ESC201A Assignment 8

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Topics

Binary numbers

Questions

1. Convert the following numbers into the number system indicated
   (a) $(1010.011)_2$ to decimal
   (b) $(FA)_{16}$ to decimal
   (c) $(101110101101)_2$ into hexadecimal
   (d) $(FA)_{16}$ to binary

Ans 1. (a) $(1010.011)_2$

$= (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (0 \times 2^0) + (0 \times 2^{-1}) + (1 \times 2^{-2}) + (1 \times 2^{-3})$

$= 8 + 0 + 2 + 0 + 0.5 + 0.25 + 0.125$

$= 10.375$

(b) $(FA)_{16}$

$= (15 \times 16^1) + (10 \times 16^0)$

$= 240 + 10$

$= 250$

(c) $(101110101101)_2$

$= (1011)(1010)(1101)$

$= (BAD)_{Hex}$

(d) $(FA)_{16}$

$= (1111)(1010)$

$= 11111010$

2. Convert the decimal number 27.25 into a binary number.

Ans 2.

	27	remainder
2	13	1
2	6	1
2	3	0
2	1	1
2	0	1

$\therefore 27 = 11011$

0.25		
0.	5	x2
1.	0	x2

$\therefore 0.25 = (.01)_2$

$\therefore (27.25)_{10} = (11011.01)_2$

3. What is the largest decimal number that you can represent using 8bits? How many bits are required to represent decimal numbers less than or equal to 10^6?

Ans 3. Largest binary number that can be represented using 8 bits is $(11111111)_2 = 255$

Let the number of bits be n. The largest binary number using these n bits is

$$2^{n-1} + 2^{n-2} + 2^{n-3} + \ldots + 2^1 + 1$$

$$= 2^n - 1 \quad \text{(Sum of a GP)}$$

From the given condition,

$$2^n - 1 \geq 10^6$$

$$\therefore n \geq \log_2(10^6 + 1)$$

$$\therefore n = 20.$$

4. Determine the number system in which the following arithmetic operations have been carried out. Give justifications for your answer.
   (a) 24+17=40

∴ (a) $24 + 17 = 40$

Let the number system be x.

$\therefore 2x^1 + 4x^0 + 1x^1 + 7x^0 = 4x^1 + 0$

$\Rightarrow 2x + 4 + x + 7 = 4x$

$\Rightarrow 11 = 4x - 3x$

or $x = 11$

(b) 22 x 5 = 132

(b) $22 \times 5 = 132$

Let the number system be x.

$\therefore (2x^1 + 2x^0)(5x^0) = (1x^2 + 3x^1 + 2x^0)$

$\Rightarrow (2x + 2)(5) = (x^2 + 3x + 2)$

$\Rightarrow 10x + 10 = x^2 + 3x + 2$

$\Rightarrow x^2 - 7x - 8 = 0$

$\Rightarrow x^2 - 8x + x - 8 = 0$

$\Rightarrow x(x - 8) + 1(x - 8) = 0$

$\Rightarrow (x + 1)(x - 8) = 0$

$x = -1$ is inadmissible.

$\therefore x = 8$

5. Obtain 1's and 2's complement of the following binary numbers:
   (a) 10000000
   (b) 10101010
   (c) 01110101
   (d) 10011100

(a) 10000000

1's complement : 01111111

2's complement : 10000000

(b) 10101010

1's complement : 01010101

2's complement : 01010110

(c) 01110101

1's complement : 10001010

2's complement : 10001011

(d) 10011100

1's complement : 01100011

2's complement : 01100100

6. (a) What is the minimum number of bits required to represent -32 in 2's complement form?

Ans6. (a) $(32)_{10} = (100000)_2 = (0100000)_2$ — ↳ for sign.

For -32, take 2's complement of +32

$(-32)_{10} = (1100000)_2$ — Six bits

(b) 11011111 is a number in 2's complement. Is it positive or negative? What is its magnitude?

(b) Since the sign bit is 1, the number is negative. 2's complement of 11011111 is 00100001

$(00100001)_2 = (33)_{10}$

7. Carry out the following four operations using 8bit 2's complement representation:

$\pm 24 \pm 32$

Verify that operations have been properly carried out.

Ans 7. $(+32)_{10} = (00100000)_2$

$(-32)_{10} = (11100000)_2$

$(+24)_{10} = (00011000)_2$

$(-24)_{10} = (1110\,1000)_2$

+32 +24:

$$\begin{array}{r} 00100000 \\ +\ 00011000 \\ \hline 00111000 \end{array}$$

= 56

-32+24

$$\begin{array}{r} 11100000 \\ +\ 00011000 \\ \hline 11111000 \end{array}$$

This is a negative number.
2's complement of 11111000
= 00001000 = 8

-32-24

$$\begin{array}{r} 11100000 \\ +\ 11101000 \\ \hline 111001000 \end{array}$$

This is a negative number.
2's complement of 111001000 = 00011000 = 56

+32-24

$$\begin{array}{r} 00100000 \\ +\ 11101000 \\ \hline 100001000 \end{array}$$

= 8