MSO 201A : Problem Set 3

Answers

(1) (a), (b), (c)

verify that $Y^{-1}(-\infty, x] \in \mathcal{F}_c \quad \forall\ x \in \mathbb{R}$

(2) same approach as in (1)

(3) X is not a r.v.

(4) Follow the def^n of r.v. to prove

~~(5)~~
$$F(x) = \begin{cases} 0, & x < 0 \\ 36/52, & 0 \le x < 1 \\ 40/52, & 1 \le x < 2 \\ 44/52, & 2 \le x < 3 \\ 48/52, & 3 \le x < 4 \\ 1, & x \geq 4 \end{cases}$$

(5) To prove that $F(.)$ is d.f. verify that it is right cont, non-decreasing, $F(-\infty) = 0$ & $F(\infty) = 1$.

$$P\left(-\tfrac{1}{2} < X \le 1/2\right) = \frac{2}{8}$$

$$P(X = 0) = 0$$

$$P(X = 1) = \frac{1}{4}$$

$$P(-1 \le X < 1) = \frac{3}{4}$$

$$F(x) = \frac{1}{2} F_c(x) + \frac{1}{2} F_d(x)$$

$$F_c(x) = \begin{cases} 0, & x < -1 \\ \frac{x+1}{2}, & -1 \le x < 1 \\ 1 & x \geq 1 \end{cases} \quad \& \quad F_d(x) = \begin{cases} 0, & x < -1 \\ \frac{1}{2}, & -1 \le x < 1 \\ 1, & x \geq 1 \end{cases}$$

(6)

(a) $F(.)$ is not a d.f.

(b) & (c) $F(.)$ is a d.f.; verify the n.s.c.

(7) $P(X > 6) = e^{-2}$

$P(X = 5) = 0$

$P(5 \le X \le 8) = F(8) - F(5-)$

$= \left(1 - \frac{2}{3} e^{-8/3} - \frac{1}{3} e^{-2}\right) - \left(1 - \frac{2}{3} e^{-5/3} - \frac{1}{3} e^{-1}\right)$

$= \ldots$

(8) $P(-2 \le X < 5) = \frac{1}{2}$

$P(0 < X < 5.5) = \frac{1}{8}$

$P(1.5 < X \le 5.5 \mid X > 2) = \frac{1}{4}$

(9) verify the n.s.c. for $F(.)$ to be d.f.

(10) $G(.)$ is not d.f.

(11) $K = -1$

(12) $c = 2$

$$F(x) = \frac{1}{4} F_d(x) + \frac{3}{4} F_c(x)$$

$$F_d(x) = \begin{cases} 0, & x < 0 \\ 1, & x \geq 0. \end{cases}$$

$$F_c(x) = \begin{cases} 0, & x < 0 \\ x/6, & 0 \le x < 1 \\ x^2/6, & 1 \le x < 2 \\ x/3, & 2 \le x < 3 \\ 1, & x \geq 3 \end{cases}$$

(13) $Y = X^+$ $\quad F_Y(y) = \begin{cases} 0, & y < 0 \\ F(y), & y \geq 0 \end{cases}$

$Z = |X|$

$$F_Z(z) = \begin{cases} 0, & z < 0 \\ F(z) - F(-z-), & z \geq 0 \end{cases}$$

(14) Verify the n.s.c. for $F(.)$ to be d.f.

(15) (a) $f(.)$ is not a ~~pdf~~ p.m.f.

(b) $f(.)$ is a p.m.f.

(16) $f(.)$ is not a p.m.f.

(~~16~~) (17)

$X = x$	-3	-2	-1	0	1	2	3
$P(X = x)$	$1/9$	$1/9$	$1/9$	$3/9$	$1/9$	$1/9$	$1/9$

(18) X : total no. of clocks working after 300 days

Possible values 0, 1, 2, 3

$$P(X=3) = P(ABC) = P(A)\,P(B)\,P(C) = .95 \times .9 \times .8$$

$$P(X=0) = P(A^c B^c C^c) = P(A^c)\,P(B^c)\,P(C^c)$$

$$= \ldots$$

$$P(X=1) = P(A^c B^c C \cup A^c B C^c \cup A B^c C^c)$$

$$= \ldots$$

$$P(X=2) = P(ABC^c \cup AB^cC \cup A^cBC)$$

$$= \ldots$$

d.f. has 4 pts of jump discontinuities at 0, 1, 2, 3

(19) $$P(2 < X < 3) = F(3) - F(2)$$

$$P(X > 5) = 1 - F(5)$$

$$F(x) = \theta^2 \int_0^x t\, e^{-tx}\, dt = \ldots \quad \forall x > 0$$

(20) $$C = \frac{\lambda^2}{1+\lambda}$$

(21) $$F(x) = \begin{cases} 0, & x < -3 \\ \dfrac{x^3+27}{54}, & -3 \le x < 3 \\ 1, & x \geq 3 \end{cases}$$

$$P(|X| < 1) = \frac{1}{27} \; ; \; P(X^2 < 9) = 1$$