ESc201 : Introduction to Electronics

Logic Gates and Minimization

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3-variable minimization (recap)

$$f = \overline{x}.\overline{y}.\overline{z} + \overline{x}.y.z + x.y.z + x.\overline{y}.z$$

x \ yz	00	01	11	10
0	1	0	1	0
1	0	1	1	0

$$y.z$$

$$x.z$$

$$f = \overline{x}.\overline{y}.\overline{z} + y.z + x.z$$

Idea is to cover all the 1's with as few and as simple terms as possible

3-variable minimization (recap)

$$f = \bar{x}.\bar{y}.\bar{z} + \bar{x}.y.\bar{z} + x.y.z + x.\bar{y}.z$$

x \ yz	00	01	11	10
0	1	0	0	1
1	0	1	1	0

$$\bar{x}.\bar{z}$$

$$x.z$$

$$f = \bar{x}.\bar{z} + x.z$$

3-variable minimization (recap)

x \ yz	00	01	11	10
0	0	0	0	0
1	1	1	1	1

$$f = x.\bar{y}.\bar{z} + x.\bar{y}.z + x.y.z + x.y.\bar{z}$$

$$x.\bar{y}$$

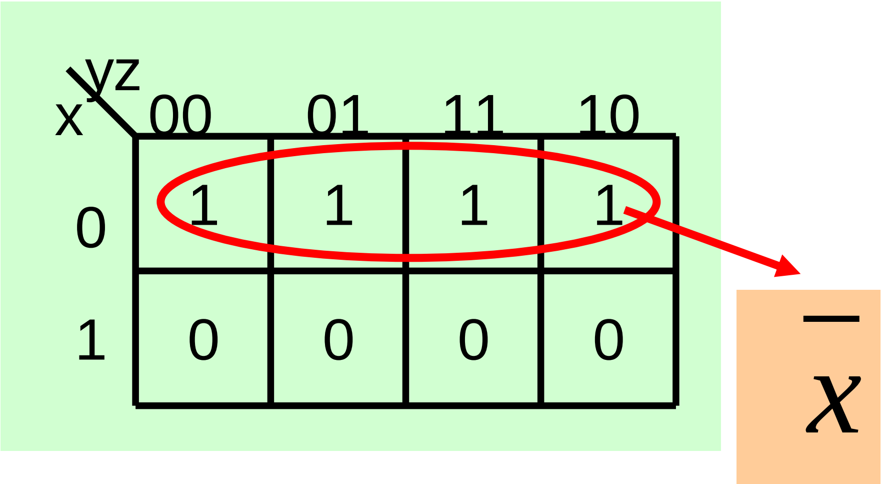
$$x.y$$

$$f = x.\bar{y} + x.y$$

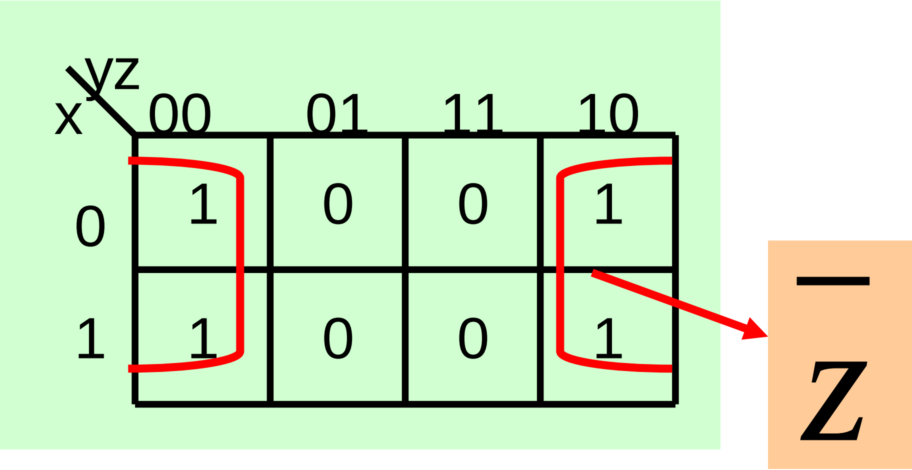
x \ yz	00	01	11	10
0	0	0	0	0
1	1	1	1	1

$$f = x.(\bar{y} + y) = x$$

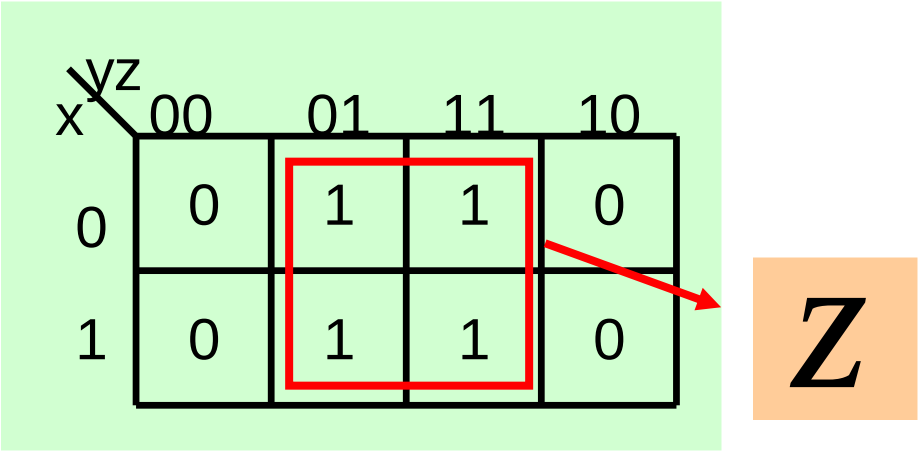
$$x$$



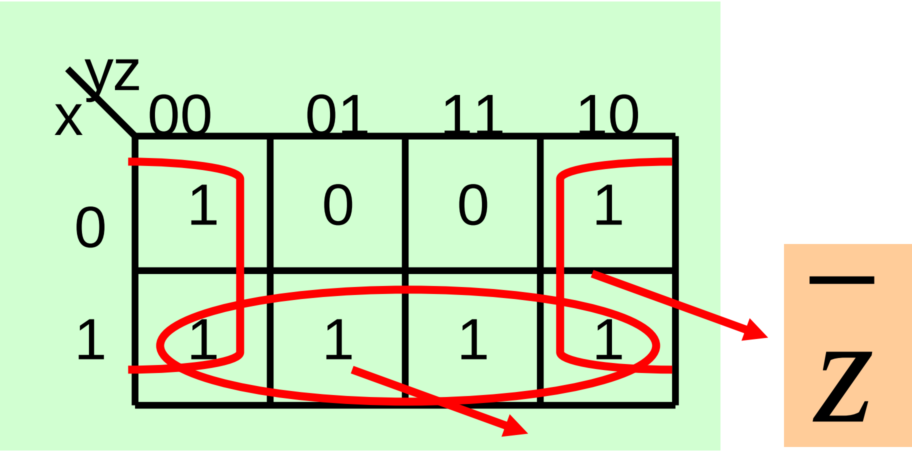
yz
x
00 01 11 10
0 1 1 1 1
1 0 0 0 0
$\bar{x}$



yz
x
00 01 11 10
0 1 0 0 1
1 1 0 0 1
$\bar{z}$



yz
x
00 01 11 10
0 0 1 1 0
1 0 1 1 0
z



yz
x
00 01 11 10
0 1 0 0 1
1 1 1 1 1
$\bar{z}$

$$f = x + \bar{z}$$

x

4-variable minimization

wx \ yz	00	01	11	10
00	1	0	1	0
01	0	1	1	0
11	1	0	0	1
10	1	0	0	0

$$\overline{w}.y.z$$

$$\overline{w}.x.z$$

$$w.\overline{y}.\overline{z}$$

$$f = \overline{w}.y.z + \overline{w}.x.z + w.\overline{y}.\overline{z} + \overline{w}.\overline{x}.\overline{y}.\overline{z} + w.x.y.\overline{z}$$

Is this the simplest expression ?

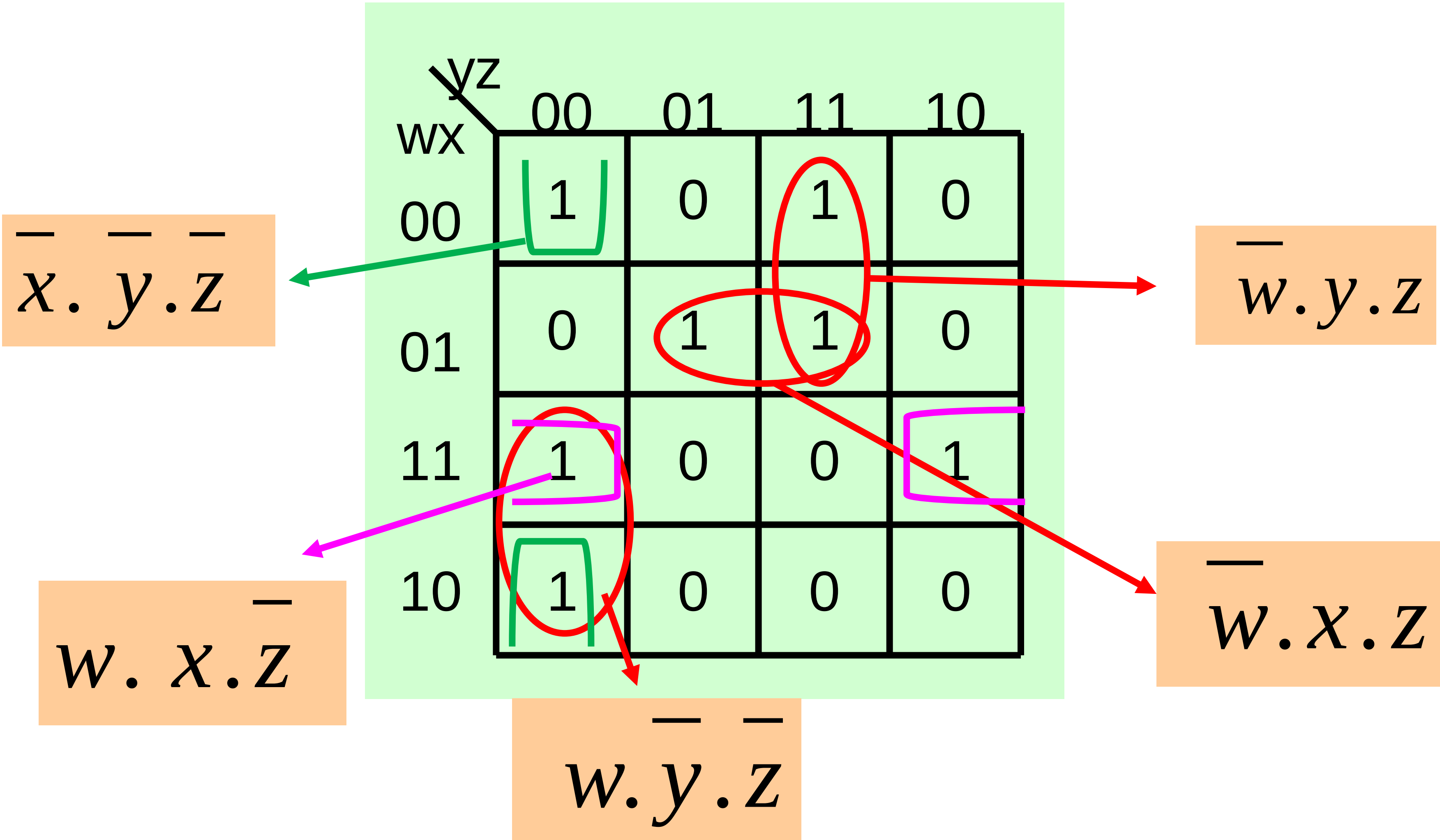
wx \ yz	00	01	11	10
00	1	0	1	0
01	0	1	1	0
11	1	0	0	1
10	1	0	0	0

$$w.\,x.\bar{y}.\bar{z}+w.\,x.\,y.\bar{z}=w.\,x.\bar{z}$$

wx \ yz	00	01	11	10
00	1	0	1	0
01	0	1	1	0
11	1	0	0	1
10	1	0	0	0

$$w.\,\bar{x}.\bar{y}.\bar{z}+\bar{w}.\,\bar{x}.\bar{y}.\bar{z}=\bar{x}.\,\bar{y}.\bar{z}$$

4-variable minimization



$$f = \bar{w}.y.z + \bar{w}.x.z + w.\bar{y}.\bar{z} + w.x.\bar{z} + \bar{x}.\bar{y}.\bar{z}$$

Is this the best that we can do ?

Cover the 1's with minimum number of terms

wx \ yz	00	01	11	10
00	1	0	1	0
01	0	1	1	0
11	1	0	0	1
10	1	0	0	0

wx \ yz	00	01	11	10
00	1	0	1	0
01	0	1	1	0
11	1	0	0	1
10	1	0	0	0

$$\text{f} = \overline{w}.y.z + \overline{w}.x.z + w.\overline{y}.\overline{z} + w.x.\overline{z} + \overline{x}.\overline{y}.\overline{z}$$

$$\text{f} = \overline{w}.y.z + \overline{w}.x.z + w.x.\overline{z} + \overline{x}.\overline{y}.\overline{z}$$

4-variable minimization

wx \ yz	00	01	11	10
00	1	0	0	0
01	1	1	0	0
11	0	0	0	0
10	1	0	0	1

$$f = \bar{w}.x.\bar{y} + w.\bar{x}.\bar{z} + \bar{w}.\bar{y}.\bar{z}$$

wx \ yz	00	01	11	10
00	1	0	0	0
01	1	1	0	0
11	0	0	0	0
10	1	0	0	1

$$f = \bar{w}.x.\bar{y} + w.\bar{x}.\bar{z} + \bar{x}.\bar{y}.\bar{z}$$

Groups of 4

wx \ yz	00	01	11	10
00	0	1	0	0
01	1	1	1	1
11	0	1	0	0
10	0	1	0	0

$\overline{w}.x$

$\overline{y}.z$

wx \ yz	00	01	11	10
00	0	0	0	0
01	0	1	1	0
11	0	1	1	0
10	0	1	1	0

$x.z$

$w.z$

wx \ yz	00	01	11	10
00	0	0	0	0
01	1	0	0	1
11	1	0	0	1
10	0	0	0	0

$x.\bar{z}$

wx \ yz	00	01	11	10
00	0	1	1	0
01	0	0	0	0
11	0	0	0	0
10	0	1	1	0

$\bar{x}.z$

wx \ yz	00	01	11	10
00	1	0	0	1
01	0	0	0	0
11	0	0	0	0
10	1	0	0	1

$\bar{x}.\bar{z}$

wx \ yz	00	01	11	10
00	1	0	1	0
01	0	0	0	0
11	0	0	0	0
10	1	0	1	0

??

Groups of 8

wx \ yz	00	01	11	10
00	0	1	1	0
01	0	1	1	0
11	0	1	1	0
10	0	1	1	0

z

wx \ yz	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	1	1	1	1
10	0	0	0	0

x

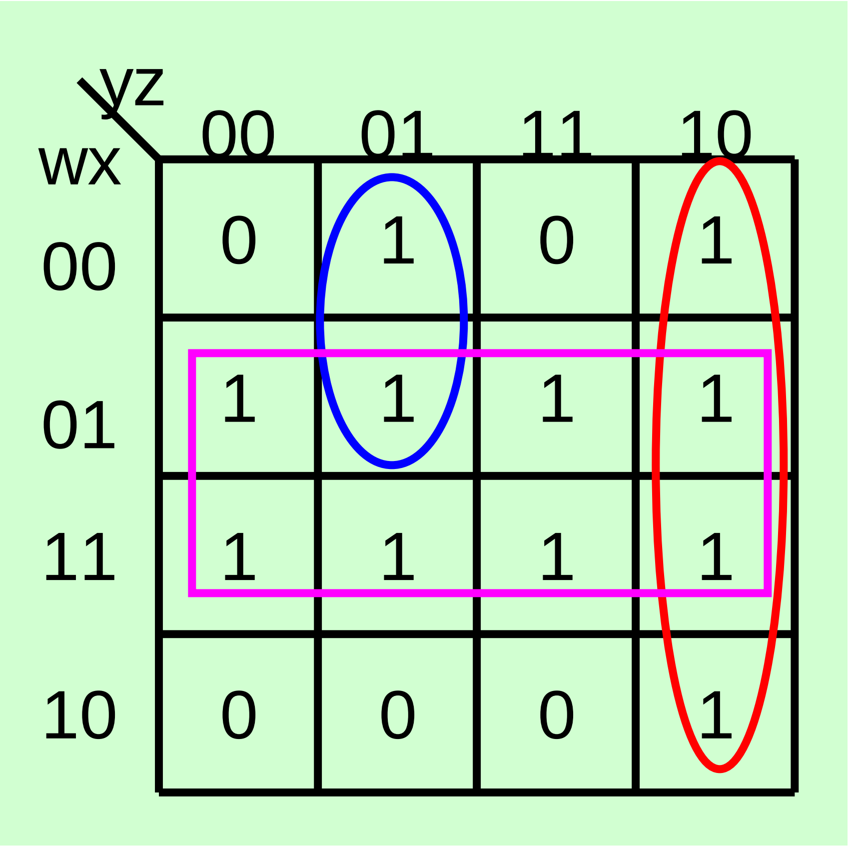
wx \ yz	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	1	0	0	1
10	1	0	0	1

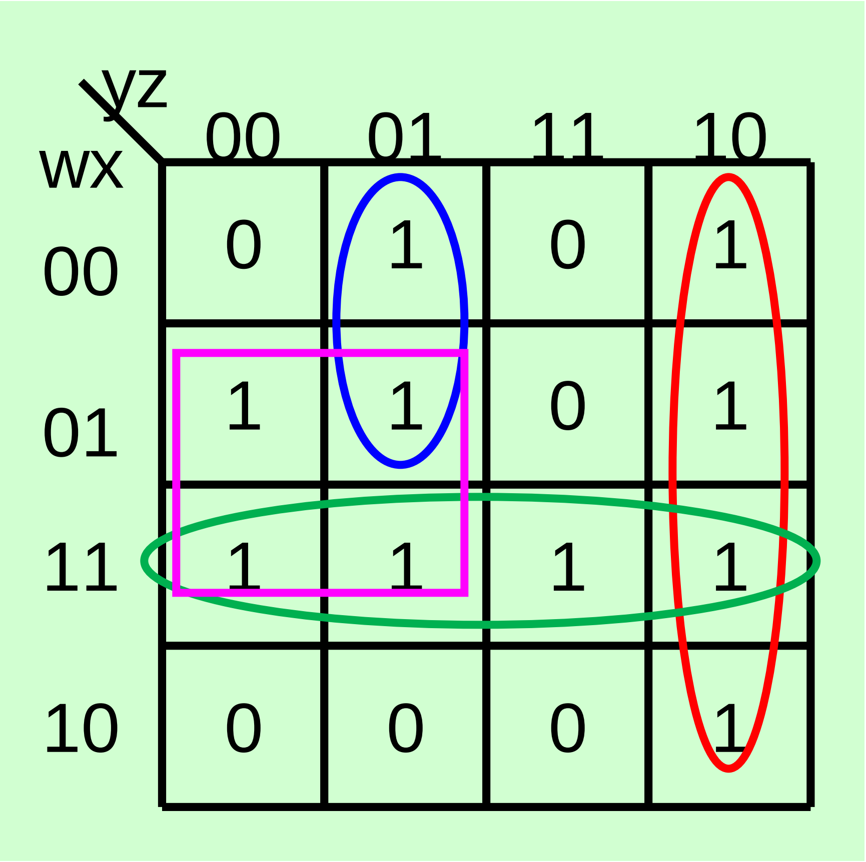
$\overline{z}$

wx \ yz	00	01	11	10
00	1	1	1	1
01	0	0	0	0
11	0	0	0	0
10	1	1	1	1

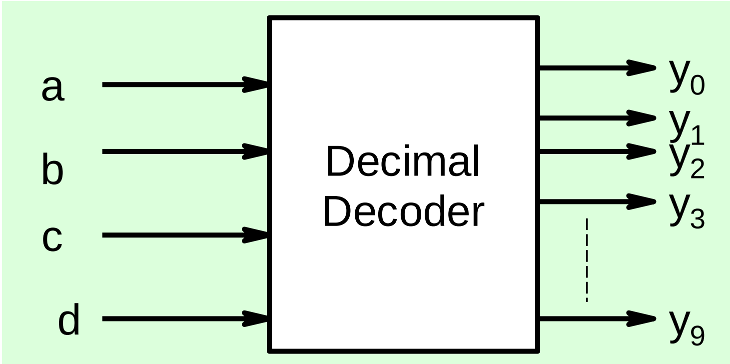
$\overline{x}$

Examples


yz
wx
00 01 11 10
00 0 1 0 1
01 1 1 1 1
11 1 1 1 1
10 0 0 0 1


yz
wx
00 01 11 10
00 0 1 0 1
01 1 1 0 1
11 1 1 1 1
10 0 0 0 1

Don’t care terms



Y_3

ab \ cd	00	01	11	10
00	0	0	1	0
01	0	0	0	0
11	x	x	x	x
10	0	0	x	x

$$y_3 = \overline{a}.\overline{b}.c.d$$

a	b	c	d	$y_0y_1y_2y_3y_4y_5y_6y_7y_8y_9$
0	0	0	0	1 0 0 0 0 0 0 0 0 0
0	0	0	1	0 1 0 0 0 0 0 0 0 0
0	0	1	0	0 0 1 0 0 0 0 0 0 0
0	0	1	1	0 0 0 1 0 0 0 0 0 0
0	1	0	0	0 0 0 0 1 0 0 0 0 0
0	1	0	1	0 0 0 0 0 1 0 0 0 0
0	1	1	0	0 0 0 0 0 0 1 0 0 0
0	1	1	1	0 0 0 0 0 0 0 1 0 0
1	0	0	0	0 0 0 0 0 0 0 0 1 0
1	0	0	1	0 0 0 0 0 0 0 0 0 1
1	0	1	0	x x x x x x x x x x
1	0	1	1	x x x x x x x x x x
1	1	0	0	x x x x x x x x x x
1	1	0	1	x x x x x x x x x x
1	1	1	0	x x x x x x x x x x
1	1	1	1	x x x x x x x x x x

Don’t care terms can be chosen as 0 or 1. Depending on the problem, we can choose the don’t care term as 1 and use it to obtain a simpler Boolean expression

Y_3

ab \ cd	00	01	11	10
00	0	0	1	0
01	0	0	0	0
11	x	x	x	x
10	0	0	x	x

$$y_3 = \bar{b}.c.d$$

Don’t care terms should only be included in encirclements if it helps in obtaining a larger grouping or smaller number of groups.

SOP/POS form (recap)

x_1	x_2	x_3	y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

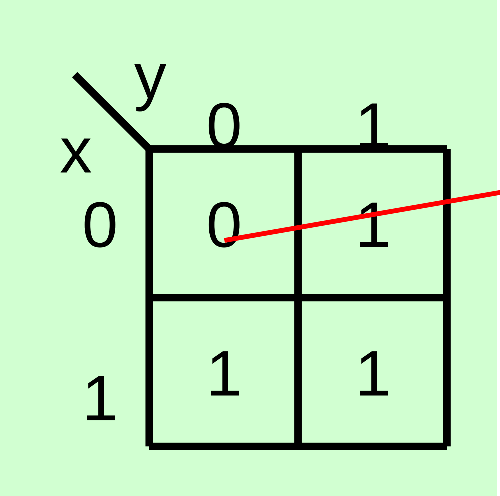
$$y = \overline{x_1}.\overline{x_2}.x_3 + \overline{x_1}.x_2.x_3 + x_1.\overline{x_2}.x_3 + x_1.x_2.x_3$$

Sum of Products (SOP) form

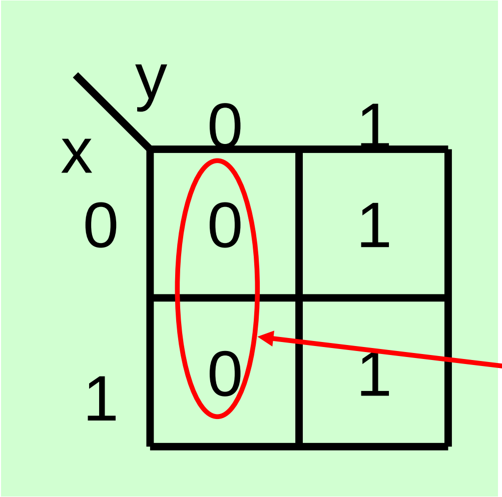
$$y = (x_1 + x_2 + x_3).(x_1 + \overline{x_2} + x_3).(\overline{x_1} + x_2 + x_3).(\overline{x_1} + \overline{x_2} + x_3)$$

Product of Sum (POS) form

Minimization of Product of Sum Terms using Kmap



$$f = x + y$$



$$f = y$$

x \ y	0	1
0	1	0
1	1	0

$$\Rightarrow f = \overline{y}$$

x \ y	0	1
0	1	1
1	0	0

$$\Rightarrow f = \overline{x}$$

x \ yz	00	01	11	10
0	1	0	0	1
1	0	1	1	0

$$\overline{x} + z$$

$$x + \overline{z}$$

$$f = (\overline{x} + z).(x + \overline{z})$$

$$\Rightarrow f = \overline{x}\,.\,\overline{z} + x\,.\,z$$

$$x + y + z$$

$$x + \overline{y} + \overline{z}$$

wx \ yz	00	01	11	10
00	0	1	0	1
01	1	1	1	1
11	1	0	1	1
10	0	0	0	0

$$\overline{w} + y + \overline{z}$$

$$\overline{w} + x$$

$$f = (x + y + z).(x + \overline{y} + \overline{z}).(\overline{w} + y + \overline{z}).(\overline{w} + x)$$

Example

Obtain the minimized PoS by suitably using don’t care terms

wx \ yz	00	01	11	10
00	1	x	0	1
01	1	0	1	1
11	0	x	1	1
10	1	x	1	x

$$f = (w + x + \overline{z}).(\overline{w} + \overline{x} + y).(y + \overline{z})$$

Design Flow

System Description

↓

Truth Table

↓

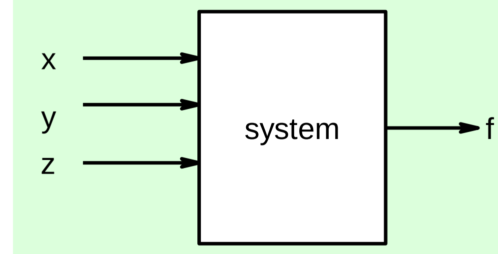
Boolean Expression

↓

Minimized Boolean Expression

↓

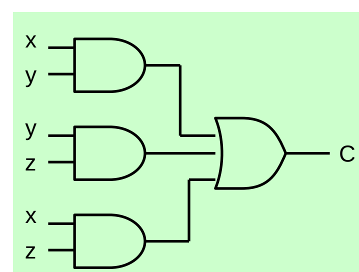
Gate Netlist



x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

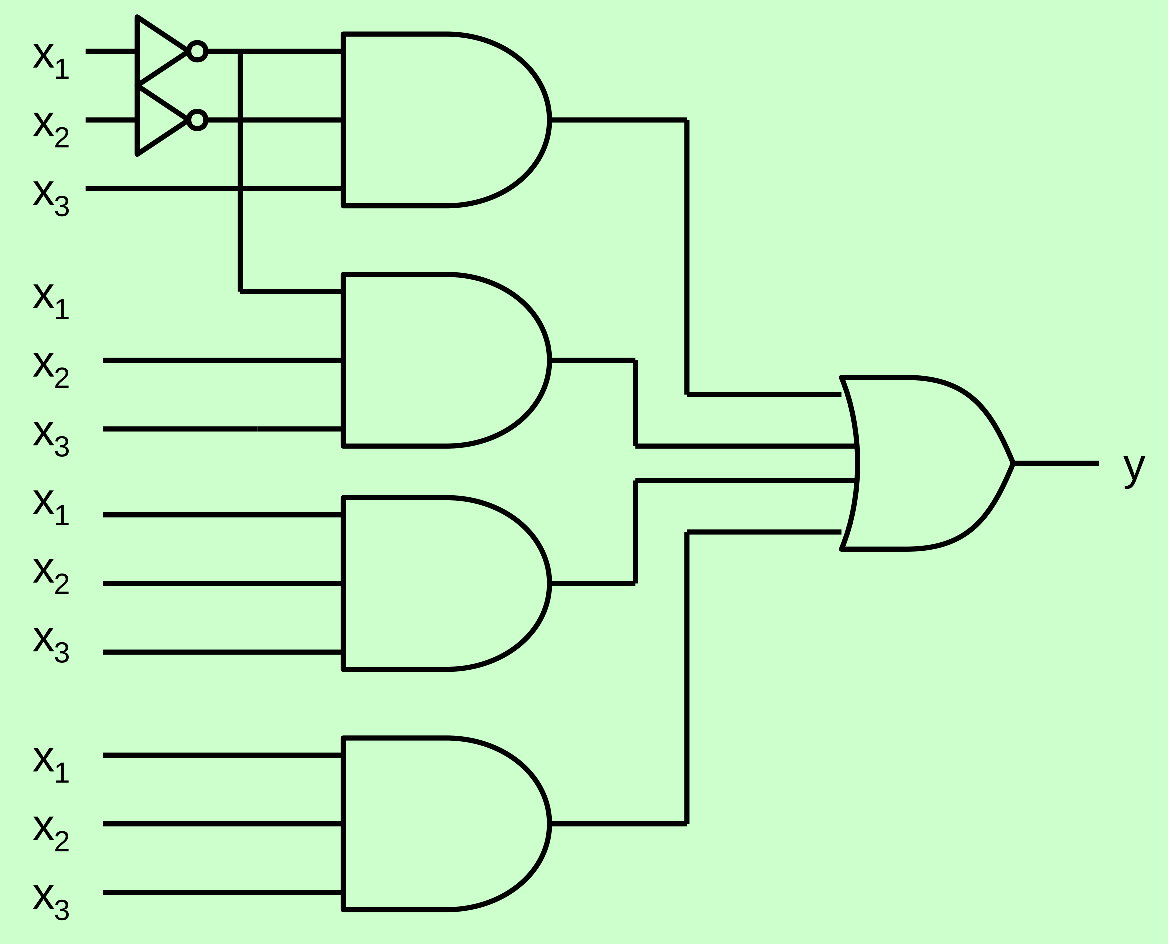
$$\mathrm{f} = \overline{x}.\overline{y}.z + \overline{x}.y.z + x.\overline{y}.z + x.y.z$$

$$\Rightarrow \mathrm{f} = \overline{x}\,.\overline{z} + x.z$$



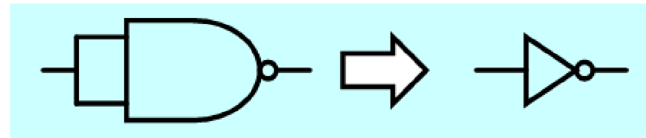
Mapping of Boolean expression to a Network of gates available in the library

$$y = \overline{x_1} . \overline{x_2} . x_3 + \overline{x_1} . x_2 . x_3 + x_1 . \overline{x_2} . x_3 + x_1 . x_2 . x_3$$



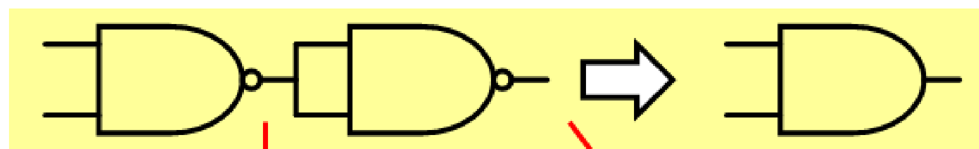
Implementation using only NAND gates

NAND to Inverter



$$\overline{x.x} = \overline{x}$$

NAND to AND

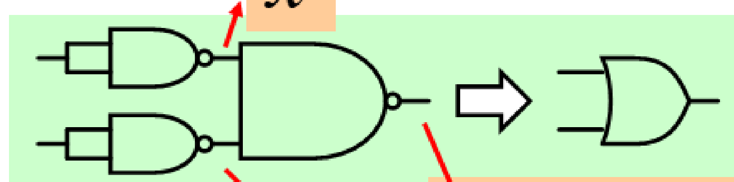


$\overline{x.y}$

$x.y$

NAND to OR

$\overline{x}$

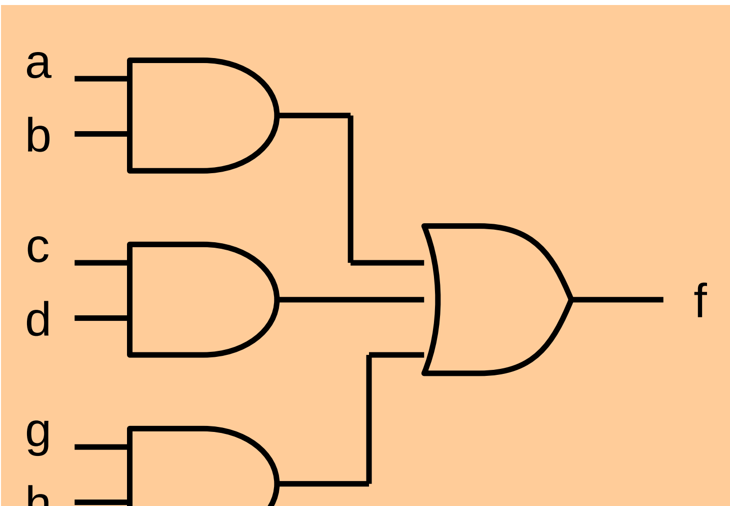


$\overline{y}$

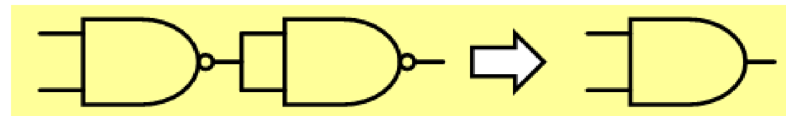
$$f = \overline{\overline{x}.\overline{y}} = x + y$$

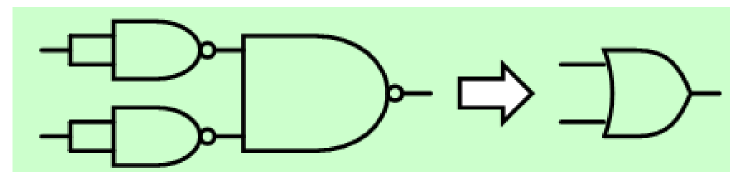
Implementation using only NAND gates

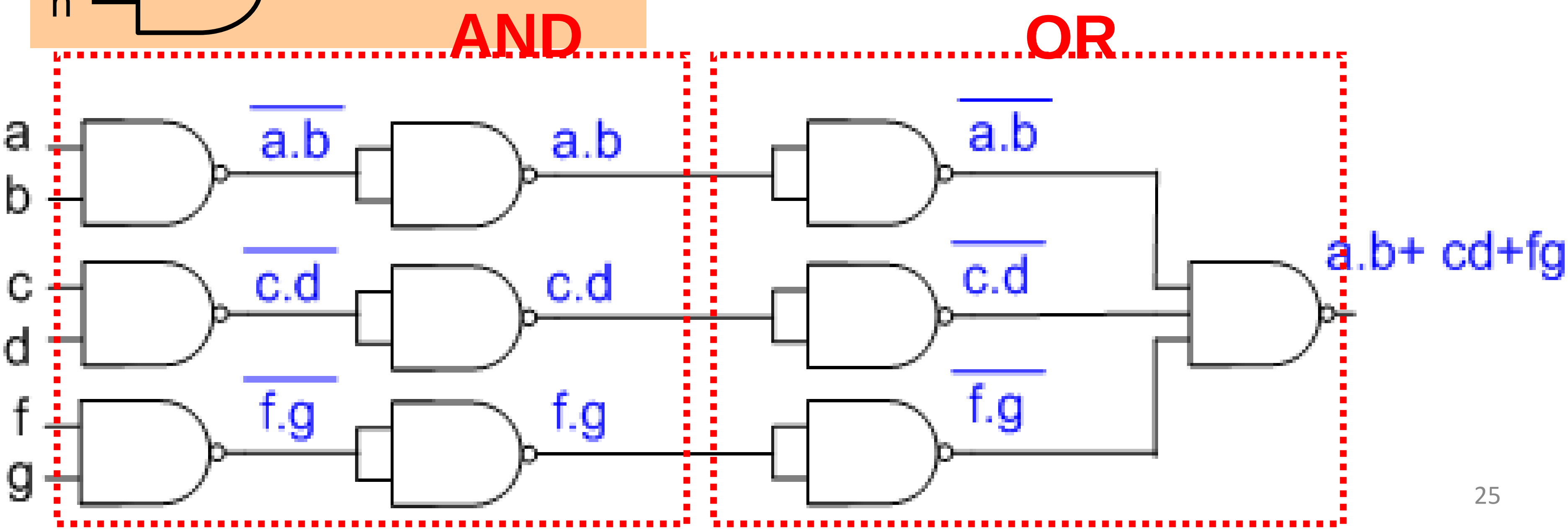
A SoP expression is easily implemented with NAND gates.

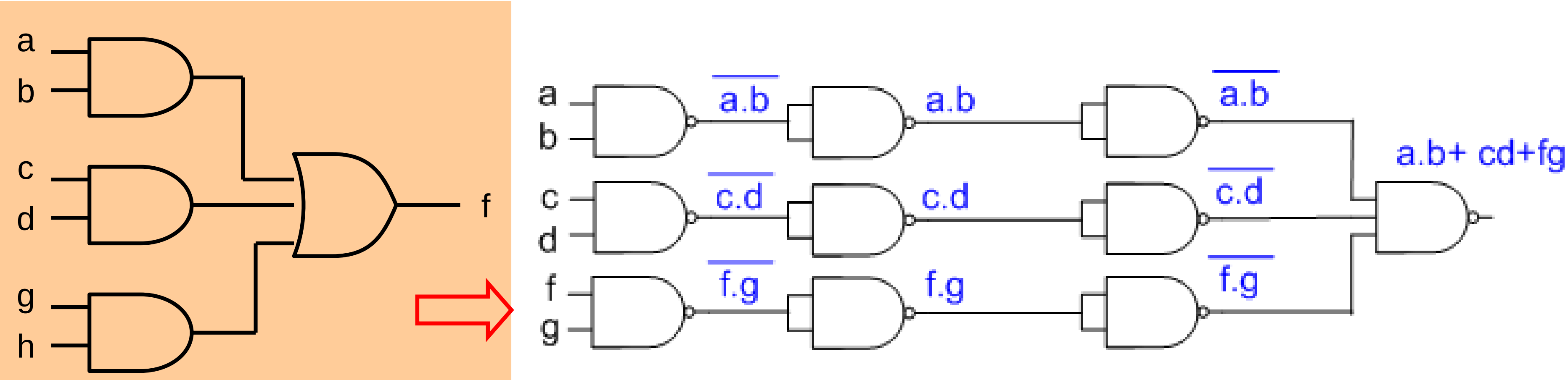


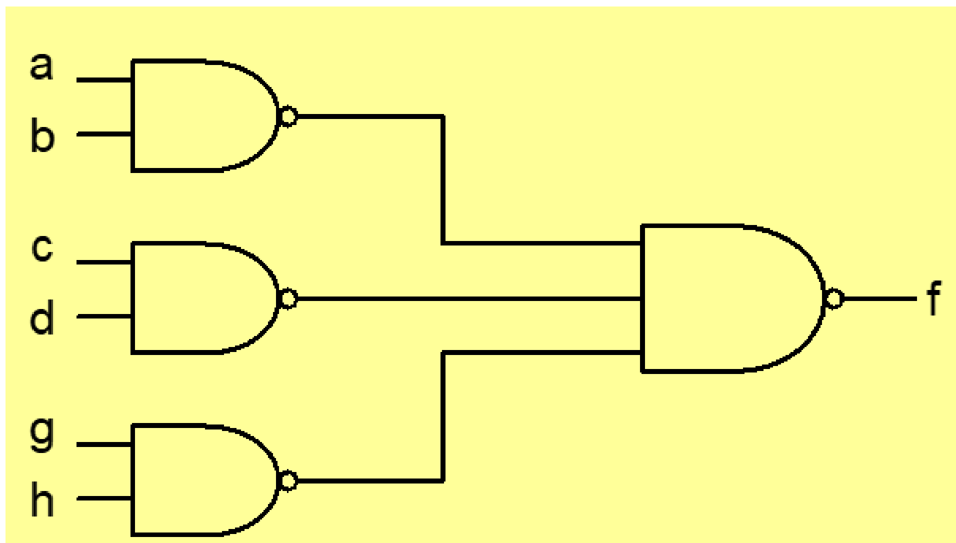
$$f = a.b + c.d + f.g$$







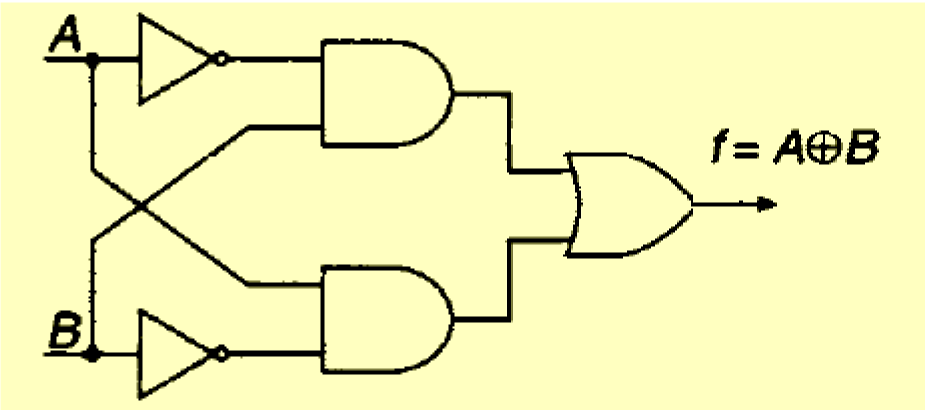


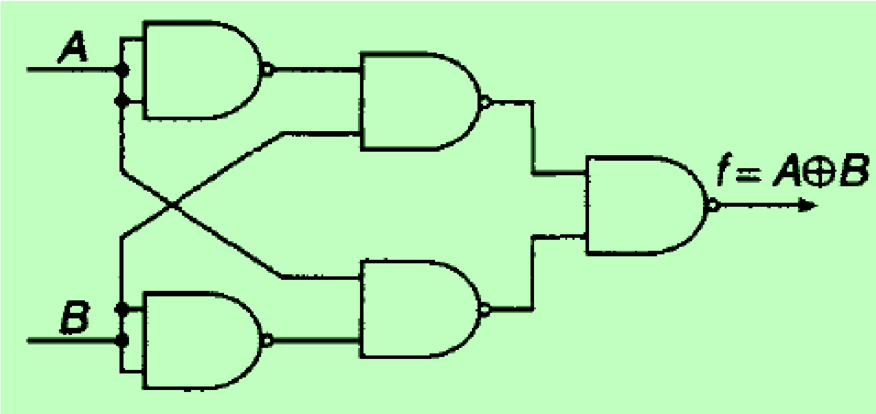


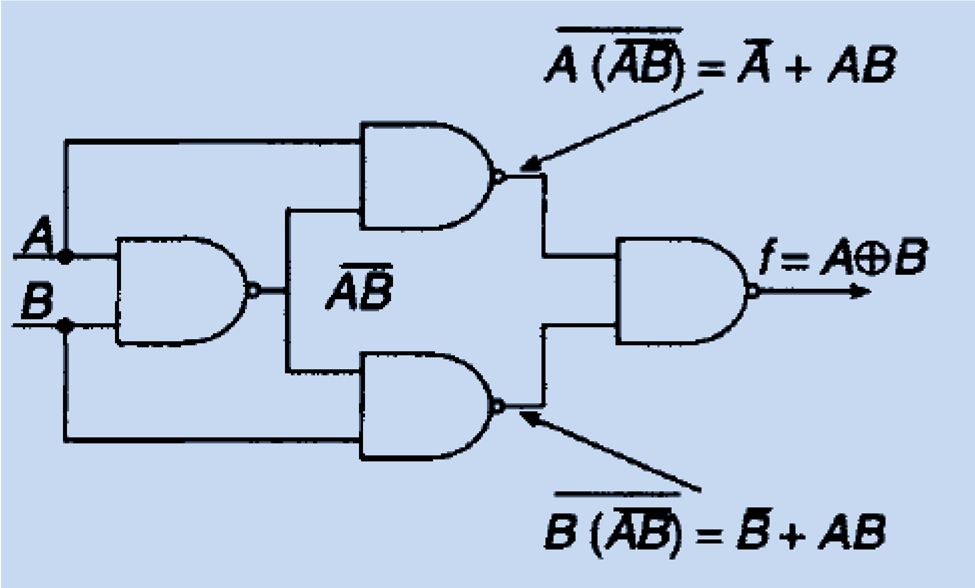
There is a one-to-one mapping between AND-OR network and NAND network

Often there is lot of further optimization that can be done

Consider implementation of XOR gate $f = \overline{A}.B + A.\overline{B}$

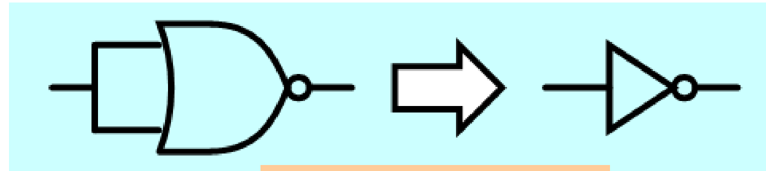






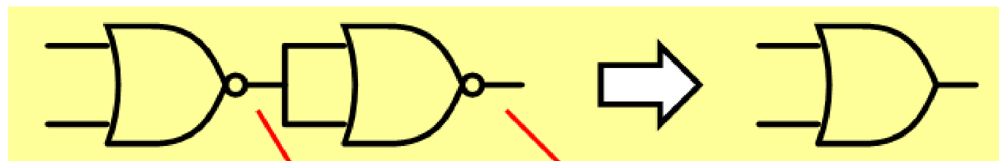
Implementation using only NOR gates

NOR to Inverter



$$\overline{x + x} = \overline{x}$$

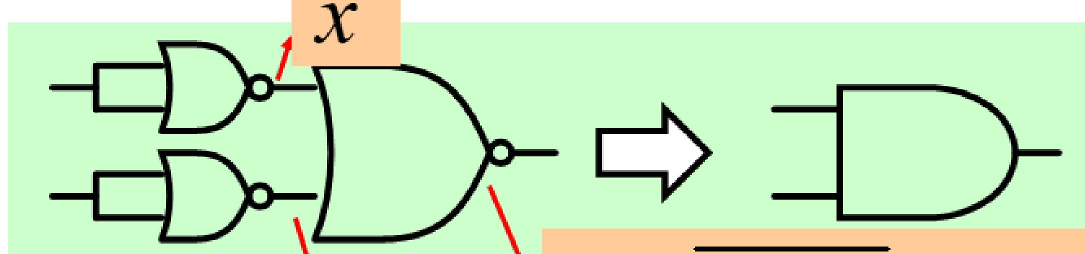
NOR to OR



$$\overline{x + y} \qquad x + y$$

NOR to AND

$$\overline{x}$$

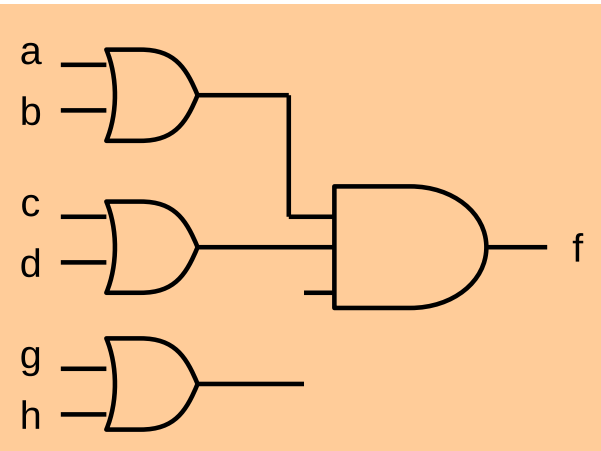


$$\overline{y}$$

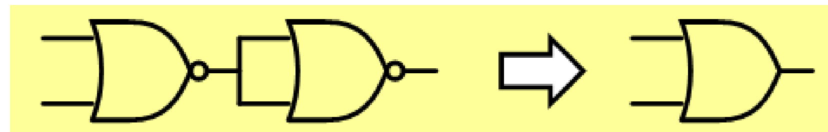
$$f = \overline{\overline{x} + \overline{y}} = x . y$$

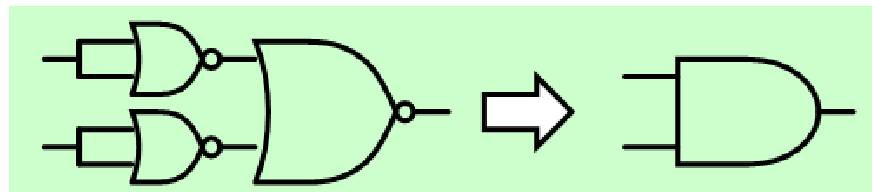
Implementation using only NOR gates

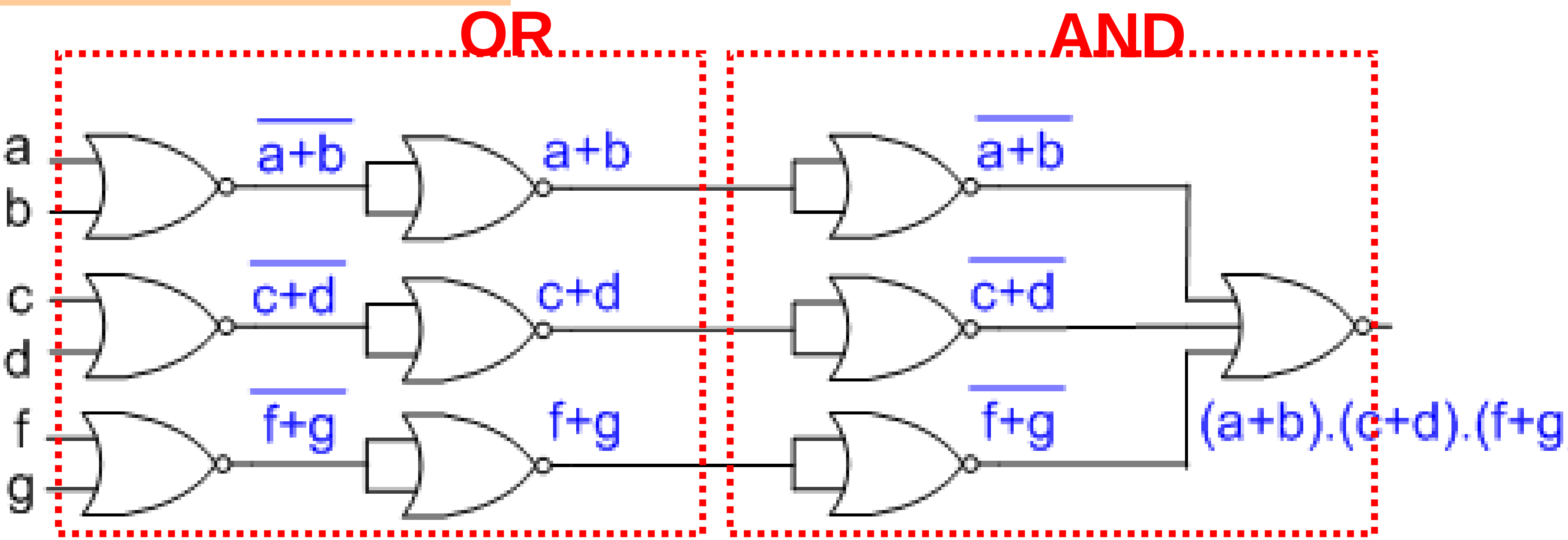
To implement using NOR gates, it is easiest to start with minimized Boolean expression in POS form



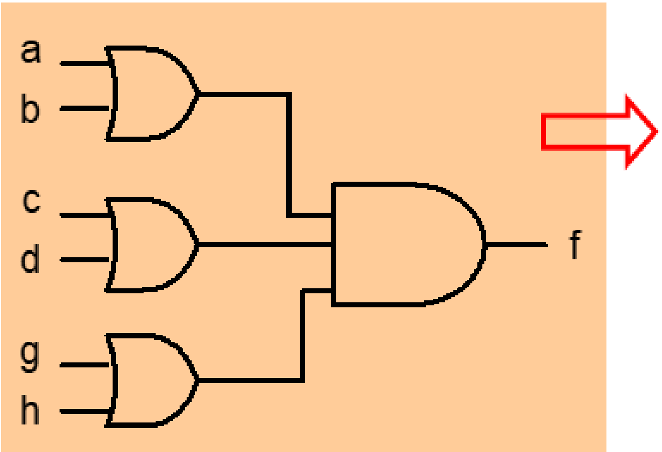
$$f = (a+b).(c+d).(f+g)$$

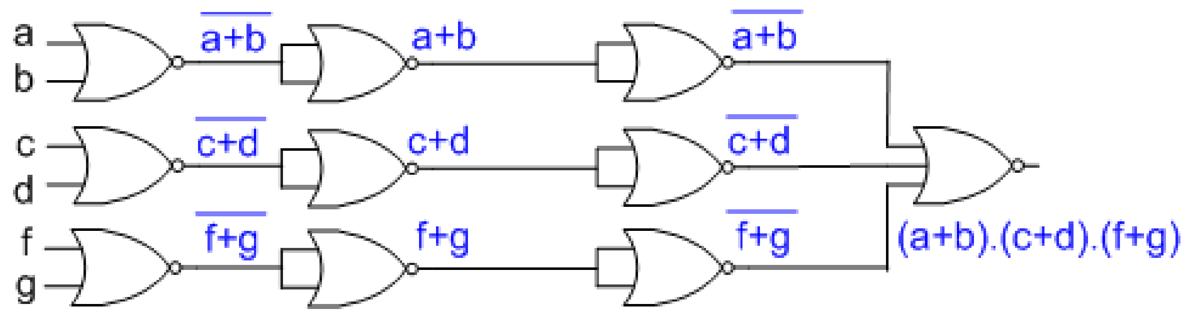


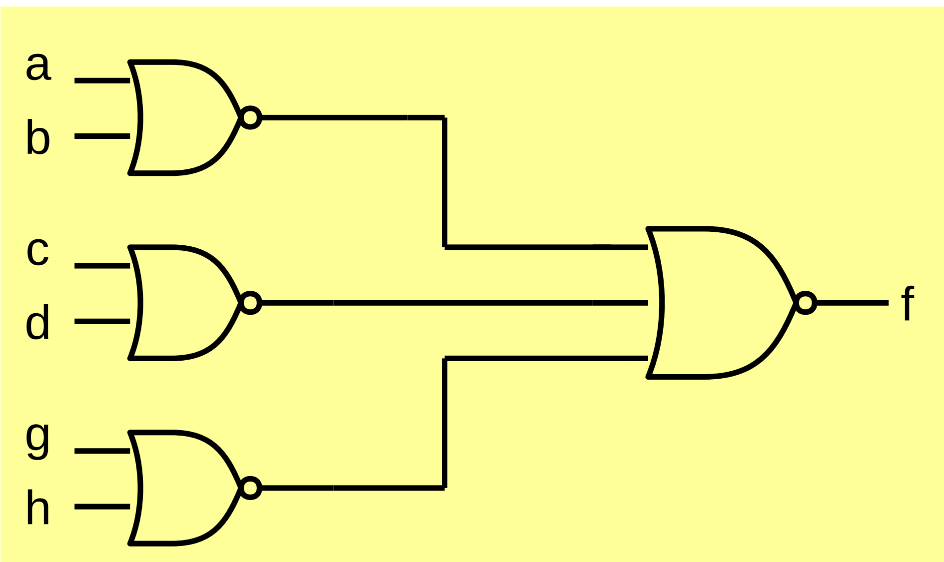




$$f = (a+b).(c+d).(f+g)$$







There is a one-to-one mapping between OR-AND network and NOR network

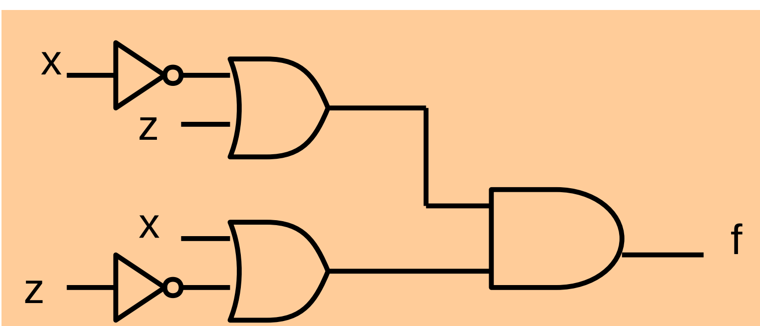
To implement SoP expression using NOR gates, determine first the corresponding PoS expression and then follow the procedure outlined earlier

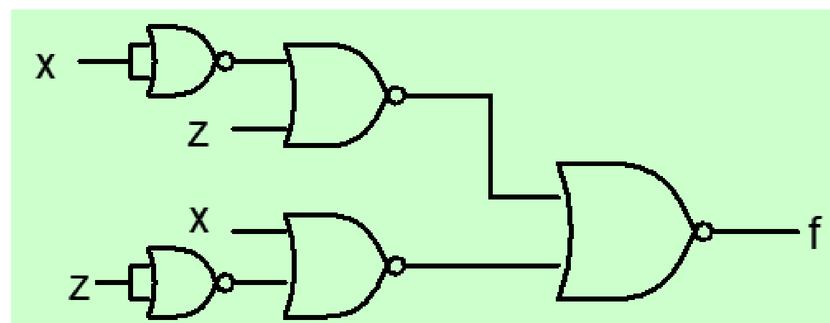
Implement $f(x,y,z) = \bar{x}.\bar{z} + x.z$ using NOR gates

$\Downarrow$

x \ yz	00	01	11	10
0	1	0	0	1
1	0	1	1	0

$$\Rightarrow f = (\bar{x} + z).(x + \bar{z})$$





Similarly PoS expression can be implemented as NAND network by first converting it to SoP expression and then following the procedure outlined earlier

How do we get the chocolate?