ESc201 : Introduction to Electronics

Number System and Logic Gates

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Binary Addition (recap)

```
  0
  0
  0
```

```
  1     0
  0     1
  1     1
```

```
   1
   1
 1 0
```

```
   1
   1
   1
 1 1
```

```
   1 0 1
   1 1 0
 1 0 1 1
```

```
     1 1 0 1
  +  1 1 1 0
   1 1 0 1 1
```

Complement of a binary number (recap)

Binary system: 1's complement / 2's complement

1's complement of n-bit number x is $2^n - 1 - x$

2's complement of n-bit number x is $2^n - x$

1's complement of 1011 ? $2^4 - 1 - 1011$ $1111 - 1011 = 0100$

1's complement is simply obtained by flipping a bit (changing 1 to 0 and 0 to 1)

$$\text{1's complement of } 1001101 = ?$$

$$0110010$$

$$2\text{'s complement of } 1010 = 1\text{'s complement of } 1010+1$$

$$= 0101 + 1 = 0110$$

$$2\text{'s complement of } 110010 =$$

Leave all least significant 0’s as they are, leave first 1 unchanged and then flip all subsequent bits

$$001110$$

$$1011 \rightarrow 0101$$

$$101101100 \rightarrow 010010100$$

Arithmetic Including Negative Numbers

- A digital system has finite number of bits
- For n bits available, 2^n unique numbers can be represented

- There is need to be able to represent negative number
- We would like a link between negative and positive number
  - Likely to make the math easy
- We would like to have a unique representation of zero

- We would like to do arithmetic (addition and subtraction)
- Positive and negative numbers are generated during arithmetic operations

- Finite size available to represent numbers will bring in constraints
- But we want to optimise as much as possible within the constraints

Representing Positive and Negative Numbers

Extra bit needed to carry sign information
"MSB" is often the sign bit

One option
Sign bit = 0 represents non-negative nos.
Sign bit = 1 represents negative numbers

decimal	Signed Magnitude
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
-0	1000
-1	1001
-2	1010
-3	1011
-4	1100
-5	1101
-6	1110
-7	1111

decimal	Signed 1's complement
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
-7	1000
-6	1001
-5	1010
-4	1011
-3	1100
-2	1101
-1	1110
0	1111

decimal	Signed 2's complement
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
-8	1000
-7	1001
-6	1010
-5	1011
-4	1100
-3	1101
-2	1110
-1	1111

magnitude

Unique zero representation

$2^{n-1} - 1$ positive nos.

2's comp. of 0 and -8 are themselves

2^{n-1} negative nos.

Arithmetic with 2's Complement

2's complement representation of numbers with *n* bits: $b_{n-1}\ b_{n-2}\ b_{n-3}\ \ldots\ b_2\ b_1\ b_0$

- There are *n* bits; 2^n unique numbers can be represented
  - Zero, $2^{n-1}-1$ positive and 2^{n-1} negative numbers are represented
- Place value based binary representation

  for LSB for MSB
  - Weights for bits $(b_0, b_1, b_2, \ldots, b_{n-2}, b_{n-1})$: $(+2^0, +2^1, +2^2 \ldots +2^{n-2}, -2^{n-1})$
  - All positions have positive weights, except MSB (bn-1) which is negative
- The negative of a number A is represented by its 2's complement
  - Negative of the negative of the number is the number itself
- To evaluate **A – B**, one can following the following algorithm
  - Find **–B** by taking 2' complement of **B**
  - Then **A – B = A + (-B) = A +** (2's complement of **B**)

Example

Adding or subtracting numbers with addition operation alone

To get a negative number, 2’s complement of positive number is taken

$$\begin{array}{r} +5 \\ \underline{+2} \\ +7 \end{array} \qquad \begin{array}{r} 0\,1\,0\,1 \\ \underline{+\,0\,0\,1\,0} \\ \underline{0\,1\,1\,1} \end{array} \qquad \begin{array}{r} +5 \\ \underline{-2} \\ \underline{+3} \end{array} \qquad \begin{array}{r} 0\,1\,0\,1 \\ \underline{+\,1\,1\,1\,0} \\ \underline{0\,0\,1\,1} \end{array}$$

2’s comp. of +2

2’s comp. of +5

$$\begin{array}{r} -5 \\ \underline{+2} \\ \underline{-3} \end{array} \qquad \begin{array}{r} 1\,0\,1\,1 \\ \underline{+\,0\,0\,1\,0} \\ \underline{1\,1\,0\,1} \end{array} \qquad \begin{array}{r} -5 \\ \underline{-2} \\ \underline{-7} \end{array} \qquad \begin{array}{r} 1\,0\,1\,1 \\ \underline{+\,1\,1\,1\,0} \\ \underline{1\,0\,0\,1} \end{array}$$

2’s comp. of +5

2’s comp. of +2

2’s complement is 0011 = 3

2’s complement is 0111 = 7

Boolean Algebra

Algebra on Binary numbers

A variable x can take two values {0,1}

0 → False, No, Low voltage

1 → True, Yes, High voltage

Basic operations:

AND: $y = x_1 . x_2$

y is 1 if and only if both x_1 and x_2 are 1, otherwise zero

Truth Table

x_1	x_2	y
0	0	0
0	1	0
1	0	0
1	1	1

Basic operations:

$$\text{OR:} \quad y = x_1 + x_2$$

y is 1 if either x_1 or x_2 is 1. y= 0 if and only if both variables are zero

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	1

$$\text{NOT:} \quad y = \bar{x}$$

x	y
0	1
1	0

Boolean Algebra

Basic Postulates

P1.a: $x + 0 = x$	P1.b: $x \cdot 1 = x$	Identity element
P2.a: $x + y = y + x$	P2.b: $x \cdot y = y \cdot x$	Commutative
P3.a: $x.(y+z) = x.y+x.z$	P3.b: $x+y.z = (x+y).(x+z)$	Distributive
P4.a: $x + \overline{x} = 1$	P4.b: $x \cdot \overline{x} = 0$	Complement

Basic Theorems

T1.a: $x + x = x$	T1.b: $x \cdot x = x$
T2.a: $x + 1 = 1$	T2.b: $x \cdot 0 = 0$
T3.a: $\overline{(\overline{x})} = x$	T4.b: $x \cdot (y.z) = (x.y).z$
T4.a: $x + (y+z) = (x+y)+z$	T5.b: $\overline{(x.y)} = \overline{x} + \overline{y}$ (DeMorgan
T5.a: $\overline{(x+y)} = \overline{x} \cdot \overline{y}$ (DeMo	T6.b: $x.(x+y) = x$
T6.a: $x+ x.y = x$	

Proving Theorems

P1.a: $x + 0 = x$
P2.a: $x + y = y + x$
P3.a: $x.(y+z) = x.y+x.z$
P4.a: $x + \overline{x} = 1$

P1.b: $x . 1 = x$
P2.b: $x . y = y . x$
P3.b: $x+y.z = (x+y).(x+z)$
P4.b: $x . \overline{x} = 0$

Prove T1.a: $x + x = x$

$x + x = (x+x). 1$ (P1.b)

$= (x+x). (x+\overline{x})$ (P4.a)

$= x + x.\overline{x}$ (P3.b)

$= x + 0$ (P4.b)

$= x$ (P1.a)

Prove T1.b: $x . x = x$

$x . x = x.x+ 0$ (P1.a)

$= x.x + x.\overline{x}$ (P4.b)

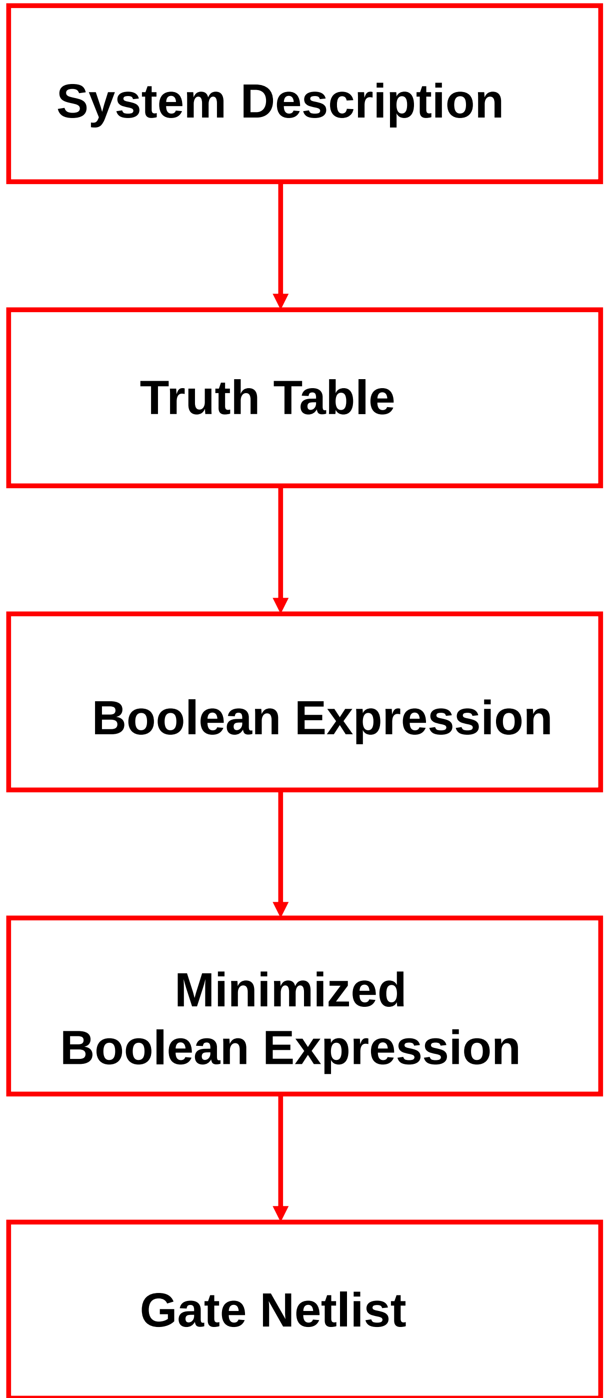
$= x . (x+\overline{x})$ (P3.a)

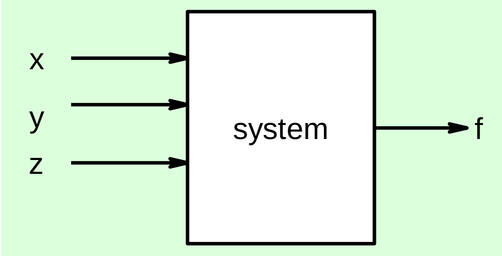
$= x . 1$ (P4.a)

$= x$ (P1.b)

How do we get the chocolate?

Design Flow





x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

$$\mathrm{f} = \overline{x}.\overline{y}.z + \overline{x}.y.z + x.\overline{y}.z + x.y.z$$

$$\Rightarrow \mathrm{f} = \overline{x}\,.\overline{z} + x.z$$