27.09.2024

**Greedy algorithms:**
- rule based.
- nontrivial: what rule.?
- proof:

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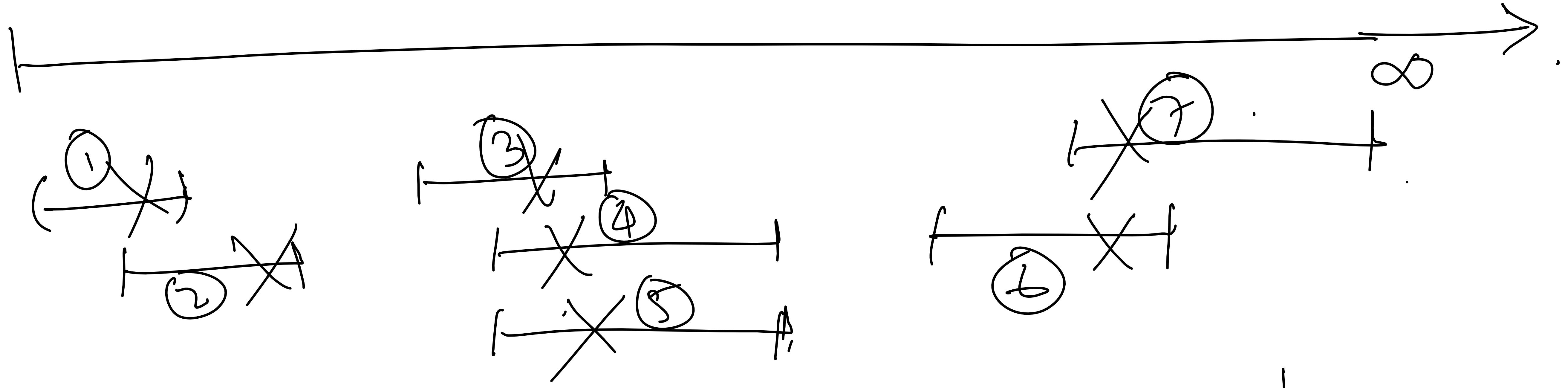
## Interval Scheduling:

| jobs | start | finish |
|---|---|---|
| i | $s(i)$ | $f(i)$ |
| ⋮ | ⋮ | ⋮ |



1 3 6

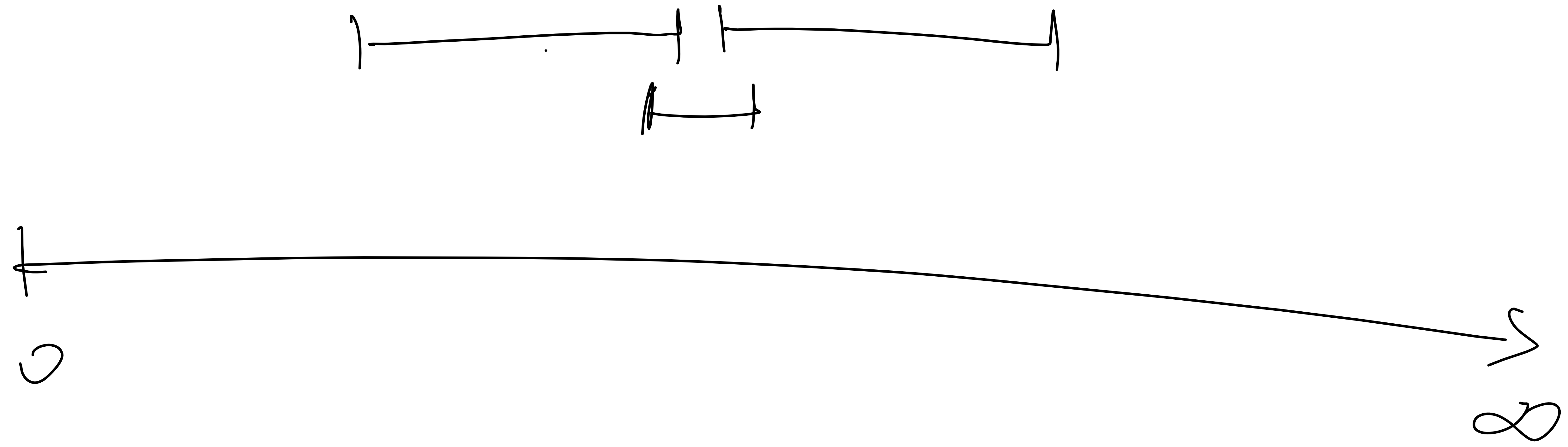
$\boxed{1 \quad 3 \quad 6}$

#3

Max. subset of jobs that don't interfere with each other.

Rule 1: Select min-duration job first.

- Select the first job.
- remove ~~of~~ all interfering job.




Counter example

Rule 2: Select the job that interferes with the least # of other jobs.

leave as an exercise

– find a counterexample!

Rule 3: Select the job whose finish time is least

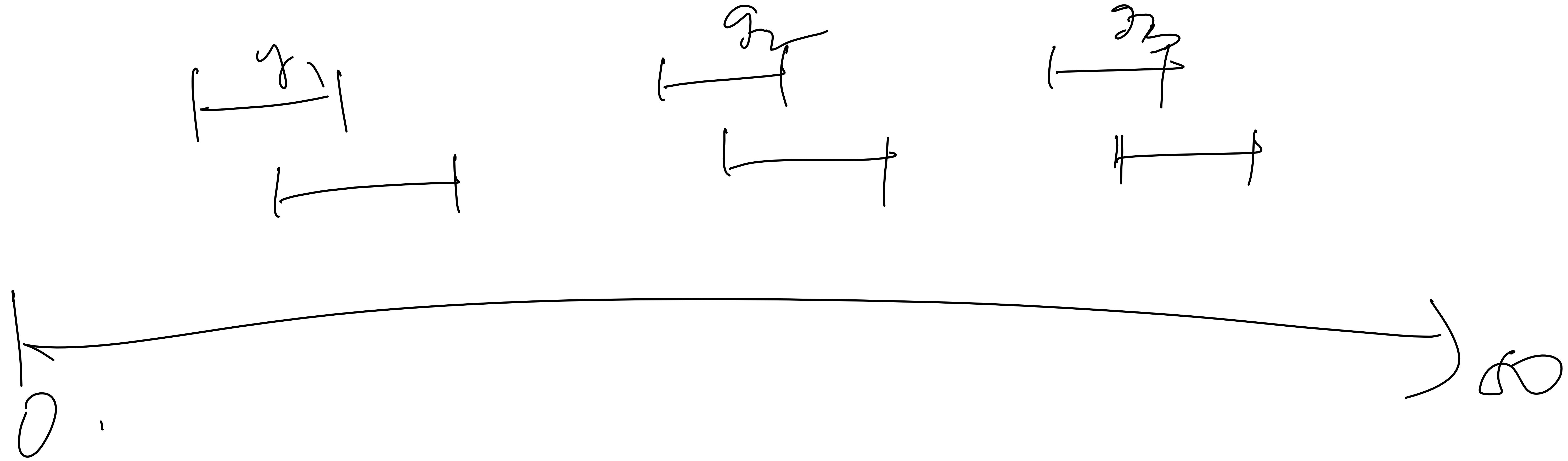
– this is going to work.

We'll prove the following:

If $\overbrace{O_1, O_2, \ldots, O_k}^{\text{sorted wlog}}$ be any set of "good jobs" → do not intersect.

Let $\overbrace{g_1, g_2, \ldots, g_m}^{\text{sorted wlog}}$ be the rule 3 solution.

$m \geq k$.




For contradiction, assume $\underline{m < k}$.

Claim: $\forall\ 1 \le j \le m$.

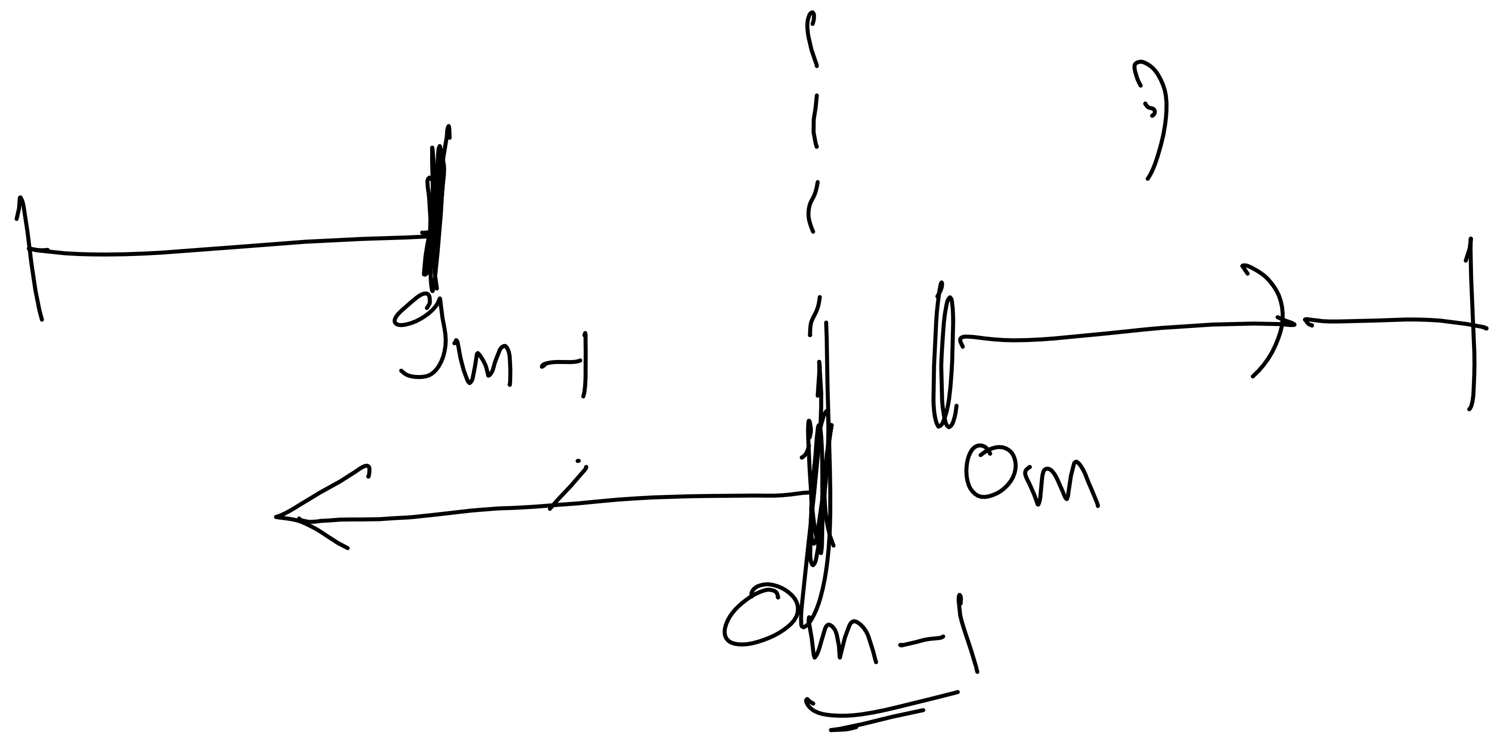
$$f(g_j) \le f(o_j).$$

proof: by Induction

$j=1$ by def<sup></sup>n $f(g_1) \le f(o_1)$

since $g_1$ finishes first among all jobs.

I.H: $1 \le j \le m-1$



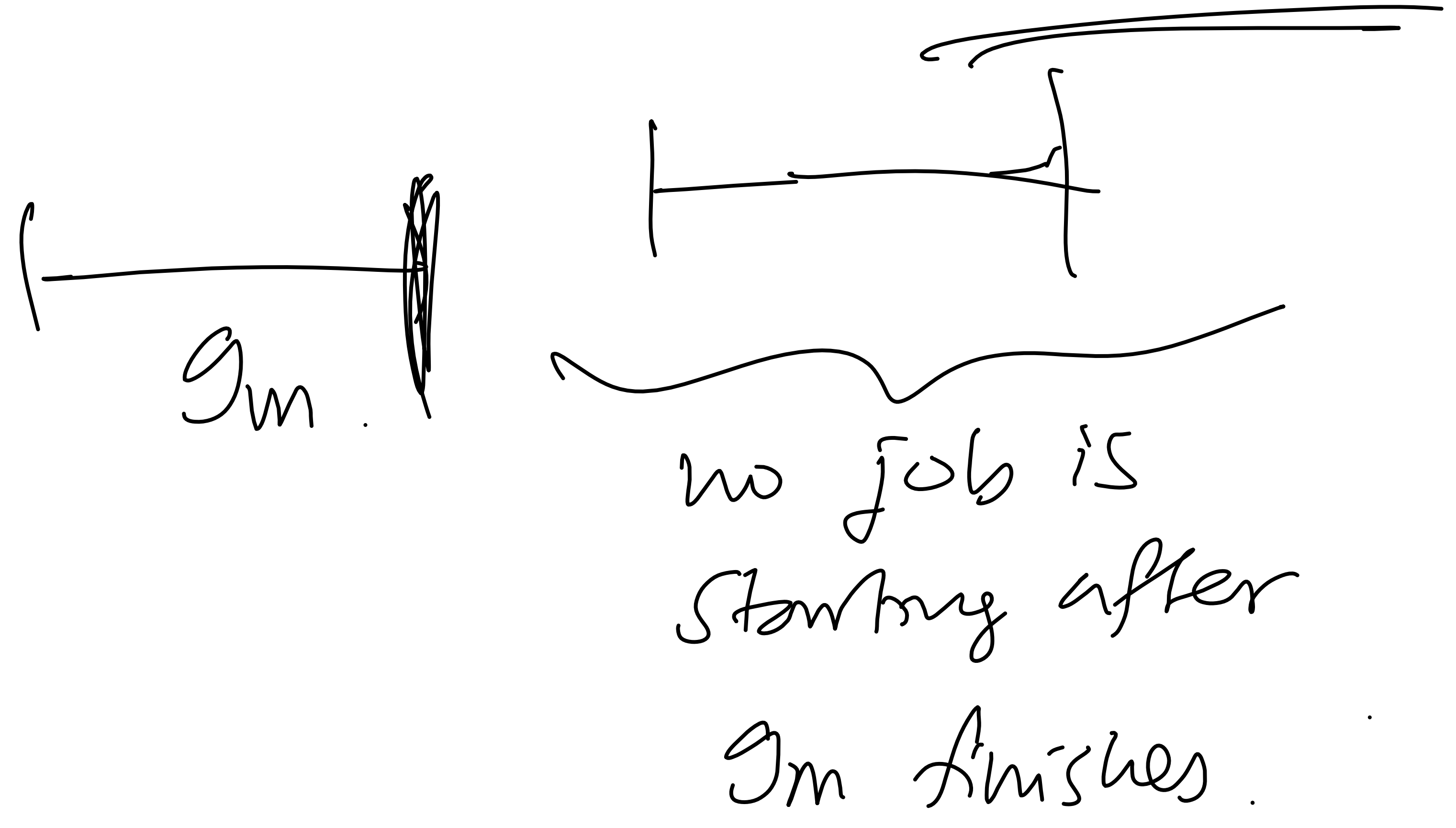

$\Rightarrow$ $f(g_m) \le f(O_m)$.

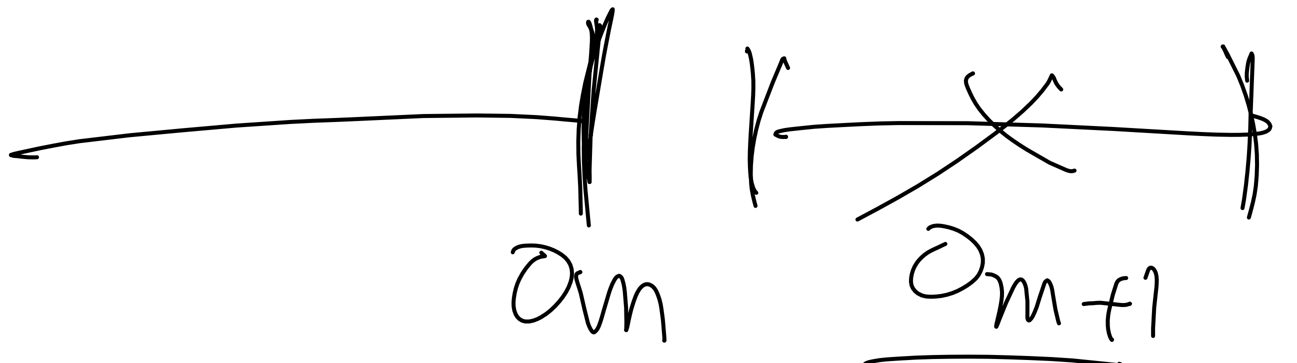
immediate because $O_m$ is a valid candidate to be put after $g_{m-1}$.

&

we are selecting least finish time first after $g_{m-1}$ & its conflicts.

leads to contradiction as follows if $m < k$.




D

Contradict. $\square$

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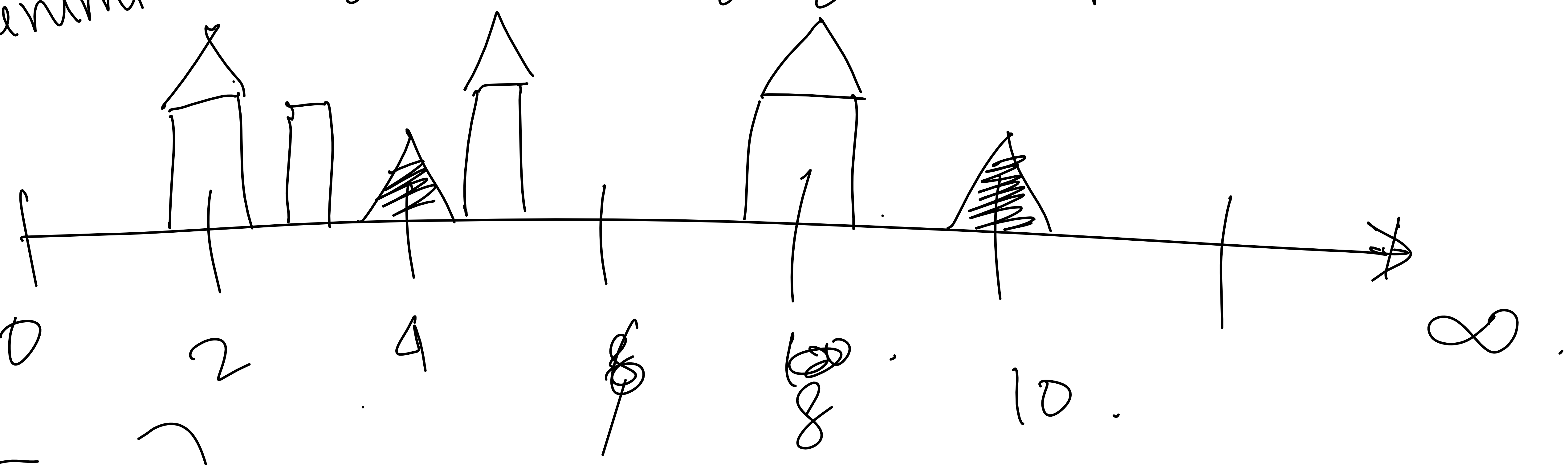
- sort all jobs by finish time.
- keep on picking jobs by rule 3's strategy.

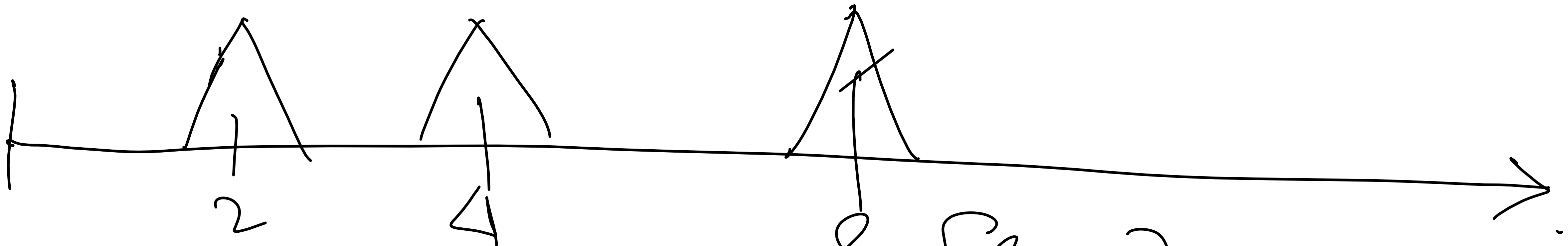
time complexity $= O(n\log n)$.

# Mobile Tower Placement

minimize # Towers going to be placed.




$d = 2$ [ ]




[2, 6]. 8. [6, 10].

[0, 4] → includes both end points.

<u>Input:</u> location of houses. $\in [0, \infty)$.

range $d$.

<u>Output:</u> # of min. towers

~~#~~ location of towers.

Rule 1: Place tower at first uncovered house.

- counterexample.

Rule 2:


Place tower at distance d right of first uncovered house.

How to prove that Rule 2 works?

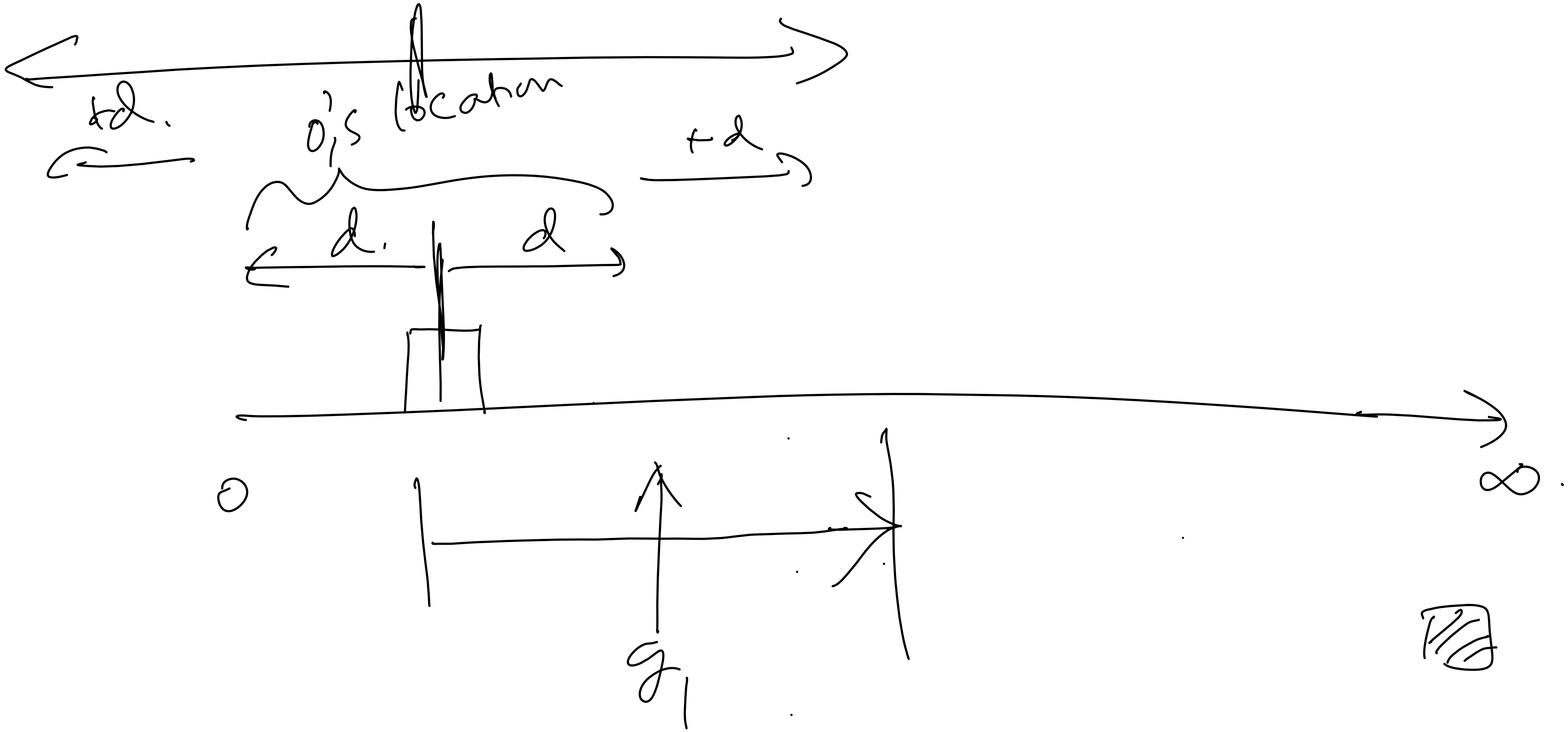
Exchange argument.

$\cancel{o_1}, \ldots, o_k$

be an optimal set of towers (sorted wlog).

Claim: $\underbrace{g_1}_{\text{rule2 first chosen tower.}}, o_2, \ldots, o_k$ will also be optimal.

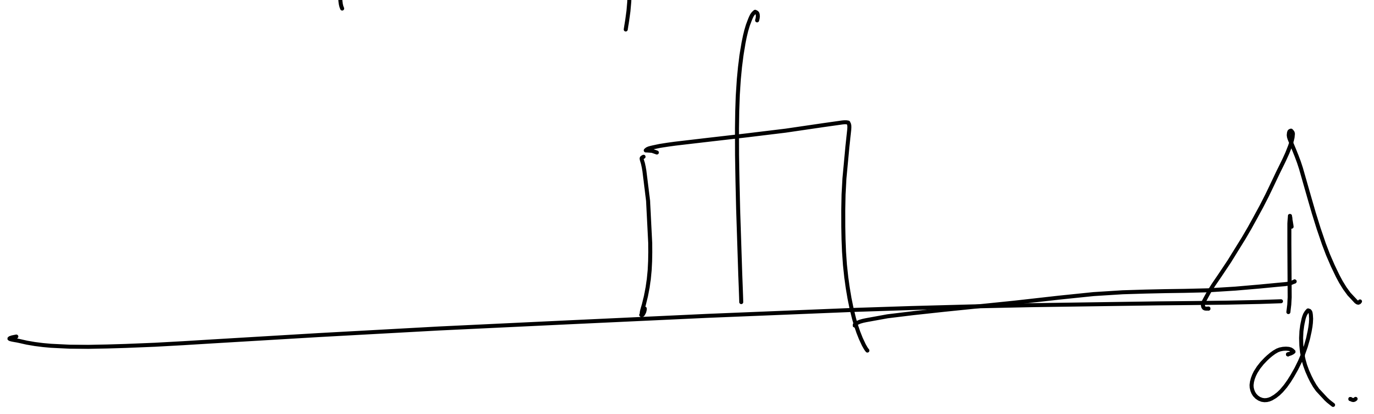
pf. It suffices to show that $g_1$ covers whatever $O_1$ was exclusively covering.




$\square$

We'll prove ~~by induction~~ that greedy rule 2. is optimal. by induction on $n \rightarrow$ (# of houses)

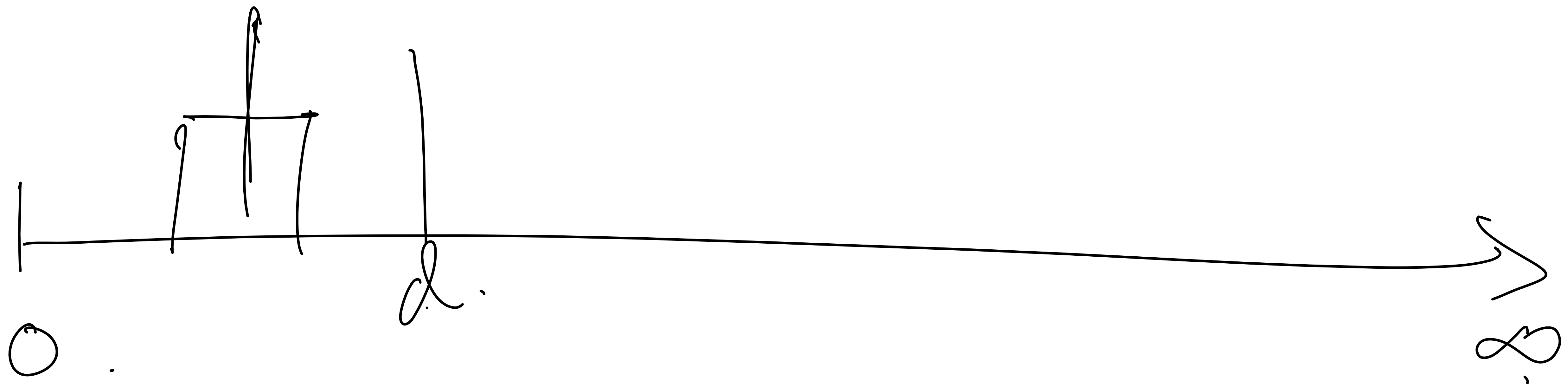
<u>base case</u>: $n = 1$




greedy rule 2 is working.

<u>I.H</u>: we'll assume greedy rule 2 is best for $(n-1)$ houses.

I.S. Show it for $n$ houses.



Optimal set: $\{O_1, \ldots, O_k\}$. $k \geq 1$

Also an optimal solution: $\{\cancel{O_1}, \ldots, O_k\}$.

$(k-1)$

- remove $g_1$ & its covered houses.

by induction. greedy is best for leftover houses.

$\{ g_2, \ldots, g_{k} \}$

optimal for the leftover houses.

Greedy rule 3 Solution $\rightarrow$ $\{ g_1, g_2, \ldots, g_k \}$

must also be optimal. since optimal size = k.