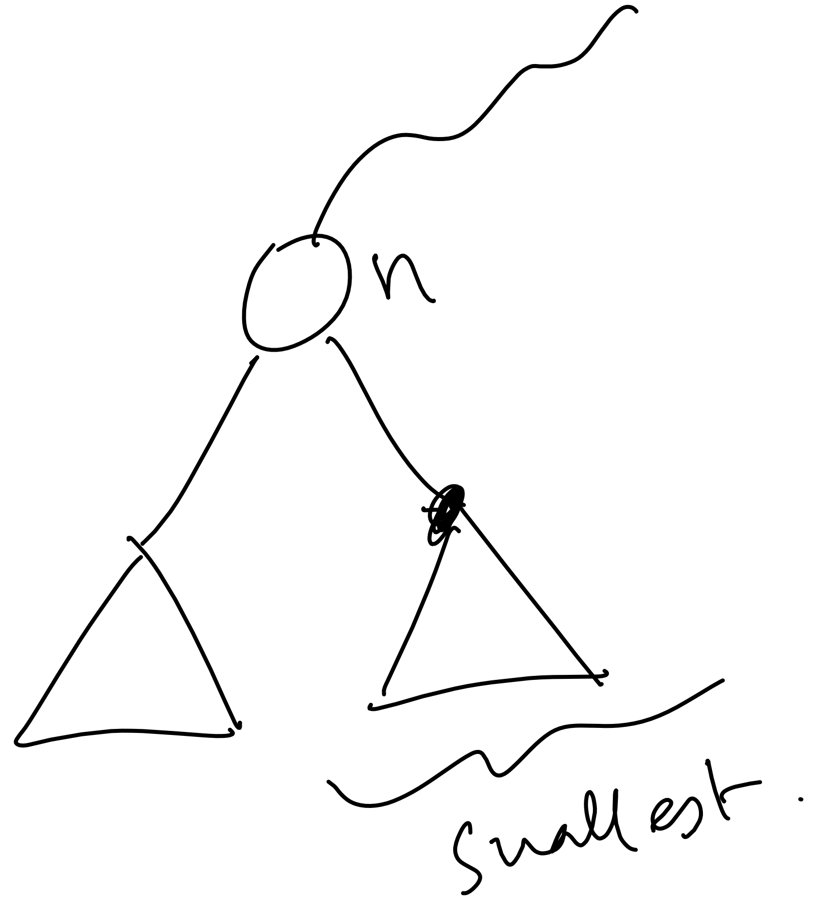
13.09.2024

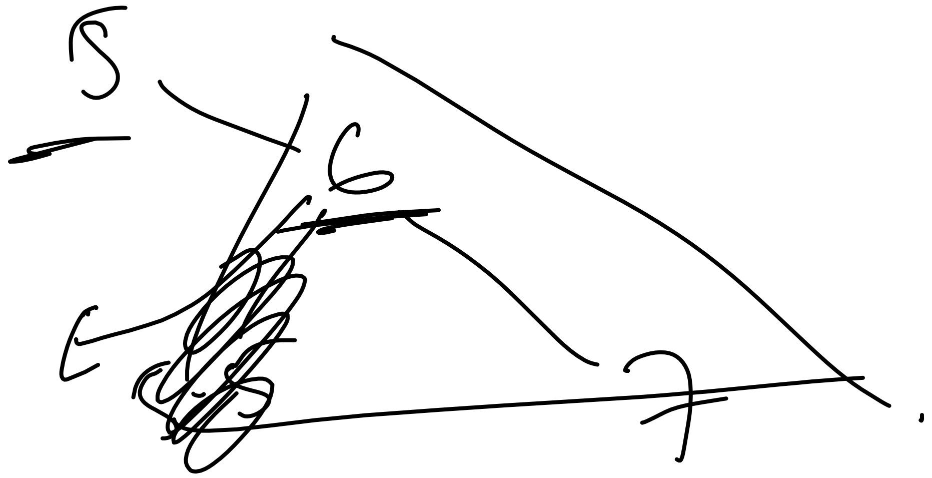
Procedure Delete (node n).



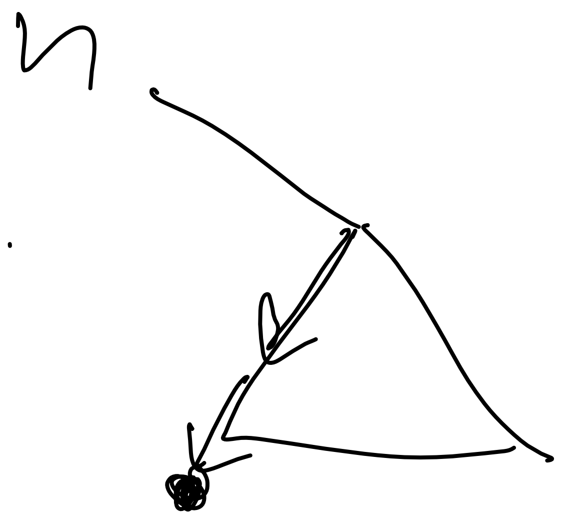
① successor is a left child

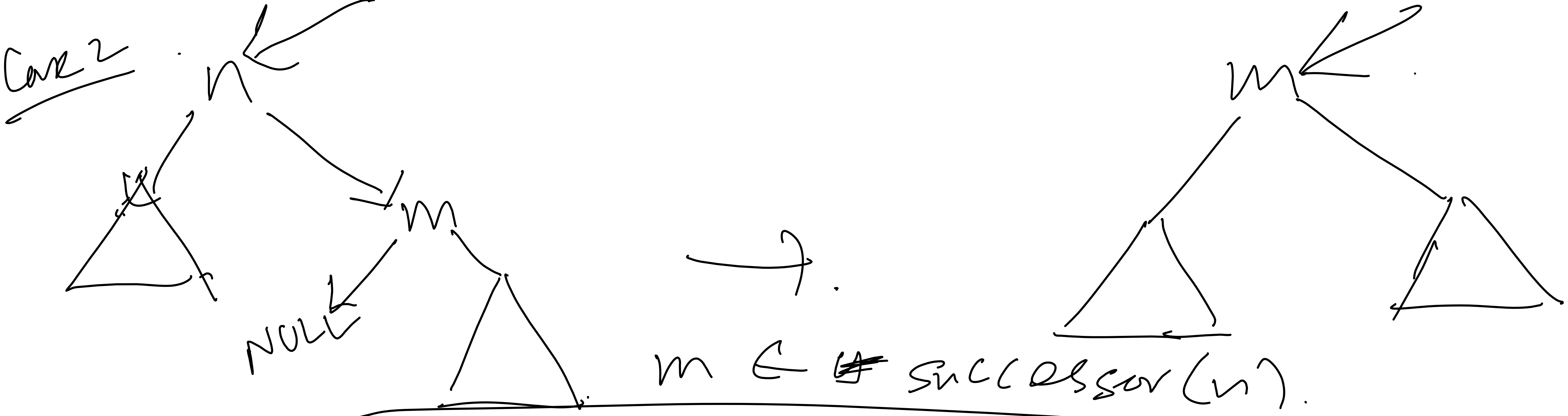
② successor is a right child.

②



①.

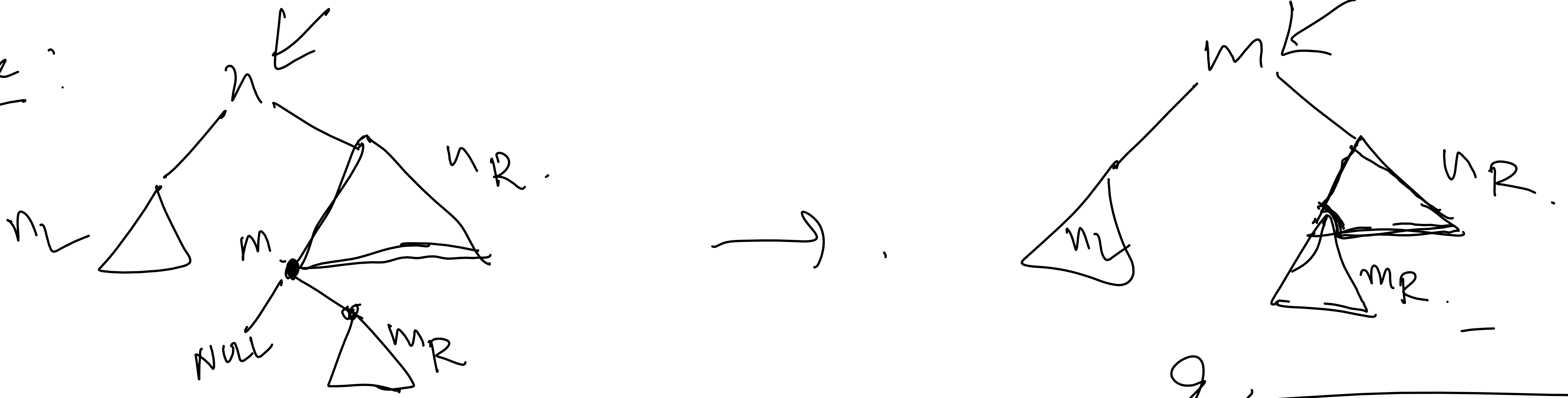




m ← ~~#~~ successor(n).

```
m.left ← n.left.
n.left.parent ← m
if (n.parent = NULL) { return }
if (n = n.parent.left)
        n.parent.left ← m
        m.parent ← n.parent
else
        n.parent.right ← m
        m.parent ← n.parent.
```

Case:



$n_L < n < n_R, m, m_R$.

$m < n_R \setminus m$.

$m_R > m$.

Exercise: Check BST ordering is preserved.

Procedure

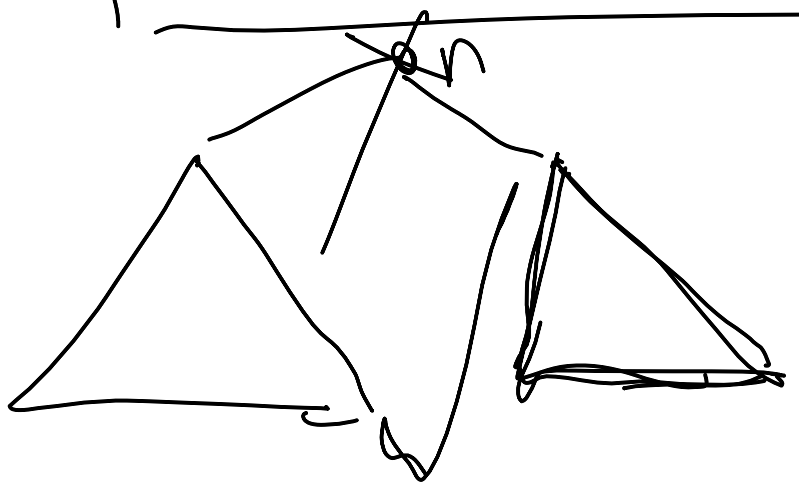
m.left ← n.left

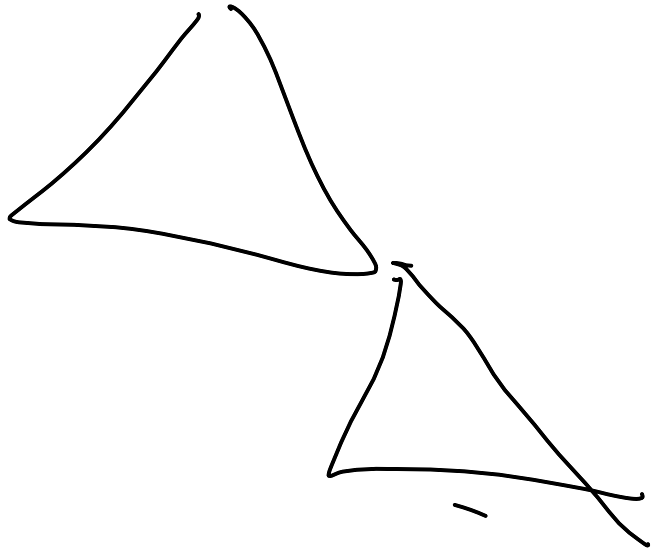
n.left.parent ← m.

m.right.parent ← m.parent

m.parent.(left/~~right~~) ← m.right

```
m.right ← n.right.
n.right.parent ← m.
if (n.parent=NULL)    return.
m.parent ← m.parent.
m.parent.(left/right) ← m.
```





Skewed.

BST

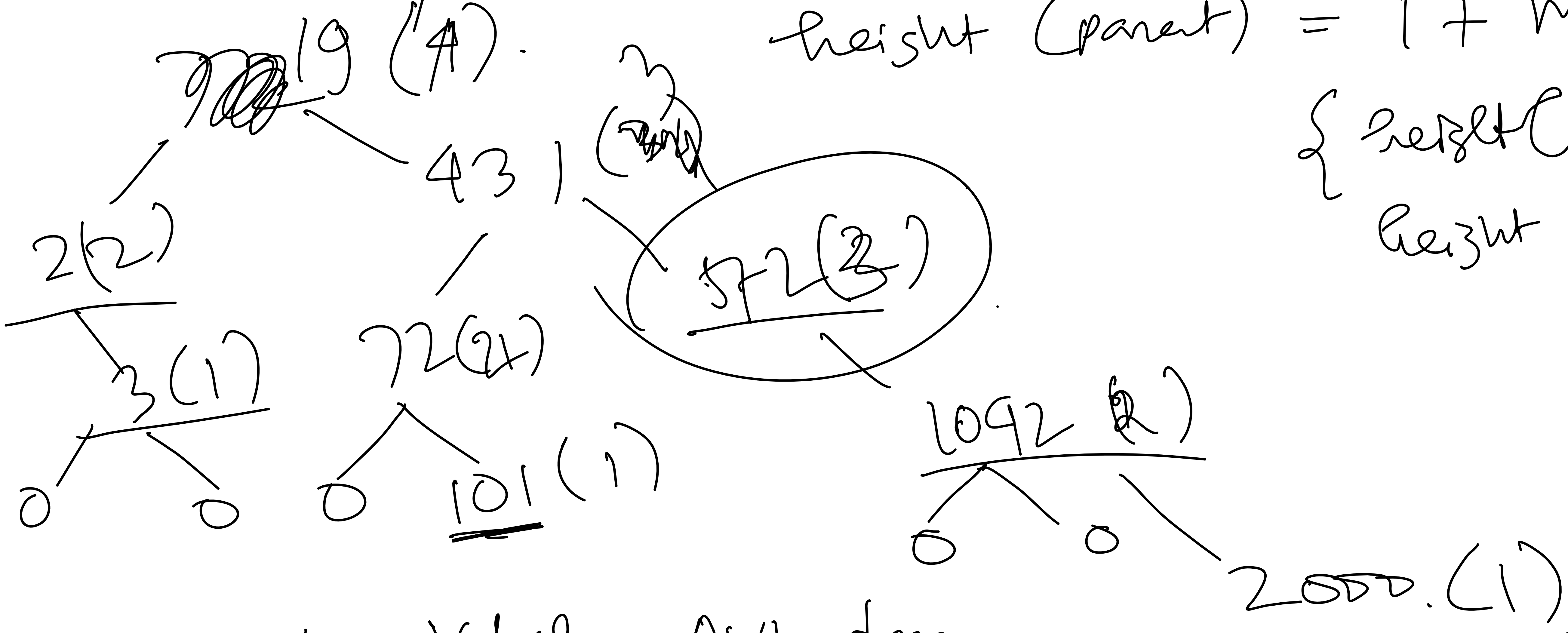
Tree Rotations & AVL Tree

- improvement to BST
- height is small.

Defn (height of BST)

height(NULL) = 0.

height (parent) = 1 + max {height(left_child), height (right_child)}



Invalid AVL tree., violation at 572.

```
Struct Node {
    int key;
    struct Node* left;
    "            *right;
    "            parent;
       int       height
};
```

```
while( temp ≠ NULL )
    temp ← temp. parent.
    old_height = temp. height.
    temp.height =
     1 + max{ temp.left.height
              temp.right.height }.
    if (old_height
        = temp. height)

        break.
```

Defn [AVL Tree] : A BST with the following additional property:

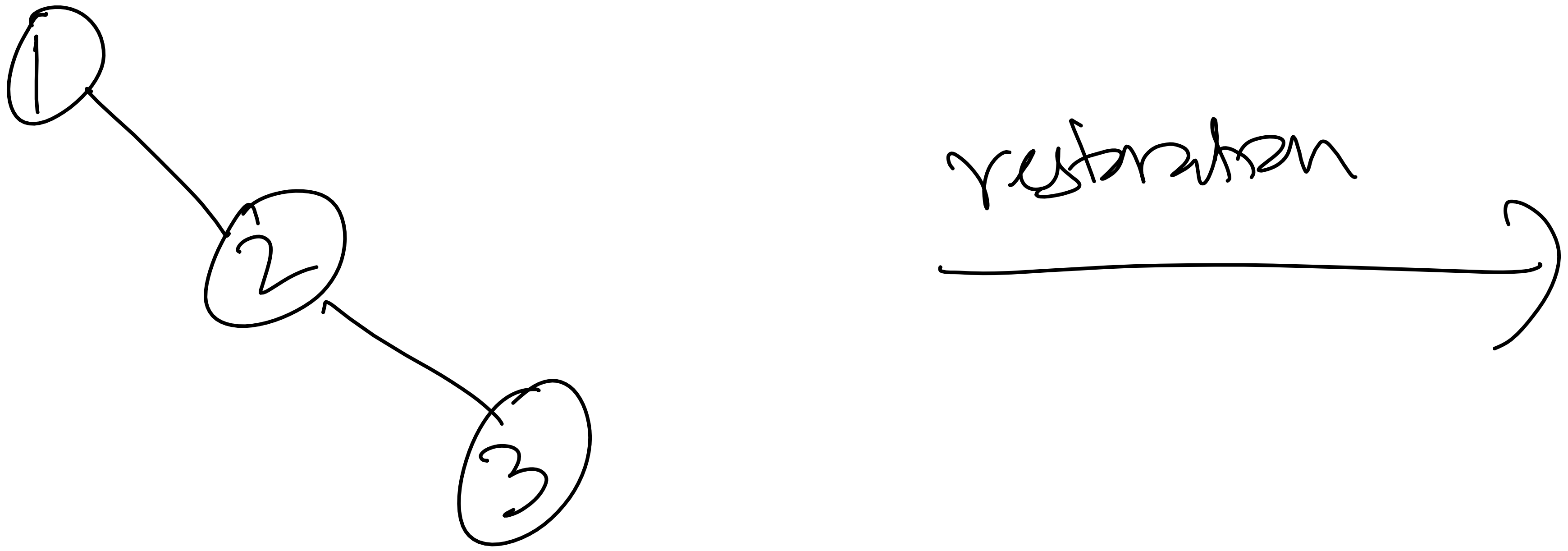
- $|\text{height}(\text{node.left}) - \text{height}(\text{node.right})| \leq 1$

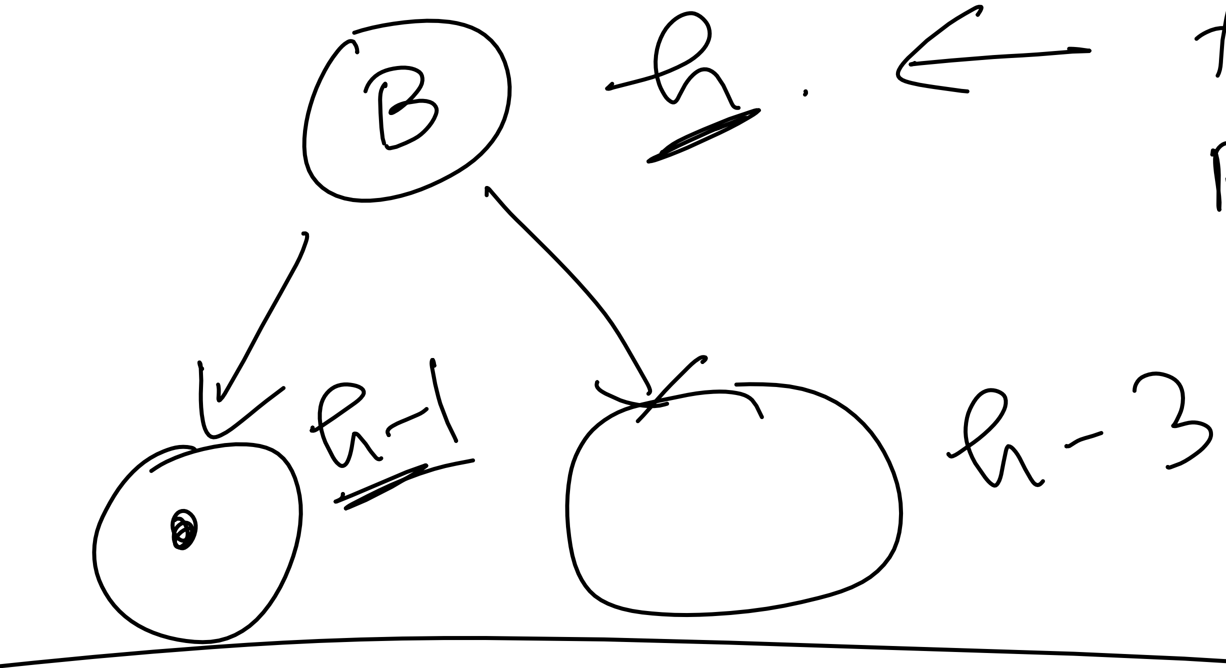
  $\forall$ node $\in$ Tree.

- Make a pass from newly inserted node to the root of tree.

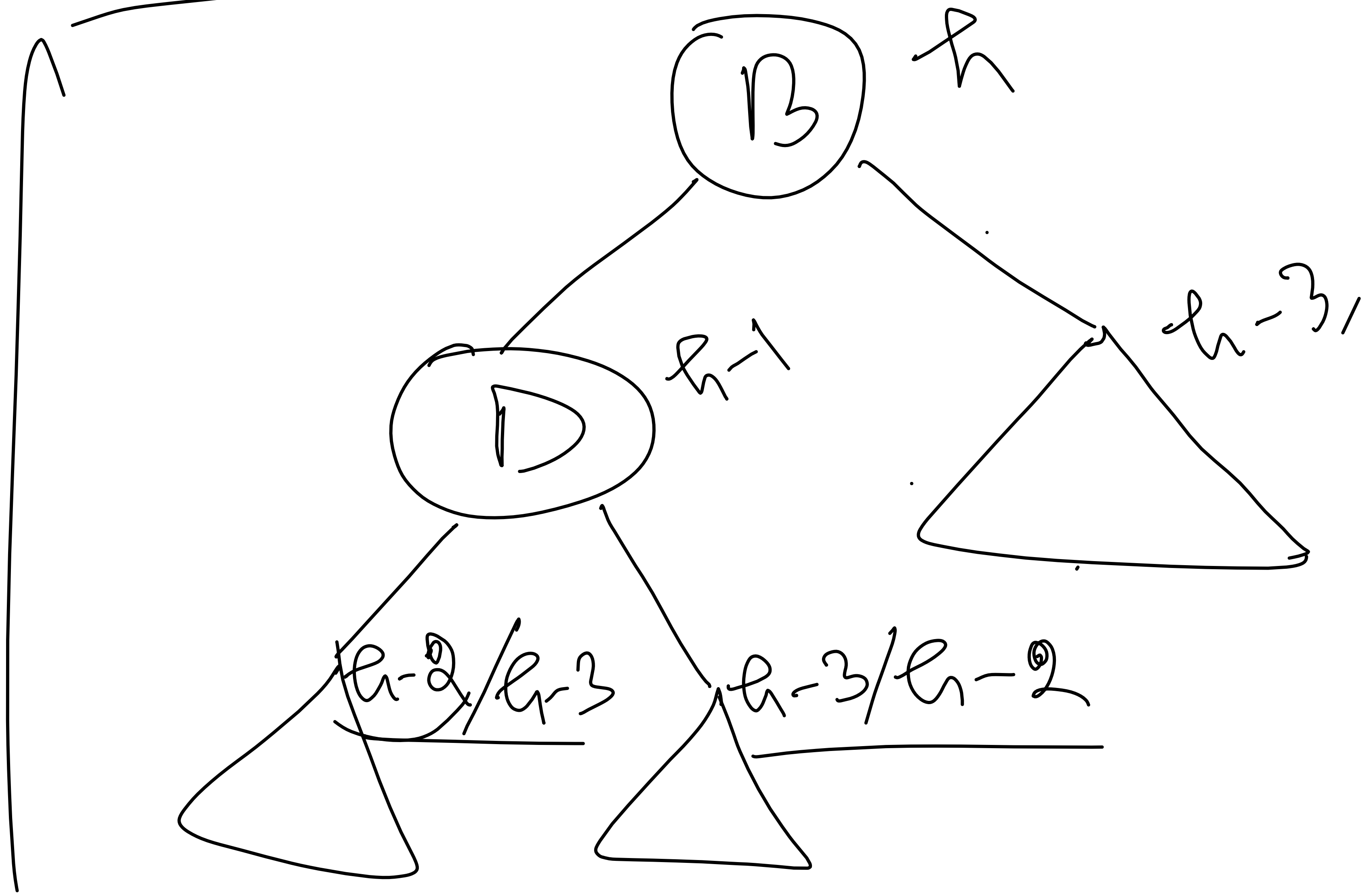
  Any time it violates AVL property, break & return the node.

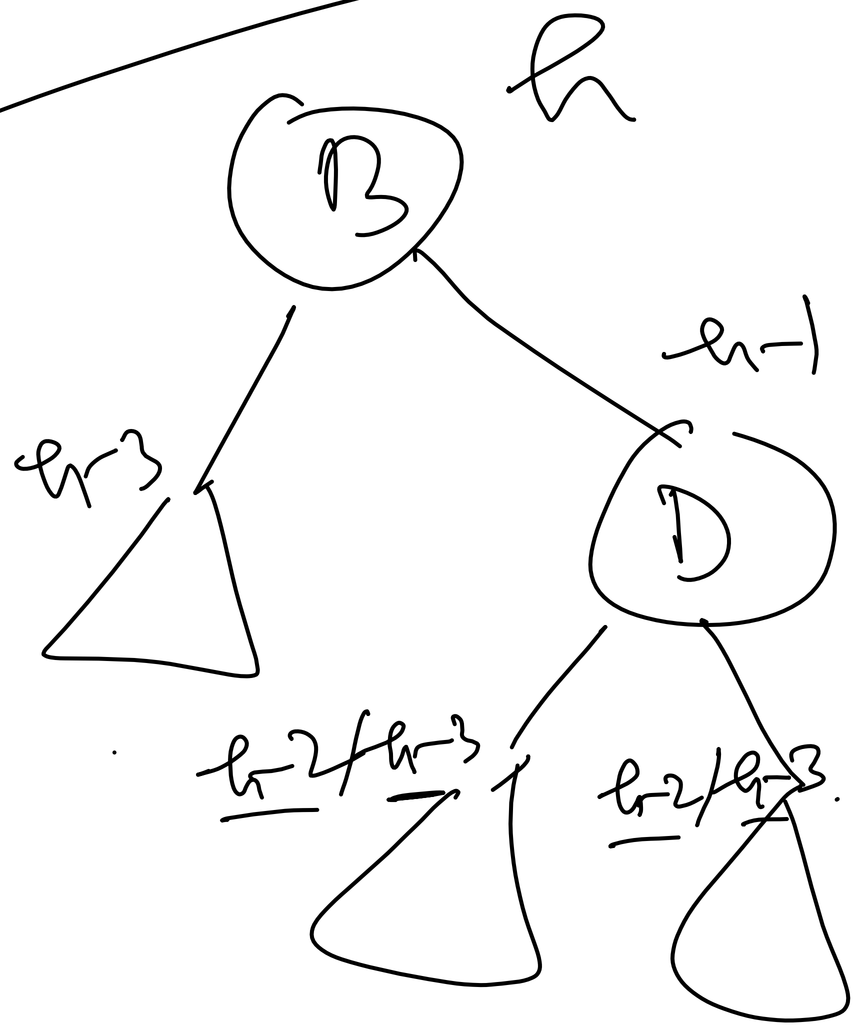
Restore AVL property





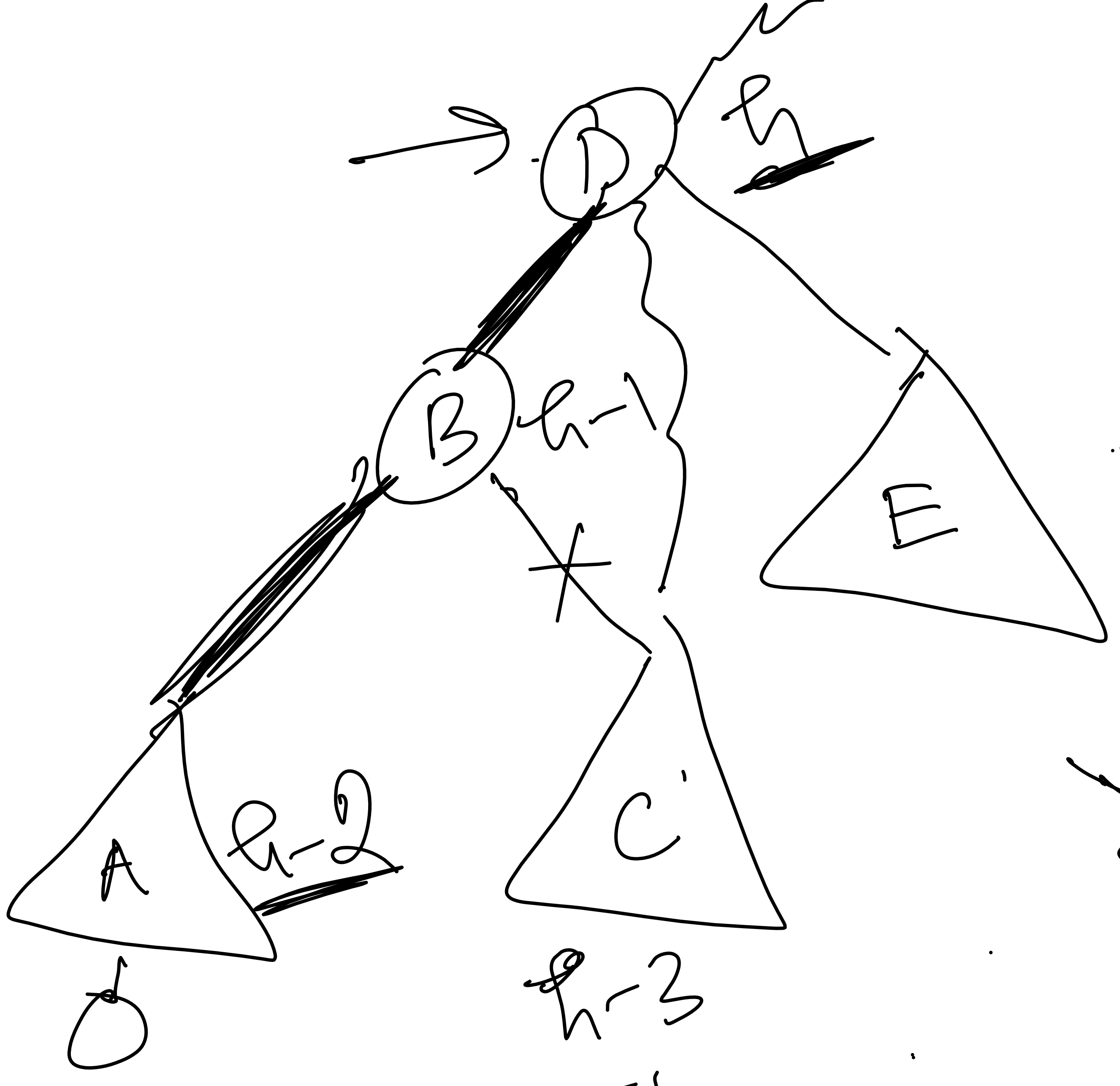
first node where AVL property is violated.



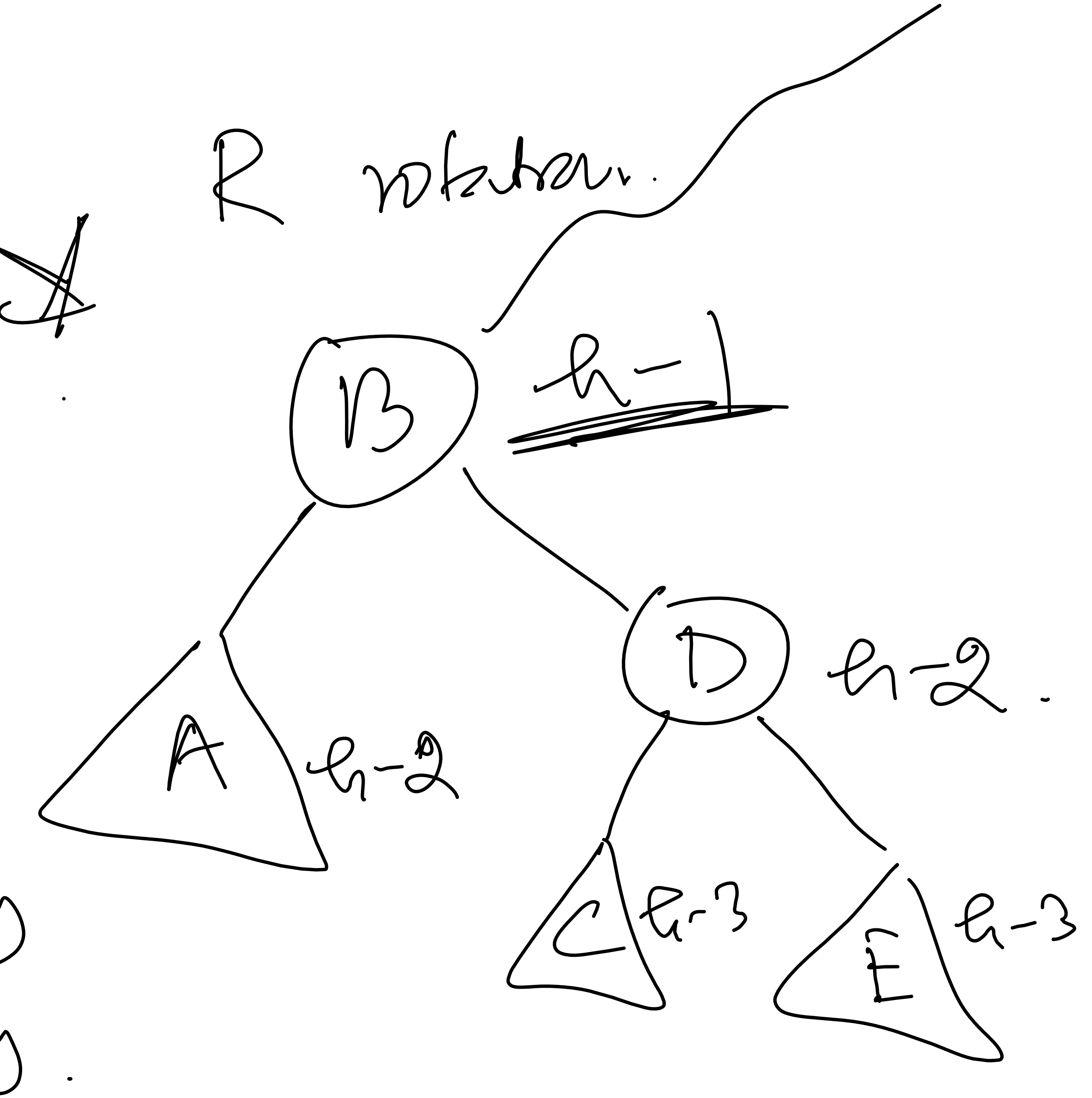


Case 1

(a)



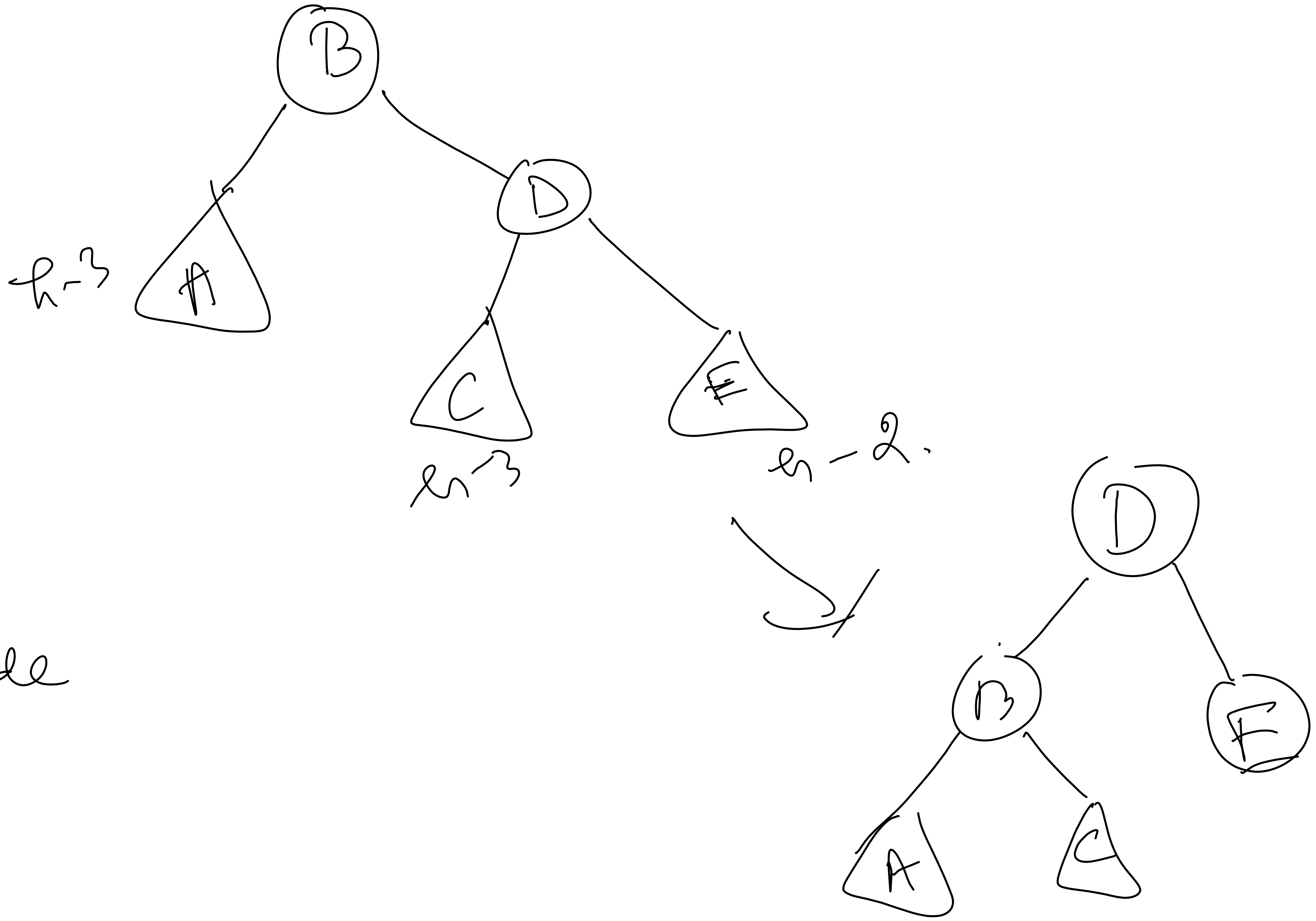
R rotation.



A < B < C < D < E

- preserves BST property
- restores AVL property.

(b)



Exercise

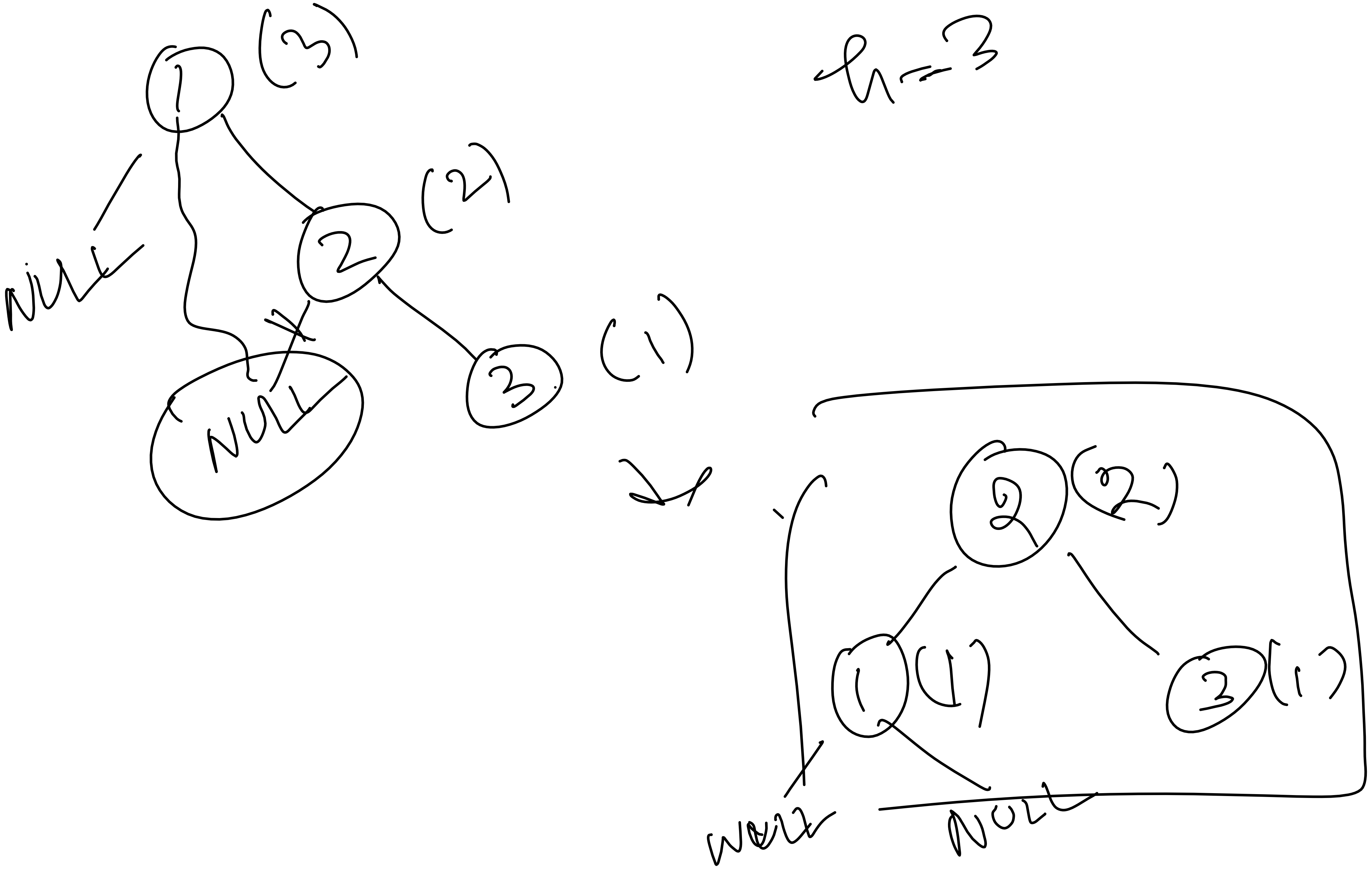
Pseudocode

Remark: Insert followed by rotation will never increase the height.

Example

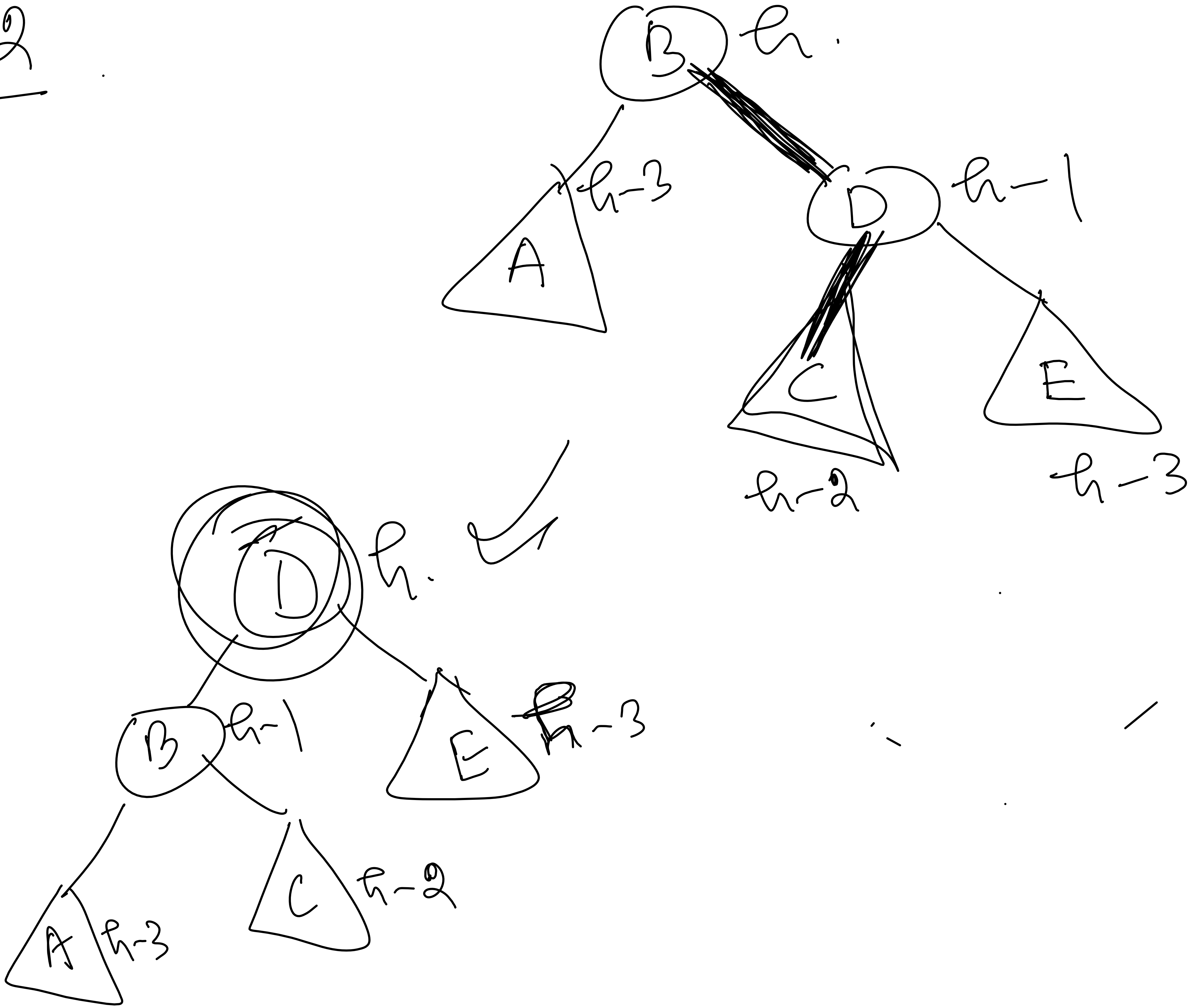
1,3

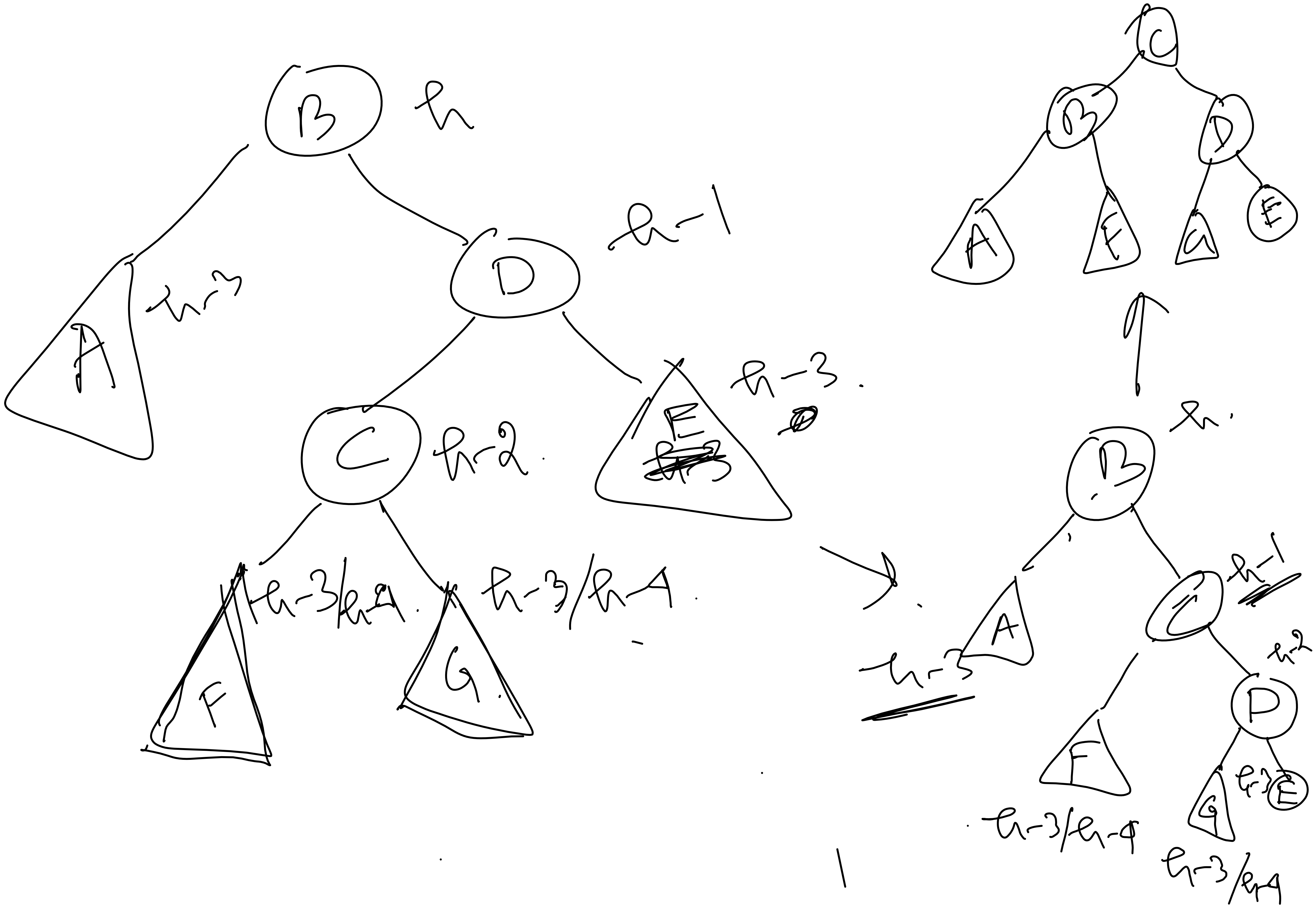
h=3



Case 2

(a)





B
h
A
h-3
D
h-1
C
h-2
E
h-3
F
h-3/h-4
G
h-3/h-4
C
B
D
A
F
G
E
B
h
A
h-3
C
h-1
F
h-3/h-4
P
h-2
G
h-3/h-4
E
h-3