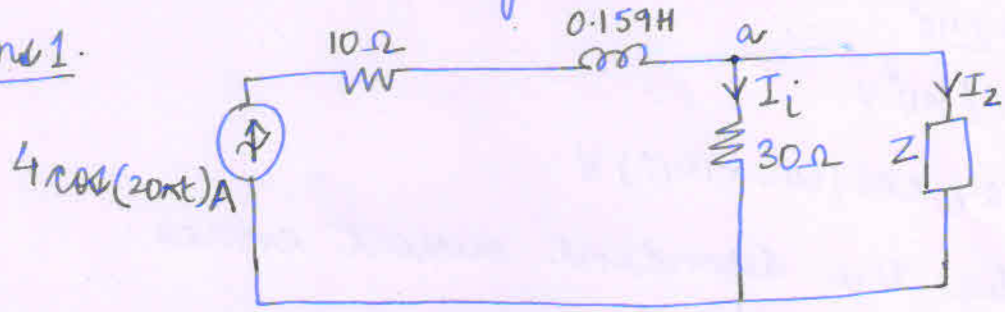
ESC201

Assignment 4 Solutions

Ans 1.



$I_0 = (2.56 + j1.92) A$

Applying KCL at node A:

$$4\cos(20\pi t) = I_i + I_Z$$

or $I_Z = 4 - (2.56 + j1.92)\ A$

$= (1.44 - j1.92)\ A$

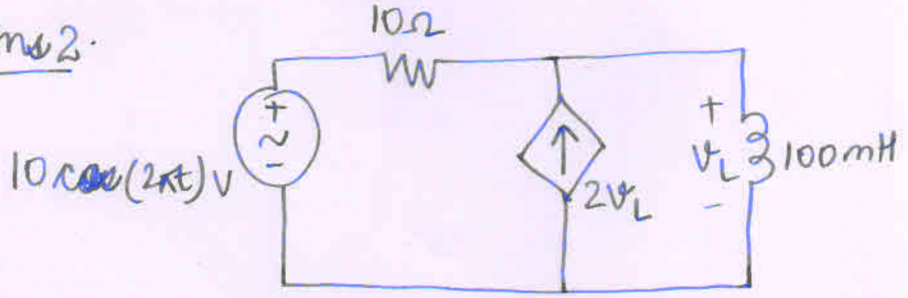
Since 30Ω and Z are in parallel,

$I_Z \times Z = I_i \times 30$

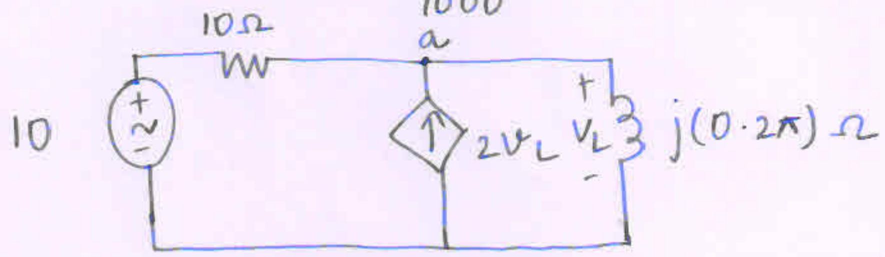
$\therefore Z = \dfrac{I_i \times 30}{I_Z} = \dfrac{(2.56 + j1.92) \times 30}{(1.44 - j1.92)}\ \Omega$

$= \dfrac{30 \times 3.2 \angle 36.87^\circ}{2.4 \angle -53.13^\circ}\ \Omega = 40 \angle 90^\circ\ \Omega = j40\ \Omega$

Ans 2.



$\omega = 2\pi \Rightarrow X_L = j \dfrac{\times 2\pi \times 100}{1000}\ \Omega = j(0.2\pi)\ \Omega$



Applying nodal analysis at node a:

$$\frac{v_L - 10}{10} - 2v_L + \frac{v_L}{j(0.2\pi)} = 0$$

$$\Rightarrow \left(\frac{v_L}{10} - 2v_L - \frac{j}{0.2\pi} v_L\right) = 1$$

$$\Rightarrow v_L = \frac{-1\angle 0^\circ}{2.477\angle 40^\circ} \text{ V}$$

$$\Rightarrow v_L = 0.404\angle 40^\circ \text{ V}$$

$$\text{or } v_L = 0.404 \cos(\omega t + 140^\circ) \text{ V}$$

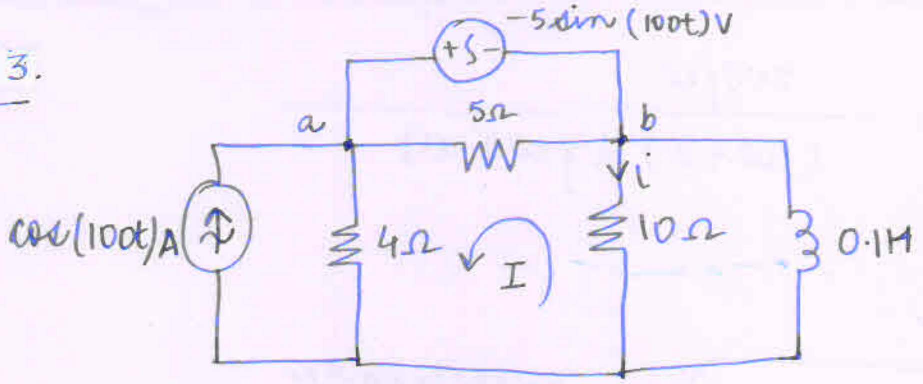
Power supplied by the dependent current source:

$$P = \frac{1}{2} \times 2v_L \times v_L$$

$$\Rightarrow P = v_L^2$$

$$\Rightarrow P = 163\text{mW}$$

Ans 3.



Current through 5Ω resistor $= i_{5\Omega} = \frac{-5j}{5} A = -j\ A$

Applying KCL at node a,

$$-j + 1 = i_{4\Omega} + i_{5\Omega} \qquad (1)$$

Applying KCL at node b,

$$i_{10\Omega} + i_{0.1H} - j = i_{5\Omega} \qquad (2)$$

from (1) and (2),

$$-j + 1 - i_{4\Omega} = i_{10\Omega} + i_{0.1H} - j$$

$$1 = i_{4\Omega} + i_{10\Omega} + i_{0.1H} \qquad (3)$$

Applying KVL in loop I

$$10\, i_{10\Omega} - 5j = 4\, i_{4\Omega}$$

$$\Rightarrow i_{4\Omega} = \frac{10 i_{10\Omega} - 5j}{4} \qquad (4)$$

From (3) and (4),

$$1 = \frac{7}{2} i_{10\Omega} - \frac{5}{4} j + i_{0.1H} \qquad (5)$$

Also, $i_{0.1H} = \left(\frac{10}{10 + j10}\right)(i_{10\Omega} + i_{0.1H}) = \frac{i_{10\Omega}}{1+j} + \frac{i_{0.1H}}{1+j}$

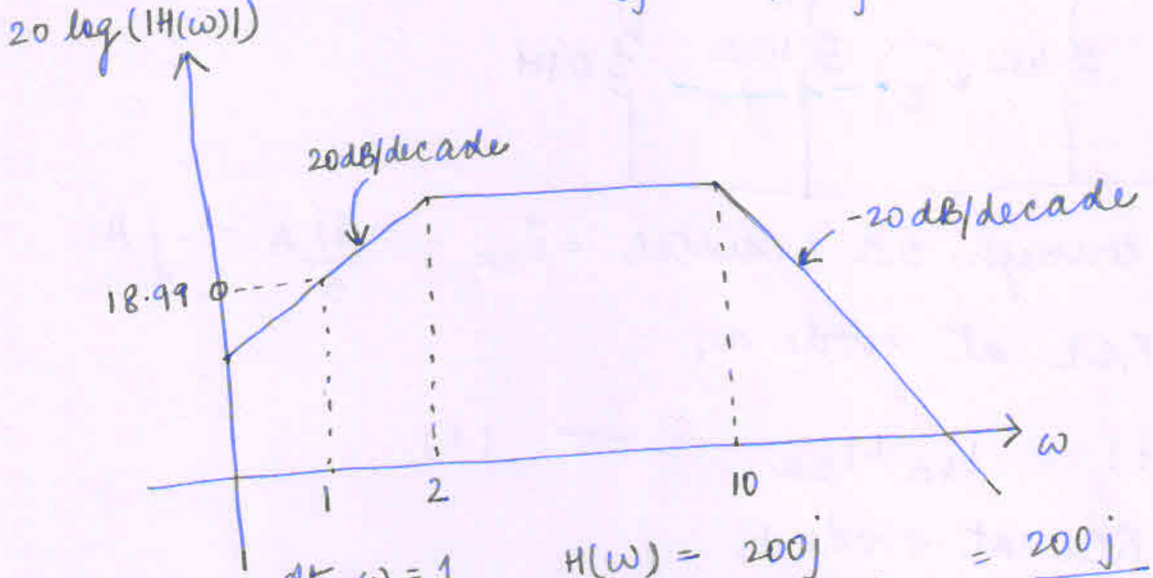
$$\Rightarrow i_{0.1H} = -j\, i_{10\Omega}\ A \qquad (6)$$

Substituting (6) in (5),

$$1 = 3.5\, i_{10\Omega} - 1.25j - j\, i_{10\Omega}$$

$$\Rightarrow i_{10\Omega} = \frac{1 + j2.5}{3.5 - j} A = 0.44 \angle 67.28°$$

Ans 4 (a) $H(\omega) = \dfrac{200j\omega}{(j\omega+2)(j\omega+10)}$

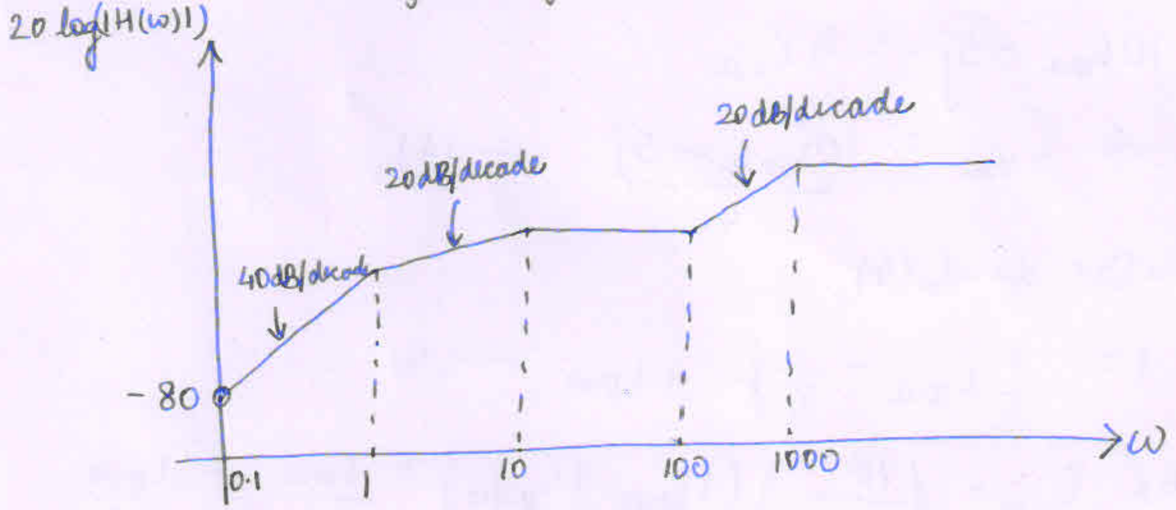


At $\omega = 1$, $H(\omega) = \dfrac{200j}{(2+j)(10+j)} = \dfrac{200j}{19+j12}$

$$|H(\omega)| = \frac{200}{\sqrt{19^2+12^2}} = 8.9$$

$$20\log|H(\omega)| = 20\log(8.9) = 18.99$$

(b) $H(j\omega) = \dfrac{(j\omega)^2 (j\omega+100)}{(j\omega+1)(j\omega+10)(j\omega+1000)}$



At $\omega = 0.1$,

$$|H(\omega)| = \left|\frac{(j(0.1))^2 (j0.1+100)}{(j0.1+1)(j0.1+10)(j0.1+1000)}\right| = 10^{-4}$$

$$20\log(|H(\omega)|) = -80\log(10) = -80$$

Ans 5. Transfer function:

$$H(\omega) = \frac{A(j\omega)^2}{\left(1+\frac{j\omega}{0.1}\right)\left(1+j\omega\right)\left(1+\frac{j\omega}{10}\right)\left(1+\frac{j\omega}{100}\right)}$$

$$H(\omega=1) = 17\ dB$$

$$\Rightarrow 17 = 20\log\left|\frac{A}{(1+j10)(1+j)(1+0.1j)(1+0.01j)}\right|$$

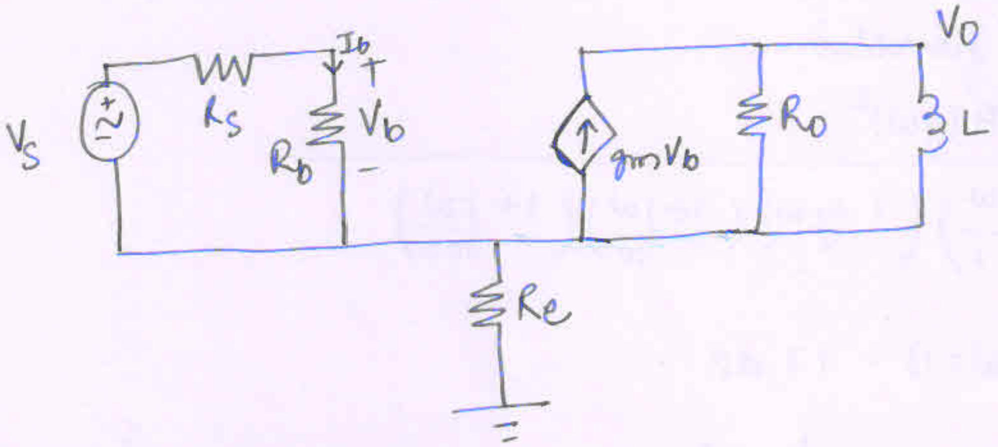
$$= 20\log\left(\frac{A}{10\times1\times1\times1}\right)$$

$$\Rightarrow 20\log A = 17+20 = 37$$

$$\therefore\quad A = 10^{37/20} = 70.79$$

$$\therefore H(\omega) = \frac{7079\,(j\omega)^2}{(j\omega+0.1)(j\omega+1)(j\omega+10)(j\omega+100)}$$

Ans 6.



$$V_b = \frac{R_b \cdot V_S}{R_b + R_S}$$

$$I_b = \frac{V_b}{R_b} = \frac{V_S}{R_S + R_b}$$

Applying nodal

$$-g_m V_b + \frac{V_0}{R_0} + \frac{V_0}{j\omega L} = 0$$

$$\therefore V_0\left(\frac{1}{R_0} + \frac{1}{j\omega L}\right) = g_m V_b$$

$$\Rightarrow V_0\left(\frac{j\omega L + R_0}{j\omega L R_0}\right) = g_m V_b$$

$$\Rightarrow V_0 = \frac{j\omega g_m R_0 L}{j\omega L + R_0} \cdot V_b$$

$$\Rightarrow V_0 = \frac{j\omega L R_0 g_m}{j\omega L + R_0} \cdot \frac{R_b V_s}{R_b + R_S}$$

$$\therefore \frac{V_0}{V_S} = \frac{j\omega g_m R_0 R_b L}{(j\omega L + R_0)(R_b + R_S)}$$