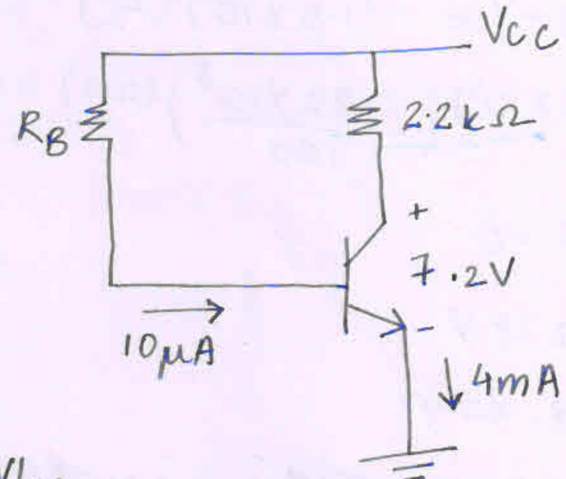
ESC201 Assignment 7 Solutions

Ans 1.



Applying KVL,

$$V_{CC} - (4\times10^{-3})(2.2\times10^{3}) - 7.2 = 0$$

$$\Rightarrow V_{CC} = 8.8 + 7.2\ \text{V}$$

$$\Rightarrow V_{CC} = 16\text{V}$$

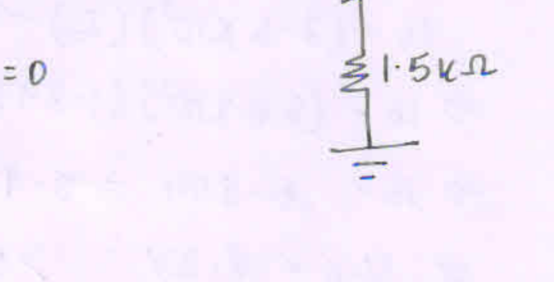
Applying KVL,

$$V_{CC} - (10\times10^{-6})(R_B) - V_{BE} = 0$$

$$\Rightarrow V_{CC} = (10\times10^{-6})(R_B) + V_{BE}$$

or $R_B = \dfrac{16-0.7}{10}\ \text{M}\Omega = 1.53\ \text{M}\Omega$

Ans 2. Assume the transistor to be in forward active mode.



Applying KVL,

$$20 - (510\times10^{3})(I_B) - V_{BE} - (1.5\times10^{3})(I_E) = 0$$

$$\Rightarrow 20 - (510\times10^{3})\left(\frac{I_C}{\beta}\right) - V_{BE} - (1.5\times10^{3})\left(\frac{I_C}{\alpha}\right) = 0$$

$$\Rightarrow 20 - (510\times10^{3})\left(\frac{I_C}{100}\right) - 0.7 - (1.5\times10^{3})\left(\frac{I_C}{\beta}\right)(\beta+1) = 0$$

$$\Rightarrow 20 - 5100I_C - 0.7 - \left(\frac{1.5\times10^{3}}{100}\times101\right)I_C = 0$$

$$\Rightarrow 19.3 = 5100I_C + 1515I_C$$

or $6615\,I_C = 19.3$

$$\Rightarrow I_C = \frac{19.3}{6615}\ \text{A} = 2.917\ \text{mA} \approx 2.92\ \text{mA}$$

Applying KVL,

$$20-(2.92\times10^{-3})(2.4\times10^{3})-V_{CE}-(1.5\times10^{3})(I_E)=0$$

$$\Rightarrow 20-7.008-V_{CE}-(1.5\times10^{3})\left(\frac{2.92\times10^{-3}}{100}\right)(101)=0$$

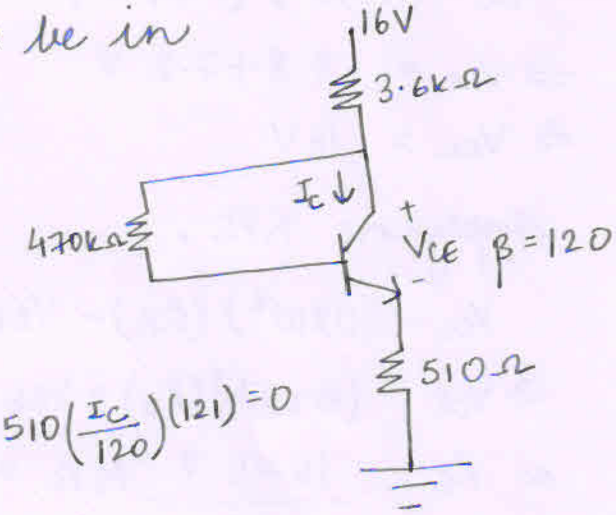
$$\Rightarrow 12.992-V_{CE}-4.4238=0$$

$$\cancel{\Rightarrow}\text{ or } V_{CE}=12.992-4.4238\ V$$

$$\Rightarrow V_{CE}=8.5682V \approx 8.57V$$

Since $V_{CE} > 0.2V$, our assumption is correct.

Ans 3. Assume the transistor to be in forward active mode.



Applying KVL,

$$16-(3.6\times10^{3})I_C-(470\times10^{3})I_B-V_{BE}-510I_E=0$$

$$\Rightarrow 16-(3.6\times10^{3})I_C-(470\times10^{3})\left(\frac{I_C}{120}\right)-0.7-510\left(\frac{I_C}{120}\right)(121)=0$$

$$\Rightarrow \overset{15.3}{\cancel{16}}=(3.6\times10^{3}+3916.66+514.25)I_C$$

$$\Rightarrow 15.3=8030.91\,I_C$$

$$\text{or } I_C \approx 1.89\ mA$$

Applying KVL,

$$16-(3.6\times10^{3})(I_C)-V_{CE}-510I_E=0$$

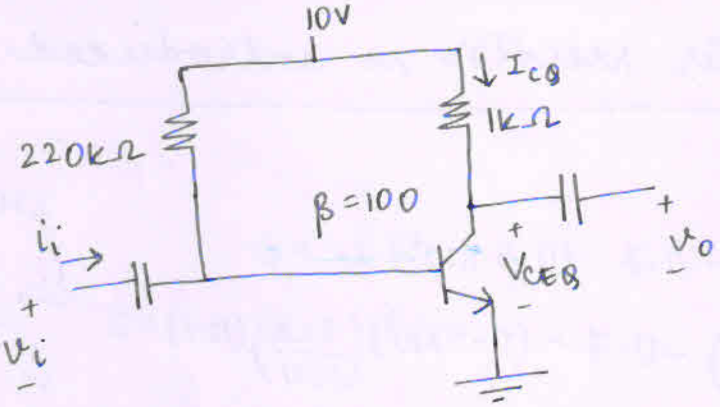
$$\Rightarrow 16-(3.6\times10^{3})(1.89\times10^{-3})-V_{CE}-510\left(\frac{1.89\times10^{-3}}{120}\right)(121)=0$$

$$\Rightarrow 16-6.804-0.972=V_{CE}$$

or $V_{CE}=8.2V$ $>0.2V \Rightarrow$ Our assumption is correct.

Ans 4.

Assume the transistor to be in forward active mode.



Applying KVL,

$$10-(220\times10^3)I_B - V_{BE} = 0$$

$$\Rightarrow 10-(220\times10^3)\left(\frac{I_C}{\beta}\right) - 0.7 = 0$$

$$\text{or } \left(\frac{220\times10^3}{100}\right) I_C = 9.3$$

$$\Rightarrow I_C = 4.227\text{ mA} \approx 4.23\text{ mA}$$

$$\boxed{\alpha = \frac{\beta}{\beta+1}}$$

Applying KVL,

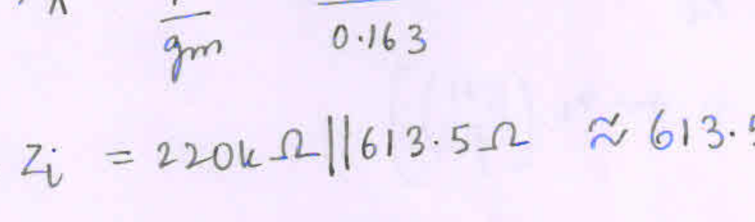
$$10-(10^3)(I_C) - V_{CEQ} = 0$$

$$\Rightarrow 10-(10^3)(4.23\times10^{-3}) = V_{CEQ}$$

or $V_{CEQ} = 5.77\text{V} > 0.2\text{V}$ → correct assumption of forward active mode

$$g_m = \frac{\alpha I_E}{V_T} = \frac{I_C}{V_T} = \frac{4.23\times10^{-3}}{26\times10^{-3}}\text{ A/V} = 0.163\text{ A/V}$$

$$A_V = -g_m R_C = -(0.163)(1\times10^3) = -163$$

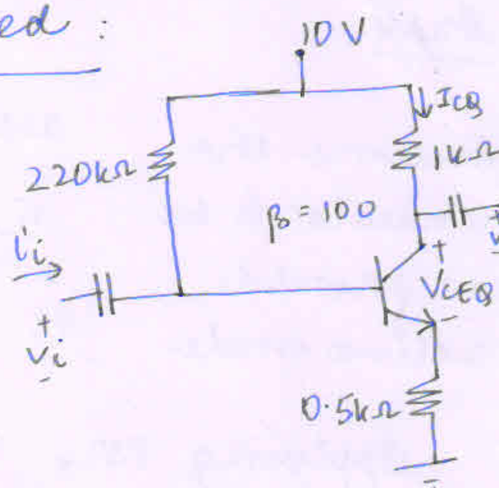


$Z_i = 220\text{k}\Omega \,||\, r_\pi$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{0.163}\,\Omega = 613.49\,\Omega \approx 613.5\,\Omega$$

$$\therefore Z_i = 220\text{k}\Omega \,||\, 613.5\,\Omega \approx 613.5\,\Omega$$

After the emitter resistor is introduced :



Applying KVL,

$$10 - (220\times10^3) I_B - 0.7 - (0.5\times10^3) I_E = 0$$

$$\Rightarrow 10 - (220\times10^3)\left(\frac{I_{CQ}}{100}\right) - 0.7 - (0.5\times10^3)\left(\frac{I_{CQ}}{100}\right)(101) = 0$$

or $(2200 + 505) I_{CQ} = 9.3$

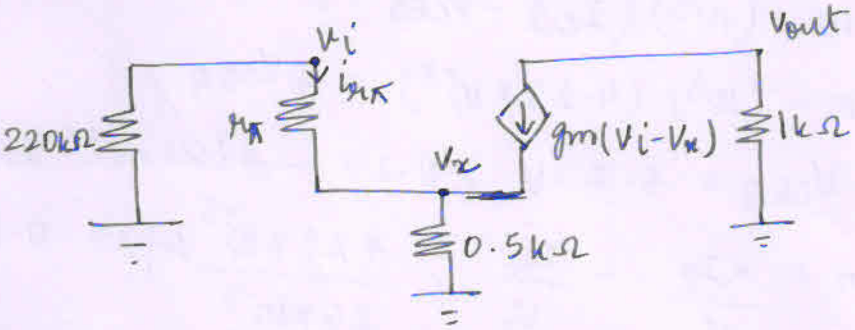
$$\Rightarrow I_{CQ} = \frac{9.3}{2705} A = 3.438 mA \approx 3.44 mA$$

Applying KVL,

$$10 - (10^3)(3.44\times10^{-3}) - V_{CEQ} - (0.5\times10^3)\left(\frac{3.44\times10^{-3}}{100}\right)(101) = 0$$

or $V_{CEQ} = 10 - 3.44 - 1.7372$ V

$\Rightarrow V_{CEQ} = 4.82$ V



Applying KCL at V_x,

$(r_E = 0.5k\Omega, r_c = 1k\Omega)$

$$\frac{v_i - V_x}{r_\pi} + g_m(v_i - V_x) = \frac{V_x}{r_E}$$

$$\Rightarrow \frac{(v_i - V_x)}{\beta} g_m + g_m(v_i - V_x) = \frac{V_x}{r_E}$$

$$\Rightarrow g_m(v_i - V_x)\left[\frac{1+\beta}{\beta}\right] = \frac{V_x}{r_E}$$

$$\Rightarrow g_m v_i \left(\frac{\beta+1}{\beta}\right) r_E = V_x\left[1 + g_m r_E\left(\frac{\beta+1}{\beta}\right)\right]$$

or $$V_x = \frac{v_i\, g_m \left(\frac{\beta+1}{\beta}\right) r_E}{1 + g_m\left(\frac{\beta+1}{\beta}\right) r_E} \quad \text{--- (1)}$$

$$V_{out} = -g_m (v_i - V_x) r_C$$

$$\Rightarrow V_{out} = -g_m \left[v_i - \frac{v_i g_m \left(\frac{\beta+1}{\beta}\right) r_E}{1 + g_m \left(\frac{\beta+1}{\beta}\right) r_E} \right] r_C \qquad \text{(from (1))}$$

$$\Rightarrow V_{out} = -g_m v_i r_C \left[\frac{1 + g_m \left(\frac{\beta+1}{\beta}\right) r_E - g_m \left(\frac{\beta+1}{\beta}\right) r_E}{1 + g_m \left(\frac{\beta+1}{\beta}\right) r_E} \right]$$

$$\Rightarrow A_V = \frac{V_{out}}{V_i} = \frac{-g_m r_C}{1 + g_m \left(\frac{\beta+1}{\beta}\right) r_E} \qquad \text{--- (2)}$$

$$g_m = \frac{\alpha I_E}{V_T} = \frac{I_C}{V_T} = \frac{3.44 \times 10^{-3}}{26 \times 10^{-3}} \text{ A/V} = 0.132 \text{ A/V}$$

$$\Rightarrow A_V = -\frac{0.132 \times 10^3}{1 + (0.132)\left(\frac{101}{100}\right)(0.5 \times 10^3)}$$

$$\Rightarrow A_V = -\frac{0.132 \times 10^3}{1 + 66.66}$$

$$\Rightarrow A_V = -1.95$$

$$i_{r\pi} = \frac{v_i - V_x}{r_\pi} = \frac{1}{r_\pi}\left[v_i - \frac{v_i g_m \left(\frac{\beta+1}{\beta}\right) r_E}{1 + g_m \left(\frac{\beta+1}{\beta}\right) r_E} \right]$$

$$\Rightarrow i_{r\pi} = \frac{v_i}{r_\pi}\left[\frac{1 + g_m \left(\frac{\beta+1}{\beta}\right) r_E - g_m \left(\frac{\beta+1}{\beta}\right) r_E}{1 + g_m \left(\frac{\beta+1}{\beta}\right) r_E} \right]$$

$$\Rightarrow i_{r\pi} = \frac{v_i}{r_\pi}\left[\frac{1}{1 + g_m \left(\frac{\beta+1}{\beta}\right) r_E} \right]$$

$$\text{or } \frac{v_i}{i_{r\pi}} = \frac{1}{r_\pi + \frac{r_\pi g_m}{\beta}(\beta+1) r_E} = \frac{1}{r_\pi + \cancel{g_m}(\beta+1) r_E}$$

$$\therefore Z_i = \left[r_\pi + (\beta+1) r_E \right] \,||\, 220\text{k}\Omega \qquad \text{--- (3)}$$

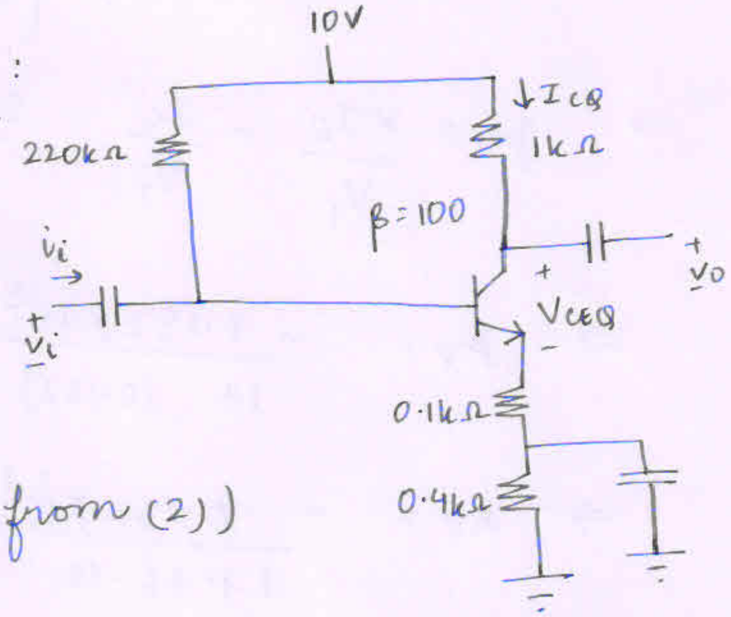
$$r_\pi = \frac{\beta}{g_m} = \frac{100}{0.132}\,\Omega = 757.57\,\Omega$$

$$\therefore Z_i = \left[757.57 + (101)(0.5\times10^3)\right] \| (220\times10^3)\,\Omega$$

$$\Rightarrow Z_i \approx (51.26\times10^3) \| (220\times10^3)\,\Omega$$

$$\Rightarrow Z_i = \frac{51.26\times220\times10^3}{51.26+220}\,\Omega$$

$$\Rightarrow Z_i = 41.57\,k\Omega$$



After circuit modification:

Now the effective emitter resistance is 0.1kΩ as 0.4 kΩ is shorted by the capacitor across it.

$$\therefore A_V = \frac{-g_m r_c}{1+(\beta+1)\left(\frac{r_E}{r_\pi}\right)} \quad \text{(from (2))}$$

$$= -\frac{0.132\times10^3}{1+(101)\left(\frac{0.1\times10^3}{757.57}\right)} = -9.21$$

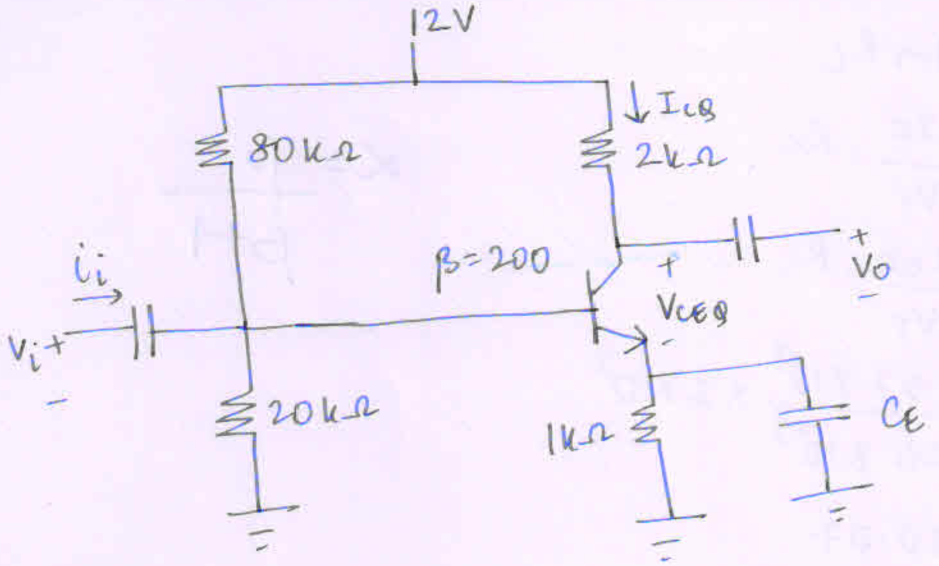
$$Z_i = \left[r_\pi + (\beta+1)r_E\right] \| 220\,k\Omega \quad \text{(from (3))}$$

$$\Rightarrow Z_i = \left[757.57 + (101)(0.1\times10^3)\right] \| (220\times10^3)\,\Omega$$

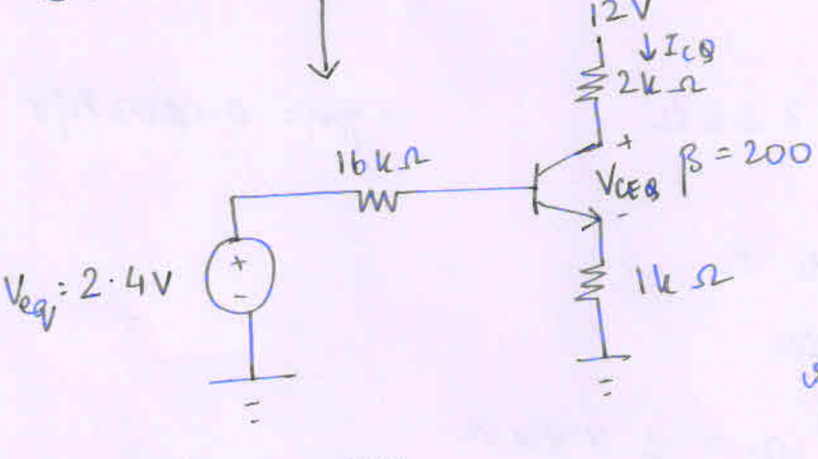
$$\Rightarrow Z_i \approx (10.86\times10^3) \| (220\times10^3)\,\Omega$$

$$\Rightarrow Z_i = \frac{10.86\times220\times10^3}{10.86+220}\,\Omega = 10.34\,k\Omega$$

Ans 5.



(a)



$$V_{eq} = \frac{20 \times 12V}{100} = 2.4V$$

$80k \| 20k = 16k$

Assume the transistor to be in forward active mode

Applying KVL,

$$2.4 - (16 \times 10^3) I_B - 0.7 - (10^3) I_E = 0$$

$$\Rightarrow 2.4 - (16 \times 10^3)\left(\frac{I_{CQ}}{\beta}\right) - 0.7 - (10^3)\left(\frac{I_{CQ}}{\beta}\right)(\beta+1) = 0$$

$$\text{or} \quad I_{CQ}\left[\frac{16 \times 10^3}{200} + \frac{10^3 \times 201}{200}\right] = 1.7$$

$$\Rightarrow I_{CQ}\left[80 + 1005\right] = 1.7$$

$$\Rightarrow I_{CQ} = 1.566\,mA \approx 1.57\,mA$$

Applying KVL,

$$12 - (2 \times 10^3)(1.57 \times 10^{-3}) - V_{CEQ} - (10^3)\left(\frac{1.57 \times 10^{-3}}{200}\right)(201) = 0$$

$$\text{or } V_{CEQ} = 12 - 3.14 - 1.57\,V$$

$\Rightarrow V_{CEQ} = 7.28\,V \approx 7.3V > 0.2V \longrightarrow$ Correct assumption of forward active mode.

(b) $A_V = -g_m R_C$

$= -\frac{\alpha I_E}{V_T} \cdot R_C$

$\alpha = \frac{\beta}{\beta+1}$

$= -\frac{I_{CQ}}{V_T} \cdot R_C$

$= -\frac{1.57 \times 10^{-3}}{26 \times 10^{-3}} \times 2 \times 10^{3}$

$= -120.07$

$Z_i = 80k \| 20k \| r_\pi$

$r_\pi = \frac{\beta}{g_m} = \frac{200 \times 26}{1.57} \Omega = 3.3 k\Omega$, $g_m = 0.0603 A/V$

$\therefore Z_i = 80k \| 20k \| 3.3k \ \Omega$

$\Rightarrow Z_i = 16k \| 3.3k \ \Omega$

$\Rightarrow Z_i = \frac{16 \times 3.3 \times 10^3}{16 + 3.3} \Omega = 2.735k \Omega$

(c) from equation (2) of Ans 4,

$A_V = \frac{-g_m R_C}{1 + \left(\frac{\beta+1}{\beta}\right) g_m R_E} = \frac{-g_m R_C}{1 + (\beta+1)\frac{R_E}{r_\pi}}$

$\Rightarrow A_V = \frac{-0.0603 \times 2 \times 10^3}{1 + \frac{(201) \times 10^3}{3.3 \times 10^3}}$

$\Rightarrow A_V = -1.948$

from equation (3) of Ans 4,

$Z_i = 80k \| 20k \| r_\pi + (\beta+1) R_E$

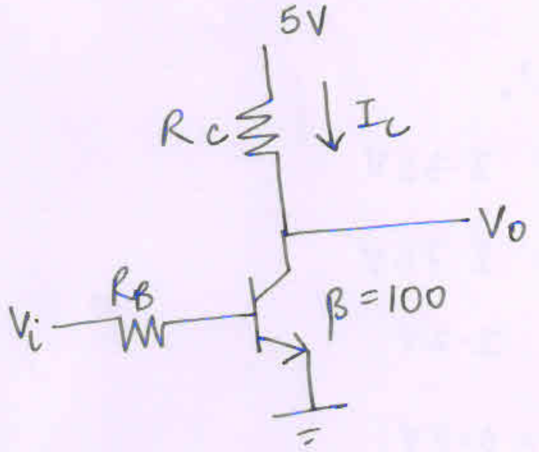
$\Rightarrow Z_i = 16k \| 3.3k + (201) 1k \ \Omega$

$\Rightarrow Z_i = 16k \| 204.3k \ \Omega$

$$\Rightarrow Z_i = \frac{16 \times 204.3 \times 10^3}{16 + 204.3} \; \Omega$$

$$\Rightarrow Z_i = 14.837 \, k\Omega$$

Ans 6.



For saturation, $V_{CE} = 0.2V$

$$\therefore R_C = \frac{5 - 0.2}{2 \times 10^{-3}} \; \Omega = 2.4 \, k\Omega$$

$$\beta = 20$$

$$\therefore I_B = \frac{2mA}{20} = 0.1 mA$$

$$\therefore R_B = \frac{5 - 0.7}{0.1 \times 10^{-3}} \; \Omega = 43 k\Omega$$

$$V_0 = V_{CC} - I_C R_C$$

$$\Rightarrow V_0 = V_{CC} - \beta I_B R_C \quad \text{——— (1)}$$

$$\Rightarrow V_0 = V_{CC} - \beta \left(\frac{V_i - 0.7}{R_B} \right) R_C$$

For $V_i < 0.7V$, the transistor is in cut-off mode

$\therefore$ For $V_i < 0.7V$, $V_0 = 5V$

When V_i increases beyond 0.7V, the transistor enters the forward active mode and continues to remain ~~till~~ in forward active mode till a value of V_i where $V_0 = 0.2V$ and the transistor goes in saturation. To find the value of V_i where $V_0 = 0.2V$, from equation(1),

$$\therefore\ 0.2 = 5 - 100\left(\frac{V_i - 0.7}{43\times10^3}\right)(2.4\times10^3)$$

$$\Rightarrow (V_i - 0.7)\,5.58 = 4.8$$

$$\Rightarrow V_i = 1.56\,V$$

e.g. From equation (1),

for $V_i = 1V$, $V_0 = 3.32V$

for $V_i = 1.1V$, $V_0 = 2.76V$

for $V_i = 1.2V$, $V_0 = 2.2V$

For $V_i > 1.56V$, $V_0 = 0.2V$