ESC201A Assignment 9

Instructor Abhishek Gupta

2023-2024 Semester I

Topics

Truth Tables, Boolean Expressions, Minimization

Questions

1. Show that the Boolean expression $x + \overline{x}.y$ is equivalent to $x + y$ using basic postulates and theorems of Boolean algebra.

$x + \bar{x}y$

$= (x + \bar{x}).(x + y)$

$= 1 \cdot (x + y)$

$= x + y$

2. Reduce the following expressions to a minimum number of literals using basic postulates and theorems of Boolean algebra.

(a) $f = (x + y).(\overline{y} + \overline{x})$

(b) $f = ABCD + \overline{A}BD + AB\overline{C}D$

(a) $f = (x + y).(\cancel{x}\bar{y} + \bar{x})$

$\Rightarrow f = x\bar{y} + x.\bar{x} + y.\bar{y} + y.\bar{x}$

$\Rightarrow f = x\bar{y} + y\bar{x}$

(b) $f = ABCD + \bar{A}BD + AB\bar{C}D$

$\Rightarrow f = ABD(C + \bar{C}) + \bar{A}BD$

$\Rightarrow f = ABD + \bar{A}BD$

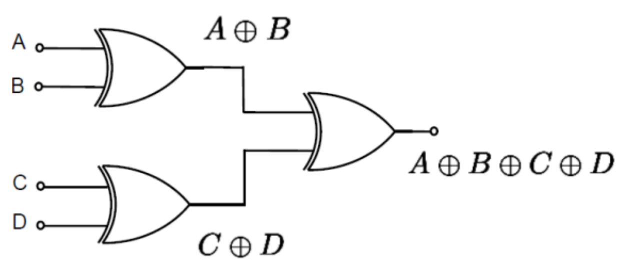
$\Rightarrow f = BD(A + \bar{A})$

$\Rightarrow f = BD$

3. Consider four-input function F(A, B, C, D) that outputs 1 whenever an odd number of its inputs are 1, (a) construct the truth table (b) write down the Boolean expressions, present an implementation of

the function using two-input XOR gate

A	B	C	D	F
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0



4. Four switches operate a lamp as follows: the lamp lights up if switches 1,3 and 4 are closed and switch 2 is open, or if 2, 4 are closed and 3 is open, or if all the switches are kept closed. Express this as a boolean function in a standard sum of product form and solve it using k- map. (Use bit '1' when switch is closed and bit '0' when switch is open).

Let four switches are represented by w, x, y, z
For closed switch variable will have value 1 and for open switch variable will have value 0.

1, 3, 4 closed ; 2 open $\rightarrow$ $w\bar{x}yz$
2, 4 closed ; 3 open $\rightarrow$ $x\bar{y}z$
All closed $\rightarrow$ $wxyz$

wx \ yz	00	01	11	10
00	0 (0)	0 (1)	0 (3)	0 (2)
01	0 (4)	1 (5)	0 (7)	0 (6)
11	0 (12)	1 (13)	1 (15)	0 (14)
10	0 (8)	0 (9)	1 (11)	0 (10)

$f = x\bar{y}z + wyz$

5. Obtain the truth table for the following function: $(x.y + z)(y + x.z)$ and write it as sum of products (SOP) and product of sums (POS).

$$f = (x \cdot y + z) \cdot (y + x \cdot z)$$

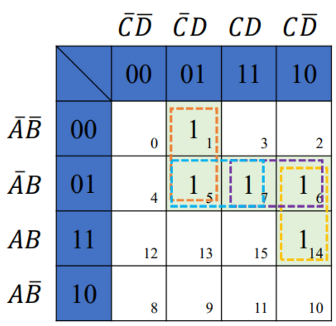
x	y	z	$(x \cdot y + z)$	$(y + x z)$	f
0	0	0	0	0	0
0	0	1	1	0	0
0	1	0	0	1	0
0	1	1	1	1	1
1	0	0	0	0	0
1	0	1	1	1	1
1	1	0	1	1	1
1	1	1	1	1	1

SOP: $f = \bar{x}yz + x\bar{y}z + xy\bar{z} + xyz$

POS: $f = (x+y+z)(x+y+\bar{z})(x+\bar{y}+z)(\bar{x}+y+z)$

6. Simplify the following 4-variable functions into sum-of-products form using K-map.
   a. $\sum(1,5,6,7,14)$

$\sum(1,5,6,7,14)$



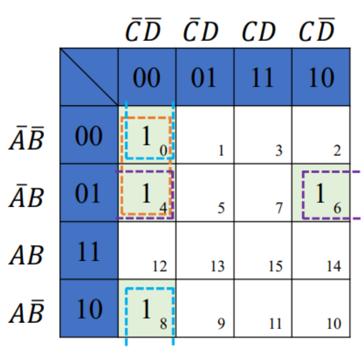
$\boxed{\bar{A}\bar{C}D + BC\bar{D} + \bar{A}BD}$

$\boxed{\bar{A}\bar{C}D + BC\bar{D} + \bar{A}BC}$

there are two answers possible.

b. $\sum(0,4,6,8)$

$\sum(0,4,6,8)$



$\bar{B}\bar{C}\bar{D} + \bar{A}B\bar{D}$

c. $\sum(0,1,4,6,8,9,14)$

$\sum(0,1,4,6,8,9,14)$

		$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
		00	01	11	10
$\bar{A}\bar{B}$	00	1_{0}	1_{1}	$_{3}$	$_{2}$
$\bar{A}B$	01	1_{4}	$_{5}$	$_{7}$	1_{6}
AB	11	$_{12}$	$_{13}$	$_{15}$	1_{14}
$A\bar{B}$	10	1_{8}	1_{9}	$_{11}$	$_{10}$

$\boxed{\bar{B}\bar{C} + \bar{A}B\bar{D} + BC\bar{D}}$

$\boxed{\bar{B}\bar{C} + \bar{A}\bar{C}\bar{D} + BC\bar{D}}$

d. $\sum(1,4,7,11,13,14)$

		$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
		00	01	11	10
$\bar{A}\bar{B}$	00	$_{0}$	1_{1}	$_{3}$	$_{2}$
$\bar{A}B$	01	1_{4}	$_{5}$	1_{7}	$_{6}$
AB	11	$_{12}$	1_{13}	$_{15}$	1_{14}
$A\bar{B}$	10	$_{8}$	$_{9}$	1_{11}	$_{10}$

$\bar{A}\bar{B}\bar{C}D + \bar{A}B\bar{C}\bar{D} + \bar{A}BCD$
$+ A\bar{B}CD + AB\bar{C}D + ABC\bar{D}$

This cannot be minimized any further.

7. Simplify the following 4-variable functions into product-of-sums form using K-map

a. $\Pi(1,3,5,7,13,15)$

$x_1x_2 \backslash x_3x_4$	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	1	0	0	1
10	1	1	1	1

$F = (x_1 + \bar{x}_4)(\bar{x}_2 + \bar{x}_4)$

b. $\Pi(1,3,6,9,11,12,14)$

$x_1x_2 \backslash x_3x_4$	00	01	11	10
00		0	0	
01				0
11	0			0
10		0	0	

$F = (x_2 + \bar{x}_4)(\bar{x}_1 + \bar{x}_2 + x_4).$
$(\bar{x}_2 + \bar{x}_3 + x_4)$

c. $\Pi(1,3,5,7,9,11,12,13,14,15,)$

$x_1x_2 \backslash x_3x_4$	00	01	11	10
00		0	0	
01		0	0	
11	0	0	0	0
10		0	0	

$F = (\bar{x}_1 + \bar{x}_2).(\bar{x}_4)$

d. $\Pi(0,1,3,4,5,7,12,13,15)$

$x_1x_2 \backslash x_3x_4$	00	01	11	10
00	0	0	0	
01	0	0	0	
11	0	0	0	
10				

$F = (x_1 + x_3)(x_1 + \bar{x}_4)(\bar{x}_2 + x_3)(\bar{x}_2 + \bar{x}_4)$

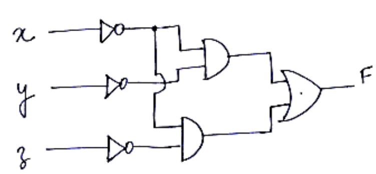
8. Design a combinational circuit with 3 inputs and 1 output

(a) The output is 1 when the binary value of the inputs is less than 3. The output is 0 otherwise

x	y	z	f
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

$x \backslash yz$	00	01	11	10
0	1	1		1
1				

$F = \bar{x}\bar{y} + \bar{x}\bar{z}$



(b) The output is 1 when the binary value of inputs is an odd number.

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

$x \backslash yz$	00	01	11	10
0		1	1	
1		1	1	

$F = z$