$T(n) \leq T(n/5) + T\left(\frac{7.2n}{10}\right) + cn$ $>$

$\left(\frac{1}{5}\right)^i \left(\frac{7.2}{10}\right)^j$

Claim: $T(n) \leq dn$ for some constant d.

choose d large enough $\geq \frac{c}{0.08}$

proof: By induction.

I.H. Claim is true for $1, \cdots, n-1$

$$T(n) \leq T(n/5) + T\left(\frac{7.2n}{10}\right) + cn$$

$$\leq d \cdot n/5 + d \cdot \frac{7.2n}{10} + cn \leq dn$$

$$0.92 dn + cn \leq dn.$$

$$\equiv \quad cn \leq 0.08 dn$$

$$\equiv \quad \boxed{d \geq \frac{c}{0.08}}$$

Therefore, $T(n) \leq dn$.

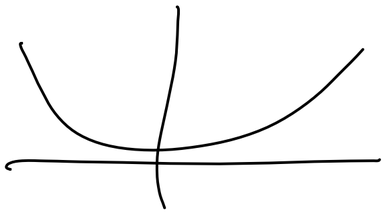
[Induction step holds] □

$$T(n) = O(n), \quad T(n) = \Omega(n)$$

$$T(n) = \Theta(n).$$

Module 9 : Graph Algorithm

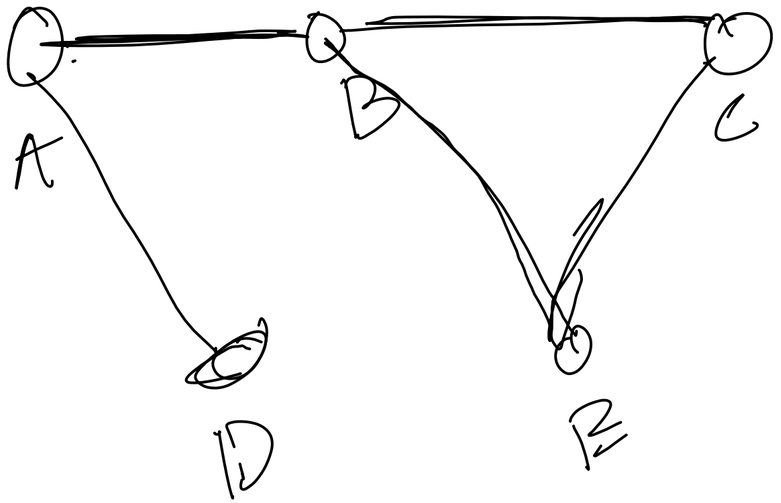
(object, relationship)



Examples : Facebook graph.

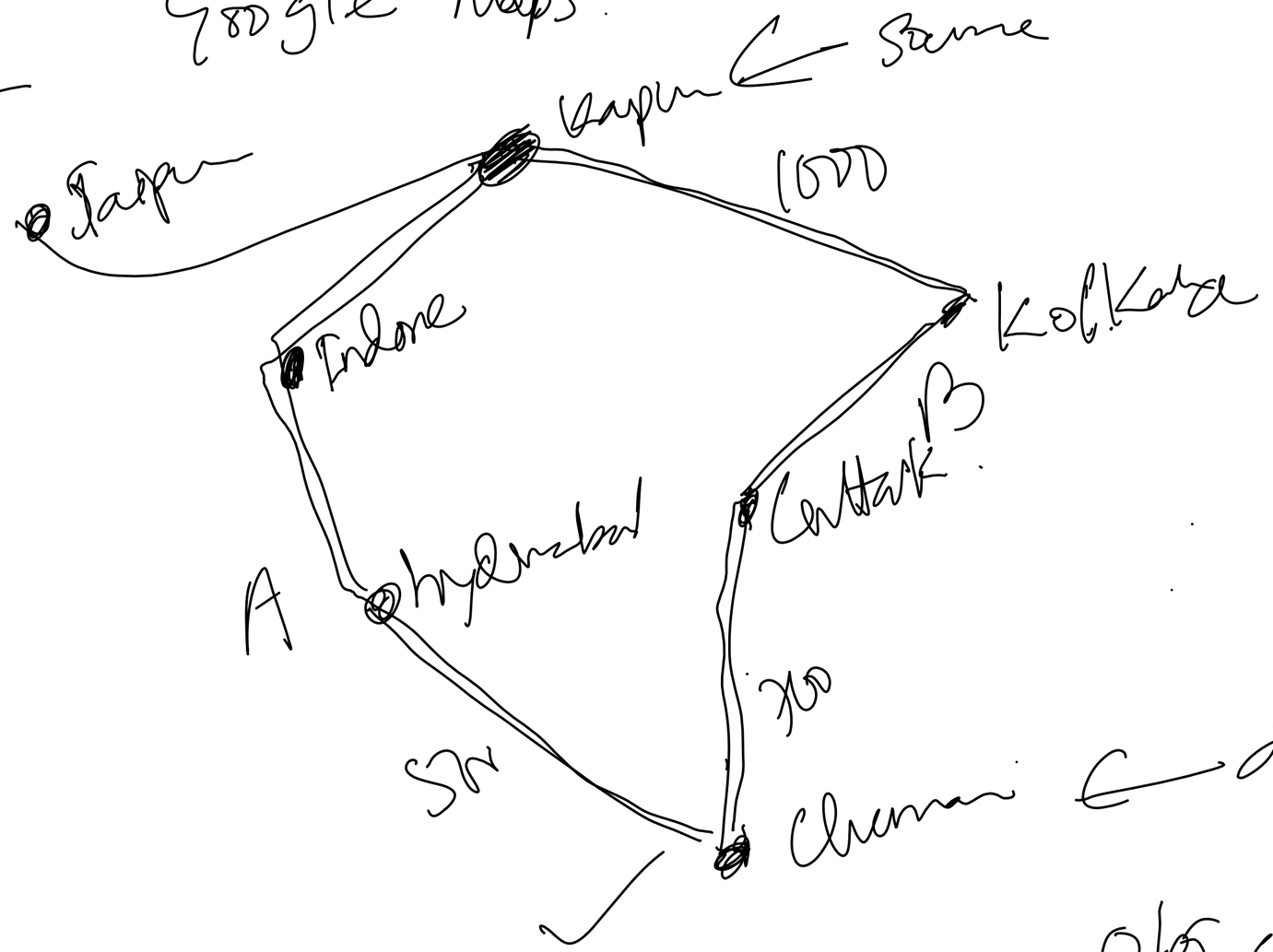
objects : users

relationship : friendship.



Example

Google maps.



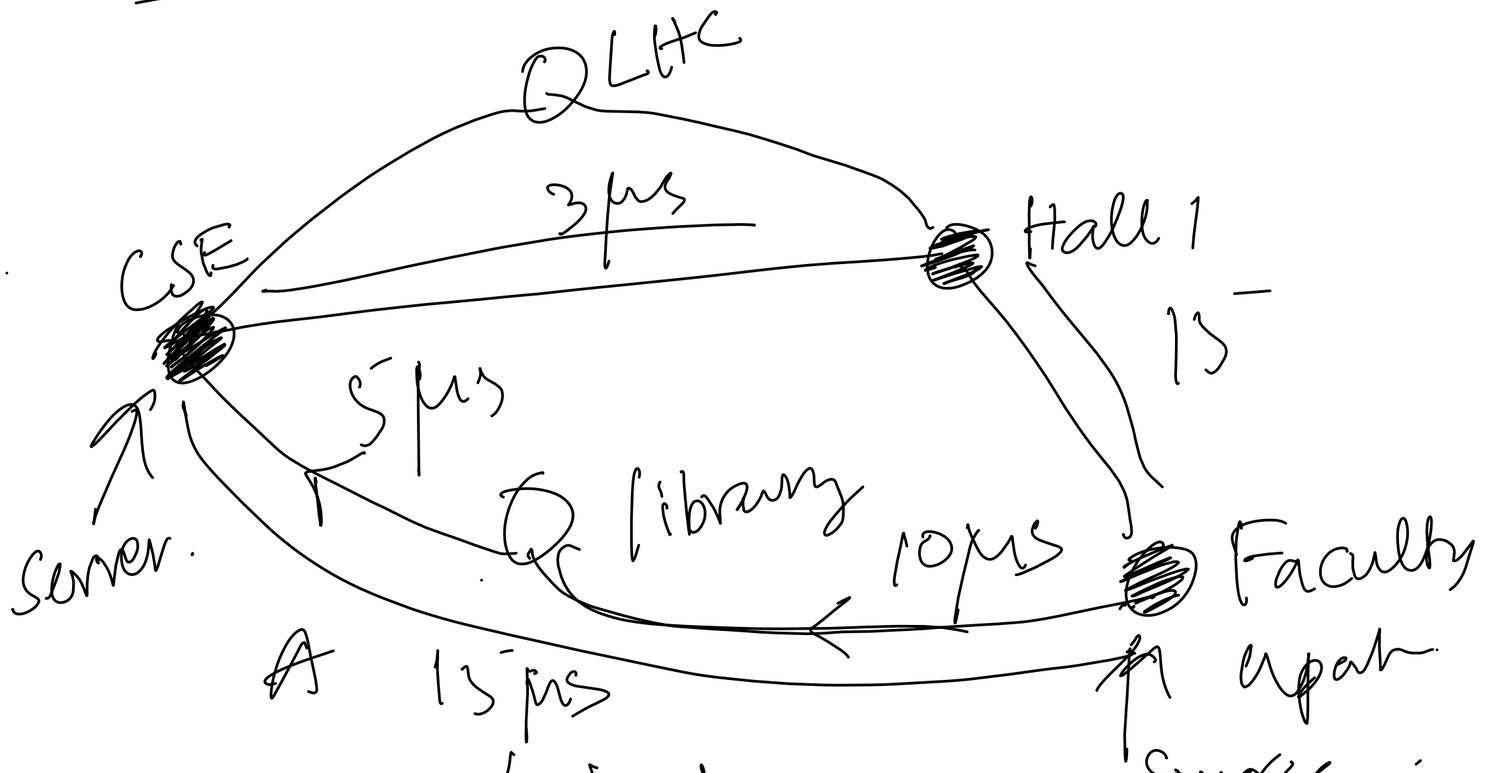
A : 200

B : 1500

Objects : cities

relationship : road.

Example Routing.



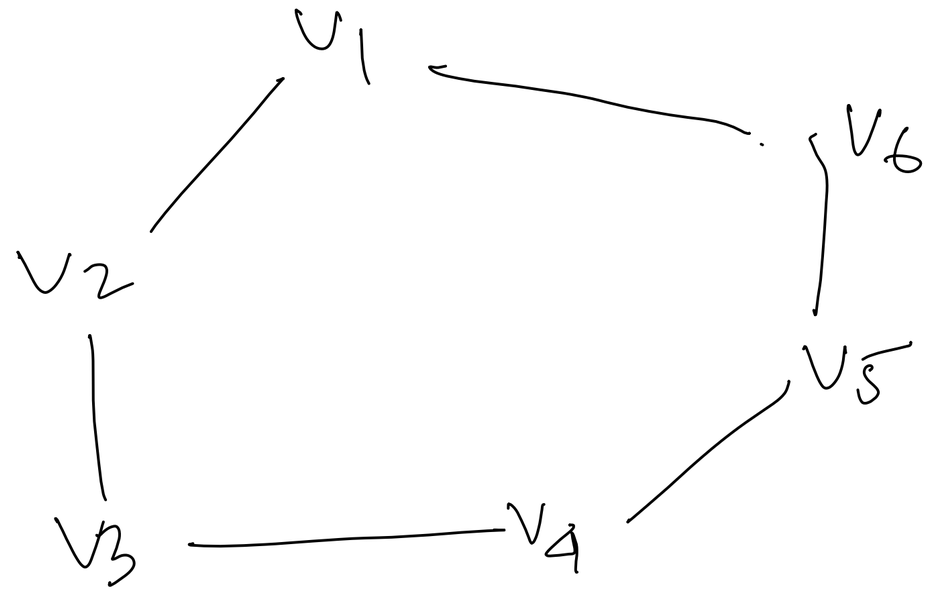
nodes: routers / clusters

relationship: μs it takes to send a packet.

$G = (V, E)$

graph / networks · nodes / vertices · edges / links

Example 1.



G

$V = \{v_1, \ldots, v_6\}$

$E \subseteq V \times V = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_5), (v_5, v_6), (v_6, v_1)\}$.

$\equiv (v_2, v_1)$

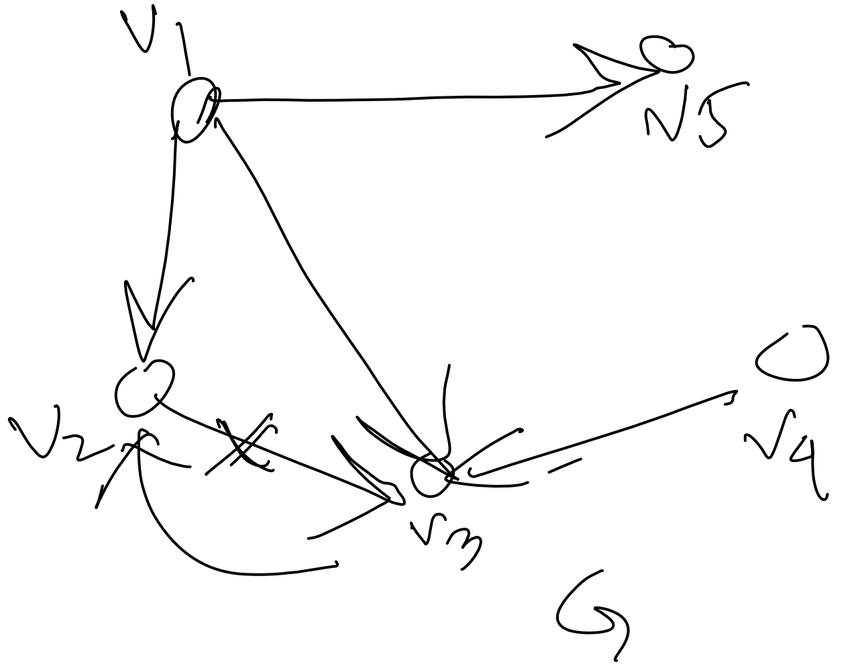
Graphs ≡ undirected graphs.

Directed graphs -

$A \longrightarrow B$

B follows A

$(A, B) \equiv (B, A)$



G

$V = \{v_1, v_2, \cdots, v_5\}$

$E \subseteq V \times V$

$= \{(v_1, v_2), (v_2, v_3), (v_3, v_2), (v_1, v_3), (v_4, v_3), (v_1, v_5)\}$

How are graphs stored inside a computer?

adjacency list

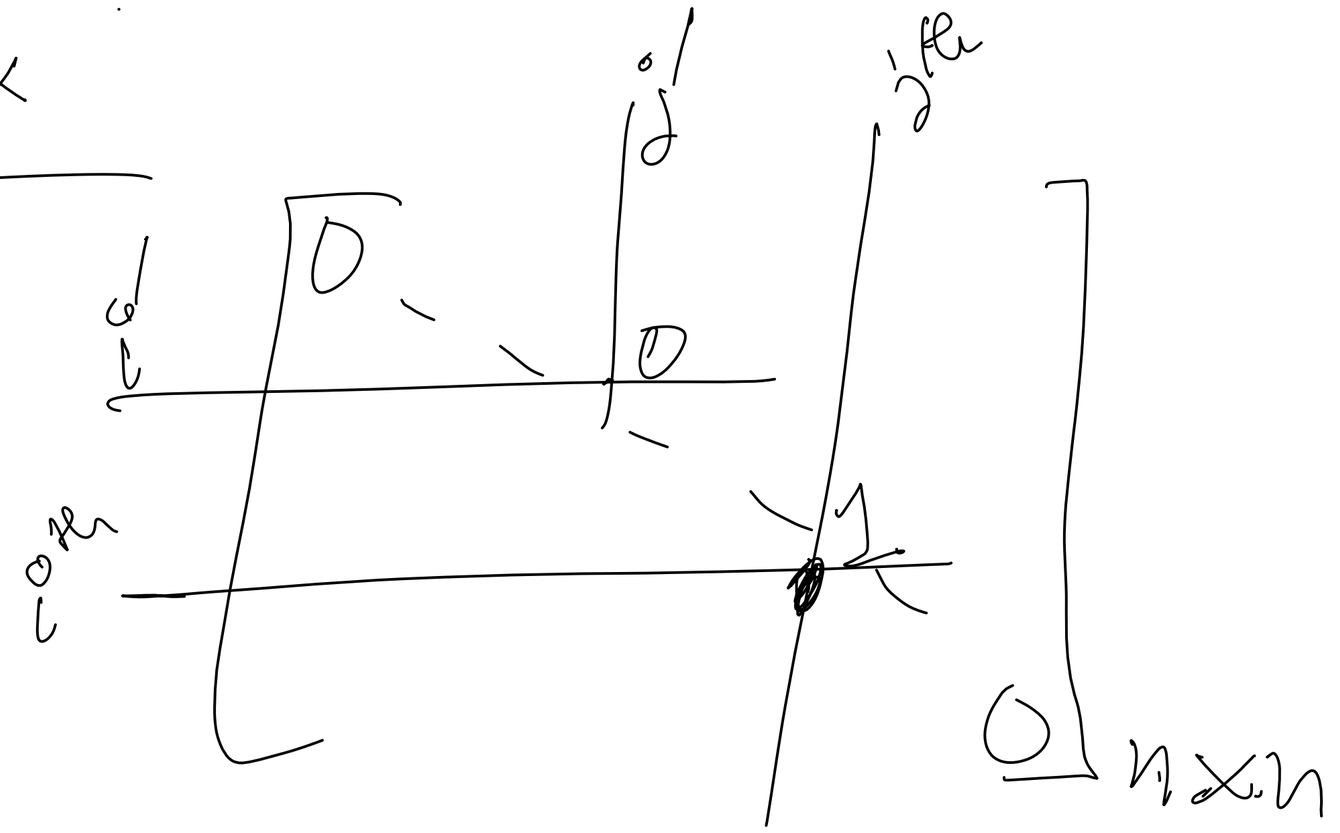
Adjacency matrix

Adjacency matrix

n nodes:

$(i,j) \notin E$

$(i,j) \in E$



	1	2	3	4	5	6
1	0	1	0	0	0	1
2	1	0	1	0	0	0
3	0	1	0	1	0	0
4	0	0	1	0	1	0
5	0	0	0	1	0	1
6	1	0	0	0	1	0

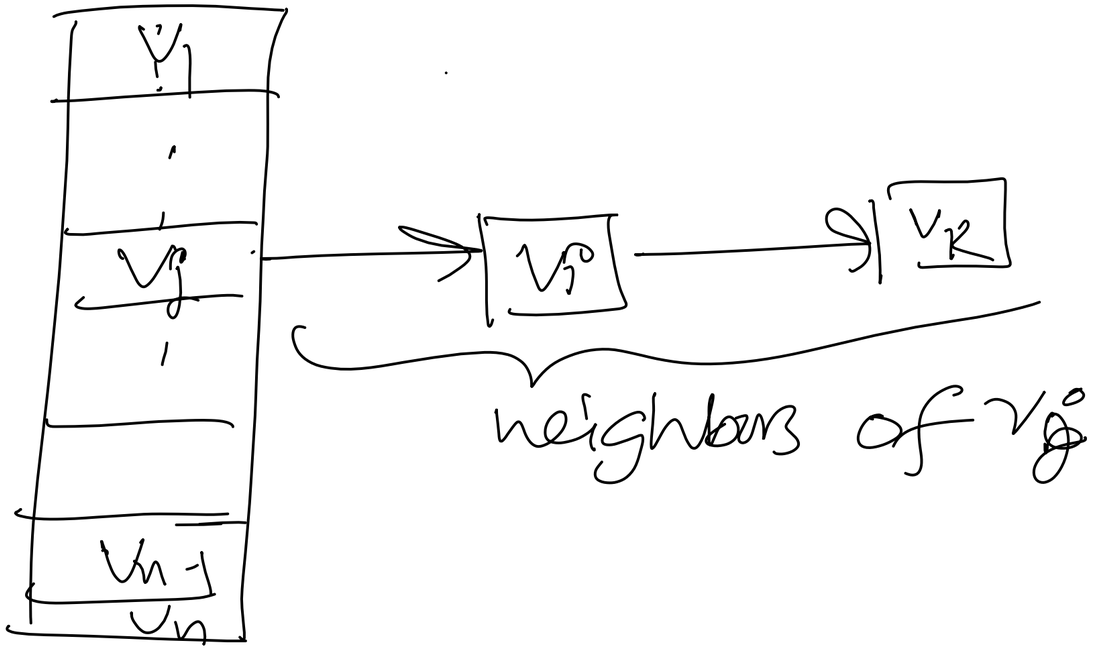
- Symmetric
- stored like a directed graph where each edge is bidirected.

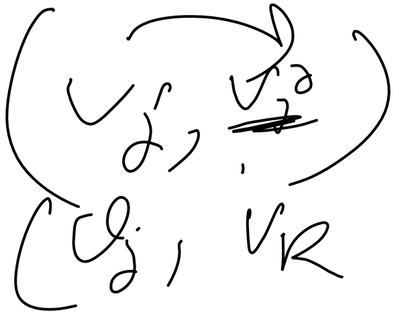
	1	2	3	4	5
1	0	1	1	0	1
2	0	0	0	0	0
3	0	1	0	0	0
4	0	0	1	0	0
5	0	0	0	0	0

no longer symmetric.

Remark: We'll focus on designing algorithms for directed graphs.
More often, they will extend to undirected graphs.

Adjacency list





$$T(m) \leq T(m/5) + T\left(\frac{7q}{2} + 3\right) + cn$$

$$q = m/5 \;;\; \frac{7q}{2} = \frac{7 \cdot m}{10}$$

$m \leq n$

$$T(n) \leq T(n/5) + T\left(\frac{7n}{10} + 3\right) + cn.$$

$$\leq \frac{7 \cdot 2n}{10}$$

for n large enough.