MSO201A: PROBABILITY & STATISTICS
Problem Set #12

[1] $X_1,...,X_n$ be a random sample from $N\left(\mu, \sigma^2\right)$ distribution. Find the Cramer-Rao Lower Bounds (CRLB) on the variances of unbiased estimators of μ and σ^2. Can you find unbiased estimators μ and σ^2 whose variances attain the respective CRLB?

[2] $X_1,....,X_n$ is a random sample from $Gamma(\alpha,\beta)$

$$f(x \mid \alpha,\beta)=\begin{cases}\dfrac{1}{\overline{|\alpha}\;\beta^{\alpha}}e^{-x/\beta}x^{\alpha-1} & \text{if } x>0\\ 0 & \text{otherwise}\end{cases}$$

α is assumed to be known. Find the Fisher Information $I(\beta)$ and the CRLB on the variances of unbiased estimators of β.

[3] $X_1,...,X_n$ be a random sample from $P(\theta),\theta\in(0,\infty)$. Find the CRLB on the variances of unbiased estimators of the following estimands: (a) $g(\theta)=\theta$, (b) $g(\theta)=\theta^2$ and (c) $g(\theta)=e^{-\theta}$.

[4] Suppose $X_1,...,X_n$ be a random sample from $B(1,\theta),\theta\in(0,1)$. Find the CRLB on the variances of unbiased estimators of the following estimands: (a) $g(\theta)=\theta^4$ (b) $g(\theta)=\theta(1-\theta)$.

[5] $X_1,...,X_n$ be a random sample from $U(0,\theta),\theta>0$. Show that (a) $\dfrac{n}{n+1}X_{(n)}$ is a consistent estimator of θ and (b) $e^{X_{(n)}}$ is consistent for e^{θ}, where $X_{(n)}=\max\left(X_1,...,X_n\right)$.

[6] $X_1,...,X_n$ be a random sample from $U\left(\theta-1/2,\theta+1/2\right),\theta\in\Re$. Show that $X_{(1)}+1/2$, $X_{(n)}-1/2$ and $\left(X_{(1)}+X_{(n)}\right)/2$ are all consistent estimators of θ, $X_{(n)}=\max\left(X_1,...,X_n\right)$ and $X_{(1)}=\min\left(X_1,...,X_n\right)$.

[7] $X_1,...,X_n$ be a random sample from

$$f(x)=\begin{cases}\dfrac{1}{2}(1+\theta x) & -1<x<1\\ 0 & \text{otherwise.}\end{cases}$$

Where, $\theta\in(-1,1)$. Find a consistent estimator for θ.

[8] $X_1,...,X_n$ be a random sample from $P(\theta)$. Find a consistent estimator of $\theta^3\left(3\sqrt{\theta}+\theta+12\right)$.

[9] Let $X_1,\ldots,X_n$ be a random sample from $Gamma(\alpha,\beta)$ with density

$$f(x)=\begin{cases}\dfrac{1}{\Gamma\alpha\,\beta^{\alpha}}\,e^{-x/\beta}x^{\alpha-1} & x>0\\ 0 & \text{otherwise.}\end{cases}$$

Where, α is a known constant and β is an unknown parameter. Show that $\sum_{i=1}^{n} X_i \big/ n\alpha$ is a consistent estimator of β.