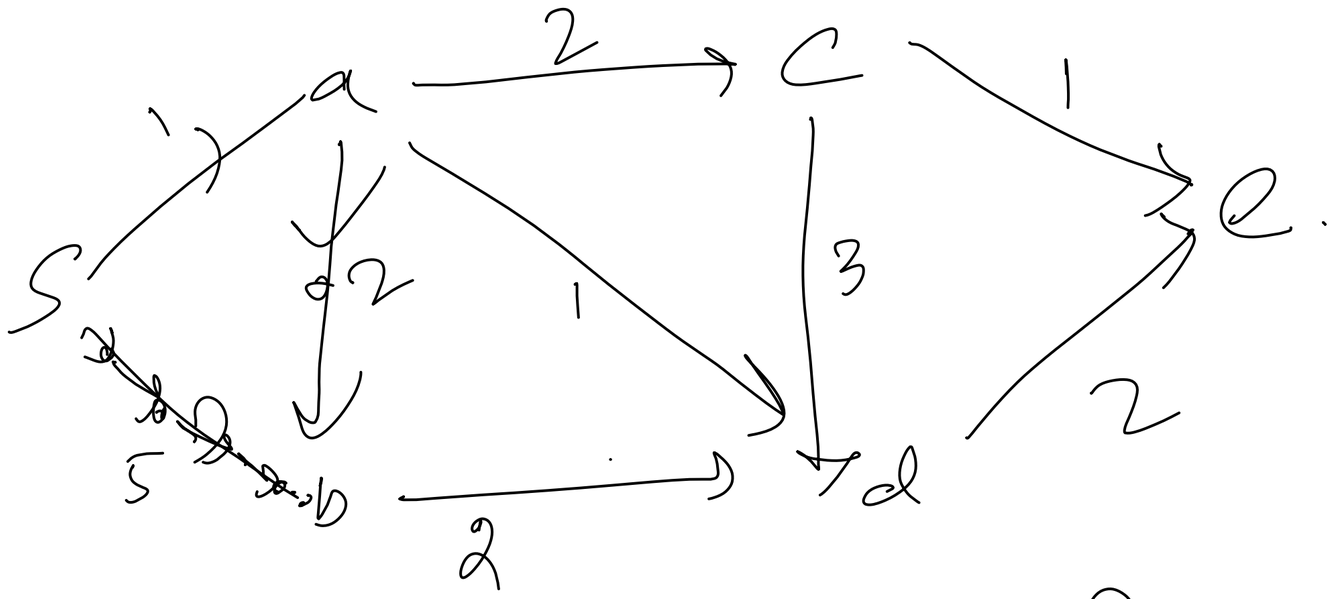
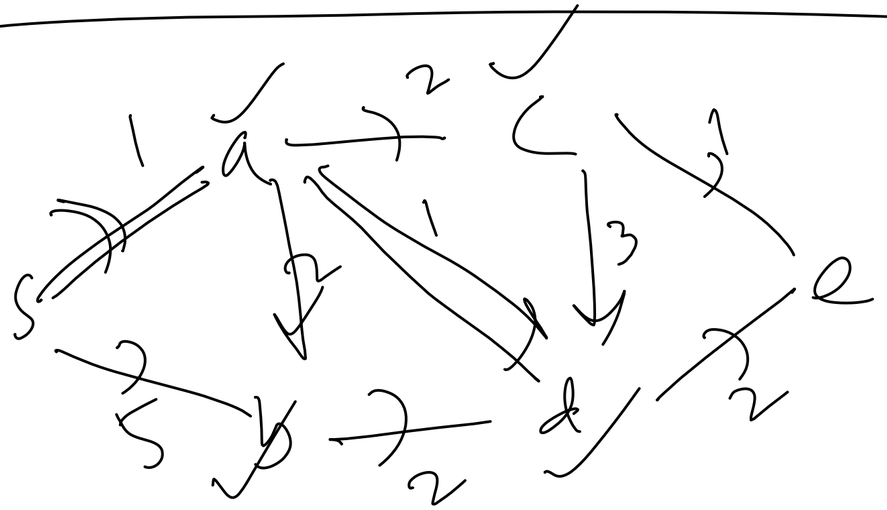
Single Source Shortest Path



Q: Give shortest paths from s to all other nodes.

(1) 1000, max wt will complexity.

(2) Real weights.



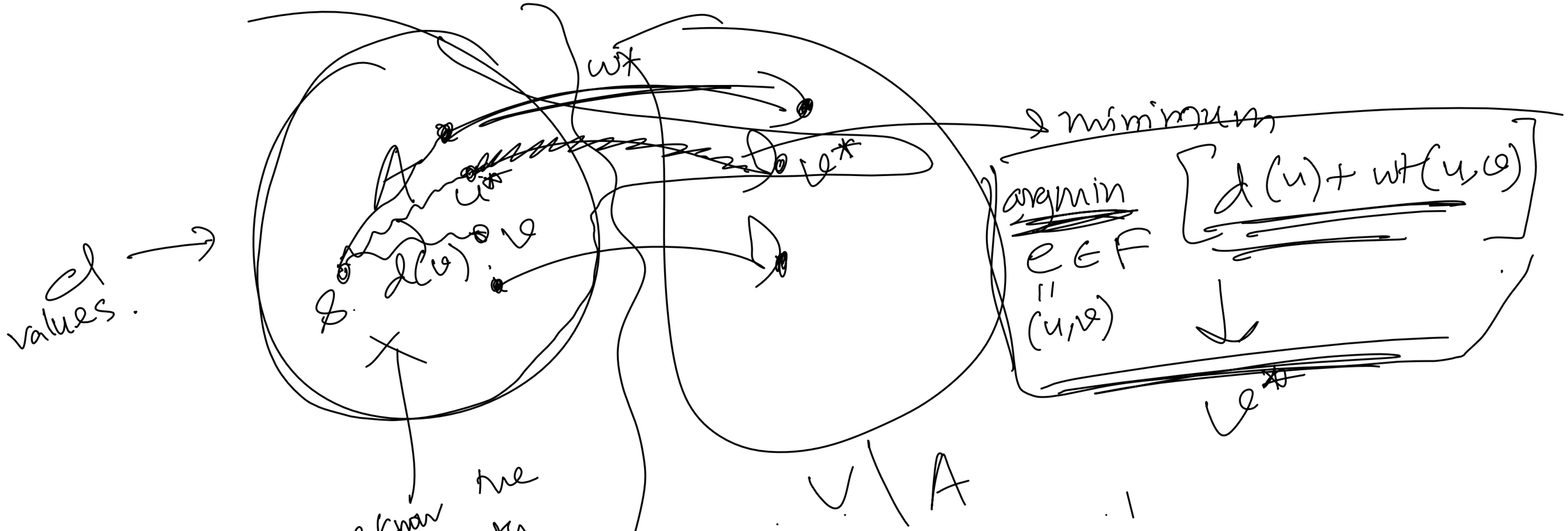
a	1
b	~~4~~ 3
c	3
d	2
e	4

~~b-3~~

c-3

e-4

assumption: wts are non-ve.



$F \subseteq E$

$\downarrow$

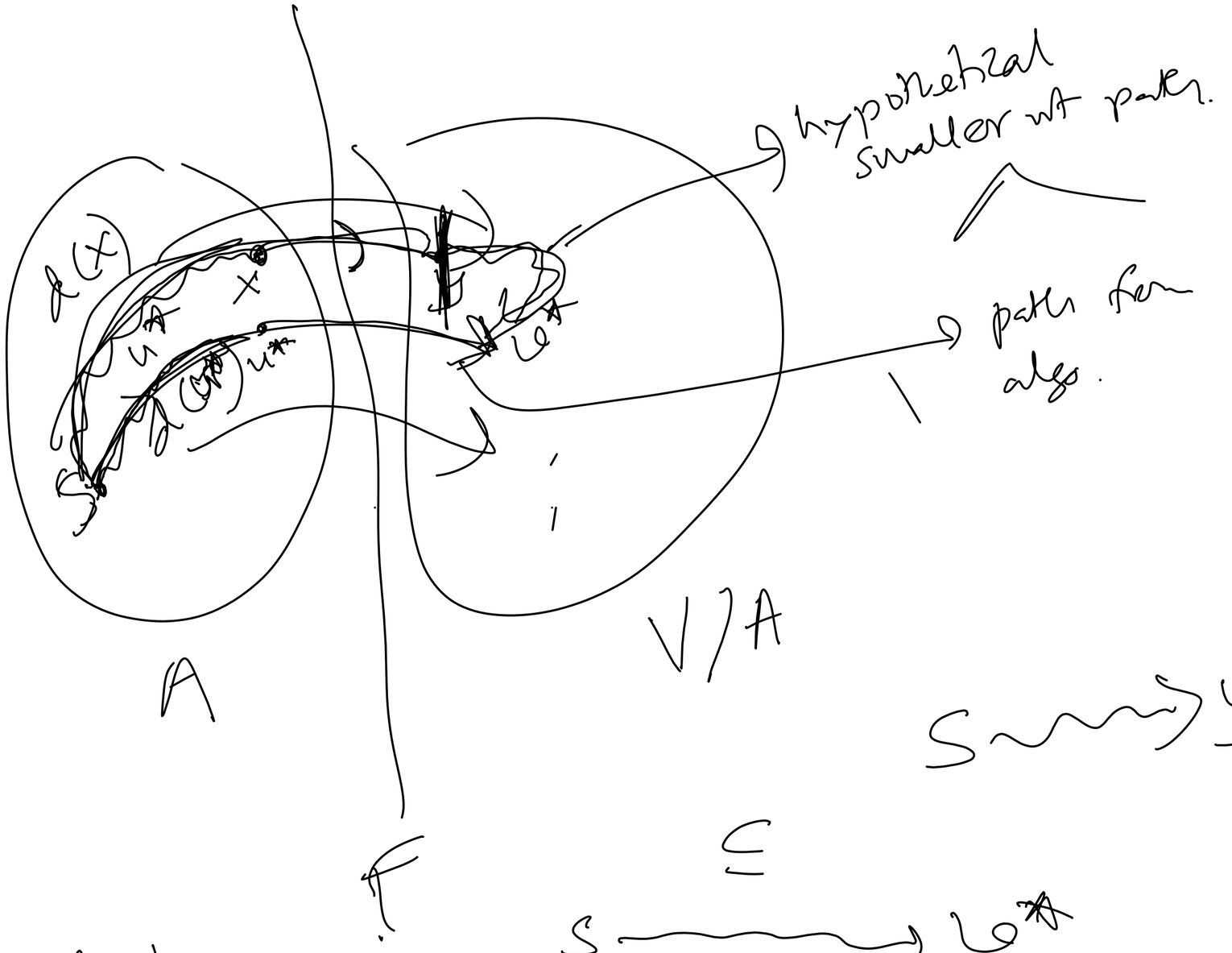
$\subseteq A \times \{V \setminus A\}$

<u>Claim:</u> $s \rightsquigarrow u^* \longrightarrow v^*$ is the shortest path to v^*

Claim: - - -

$$\cancel{d(u^*) + wt(u^*, v^*)}$$

$$\cancel{d(x) + wt(x, y) + wt(y, v^*)} \geq 0$$



$S \rightsquigarrow y$

$S \xrightarrow{\subseteq} v^*$

Proof: by contradiction.

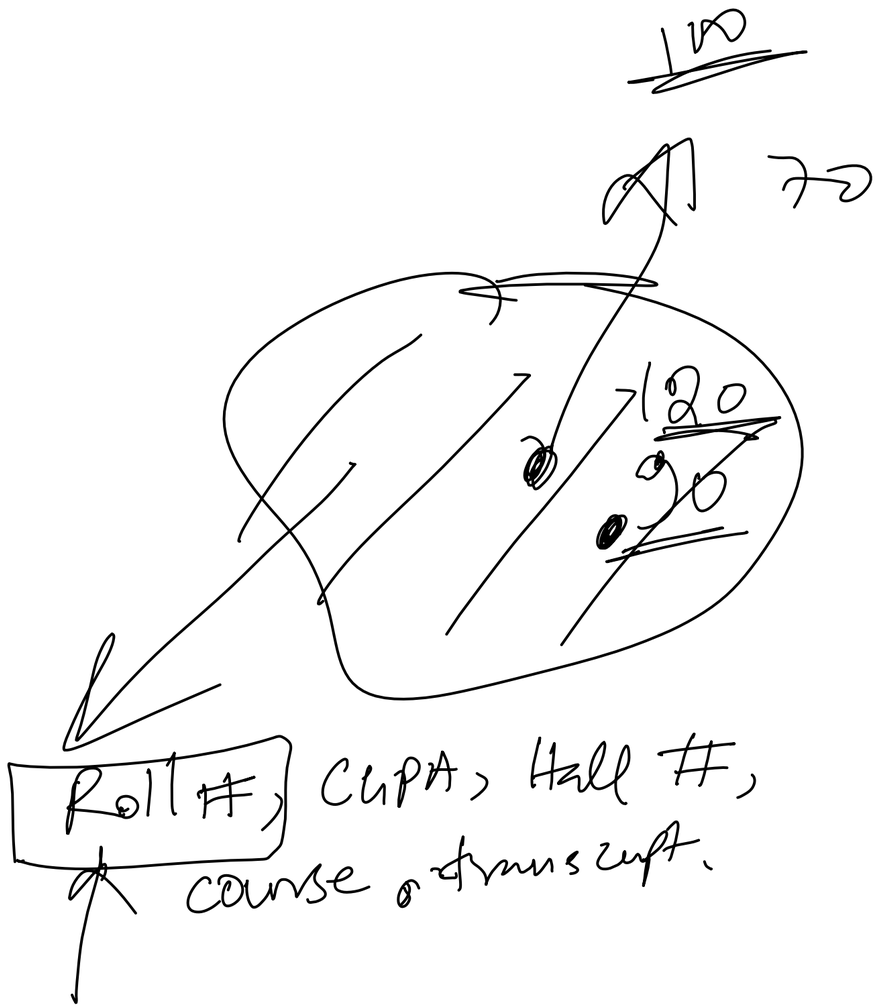
DFS – Stack

BFS – Queue

SSSP – priority queue

the item that is going to be dequeued is the minimum?

w.r.t some fixed key.



Priority queue data structure

- query a particular item → give me the key of this item
- update key to a smaller value.
- extract the minimum-key item

Procedure SSSP(G, S).

// $G = (V, E)$

$\pi, d \leftarrow$ empty array of size $|V|$

for each $v \in V$

$d(v) \leftarrow \infty$

$PQ \leftarrow$ ~~small~~ priority queue of items V, key defined by the array d.

$d(S) \leftarrow 0$.

$\pi(S) \leftarrow$ NULL

include S to A

$\Theta(n)$ while ($A \neq V$) → (run this while loop n times)

$\Theta(n)$ → $u \leftarrow PQ.$extract-min() // $d(S,u)$ is known

for each $(u, x) \in E$

~~PQ.update-key(x, $d(u) + wt(u,x)$)~~

$O(m)$

if ($d(x) > d(u) + wt(u,x)$)

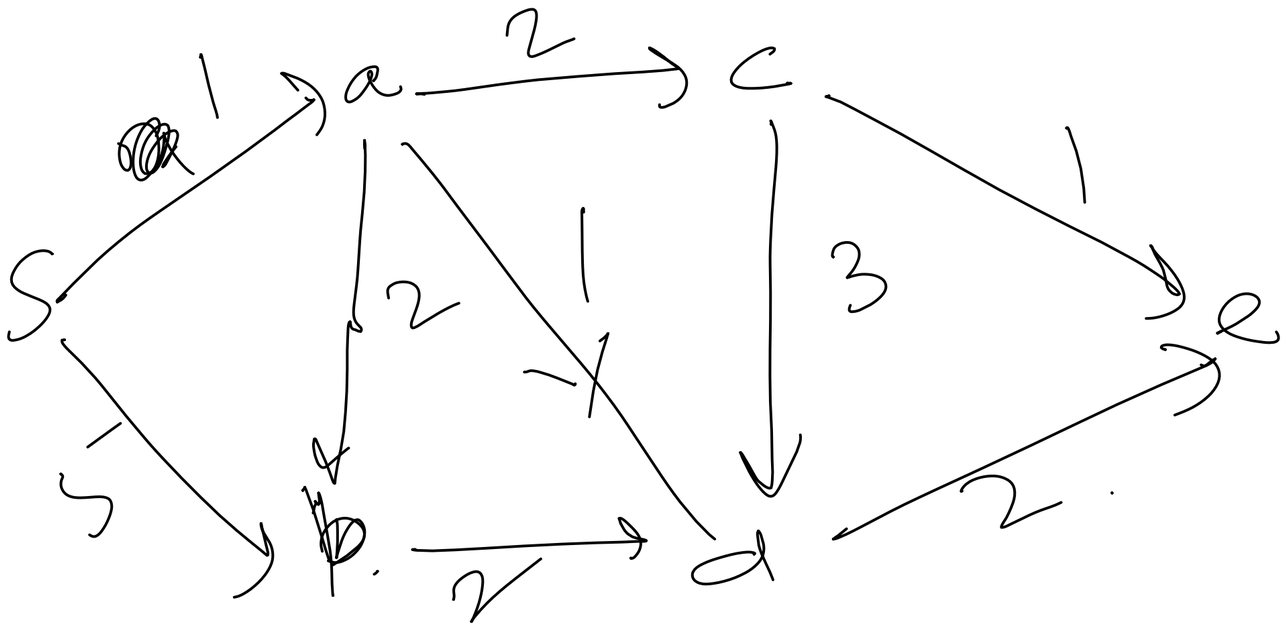
$\pi(x) \leftarrow u$

PQ.update_key(x, $d(u) + wt(u,x)$)

* Priority queue has d array inside it

this operation updates the d array.

$\Theta(n^2 + m)$ costly.



A: $\boxed{\begin{array}{l} s, a, d \\ c, b, e \end{array}}$

d array

0	s	0	
1	a	∅	1
10	b	∅	3
0	c	∅	3
1	d	∅	2
1	e	∅	4

2

Edgar Dijkstra : Dijkstra's algorithm