Cutting Cloth Problem from Dasgupta

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In this problem, you can either make a complete vertical cut or a complete horizontal cut. Therefore, it restricts some complex arrangements of the pieces. We will think of the cloth is spread in cartesian coordinates. The horizontal size is $[0, X]$ and the vertical side is $[0, Y]$.

Paremterization $DP[i, j, k, l]$ is the best profit from the sub rectangle $[i, j] \times [k, l]$. Here, i, j are X-coordinates. k, l are Y-coordinates.

Final answer $DP[0, X, 0, Y]$.

Recursion At each step we can either vertically cut at $i < a < j$ or horizontally cut at $k < b < l$. Or not cut at all. The last case happens only when the starting cloth exactly matches with some product or when the starting cloth is useless for all products.

We say that t-th product can fit $[i, j] \times [k, l]$ if any of the following two holds.

- $a_t \leq (j - i)$ and $b_t \leq (l - k)$
- $b_t \leq (j - i)$ and $a_t \leq (l - k)$

We say that t-th product can *tightly* fit $[i, j] \times [k, l]$ if any of the following two holds.

- $a_t = (j - i)$ and $b_t = (l - k)$
- $b_t = (j - i)$ and $a_t = (l - k)$

The $DP[i, j, k, l]$ for any given $i < j$ and $k < l$ is the maximum over the following choices as a, b varies over $i < a < j$ and $k < b < l$. Note that even if the current cloth tightly fits any product, it may still be more profitable to cut the cloth.

- 0 if no product fits $[i, j] \times [k, l]$
- maximum of c_t over all products t such that $a_t \times b_t$ tightly fits $[i, j] \times [k, l]$
- If $j > i + 1$ or $l > k + 1$: $\max_{i<a<j, k<b<l} \{DP[i, a, k, l] + DP[a, j, k, l], DP[i, j, k, b] + DP[i, j, b, l]\}$

Base Cases The first scenario in the above and the case when $j = i + 1, l = k + 1$ are base cases as they don't use recursion.

Runtime DP Table has size $O(X^2Y^2)$. It takes $O(X + Y + n)$ options to do each recursive step where n is the number of products. Therefore, the total time is $O(X^2Y^2(X + Y + n))$.