ESc201 : Introduction to Electronics

Number System

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Numbers

Every number system is associated with a base or radix

A positional notation is commonly used to express numbers

$$(a_5 a_4 a_3 a_2 a_1 a_0)_r = a_5 r^5 + a_4 r^4 + a_3 r^3 + a_2 r^2 + a_1 r^1 + a_0 r^0$$

The decimal system has a base of 10 and uses symbols (0,1,2,3,4,5,6,7,8,9) to represent numbers

$$(2009)_{10} = 2 \times 10^3 + 0 \times 10^2 + 0 \times 10^1 + 9 \times 10^0$$

$$(123.24)_{10} = 1 \times 10^2 + 2 \times 10^1 + 3 \times 10^0 + 2 \times 10^{-1} + 4 \times 10^{-2}$$

Numbers

An octal number system has a base 8 and uses symbols (0,1,2,3,4,5,6,7)

$$(2007)_8 = 2\times 8^3 + 0\times 8^2 + 0\times 8^1 + 7\times 8^0$$

What decimal number does it represent?

$$(2007)_8 = 2\times 512 + 0\times 64 + 0\times 8^1 + 7\times 8^0 = 1033$$

$$(2007)_8 = (1033)_{10}$$

Numbers

A hexadecimal system has a base of 16

$$(2BC9)_{16} = 2\times16^3 + B\times16^2 + C\times16^1 + 9\times16^0$$

How do we convert it into decimal number?

$$(2BC9)_{16} = 2\times4096 + 11\times256 + 12\times16^1 + 9\times16^0 = 11209$$

$$(2BC9)_{16} = (11209)_{10}$$

Number	Symbol
0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	A
11	B
12	C
13	D
14	E
15	F

A Binary system has a base 2 and uses only two symbols 0, 1 to represent all the numbers

$$(1101)_2 = 1\times 2^3 + 1\times 2^2 + 0\times 2^1 + 1\times 2^0$$

Which decimal number does this correspond to ?

$$(1101)_2 = 1\times 8 + 1\times 4 + 0\times 2^1 + 1\times 2^0 = 13$$

1 1 0 1 . 1 0 0 1

2^3 2^2 2^1 2^0 2^{-1} 2^{-2} 2^{-3} 2^{-4}

13. 5625

2^0	1
2^1	2
2^2	4
2^3	8
2^4	16
2^5	32
2^6	64
2^7	128
2^8	256
2^9	512
2^{10}	1024(K)
2^{20}	1048576(M)

2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}	2^{-6}
0.5	0.25	0.125	0.0625	0.03125	0.015625

Developing Fluency with Binary Numbers

1 1 0 0 1 = ?	25
1100001 = ?	64+32+1=97
0.101 = ?	0.5+0.125=0.625
11.001 = ?	3+0.125=3.125

Converting decimal to binary number

Convert 45 to binary number

$$(45)_{10} = b_n b_{n-1} \ldots\ldots b_0$$

$$45 = b_n 2^n + b_{n-1} 2^{n-1} \ldots\ldots + b_1 2^1 + b_0$$

Divide both sides by 2

$$\frac{45}{2} = 22.5 = b_n 2^{n-1} + b_{n-1} 2^{n-2} \ldots\ldots + b_1 2^0 + b_0 \times 0.5$$

$$22 + 0.5 = b_n 2^{n-1} + b_{n-1} 2^{n-2} \ldots\ldots + b_1 2^0 + b_0 \times 0.5$$

$$\Rightarrow \quad b_0 = 1$$

$$22+0.5 = b_n 2^{n-1} + b_{n-1} 2^{n-2} \ldots\ldots + b_1 2^0 + b_0 \times 0.5 \quad \Rightarrow \ b_0 = 1$$

$$22 = b_n 2^{n-1} + b_{n-1} 2^{n-2} \ldots\ldots + b_2 2^1 + b_1 2^0$$

Divide both sides by 2

$$\frac{22}{2} = 11 = b_n 2^{n-2} + b_{n-1} 2^{n-3} \ldots\ldots + b_2 2^0 + b_1 \times 0.5 \quad \Rightarrow \ b_1 = 0$$

$$11 = b_n 2^{n-2} + b_{n-1} 2^{n-3} \ldots\ldots + b_3 2^1 + b_2 2^0$$

$$5.5 = b_n 2^{n-3} + b_{n-1} 2^{n-4} \ldots\ldots + b_3 2^0 + 0.5 b_2 \quad \Rightarrow \ b_2 = 1$$

$$5 = b_n 2^{n-3} + b_{n-1} 2^{n-4} \ldots\ldots + b_4 2^1 + b_3 2^0$$

$$5 = b_n 2^{n-3} + b_{n-1} 2^{n-4} \ldots\ldots + b_4 2^1 + b_3 2^0$$

$$2.5 = b_n 2^{n-4} + b_{n-1} 2^{n-5} \ldots\ldots + b_4 2^0 + 0.5 b_3$$ $$\Rightarrow \; b_3 = 1$$

$$2 = b_n 2^{n-4} + b_{n-1} 2^{n-5} \ldots\ldots + b_5 2^1 + b_4 2^0$$

$$1 = b_n 2^{n-5} + b_{n-1} 2^{n-6} \ldots\ldots + b_5 2^0 + 0.5 b_4$$ $$\Rightarrow \; b_4 = 0$$

$$\Rightarrow \; b_5 = 1$$

$$(45)_{10} \;= b_5 b_4 b_3 b_2 b_1 b_0 \;= 101101$$

Converting decimal to binary number

Method of successive division by 2

45	remainder
22	1
11	0
5	1
2	1
1	0
0	1

45 = 1 0 1 1 0 1

44 = 1 0 1 1 0 0

Convert $(153)_{10}$ to octal number system

$$(153)_{10} = (b_n b_{n-1} \ldots\ldots b_0)_8$$

$$(153)_{10} = b_n 8^n + b_{n-1} 8^{n-1} \ldots\ldots b_1 8^1 + b_0$$

Divide both sides by 8

$$\frac{153}{8} = 19.125 = b_n 8^{n-1} + b_{n-1} 8^{n-2} \ldots\ldots b_1 8^0 + \frac{b_0}{8}$$

$$\Rightarrow \frac{b_0}{8} = 0.125$$

$$\Rightarrow b_0 = 1$$

153	remainder
19	1
2	3
0	2

153 = $(231)_8$

Converting decimal to binary number

Convert $(0.35)_{10}$ to binary number

$$(0.35)_{10} = 0.b_{-1}b_{-2}b_{-3}\ldots\ldots b_{-n}$$

$$0.35 = 0 + b_{-1}2^{-1} + b_{-2}2^{-2} + \ldots\ldots b_{-n}2^{-n}$$

How do we find the b_{-1} b_{-2} …coefficients?

Multiply both sides by 2

$$0.7 = b_{-1} + b_{-2}2^{-1} + \ldots\ldots b_{-n}2^{-n+1} \quad \Rightarrow \; b_{-1} = 0$$

$$0.7 = b_{-2}2^{-1} + b_{-3}2^{-2} + \ldots\ldots b_{-n}2^{-n+1}$$

$$0.7 = b_{-2}2^{-1} + b_{-3}2^{-2} + \ldots\ldots b_{-n}2^{-n+1}$$

Multiply both sides by 2

$$1.4 = b_{-2} + b_{-3}2^{-1} + \ldots\ldots b_{-n}2^{-n+2}$$

$$\Rightarrow\ b_{-2} = 1$$

Note that ½+1/4+1/8+……≤1

$$0.4 = b_{-3}2^{-1} + b_{-4}2^{-2}\ldots\ldots b_{-n}2^{-n+2}$$

$$0.8 = b_{-3} + b_{-4}2^{-1}\ldots\ldots b_{-n}2^{-n+3}$$

$$\Rightarrow\ b_{-3} = 0$$

Converting decimal to binary number

0.125 = ?

0 .	125	
		x2
0 .	25	
		x2
0 .	5	
		x2
1.	0	

$0.125 = (.001)_2$

0.8125 = ?

0 .	8125	
		x2
1 .	625	
		x2
1 .	25	
		x2
0.	5	
		x2
1.	0	

$0.8125 = (.1101)_2$

Binary numbers

Most significant bit or MSB

Least significant bit or LSB

1011000111

Binary digit = bit

This is a 10 bit number

N-bit binary number can represent numbers from 0 to 2^N -1

decimal	2bit	3bit	4bit	5bit
0	00	000	0000	00000
1	01	001	0001	00001
2	10	010	0010	00010
3	11	011	0011	00011
4		100	0100	00100
5		101	0101	00101
6		110	0110	00110
7		111	0111	00111
8			1000	01000
9			1001	01001
10			1010	01010
11			1011	01011
12			1100	01100
13			1101	01101
14			1110	01110
15			1111	01111

Converting Binary to Hex and Hex to Binary

$$(b_7 b_6 b_5 b_4 b_3 b_2 b_1 b_0)_b = (h_1, h_0)_{Hex}$$

$$b_7 2^7 + b_6 2^6 + b_5 2^5 + b_4 2^4 + b_3 2^3 + b_2 2^2 b_1 2^1 + b_0$$
$$= h_1 16^1 + h_0$$

$$\underbrace{(b_7 2^3 + b_6 2^2 + b_5 2^1 + b_4)}_{h_1} 2^4 + \underbrace{(b_3 2^3 + b_2 2^2 b_1 2^1 + b_0)}_{h_0} = h_1 16^1 + h_0$$

$$(10110011)_b = (1011)(0011) = (B3)_{Hex}$$

$$(110011)_b = (11)(0011) = (33)_{Hex}$$

$$(EC)_{Hex} = (1110)(1100) = (11101100)_b$$

Number	Symbol
0(0000)	0
1(0001)	1
2(0010)	2
3(0011)	3
4(0100)	4
5(0101)	5
6(0110)	6
7(0111)	7
8(1000)	8
9(1001)	9
10(1010)	A
11(1011)	B
12(1100)	C
13(1101)	D
14(1110)	E
15(1111)	F

Binary Addition/Subtraction

```
  0        1      0        1        1
  0        0      1        1        1
 ---      ---    ---      ---       1
  0        1      1       1 0      ---
                                   1 1
```

```
   1 0 1          1 1 0 1
   1 1 0        + 1 1 1 0
 -------        ---------
 1 0 1 1        1 1 0 1 1
```

Complement of a number

Decimal system: → 9's complement

→ 10's complement

9's complement of n-digit number x is $10^n - 1 - x$

10's complement of n-digit number x is $10^n - x$

9's complement of 85 ? $10^2 - 1 - 85$ $99 - 85 = 14$

$$\text{9's complement of } 123 = 999 - 123 = 876$$

$$\text{10's complement of } 123 = \text{9's complement of } 123 + 1 = 877$$

Complement of a binary number

Binary system: 1's complement, 2's complement

1's complement of n-bit number x is $2^n - 1 - x$

2's complement of n-bit number x is $2^n - x$

1's complement of 1011 ? $2^4 - 1 - 1011$ $1111 - 1011 = 0100$

1's complement is simply obtained by flipping a bit (changing 1 to 0 and 0 to 1)

1's complement of 1001101 = ?

0110010

$$2\text{'s complement of } 1010 = 1\text{'s complement of } 1010+1$$
$$= 0101 + 1 = 0110$$

$$2\text{'s complement of } 110010 =$$

Leave all least significant 0's as they are, leave first 1 unchanged and then flip all subsequent bits

$$001110$$

$$1011 \rightarrow 0101$$

$$101101100 \rightarrow 010010100$$