

# ESc201 : Introduction to Electronics

## Number System

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# Numbers

Every number system is associated with a base or radix

A positional notation is commonly used to express numbers

$$(a_5a_4a_3a_2a_1a_0)_r = a_5r^5 + a_4r^4 + a_3r^3 + a_2r^2 + a_1r^1 + a_0r^0$$

The **decimal system** has a **base of 10** and uses symbols (0,1,2,3,4,5,6,7,8,9) to represent numbers

$$(2009)_{10} = 2 \times 10^3 + 0 \times 10^2 + 0 \times 10^1 + 9 \times 10^0$$

$$(123.24)_{10} = 1 \times 10^2 + 2 \times 10^1 + 3 \times 10^0 + 2 \times 10^{-1} + 4 \times 10^{-2}$$

# Numbers

An **octal number system** has a **base 8** and uses symbols (0,1,2,3,4,5,6,7)

$$(2007)_8 = 2 \times 8^3 + 0 \times 8^2 + 0 \times 8^1 + 7 \times 8^0$$

What decimal number does it represent?

$$(2007)_8 = 2 \times 512 + 0 \times 64 + 0 \times 8^1 + 7 \times 8^0 = 1033$$

$$(2007)_8 = (1033)_{10}$$

# Numbers

A hexadecimal system has a base of 16

$$(2BC9)_{16} = 2 \times 16^3 + B \times 16^2 + C \times 16^1 + 9 \times 16^0$$

How do we convert it into decimal number?

$$(2BC9)_{16} = 2 \times 4096 + 11 \times 256 + 12 \times 16^1 + 9 \times 16^0 = 11209$$

$$(2BC9)_{16} = (11209)_{10}$$

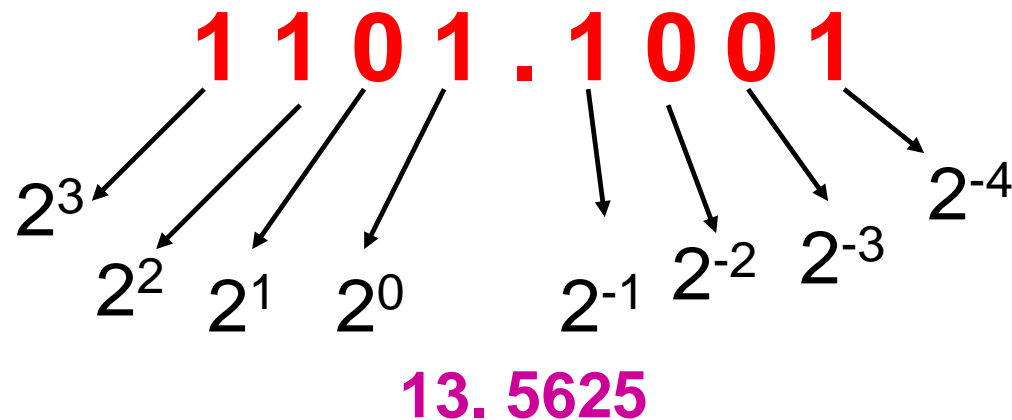
Number	Symbol
0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	A
11	B
12	C
13	D
14	E
15	F

A Binary system has a base 2 and uses only two symbols 0, 1 to represent all the numbers

$$(1101)_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$

Which decimal number does this correspond to ?

$$(1101)_2 = 1 \times 8 + 1 \times 4 + 0 \times 2^1 + 1 \times 2^0 = 13$$



$2^0$	1
$2^1$	2
$2^2$	4
$2^3$	8
$2^4$	16
$2^5$	32
$2^6$	64
$2^7$	128
$2^8$	256
$2^9$	512
$2^{10}$	1024(K)
$2^{20}$	1048576(M)

$2^{-1}$	$2^{-2}$	$2^{-3}$	$2^{-4}$	$2^{-5}$	$2^{-6}$
0.5	0.25	0.125	0.0625	0.03125	0.015625

# Developing Fluency with Binary Numbers

$$1\ 1\ 0\ 0\ 1 = ? \quad 25$$

$$1100001 = ? \quad 64+32+1=97$$

$$0.101 = ? \quad 0.5+0.125=0.625$$

$$11.001 = ? \quad 3+0.125=3.125$$

# Converting decimal to binary number

Convert 45 to binary number

$$(45)_{10} = b_n b_{n-1} \dots b_0$$

$$45 = b_n 2^n + b_{n-1} 2^{n-1} \dots + b_1 2^1 + b_0$$

Divide both sides by 2

$$\frac{45}{2} = 22.5 = b_n 2^{n-1} + b_{n-1} 2^{n-2} \dots + b_1 2^0 + b_0 \times 0.5$$

$$22 + 0.5 = b_n 2^{n-1} + b_{n-1} 2^{n-2} \dots + b_1 2^0 + b_0 \times 0.5$$

$$\Rightarrow b_0 = 1$$

$$22 + 0.5 = b_n 2^{n-1} + b_{n-1} 2^{n-2} \dots + b_1 2^0 + b_0 \times 0.5 \Rightarrow b_0 = 1$$

$$22 = b_n 2^{n-1} + b_{n-1} 2^{n-2} \dots + b_2 2^1 + b_1 2^0$$

Divide both sides by 2

$$\frac{22}{2} = 11 = b_n 2^{n-2} + b_{n-1} 2^{n-3} \dots + b_2 2^0 + b_1 \times 0.5 \Rightarrow b_1 = 0$$

$$11 = b_n 2^{n-2} + b_{n-1} 2^{n-3} \dots + b_3 2^1 + b_2 2^0$$

$$5.5 = b_n 2^{n-3} + b_{n-1} 2^{n-4} \dots + b_3 2^0 + 0.5b_2 \Rightarrow b_2 = 1$$

$$5 = b_n 2^{n-3} + b_{n-1} 2^{n-4} \dots + b_4 2^1 + b_3 2^0$$



$$5 = b_n 2^{n-3} + b_{n-1} 2^{n-4} \dots + b_4 2^1 + b_3 2^0$$

$$2.5 = b_n 2^{n-4} + b_{n-1} 2^{n-5} \dots + b_4 2^0 + 0.5b_3 \Rightarrow b_3 = 1$$

$$2 = b_n 2^{n-4} + b_{n-1} 2^{n-5} \dots + b_5 2^1 + b_4 2^0$$

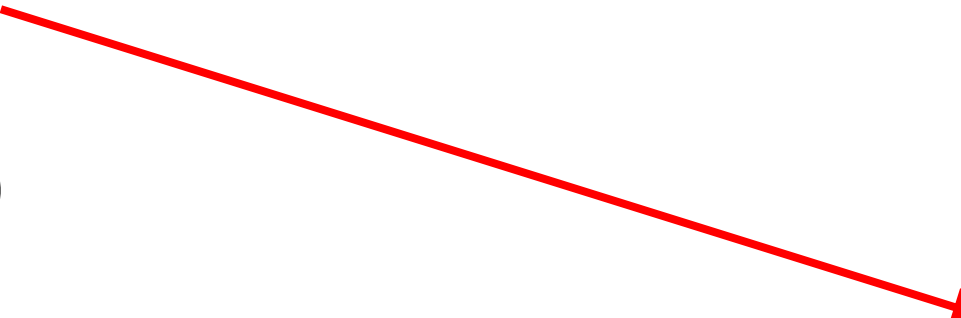
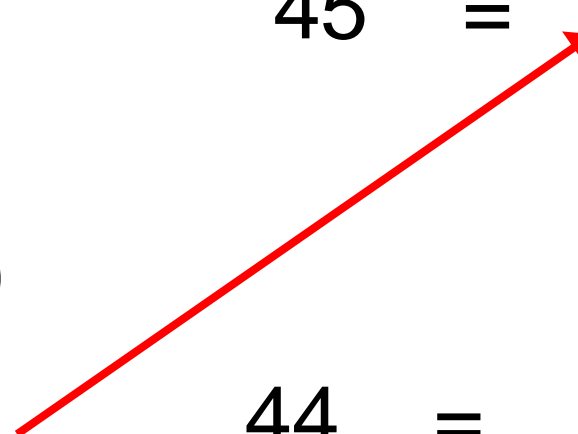
$$1 = b_n 2^{n-5} + b_{n-1} 2^{n-6} \dots + b_5 2^0 + 0.5b_4 \Rightarrow b_4 = 0$$

$$\Rightarrow b_5 = 1$$

$$(45)_{10} = b_5 b_4 b_3 b_2 b_1 b_0 = 101101$$

# Converting decimal to binary number

Method of successive division by 2

45	remainder	
22	1	
11	0	
5	1	
2	1	
1	0	
0	1	

45 = 1 0 1 1 0 1

44 = 1 0 1 1 0 0

Convert  $(153)_{10}$  to octal number system

$$(153)_{10} = (b_n b_{n-1} \dots b_0)_8$$

$$(153)_{10} = b_n 8^n + b_{n-1} 8^{n-1} \dots b_1 8^1 + b_0$$

Divide both sides by 8

$$\frac{153}{8} = 19.125 = b_n 8^{n-1} + b_{n-1} 8^{n-2} \dots b_1 8^0 + \frac{b_0}{8}$$

$$\Rightarrow \frac{b_0}{8} = 0.125$$

$$\Rightarrow b_0 = 1$$

153	remainder
19	1
2	3
0	2

$$153 = (231)_8$$

# Converting decimal to binary number

Convert  $(0.35)_{10}$  to binary number

$$(0.35)_{10} = 0.b_{-1}b_{-2}b_{-3}\dots\dots b_{-n}$$

$$0.35 = 0 + b_{-1}2^{-1} + b_{-2}2^{-2} + \dots\dots b_{-n}2^{-n}$$

How do we find the  $b_{-1}$   $b_{-2}$  ...coefficients?

Multiply both sides by 2

$$0.7 = b_{-1} + b_{-2}2^{-1} + \dots\dots b_{-n}2^{-n+1} \Rightarrow b_{-1} = 0$$

$$0.7 = b_{-2}2^{-1} + b_{-3}2^{-2} + \dots\dots b_{-n}2^{-n+1}$$

$$0.7 = b_{-2}2^{-1} + b_{-3}2^{-2} + \dots b_{-n}2^{-n+1}$$

Multiply both sides by 2

$$1.4 = b_{-2} + b_{-3}2^{-1} + \dots b_{-n}2^{-n+2} \Rightarrow b_{-2} = 1$$

Note that  $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots \leq 1$

$$0.4 = b_{-3}2^{-1} + b_{-4}2^{-2} \dots b_{-n}2^{-n+2}$$

$$0.8 = b_{-3} + b_{-4}2^{-1} \dots b_{-n}2^{-n+3} \Rightarrow b_{-3} = 0$$

# Converting decimal to binary number

$$0.125 = ?$$

	0 .	125	
			x2
	0 .	25	
			x2
	0 .	5	
			x2
0.125 = (.001) <sub>2</sub>	1 .	0	

$$0.8125 = ?$$

	0 .	8125	
			x2
	1 .	625	
			x2
	1 .	25	
			x2
	0 .	5	
			x2
0.8125 = (.1101) <sub>2</sub>	1 .	0	

# Binary numbers

Most significant bit or **MSB**

Least significant bit or **LSB**

1011000111

**B**inary digit = bit

This is a 10 bit number

N-bit binary number  
can represent numbers  
from 0 to  $2^N - 1$

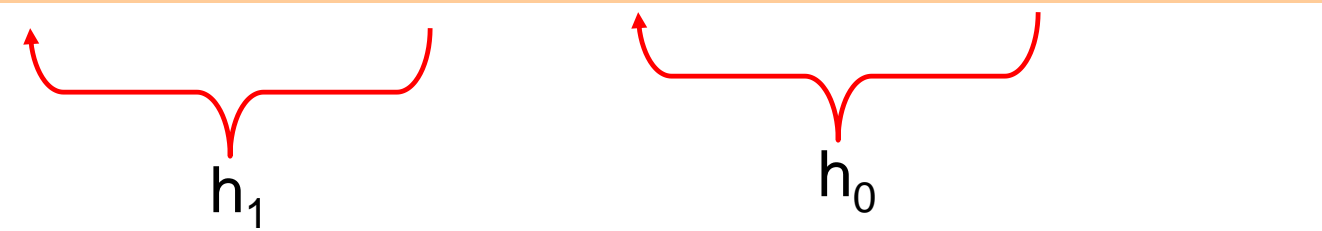
decimal	2bit	3bit	4bit	5bit
0	00	000	0000	00000
1	01	001	0001	00001
2	10	010	0010	00010
3	11	011	0011	00011
4		100	0100	00100
5		101	0101	00101
6		110	0110	00110
7		111	0111	00111
8			1000	01000
9			1001	01001
10			1010	01010
11			1011	01011
12			1100	01100
13			1101	01101
14			1110	01110
15			1111	01111

# Converting Binary to Hex and Hex to Binary

$$(b_7b_6b_5b_4b_3b_2b_1b_0)_b = (h_1, h_0)_{Hex}$$

$$b_72^7 + b_62^6 + b_52^5 + b_42^4 + b_32^3 + b_22^2b_12^1 + b_0 = h_116^1 + h_0$$

$$(b_72^3 + b_62^2 + b_52^1 + b_4)2^4 + (b_32^3 + b_22^2b_12^1 + b_0) = h_116^1 + h_0$$



$$(10110011)_b = (1011)(0011) = (B3)_{Hex}$$

$$(110011)_b = (11)(0011) = (33)_{Hex}$$

$$(EC)_{Hex} = (1110)(1100) = (11101100)_b$$

Number	Symbol
0(0000)	0
1(0001)	1
2(0010)	2
3(0011)	3
4(0100)	4
5(0101)	5
6(0110)	6
7(0111)	7
8(1000)	8
9(1001)	9
10(1010)	A
11(1011)	B
12(1100)	C
13(1101)	D
14(1110)	E
15(1111)	F



# Binary Addition/Subtraction

$$\begin{array}{r} 0 \\ \hline 0 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 1 \\ \hline 0 \\ \hline 1 \end{array} \quad \begin{array}{r} 0 \\ \hline 1 \\ \hline 1 \end{array}$$

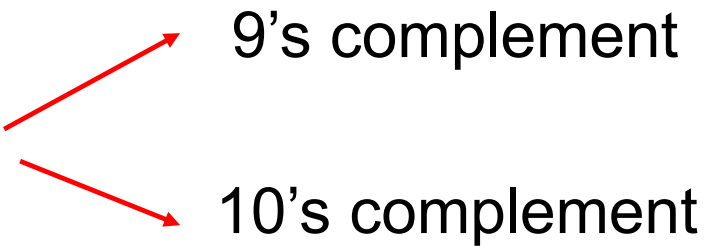
$$\begin{array}{r} 1 \\ \hline 1 \\ \hline 10 \end{array}$$

$$\begin{array}{r} 1 \\ 1 \\ \hline 1 \\ \hline 11 \end{array}$$

$$\begin{array}{r} 101 \\ 110 \\ \hline 1011 \end{array}$$

$$\begin{array}{r} 1101 \\ + 1110 \\ \hline 11011 \end{array}$$

# Complement of a number

Decimal system: 

9's complement of n-digit number x is  $10^n - 1 - x$

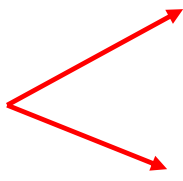
10's complement of n-digit number x is  $10^n - x$

9's complement of 85 ?  $10^2 - 1 - 85$   $99 - 85 = 14$

9's complement of 123 =  $999 - 123 = 876$

10's complement of 123 = 9's complement of 123 + 1 = 877

# Complement of a binary number

Binary system:  1's complement  
2's complement

1's complement of n-bit number x is  $2^n - 1 - x$

2's complement of n-bit number x is  $2^n - x$

1's complement of 1011 ?  $2^4 - 1 - 1011$   $1111 - 1011 = 0100$

1's complement is simply obtained by flipping a bit  
(changing 1 to 0 and 0 to 1)

1's complement of 1001101 = ?

0110010

$$\begin{aligned} \text{2's complement of } 1010 &= \text{1's complement of } 1010 + 1 \\ &= 0101 + 1 = 0110 \end{aligned}$$

2's complement of 110010 =

Leave all least significant 0's as they are, leave first 1 unchanged and then flip all subsequent bits

001110

1011  $\rightarrow$  0101

101101100  $\rightarrow$  010010100