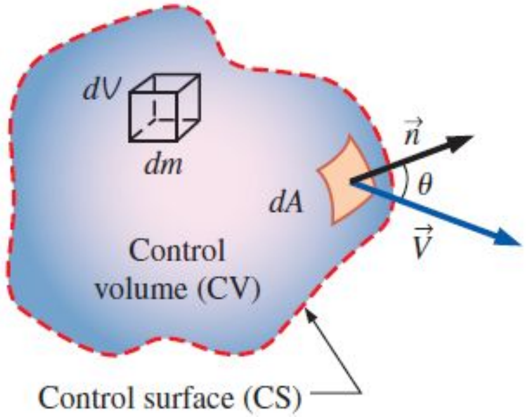


FIGURE 5–6
The differential control volume dV and the differential control surface dA used in the derivation of the conservation of mass relation.

Total mass within the CV: $$m_{CV} = \int_{CV} \rho \, dV$$

Rate of change of mass within the CV: $$\frac{dm_{CV}}{dt} = \frac{d}{dt}\int_{CV} \rho \, dV$$

Normal component of velocity: $$V_n = V\cos\theta = \vec{V}\cdot\vec{n}$$

Differential mass flow rate: $$\delta\dot{m} = \rho V_n \, dA = \rho(V\cos\theta)\, dA = \rho(\vec{V}\cdot\vec{n})\, dA$$

Net mass flow rate: $$\dot{m}_{net} = \int_{CS} \delta\dot{m} = \int_{CS} \rho V_n \, dA = \int_{CS} \rho(\vec{V}\cdot\vec{n})\, dA$$

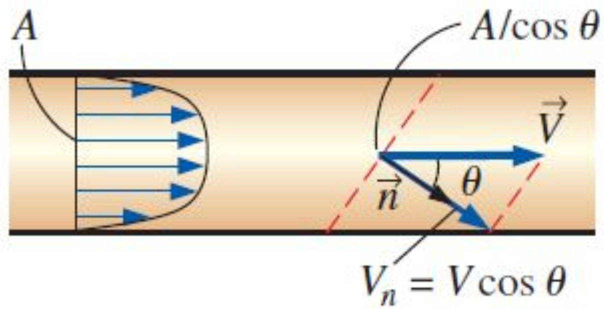
General conservation of mass: $$\frac{d}{dt}\int_{CV} \rho \, dV + \int_{CS} \rho(\vec{V}\cdot\vec{n})\, dA = 0$$

the time rate of change of mass within the control volume plus the net mass flow rate through the control surface is equal to zero.

$$\frac{d}{dt}\int_{CV} \rho \, dV + \sum_{out} \rho \left| V_n \right| dA - + \sum_{in} \rho \left| V_n \right| dA = 0$$

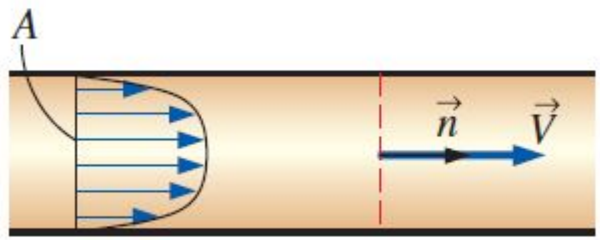
$$\frac{d}{dt}\int_{CV} \rho \, dV = \sum_{in} \dot{m} - \sum_{out} \dot{m} \quad \text{or} \quad \frac{dm_{CV}}{dt} = \sum_{in} \dot{m} - \sum_{out} \dot{m}$$

General conservation of mass in rate form



$$\dot{m} = \rho(V\cos\theta)(A/\cos\theta) = \rho VA$$

(*a*) Control surface *at an angle* to the flow



$$\dot{m} = \rho VA$$

(*b*) Control surface *normal* to the flow

FIGURE 5–7

A control surface should always be selected *normal to the flow* at all locations where it crosses the fluid flow to avoid complications, even though the result is the same.

Mass Balance for Steady-Flow Processes

During a steady-flow process, the total amount of mass contained within a control volume does not change with time (m_{CV} = constant).

Then the conservation of mass principle requires that the total amount of mass entering a control volume equal the total amount of mass leaving it.

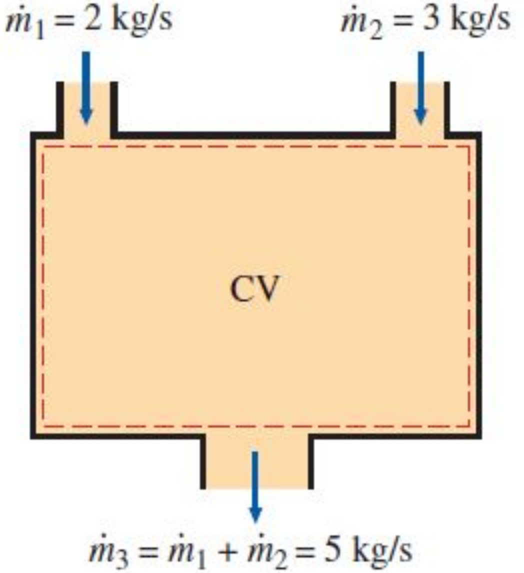


FIGURE 5–8
Conservation of mass principle for a two-inlet–one-outlet steady-flow system.

For steady-flow processes, we are interested in the amount of mass flowing per unit time, that is, *the mass flow rate.*

$$\sum_{\text{in}} \dot{m} = \sum_{\text{out}} \dot{m} \qquad (\text{kg/s})$$

Multiple inlets and exits

$$\dot{m}_1 = \dot{m}_2 \quad \rightarrow \quad \rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

Single stream

Many engineering devices such as nozzles, diffusers, turbines, compressors, and pumps involve a single stream (only one inlet and one outlet).

Special Case: Incompressible Flow

The conservation of mass relations can be simplified even further when the fluid is incompressible, which is usually the case for liquids.

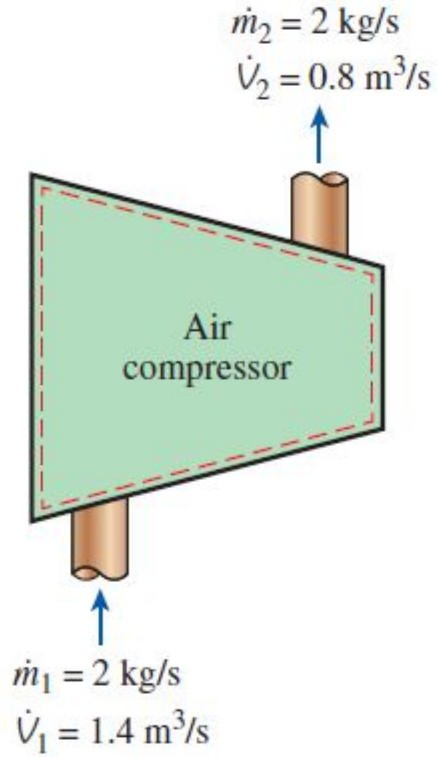


FIGURE 5–9
During a steady-flow process, volume flow rates are not necessarily conserved although mass flow rates are.

$$\sum_{\text{in}} \dot{V} = \sum_{\text{out}} \dot{V} \qquad (\text{m}^3/\text{s})$$

Steady, incompressible

$$\dot{V}_1 = \dot{V}_2 \rightarrow V_1 A_1 = V_2 A_2$$

Steady, incompressible flow (single stream)

There is no such thing as a "conservation of volume" principle.

For steady flow of liquids, the volume flow rates, as well as the mass flow rates, remain constant since liquids are essentially incompressible substances.

FLOW WORK AND THE ENERGY OF A FLOWING FLUID

Flow work, or flow energy: The work (or energy) required to push the mass into or out of the control volume. This work is necessary for maintaining a continuous flow through a control volume.

$$F = PA$$

$$W_{\text{flow}} = FL = PAL = P\mathcal{V} \qquad \text{(kJ)}$$

$$w_{\text{flow}} = Pv \qquad \text{(kJ/kg)}$$

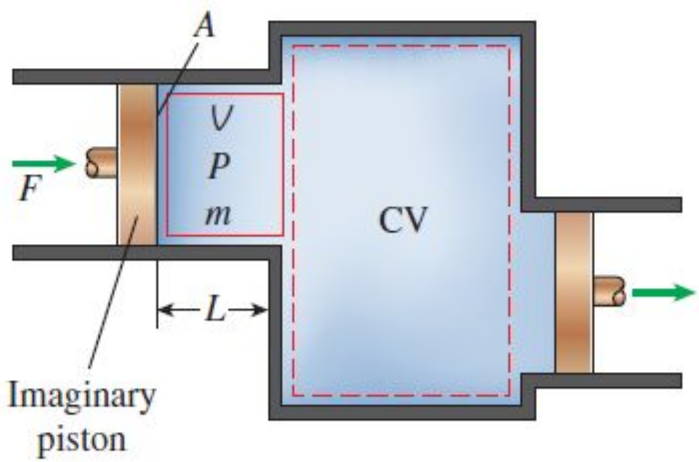


FIGURE 5–12
Schematic for flow work.

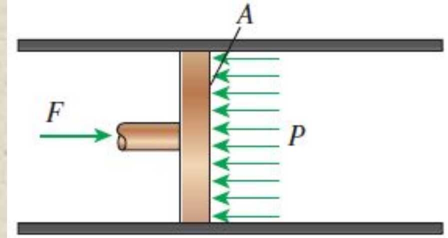


FIGURE 5–13
In the absence of acceleration, the force applied on a fluid by a piston is equal to the force applied on the piston by the fluid.

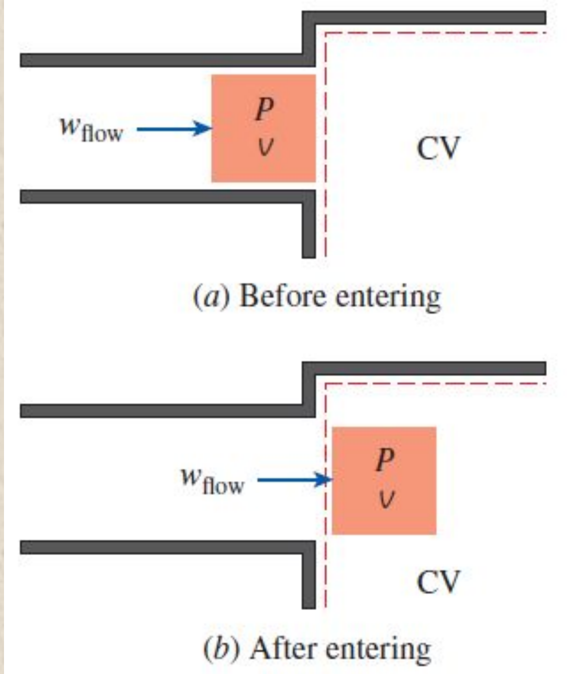


FIGURE 5–14
Flow work is the energy needed to push a fluid into or out of a control volume, and it is equal to Pv.

Total Energy of a Flowing Fluid

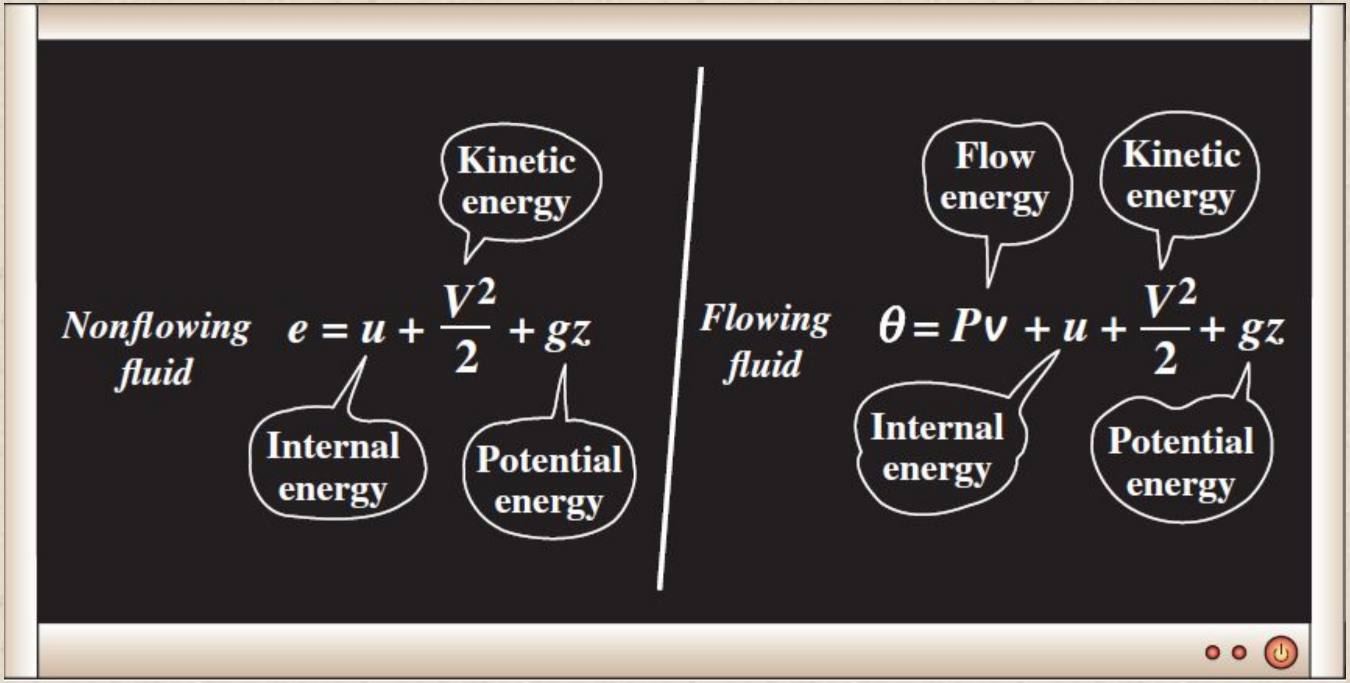
$$e = u + \mathrm{ke} + \mathrm{pe} = u + \frac{V^2}{2} + gz \qquad (\mathrm{kJ/kg})$$

$$\theta = Pv + e = Pv + (u + \mathrm{ke} + \mathrm{pe})$$

$$h = u + Pv$$

$$\theta = h + \mathrm{ke} + \mathrm{pe} = h + \frac{V^2}{2} + gz \qquad (\mathrm{kJ/kg})$$

The flow energy is automatically taken care of by enthalpy. In fact, this is the main reason for defining the property enthalpy.



The total energy consists of three parts for a nonflowing fluid and four parts for a flowing fluid.

Energy Transport by Mass

$$\textit{Amount of energy transport:} \quad E_{\text{mass}} = m\theta = m\left(h + \frac{V^2}{2} + gz\right) \qquad \text{(kJ)}$$

$$\textit{Rate of energy transport:} \quad \dot{E}_{\text{mass}} = \dot{m}\theta = \dot{m}\left(h + \frac{V^2}{2} + gz\right) \qquad \text{(kW)}$$

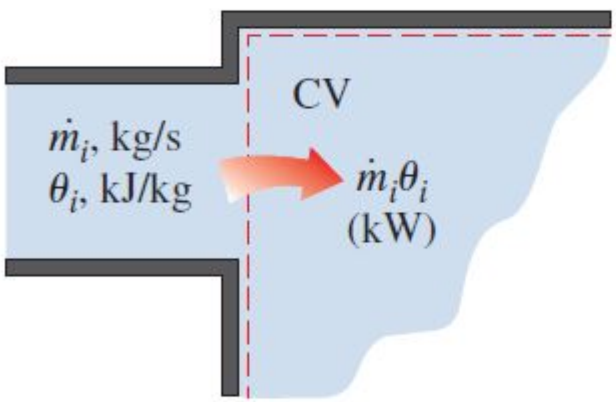


FIGURE 5–16
The product $\dot{m}_i\theta_i$ is the energy transported into control volume by mass per unit time.

When the kinetic and potential energies of a fluid stream are negligible

$$E_{\text{mass}} = mh \qquad \dot{E}_{\text{mass}} = \dot{m}h$$

When the properties of the mass at each inlet or exit change with time as well as over the cross section

$$E_{\text{in,mass}} = \int_{m_i} \theta_i \, \delta m_i = \int_{m_i} \left(h_i + \frac{V_i^2}{2} + gz_i\right) \delta m_i$$

ENERGY ANALYSIS OF STEADY-FLOW SYSTEMS

Steady-flow process: *A process during which a fluid flows through a control volume steadily*.

FIGURE 5–18

Many engineering systems such as power plants operate under steady conditions.

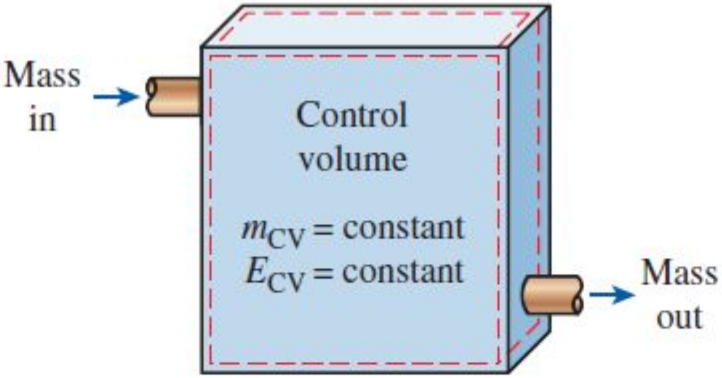


FIGURE 5–19

Under steady-flow conditions, the mass and energy contents of a control volume remain constant.

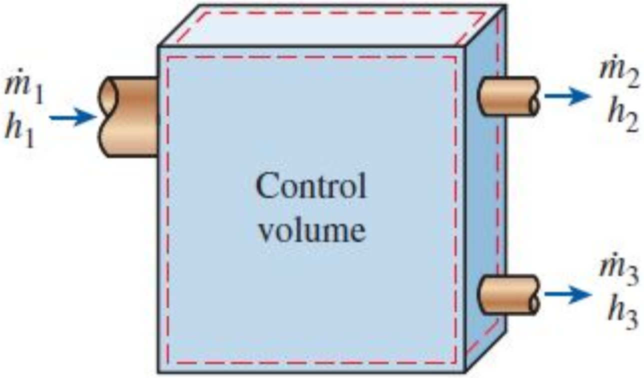


FIGURE 5–20

Under steady-flow conditions, the fluid properties at an inlet or exit remain constant (do not change with time).