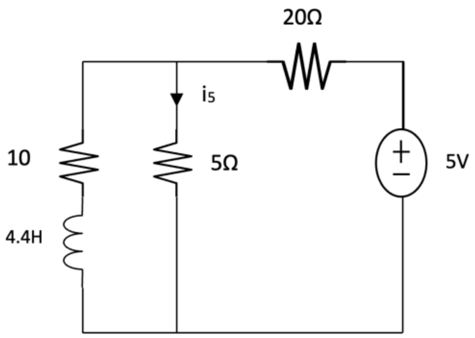
$$1 - i_5 = 2i$$

$$10 - 5i_5 - 10i - 10i_5 = 0$$

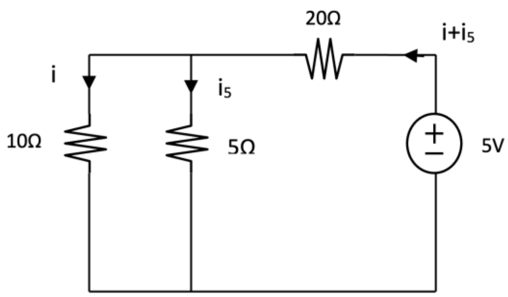
$$i = \left(\frac{1}{4}\right) A$$

$$i(0^-) = (1/4)A = i(0^+)$$

At t > 0



$t \rightarrow \infty$



$5 - 20i - 20v_5 - 5i_5 = 0$

$5 - 20i - 20i_5 - 10i = 0$

$i = 5/70$ A , $i_5 = 10/70$ A

$R_{th} = (20||5) + 10 = 14\Omega$

Therefore, τ= 4.4/14 = 0.314 s

t>0

$i(t)=i(\infty)+[i(0^+)-i(\infty)]e^{-t/\tau}$

therefore, $i(t)=(1/14)+((1/4)-(1/14))e^{-3.2t}$ A

$$v_{5\Omega} = 10i + L\left(\frac{di}{dt}\right) = \frac{5}{7} + \left(\frac{5}{2} - \frac{5}{7}\right)e^{-3.2t} + (4.4)\left(\frac{1}{4} - \frac{1}{14}\right)e^{-3.2t}(-8.2)V$$

therefore, $V_{5\Omega} = \left(\frac{10}{14}\right)(1 - e^{-3.2t})\ V$

therefore,

$i_{5\Omega} = \left(\frac{1}{7}\right)(1 - e^{-3.2t})A$

5. Express the following sinusoidal signals in the canonical form $V_m\cos(\omega t + \theta)$:

(i) $v(t) = -110cos(\omega t + 30^o)\, V$

(ii) $v(t) = 220sin(\omega t + 220^o)\, V$

(iii) $v(t) = 10sin(\omega t + 110^o) + 4cos(\omega t + 110^o)\, V$

(iv) $v(t) = 10cos(\omega t + 370^o) * 4sin(\omega t + 10^o)\, V$

Wherever needed, you may use phasors to make your task easier.

Solution:

(a) $v(t) = -110\cdot\cos(\omega t + 30°)\text{ V} = 110\cdot\cos(\omega t + 30°-180°)\text{ V} = 110\cdot\cos(\omega t - 150°)\text{ V}$

(b) $v(t) = 220\cdot\sin(\omega t + 220°)\text{ V} = 220\cdot\cos(\omega t + 220° - 90°)\text{ V} = 220\cdot\cos(\omega t + 130°)\text{ V}$

(c) $v(\text{t}) = 10\cdot\sin(\omega t + 110°) + 4\cdot\cos(\omega t + 110°)\text{ V}$

$= 10\cdot\cos(\omega t + 110° - 90°) + 4\cdot\cos(\omega t + 110°)\text{ V} = 10\cdot\cos(\omega t + 20°) + 4\cdot\cos(\omega t + 110°)\text{ V}$

$= 10\angle 20 + 4\angle 110 = 10\cdot\cos(20°) + j\,10\cdot\sin(20°) + 4\cdot\cos(110°) + j\,4\cdot\sin(110°)$

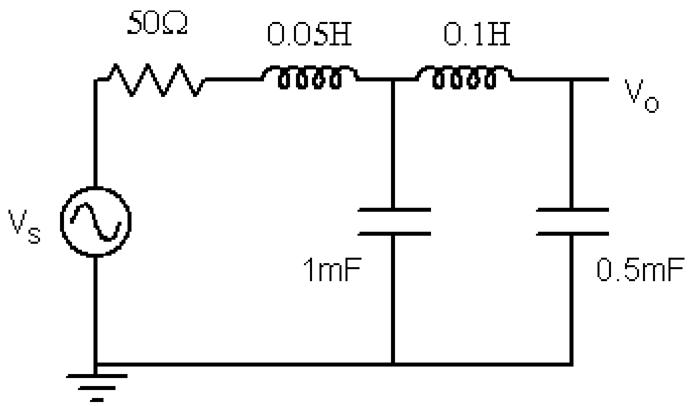
$= 8.029 + j\,7.179 = 10.77\angle 41.8° = 10.77\cdot\cos(\omega t + 41.8°)\text{ V}$

(d) $v(t) = 10.\cos(\omega t + 370^0).4.\sin(\omega t + 10^0)\text{V}$

$= 10.\cos(\omega t + 10^0).4.\cos(\omega t - 80^0)\text{V} = 20.(\cos(2\omega t - 70^0) + \cos(90^0))\text{V}$

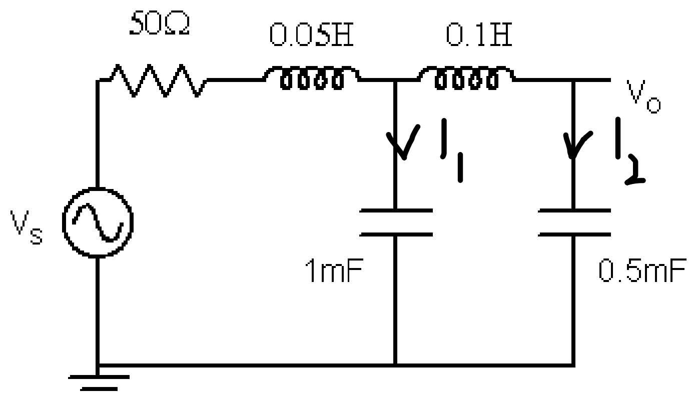
$= 20.\cos(2\omega t - 70^0)\text{V}$. [You can't solve this by converting into phasors, because it's multiplying two voltages]

6. Determine the output voltage as a function of time using the method of phasors for an input voltage of $V_s = 5cos(100t)\ V$.



Solution:

Zeq = 50 + j5 + [-10j || (j10 – j20)]
= 50 + j5 + (-j10 || -J10)
= 50 + j5 – j5
= 50 ohm



$$I = \left(\frac{5}{50}\right) cos\ (100t)\ A = 0.1 cos(100t)\ A$$

$$I_2 = \left(\frac{-j10}{-j10 + j10 - j20}\right) I = \left(-\frac{10}{-20}\right) \times 0.1\ cos(100t)$$

$$= 0.05\ cos\ (100t)\ A$$

$V_o = I_2\ (-j20) = cos(100t - 90^o)\ V$
$= sin\ (100t)\ V$