ESc201 : Introduction to Electronics

Logic Gates and Minimization

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How do we get the chocolate?

Design Flow

System Description

↓

Truth Table

↓

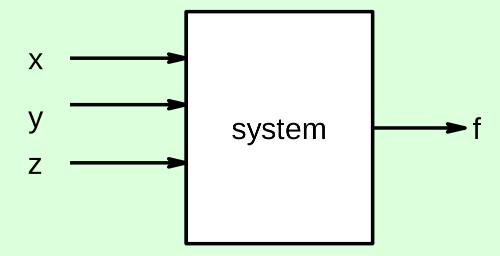
Boolean Expression

↓

Minimized Boolean Expression

↓

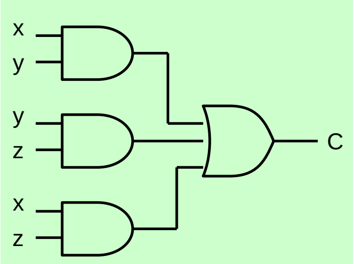
Gate Netlist



x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

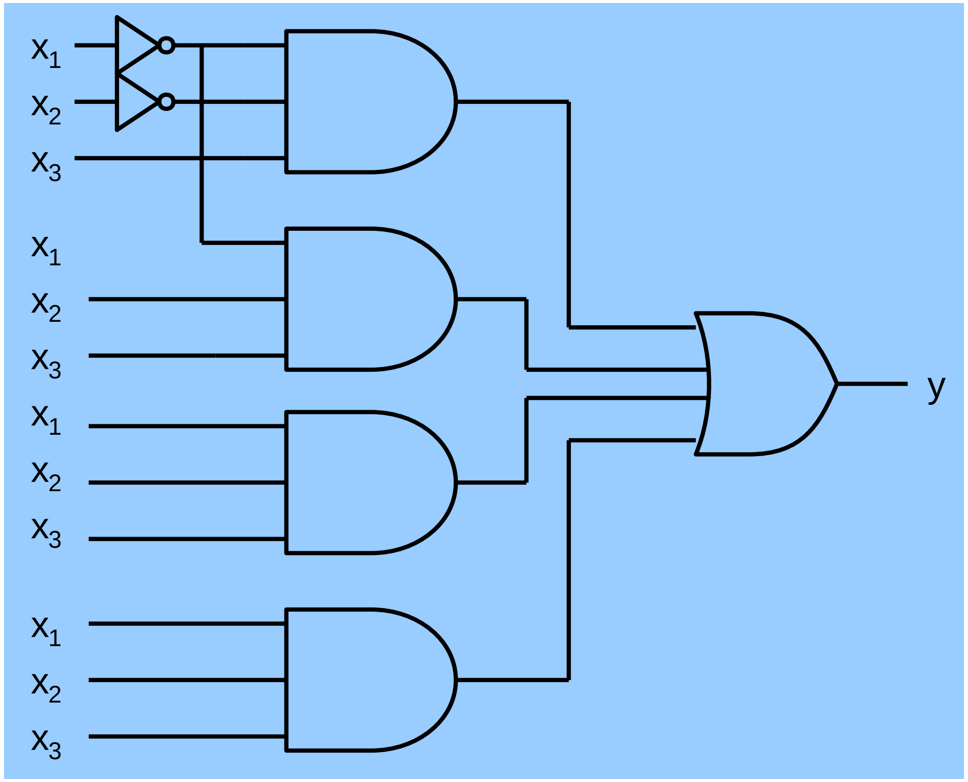
$$\mathrm{f} = \bar{x}.\bar{y}.z + \bar{x}.y.z + x.\bar{y}.z + x.y.z$$

$$\Rightarrow \mathrm{f} = \bar{x}\,.\bar{z} + x.z$$



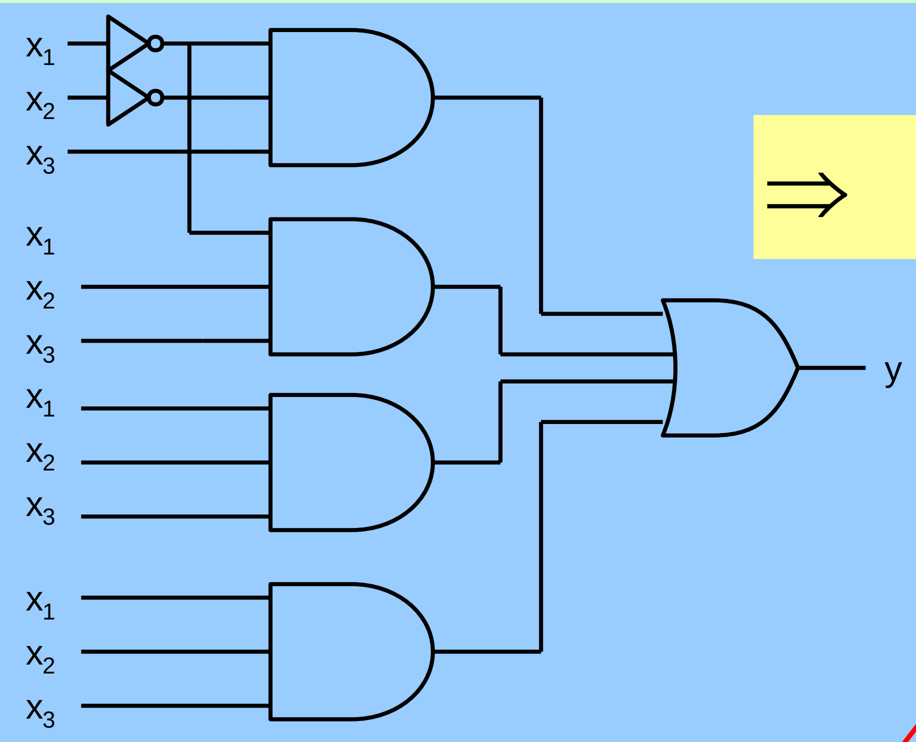
Goal of Simplification

$$y = \overline{x_1} . \overline{x_2} . x_3 + \overline{x_1} . x_2 . x_3 + x_1 . \overline{x_2} . x_3 + x_1 . x_2 . x_3$$

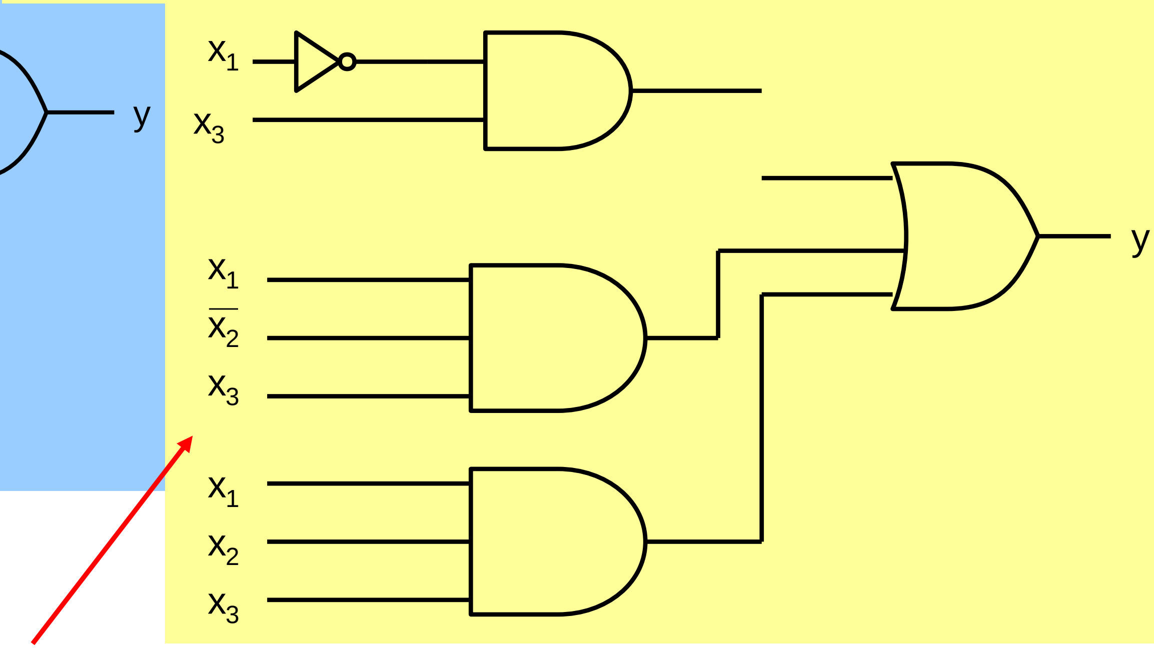


Goal of simplification is to reduce the complexity of gate circuit. This requires that we minimize the number of gates.

$$y = \overline{x_1} . \overline{x_2} . x_3 + \overline{x_1} . x_2 . x_3 + x_1 . \overline{x_2} . x_3 + x_1 . x_2 . x_3$$



$$\Rightarrow \quad y = \overline{x_1} . x_3 + x_1 . \overline{x_2} . x_3 + x_1 . x_2 . x_3$$



This circuit is simpler not just because it uses **4 gates instead of 5** but also because circuit-2 uses **one 2-input and three 3-input gates** as compared **to five 3-input gates used in circuit-1**

Minimization

$$y = \overline{x_1}.\overline{x_2}.x_3 + \overline{x_1}.x_2.x_3 + x_1.\overline{x_2}.x_3 + x_1.x_2.x_3$$

$$y = \overline{x_1}.x_3.(\overline{x_2} + x_2) + x_1.x_3.(\overline{x_2} + x_2)$$

$$y = \overline{x_1}.x_3 + x_1.x_3$$

$$y = (\overline{x_1} + x_1).x_3$$

$$y = x_3$$

Principle used: $x + \overline{x} = 1$

$$f = \bar{x}.\bar{y} + \bar{x}.y + x.\bar{y}$$

Apply the Principle: $x + \bar{x} = 1$ to simplify

$$f = \bar{x}.(\bar{y} + y) + x.\bar{y}$$

$$f = \bar{x} + x.\bar{y}$$

$$f = (\bar{x} + x)\cdot(\bar{x} + \bar{y})$$

$$f = (\bar{x}.\bar{x} + x\bar{x} + \bar{x}.\bar{y} + x.\bar{y})$$

$$f = (\bar{x} + \bar{x}.\bar{y} + x.\bar{y})$$

$$f = (\bar{x} + \bar{y}.(\bar{x} + x))$$

$$f = (\bar{x} + \bar{y})$$

Principle: $x + \bar{x} = 1$ and $x + x = x$

Need a systematic and simpler method

Karnaugh Map (K map) is a popular technique for carrying out simplification

It represents the information in problem in such a way that the two principles become easy to apply

Principle: $x + \bar{x} = 1$ and $x + x = x$

Representation of Boolean Expressions

x	y	f_1
0	0	0
0	1	1
1	0	1
1	1	0

x	y	min term
0	0	$\bar{x}\,.\,\bar{y}$ m0
0	1	$\bar{x}\,.\,y$ m1
1	0	$x\,.\,y$ m2
1	1	$x\,.\,y$ m3

$$f_1 = \bar{x}.\,y + x.\bar{y}$$

$$f_1 = m_1 + m_2$$

$$f_1 = \sum(1,2)$$

$$f_2 = \sum(0,2,3) = ?$$

$$f_2 = \bar{x}.\,\bar{y} + x.\bar{y} + x\,.\,y$$

A minterm is a product that contains all the variables used in a function

Three variable functions

x	y	z	min terms	
0	0	0	$\bar{x}.\bar{y}.\bar{z}$	m0
0	0	1	$\bar{x}.\bar{y}.z$	m1
0	1	0	$\bar{x}.y.\bar{z}$	m2
0	1	1	$\bar{x}.y.z$	m3
1	0	0	$x.\bar{y}.\bar{z}$	m4
1	0	1	$x.\bar{y}.z$	m5
1	1	0	$x.y.\bar{z}$	m6
1	1	1	$x.y.z$	m7

$$f_2 = \sum(1,4,7) = ?$$

$$f_2 = \bar{x}.\bar{y}.z + x.\bar{y}.\bar{z} + x.y.z$$

Product of Sum Terms Representation

x	y	f_1
0	0	0
0	1	1
1	0	1
1	1	0

x	y	Max term
0	0	$x + y$ M0
0	1	$x + \overline{y}$ M1
1	0	$\overline{x} + y$ M2
1	1	$\overline{x} + \overline{y}$ M3

A maxterm is a sum that contains all the variables used in a function

$$f_1 = (x+y)\cdot(\overline{x}+\overline{y})$$

$$f_1 = M_0 \cdot M_3$$

$$f_1 = \prod (M_0, M_3)$$

x	y	z	Max. terms	
0	0	0	$x + y + z$	M0
0	0	1	$x + y + \bar{z}$	M1
0	1	0	$x + y + z$	M2
0	1	1	$x + \bar{y} + \bar{z}$	M3
1	0	0	$\bar{x} + y + z$	M4
1	0	1	$\bar{x} + y + \bar{z}$	M5
1	1	0	$\bar{x} + \bar{y} + z$	M6
1	1	1	$\bar{x} + \bar{y} + \bar{z}$	M7

$$f_1 = \Pi(1,5,7) = ?$$

$$f_2 = (x + y + \bar{z}).(\bar{x} + y + \bar{z}).(\bar{x} + \bar{y} + \bar{z})$$

Recall K map

Karnaugh Map (K map) is a popular technique for carrying out simplification

It represents the information in problem in such a way that the two principles become easy to apply

Principle: $x + \bar{x} = 1$ and $x + x = x$

K-map representation of truth table

x	y	min term	
0	0	$\bar{x} . \bar{y}$	m0
0	1	$\bar{x} . y$	m1
1	0	x . y	m2
1	1	x . y	m3

x \ y	0	1
0	m_0	m_1
1	m_2	m_3

x	y	f_1
0	0	0
0	1	1
1	0	1
1	1	0

x \ y	0	1
0	0	1
1	1	0

$$f_2 = \sum(1,2,3)$$

x \ y	0	1
0	0	1
1	1	1

x \ y	0	1
0	1	0
1	0	1

$$f = \bar{x}.\bar{y} + x.y$$

3-variable K-map representation

x	y	z	min terms	
0	0	0	$\bar{x} . \bar{y} . \bar{z}$	m0
0	0	1	$\bar{x} . \bar{y} . z$	m1
0	1	0	$\bar{x} . y . \bar{z}$	m2
0	1	1	$\bar{x} . y . z$	m3
1	0	0	$x . \bar{y} . \bar{z}$	m4
1	0	1	$x . \bar{y} . z$	m5
1	1	0	$x . y . \bar{z}$	m6
1	1	1	$x . y . z$	m7

x \ yz	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

3-variable K-map representation

x	y	z	min terms	
0	0	0	$\bar{x} \cdot \bar{y} \cdot \bar{z}$	m0
0	0	1	$\bar{x} \cdot \bar{y} \cdot z$	m1
0	1	0	$\bar{x} \cdot y \cdot \bar{z}$	m2
0	1	1	$\bar{x} \cdot y \cdot z$	m3
1	0	0	$x \cdot \bar{y} \cdot \bar{z}$	m4
1	0	1	$x \cdot \bar{y} \cdot z$	m5
1	1	0	$x \cdot y \cdot \bar{z}$	m6
1	1	1	$x \cdot y \cdot z$	m7

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

x \ yz	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

x \ yz	00	01	11	10
0	0	1	1	0
1	0	1	1	0

What is the function represented by this K map?

x \ yz	00	01	11	10
0	1	0	1	0
1	0	1	1	0

$$\mathrm{f} = \bar{x}.\bar{y}.\bar{z} + \bar{x}.y.z + x.\bar{y}.z + x.y.z$$

4-variable K-map representation

w	x	y	z	min terms
0	0	0	0	m_0
0	0	0	1	m_1
0	0	1	0	m_2
0	0	1	1	m_3
⋮	⋮	⋮	⋮	⋮
1	1	1	0	m_{14}
1	1	1	1	m_{15}

wx \ yz	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

wx \ yz	00	01	11	10
00	1	0	1	0
01	0	1	1	0
11	1	0	0	1
10	1	0	0	0

$$f = \overline{w}.\overline{x}.\overline{y}.\overline{z} + \overline{w}.\overline{x}.y.z + \overline{w}.x.\overline{y}.z + \overline{w}.x.y.z + w.x.\overline{y}.\overline{z} + w.x.y.\overline{z} + w.\overline{x}.\overline{y}.\overline{z}$$

Minimization using Kmap

$$f_2 = \sum(2,3)$$

$$f = x.\overline{y} + x.y$$

$$f = x.(\overline{y} + y)$$

$$f = x$$

x \ y	0	1
0	0	0
1	1	1

Combine terms which differ in only one bit position. As a result, whatever is common remains.

x \ y	0	1
0	0	1
1	0	1

$$f = \bar{x}.y + x.y$$

$$f = (\bar{x} + x).y$$

$$\Rightarrow f = y$$

x \ y	0	1
0	1	0
1	1	0

$$\Rightarrow f = \bar{y}$$

x \ y	0	1
0	1	1
1	0	0

$$\Rightarrow f = \bar{x}$$

$$f_2 = \sum(1, 2, 3)$$

x \ y	0	1
0	0	1
1	1	1

$$f = x \cdot \bar{y} + x \cdot y + \bar{x} \cdot y$$

$$= x \cdot \bar{y} + x \cdot y + \bar{x} \cdot y + x \cdot y$$

$$= x \cdot (\bar{y} + y) + (\bar{x} + x) \cdot y$$

$$= x + y$$

The idea is to cover all the 1's with as few and as simple terms as possible

3-variable minimization

$$f = \overline{x}.\overline{y}.\overline{z} + \overline{x}.y.z + x.y.z + x.\overline{y}.z$$

x \ yz	00	01	11	10
0	1	0	1	0
1	0	1	1	0

$y.z$

$x.z$

$$f = \overline{x}.\overline{y}.\overline{z} + y.z + x.z$$

3-variable minimization

$$f = \bar{x}.\bar{y}.\bar{z} + \bar{x}.y.\bar{z} + x.y.z + x.\bar{y}.z$$

x \ yz	00	01	11	10
0	1	0	0	1
1	0	1	1	0

$$\bar{x}.\bar{z}$$

$$x.z$$

$$f = \bar{x}.\bar{z} + x.z$$

3-variable minimization

x \ yz	00	01	11	10
0	0	0	0	0
1	1	1	1	1

$$f = x.\bar{y}.\bar{z} + x.\bar{y}.z + x.y.z + x.y.\bar{z}$$

$$x.\bar{y}$$

$$x.y$$

$$f = x.\bar{y} + x.y$$

x \ yz	00	01	11	10
0	0	0	0	0
1	1	1	1	1

$$f = x.(\bar{y} + y) = x$$

$$x$$

x \ yz	00	01	11	10
0	1	1	1	1
1	0	0	0	0

$\overline{x}$

x \ yz	00	01	11	10
0	1	0	0	1
1	1	0	0	1

$\overline{z}$

x \ yz	00	01	11	10
0	0	1	1	0
1	0	1	1	0

z

x \ yz	00	01	11	10
0	1	0	0	1
1	1	1	1	1

$\overline{z}$

x

$$\mathrm{f} = x + \overline{z}$$

Can we do this ?

x \ yz	00	01	11	10
0	0	0	0	0
1	1	1	1	0

Note that each encirclement should represent a single product term. In this case it does not.

$$
\begin{aligned}
f &= x.\bar{y}.\bar{z} + x.\bar{y}.z + x.y.z \\
&= x.\bar{y} + x.z
\end{aligned}
$$

We do not get a single product term. In general we cannot make groups of 3 terms.

Can we use kmap with the following ordering of variables?

x \ yz	00	01	10	11
0	0	0	0	0
1	0	1	1	0

Can we combine these two terms into a single term ?

$$
\begin{aligned}
\mathrm{f} &= x.\bar{y}.z + x.y.\bar{z} \\
&= x.(\bar{y}.z + y.\bar{z})
\end{aligned}
$$

Note that no simplification is possible.

x \ yz	00	01	10	11
0	0	1	0	1
1	0	0	0	0

These two terms can be combined into a single term but it is not easy to show that on the diagram.

$$
\begin{aligned}
f &= \bar{x}.\bar{y}.z + \bar{x}.y.z \\
&= \bar{x}.(\bar{y} + y).z = \bar{x}.z
\end{aligned}
$$

Kmap requires information to be represented in such a way that it is easy to apply the principle $x + \bar{x} = 1$