MS0201A : Problem Set 2

Answers

(1) (a) $P(.)$ is a prob measure

(b) $P(.)$ is a prob measure

(c) $P(.)$ is not a prob measure

Second part - (a)

$$P(E) = 1 - e^{-\lambda}\left(1 + \lambda + \frac{\lambda^2}{2}\right)$$

$$P(F) = \sum_{1}^{2} \frac{e^{-\lambda}\lambda^x}{x!}$$

$$P(E \cup F) = \sum_{1}^{\infty} e^{-\lambda}\lambda^x/x! = 1 - e^{-\lambda}$$

(2) (a) & (b) : $P(.)$ is a prob measure

(c) : $P(.)$ is not a prob measure

(3) $P(A) + P(B) - 2P(AB)$

(4) use properties of $P(.)$

(5) Done in class

(6) (a) $c = e^{-2}$

(b) $$P(A) = \sum_{2}^{4} \frac{e^{-2}2^x}{x!}, \quad P(B) = \sum_{3}^{\infty} \frac{e^{-2}2^x}{x!}$$

$$P(C) = \sum_{0}^{\infty} e^{-2}\frac{2^{2x+1}}{(2x+1)!}$$

$$P(B \cap C) = P(\{3\}) + P(\{5\}) + \cdots$$

$$- - -$$

(7) reqd prob

$$= 1 - \left(P(A_1) + P(A_2) + P(A_3) - P(A_1A_2) - P(A_1A_3) - P(A_2A_3) + P(A_1A_2A_3)\right)$$

$$P(A_i) = \left(\frac{4}{5}\right)^6 \ \forall i; \quad P(A_iA_j) = \left(\frac{3}{5}\right)^6 \ \forall i \neq j; \quad P(A_1A_2A_3) = \left(\frac{2}{5}\right)^6$$

(8) $\text{req prob} = 1 - \frac{1}{2!} + \frac{1}{3!} - \cdots + (-1)^{n-1} \frac{1}{n!}$

(9) (a) $\text{req prob} = \sum_{i=0}^{n} (-1)^i \frac{1}{i!}$

(b) $\text{req prob} = \sum_{i=0}^{n} (-1)^i \frac{1}{i!\ (n)_i} \rightarrow \left(\frac{n!}{(n-i)!} \right)$

(10) use properties & def^n of cond^{n}al prob

(11)

(a) true

(b) false

(c) false

(*) Condition that $0 < P(B) < 1$ is missed out in the problem statement; this is required.

(12)

(a) false

(b) false

(c) true

(d) false

(13) (a) $\binom{4}{3} \left(\frac{1}{2}\right)^3 \cdot \frac{1}{2}$

(b) $\frac{3}{8}$

(14) $\frac{p_A}{p_A + p_B}$

(15) $1 - \prod_{i=1}^{n} (1 - p_i)$

(16) $\dfrac{90 \times 89 \times 88}{100 \times 99 \times 98}$

(17) $\dfrac{\frac{1}{2} \times \frac{2}{3}}{\frac{1}{2} \times \frac{2}{3} + \frac{2}{3} \times \frac{1}{3}}$

(18) $\dfrac{\frac{13}{27} \times \frac{1}{3}}{\frac{13}{27} \times \frac{1}{3} + \frac{14}{27} \times \frac{2}{3}}$

(19) $\frac{1}{3}$

(20) (a) $\prod_{1}^{4} P(C_i)$

(b) $\prod_{1}^{4} P(C_i^c)$

(c) $P(C_1) \prod_{2}^{4} P(C_i^c) + P(C_2) \prod_{\substack{i=1 \\ i \neq 2}}^{4} P(C_i^c) + P(C_3) \prod_{\substack{i=1 \\ i \neq 3}}^{4} P(C_i^c) + P(C_4) \prod_{i=1}^{3} P(C_i^c)$

(d) $1 - \prod_{1}^{4} P(C_i^c)$

(21) use that $1 - x < e^{-x}$ for $0 < x < 1$

(22) One counter example given in class; try to find some other

(23) If it is not true that "C carries negative info about A". think of a counter example

(24) $\frac{2}{7} + \frac{2}{3} \times \frac{4}{7} + \frac{1}{3} \times \frac{1}{7}$

(25)

(a) $(0.9)^2 \left(0.8^2 + 0.8 - 0.8 \times 0.8 \right)$

(b) $\dfrac{0.9 \times 0.9 \times 0.2 \times 0.8}{(\quad)}$

(26)

(a) $0.2 \times 0.6 \times 0.8 \times 0.9$

(b) $0.6 \times 0.8 \times 0.9$

(27)

$$\frac{0.1 \times 1}{\left(0.76 \times 0 + 0.09 \times \frac{1}{5} + 0.02 \times \frac{2}{5} + 0.01 \times \frac{3}{5} + 0.02 \times \frac{4}{5} + 0.10 \times 1\right)}$$