07.09.2024

Pairing Property :

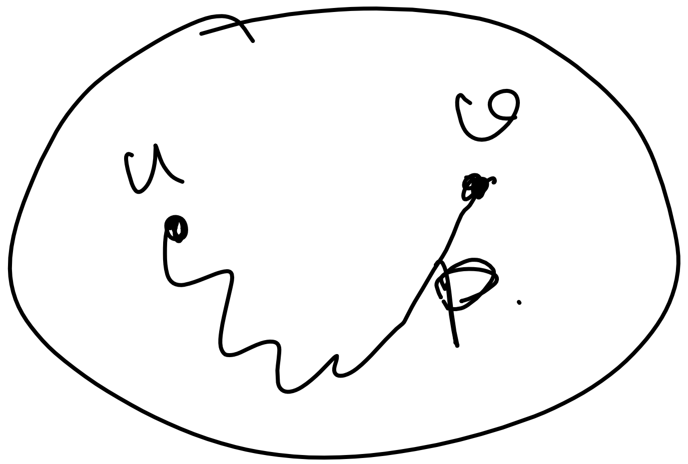
$T_1 \quad \cancel{F} \quad T_2$.

$\{e_1, \ldots, e_\ell\} \in T_1 \setminus T_2$

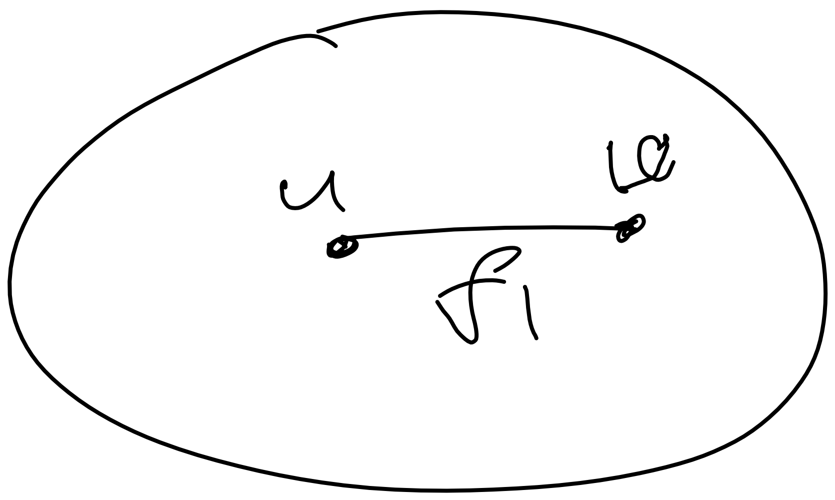
$\{f_1, \ldots, f_\ell\} \in T_2 \setminus T_1$

$\{\langle e_1, f_1 \rangle, \ldots, \langle e_\ell, f_\ell \rangle\}$

$T_1 \setminus \{e_1\} \cup \{f_1\}$. must be a spanning tree.

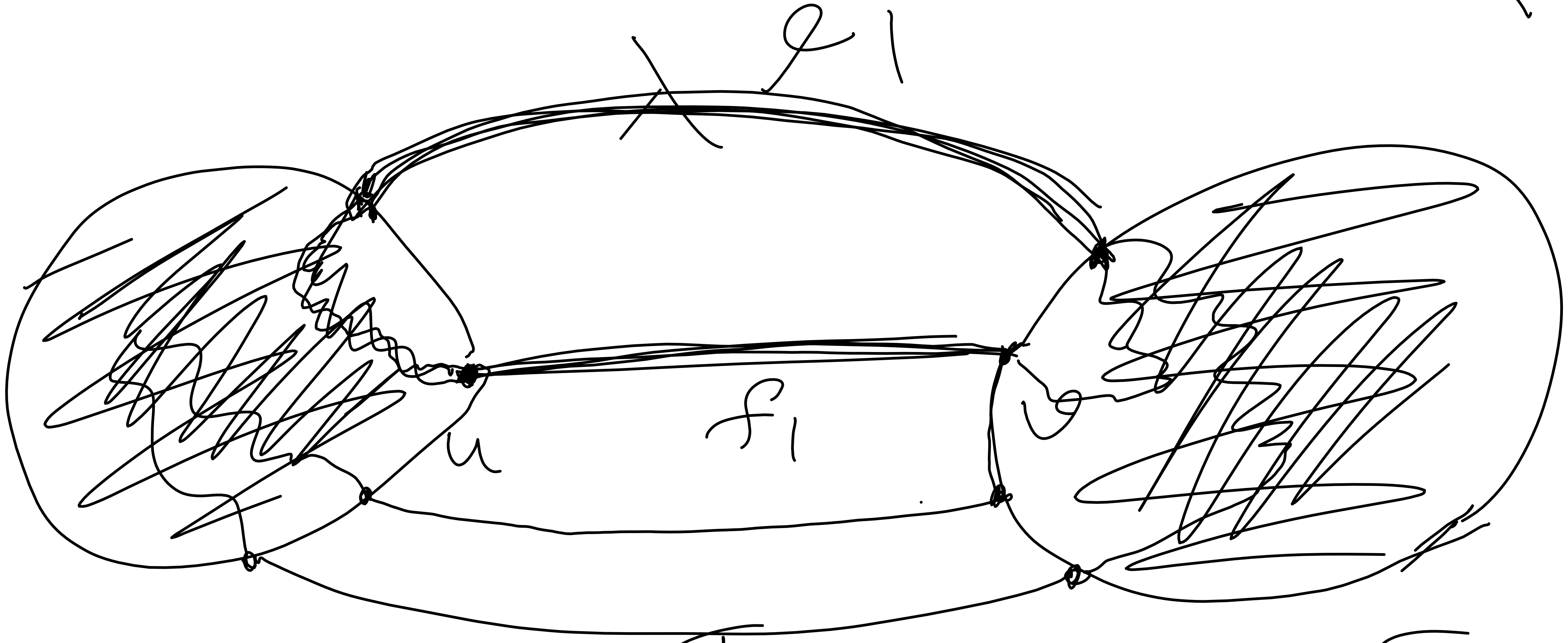


T_1



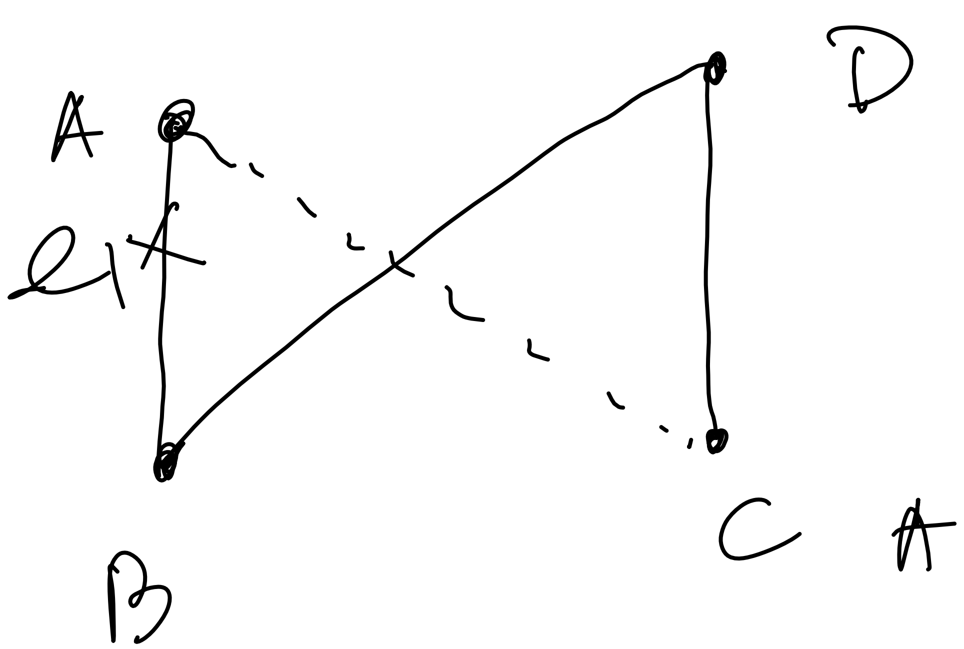
T_2.

$e_1 \in T_1 \setminus T_2$



T_2.

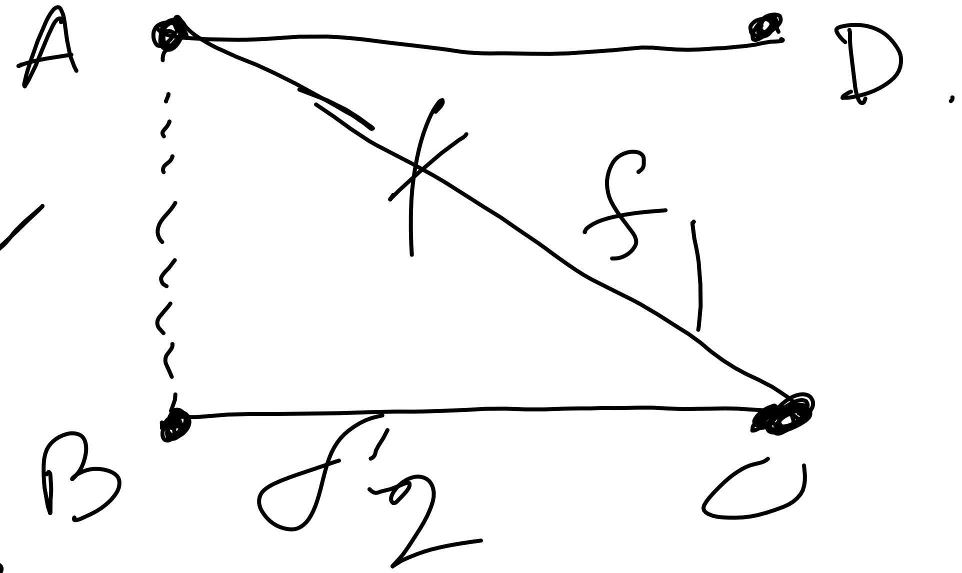
$T_2 \cup \{e_1\}$



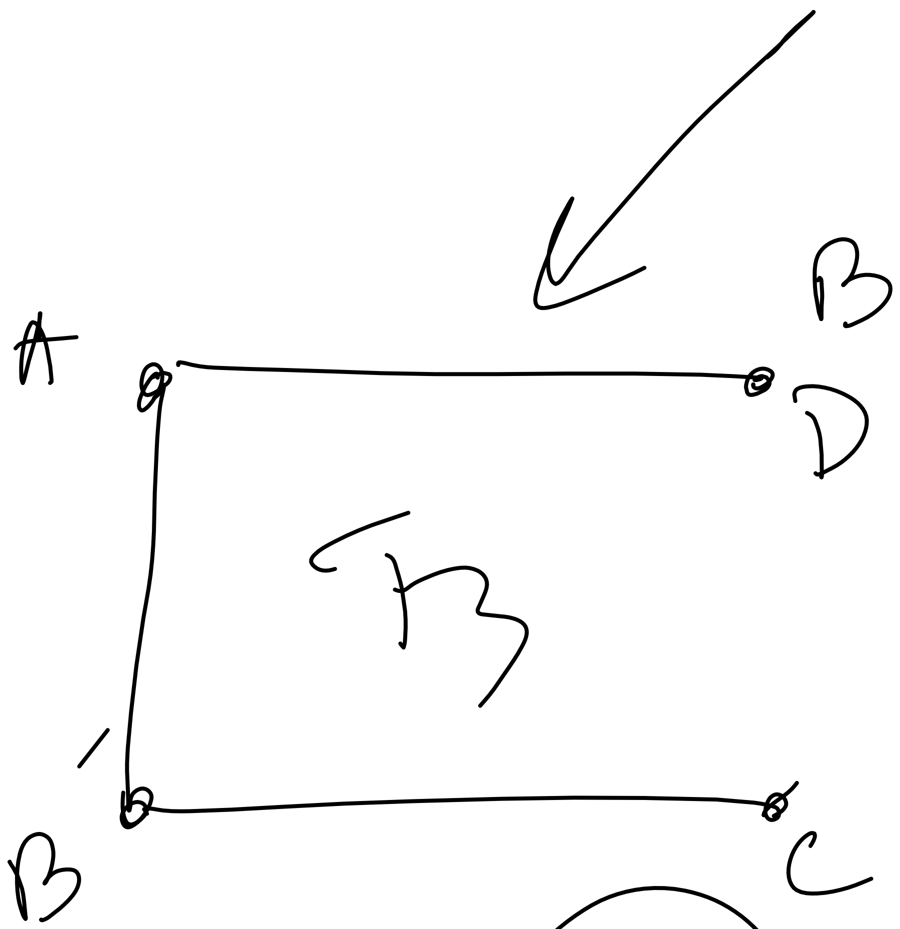
T_1

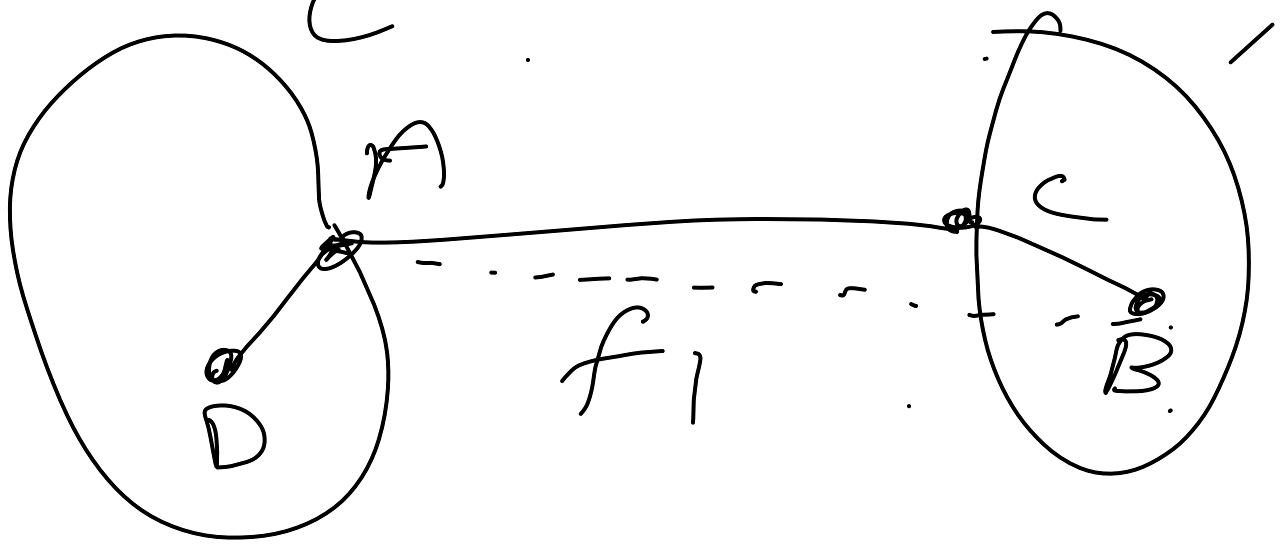
~~ABCD~~

ABDC



T_2

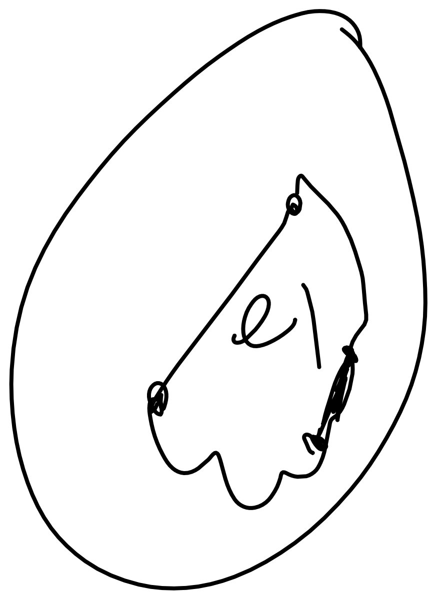




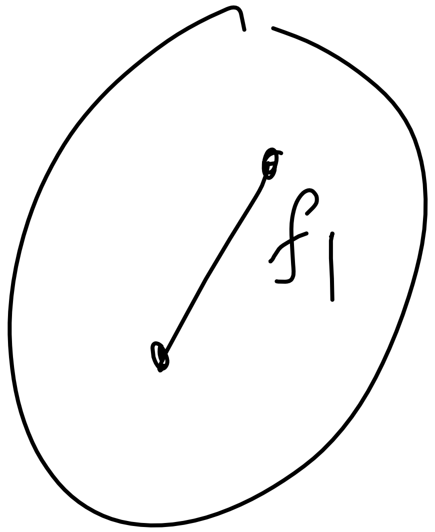
T_2

- $T_1 \setminus \{e_1\} \cup \{f_1\}$
- $T_2 \setminus \{f\} \cup \{e_1\}$

Spanning trees.



T_1



T_2.

$T_1 \cup \{f_1\} \setminus \{e_1\}$ is a spanning tree.

- connected. (cycle edge is gone)
- $(n-1)$ edges.

$T_1 \cup \{f_1\}$.

$T_2 \cup \{e_1\} \setminus \{f_1\}$ is also spanning tree.

- we are taking out a cycle edge $\Rightarrow$ connectivity doesn't change
- $(n-1)$ edges.

$T_3 = T_2 \cup \{e_1\} \setminus \{f_1\} = T_1$

by induction $\exists$ a pairing $(\ell - 1)$ $\longrightarrow$ $((e_2, f_2), \ldots, (e_\ell, f_\ell))$

Correctness of Kruskal Algorithm:

Claim: Let T^* be the spanning tree returned from Kruskal's algorithm.

Let T be any other spanning tree.

$$\underline{wt(T^*)} \; \underline{\leq} \; \underline{wt(T)}.$$

Proof.

$$\{e_1, \ldots, e_\ell\} \in T^* \setminus T$$

$$\{f_1 \ldots f_\ell\} \in T \setminus T^*$$

$$\forall i \quad \boxed{wt(e_i) \leq wt(f_i)}$$

$T^* \setminus \{e_i\} \cup \{f_i\}$. span tree.

suppose for contradiction $wt(f_i) < wt(e_i)$

— — — f_i — — — . e_i . — —

Dynamic Programing (DP).

D&C.	D.P
(1) subproblem are of smaller size.	$n \to n-1$
(2) top-down	$\leftrightarrow$ bottom-up.

Fib(n) , R'Fib(n).

$\boxed{\text{Edit distance}}$ computation

Auto correct.

t t

~~Lect~~ Lec~~th~~ure. → Lecture. → 2

- minimum # operations needed to convert one word into another word.
  - delete
  - insert
  - replace.

Symmetric.

w2: FLAVORY

w1: FAVORITE

Alignment:

||

edit distance

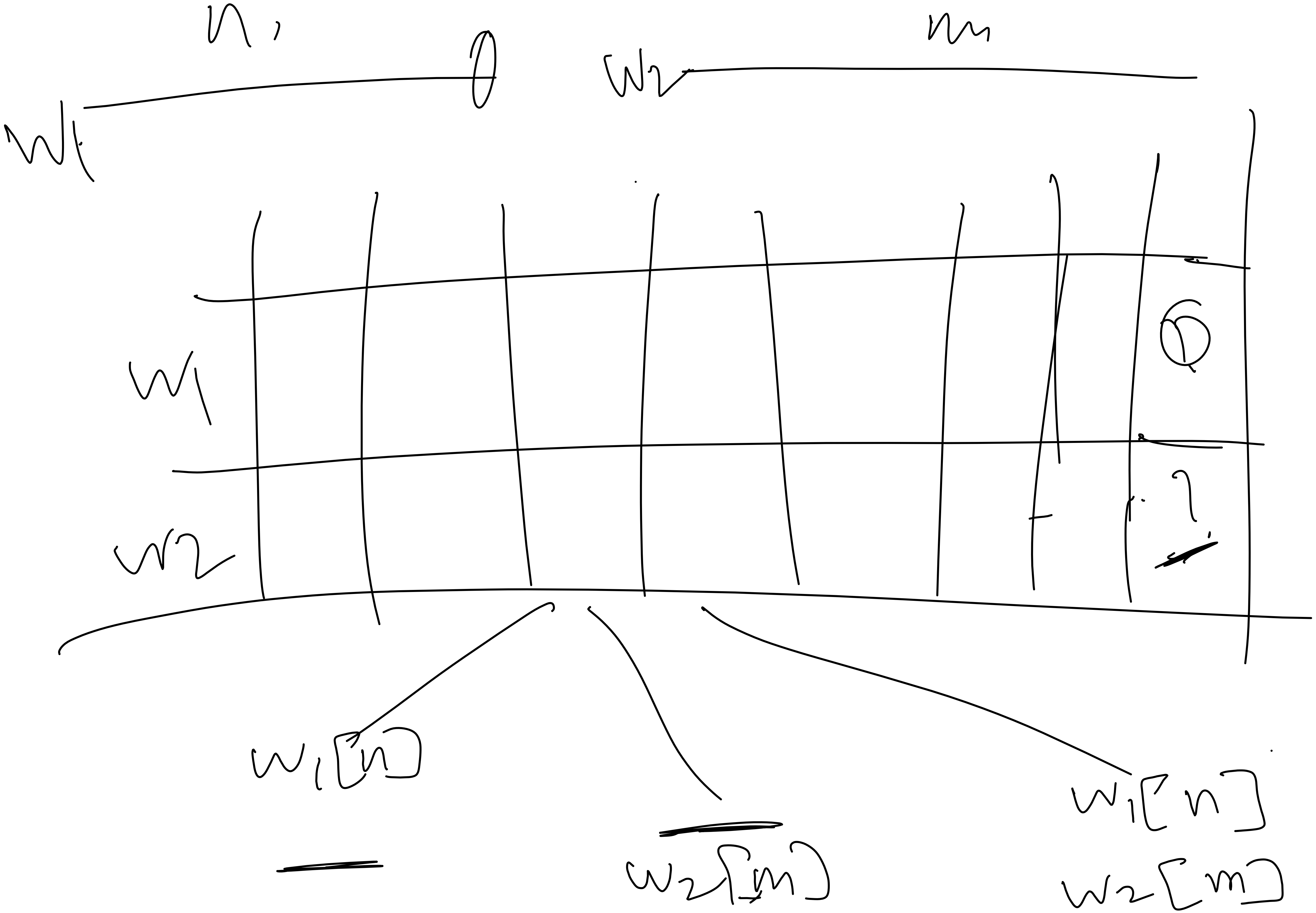
F	_	A	V	O	R	I	T	E
F	L	A	V	O	R	Y	_	_
	1					2	3	

9 |

9

spaces

edit: 3.
dist



n
0
m
W1
W2
W1
W2
w1[n]
w2[m]
w1[n]
w2[m]

1 — n
w_1

1 — m
w_2

1

$w_1[n]$

$ED(n, m)$

$= 1 + ED(n-1, m)$

Edit Distance between

$w_1[1 \dots i]$, $w_2[1 \dots j]$

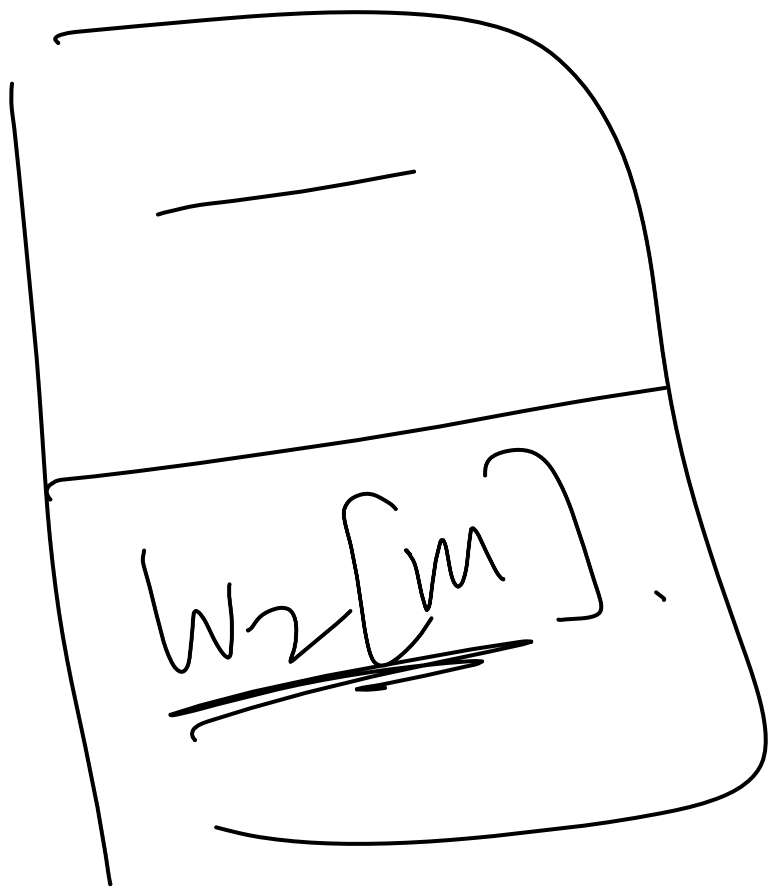
$ED(i, j)$

$i > 0$, $j > 0$.

$i = 0$ / $j = 0$

$\emptyset$ string.

2



$$ED(n, m) = 1 + ED(n, m-1)$$

(3)

$w_1[n]$

$w_2[m]$

a

$w_1[n] = w_2(m)$

$ED(n, m) = ED(n-1, m-1)$

b.

$w_1(n) \neq w_2(m)$

$ED(n, m) = ED(n-1, m-1) + 1$

$$ED(n, m) = \min \begin{cases} ED(n-1, m) + 1 \\ ED(n, m-1) + 1 \\ ED(n-1, m-1) + \delta(n, m) \end{cases}$$

$$\delta(i, j) = 0 \quad \text{if} \quad w[i] = w[j]$$

$$\quad\quad\;\; = 1 \quad \text{if} \quad w[i] \neq w[j].$$

$$T(n,m) = T(n-1,m) + T(n,m-1) + T(n-1,m-1) + \underbrace{\Theta(1)}_{c}$$

$$\geq 3\cancel{2}\,T(n-1,m-1) + c.$$

$$\geq \cancel{2}\,3^{\cancel{2}\min\{n,m\}-} + c$$