09.08.2024

Procedure RMultiply (X, Y)

// X, Y are n-bit inputs.

A) $X_L \leftarrow n/2$ LSB places of X

$X_R \leftarrow n/2$ remaining places.

$Y_L \leftarrow$ 1 - - - .

$Y_R \leftarrow$ - . . .

$A \leftarrow$ RMultiply (X_L, Y_L)

$B \leftarrow$ RMultiply (X_R, Y_R)

$C \leftarrow \text{RMultiply}(X_L, Y_R)$

$D \leftarrow \text{RMultiply}(X_R, Y_L)$.

Result $\leftarrow$ A

Result $\leftarrow$ Result + leftshift of C by $n/2$ bits

Result $\leftarrow$ Result + left shift of D by $n/2$ bits.

Result $\leftarrow$ Result + left shif of B by n bits.

Return Result.

Correctness!

Stopping condition missing.

-- RMultiply (X, Y).

(*) if ~~(~~ (length (x) = 1)

if (X = 0 or Y = 0) return 0

~~end if~~

else return 1

end if.

Runningtime: "recurrence relation"

$$T(n) \leq \boxed{4T(n/2)} + cn$$

$$\leq 4\left(4T(n/4) + c\frac{n}{2}\right) + cn$$

$$= 4^2 T(n/4) + c[n + 2n]$$

$$\leq 4^2\left(4T(n/8) + c\, n/4\right) + c[n+2n]$$

$$= 4^3 T(n/8) + c[n + 2n + 4n]$$

$$\ldots$$

$$\leq 4^{i}\, T(\underbrace{n/2^{i}}_{1}) + cn\left[1+2+4+\dots+2^{i-1}\right].$$

$$i = \log n$$

$$\leq 4^{\log_2 n}\, T(1) + cn\left[1+2+\dots+2^{\log_2 n - 1}\right]$$

$$= (\underbrace{2^{\log_2 n}}_{n})^{2}\, \Theta(1) + cn\left[n-1\right]$$

$$= \Theta(n^2).$$

$$\rightarrow \boxed{(X_L Y_R + X_R Y_L)}$$

$$= \overset{3}{\left[\underbrace{(X_L + X_R)}_{X'} \cdot \underbrace{(Y_L + Y_R)}_{Y'}\right]} - \overset{1}{X_L \cdot Y_L} - \overset{2}{X_R Y_R}$$

$\cancel{\times 2}$

Procedure RMultiply2 (X, Y)

— — — — -

2 Add

2 Subtraction $\Big\} \; \Theta(n)$

$$T(n) = 3\,T(n/2) + c \cdot n$$

$$T(n) = 2\,T(n/2) + \cancel{T(n/2 + 1)} + cn.$$

$$\approx T(n/2)$$

$$\leq$$

issue with 1-bit extra

$$\underbrace{b' \ X'}_{n/2+1 \text{ bits}} \qquad \underbrace{a' \ Y'}_{n/2+1 \text{ bit}}$$

$$\downarrow$$

$$(b' \cdot 2^{n/2} + X')(a' \cdot 2^{n/2} + Y')$$

$$T(n) = a'b' 2^{n} + a'X' 2^{n/2} + b'Y' 2^{n/2} + X', Y' \rightarrow n/2.$$

$$T(n) = 3T(n/2) + dn.$$

$$\downarrow$$

$$= 3\left[3T(n/4) + d\,\tfrac{n}{2}\right] + dn.$$

$$= 3^2\,T(n/4) + dn\left[1 + \tfrac{3}{2}\right]$$

$$\vdots$$

$$= 3^j\,T(n/2^j) + dn\left[1 + \tfrac{3}{2} + \cdots + \left(\tfrac{3}{2}\right)^{j-1}\right]$$

$j = \log_2 n$ $\boxed{\text{Karatsuba}} \to O(n \log n)$

$$T(n) \le 3^{\log_2 n} T(1) + dn\left[1 + \left(\frac{3}{2}\right) + \cdots + \left(\frac{3}{2}\right)^{\log_2 n - 1}\right]$$

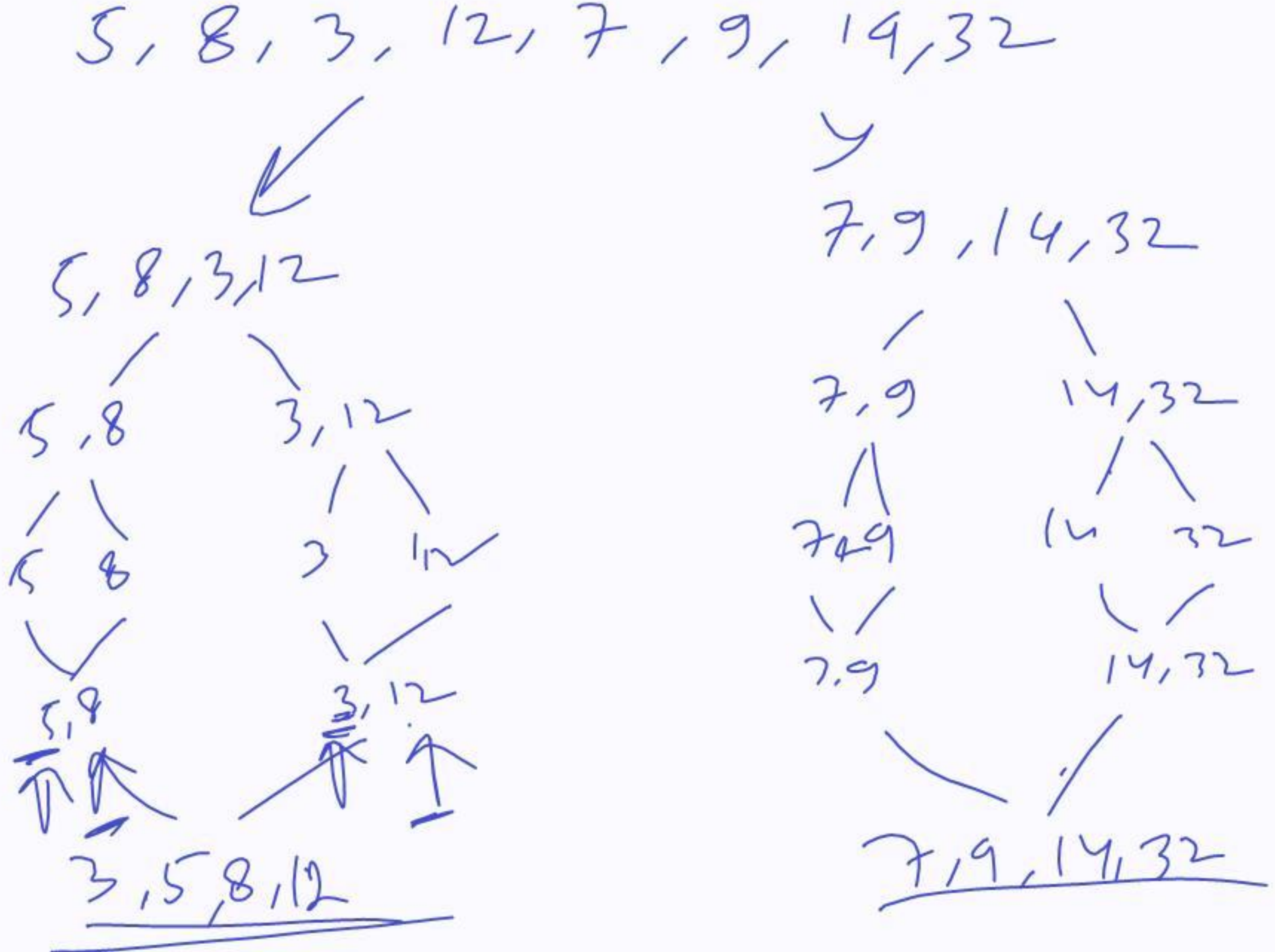
$$= 2^{\log_2 3 \cdot \log_2 n} \cdot \Theta(1) + dn\left[\left(\frac{3}{2}\right)^{\log_2 n} - 1\right]$$

$$= \Theta\left(n^{\log_2 3}\right) + dn\left(n^{\log_2 \frac{3}{2}} - 1\right)$$

$$= O\left(n^{1.6}\right) = o\left(n^2\right)$$

little oh

Mergesort



5, 8, 3, 12, 7, 9, 14, 32
5, 8, 3, 12
7, 9, 14, 32
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14, 32
3, 5, 8, 12
7, 9, 14, 32

[3, 5, 8, 12

[7, 9, 14, 32

Merge

[3, 5, 7, 8, 9, 12, 14, 32

n = power of 2.

Procedure Merge Sort (A, n)

If n = 1 return A

$B \leftarrow A[1 \cdots n/2]$

$C \leftarrow C[n/2+1 \cdots n]$

B ← Merge sort [B] ✓

C ← MergeSort [C] ✓

return Merge $(B, \frac{n}{2}, 1, C, \frac{n}{2}, 1)$

Size of B indexB index C.

Procedure merge($\underline{A}$, n, i, $\underline{B}$, m, j)

if ($n = \emptyset$ or $n = i$) return B. end if

if ($m = 0$ or $m = j$) return A

else {

if $(A[i] < B[j])$

return $A[i] \circ \text{Merge}(A, n, i+1, B, m, j)$

append.

append.

else

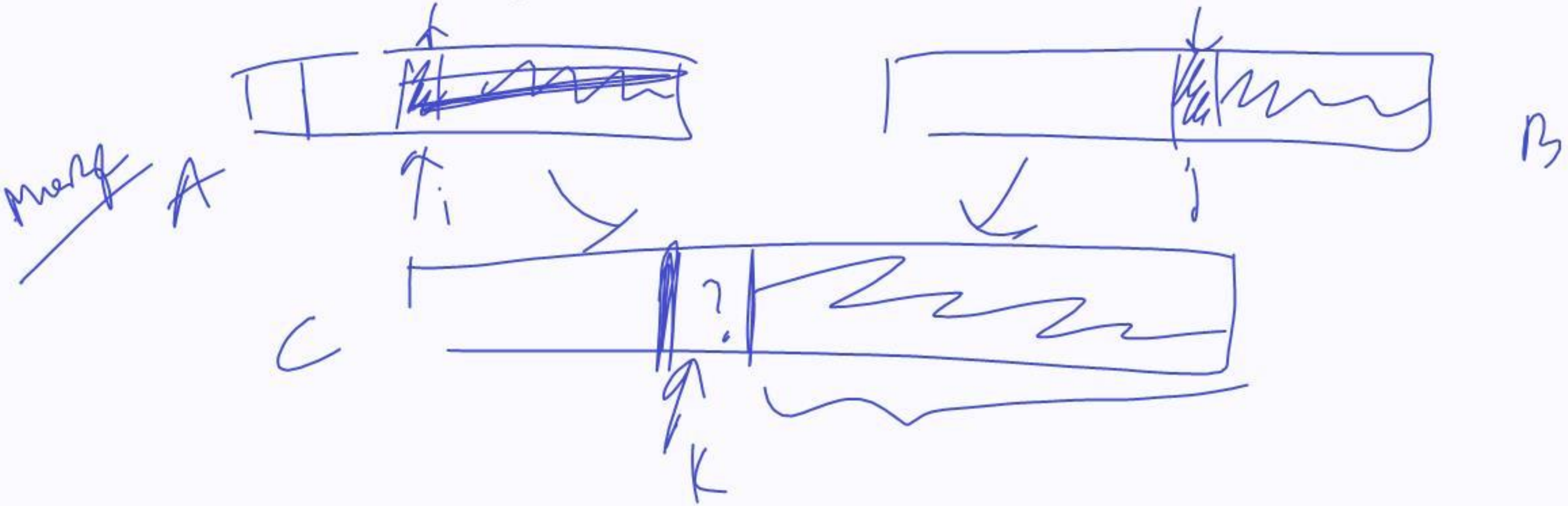
return $B[j] \circ \text{Merge}(A, n, i, B, m, j+1)$

Correctness : Proof by Induction.

Assume mergesort works correctly for any

$j < n$.

we are trying to prove it for $\boxed{j = n}$ final array



by our choice $C[k] \leq A[i]$ & $C[k] \leq B[j]$

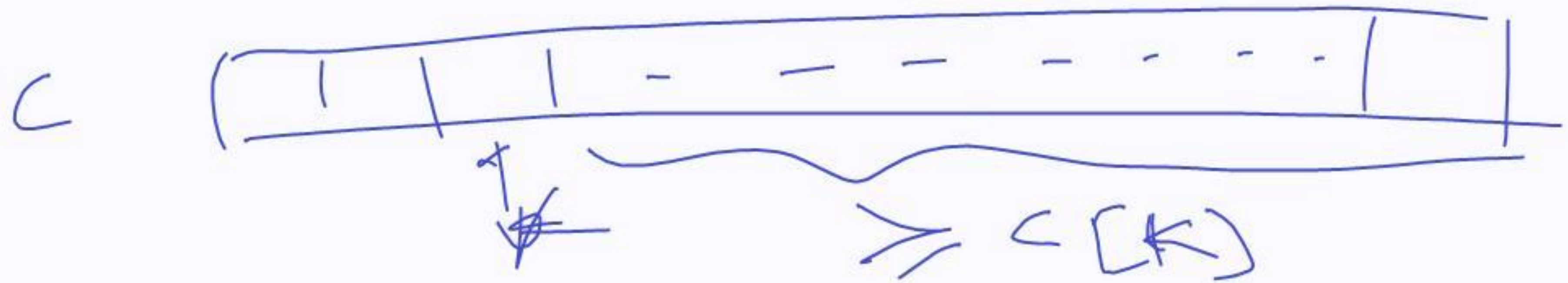
Therefore, it implies

$$C[k] \leq A[i'] \quad \forall \; i' \geq i$$

&

$$C[k] \leq B[j'] \quad \forall \; j' \geq j$$

Since we assumed A, B are sorted.



All items from either A or B

appear once in C.

=) trivial Array array is sorted.

2) Induction step holds for $j = n$

Time complexity ;

$$T(n) = \overbrace{2T(n/2) + \underline{cn} + \text{(time complexity of merge)}}$$

$$\leq d \cdot \left(\underset{\text{number}}{\#} \text{ itemes in total of } \underline{A}, \underline{B} \right)$$

$$= 2T(n/2) + \underline{f \cdot n}$$