

$$(1) X \sim U(0,1)$$

$$F_X(x) = \begin{cases} 0 & x < 0 \\ x & 0 \leq x \leq 1 \\ 1 & x > 1 \end{cases}$$

Problem set #6

Solutions using d.f. Method

$$(a) Y = \sqrt{X}$$

$$\begin{aligned} \text{d.f. of } Y : F_Y(y) &= P(\sqrt{X} \leq y) \\ &= P(X \leq y^2) = \begin{cases} 0 & y < 0 \\ y^2 & 0 \leq y \leq 1 \\ 1 & y > 1 \end{cases} \end{aligned}$$

$$\text{p.d.f. of } Y \text{ is } f_Y(y) = \begin{cases} 0, & y < 0 \\ 2y, & 0 \leq y \leq 1 \\ 1, & y > 1 \end{cases} = \begin{cases} 2y, & 0 \leq y \leq 1 \\ 0, & \text{o/w.} \end{cases}$$

$$(b) Y = X^2$$

$$\begin{aligned} \text{d.f. of } Y : F_Y(y) &= P(X^2 \leq y) \\ &= \begin{cases} 0, & y < 0 \\ P(-\sqrt{y} \leq X \leq \sqrt{y}), & 0 \leq y \leq 1 \\ 1, & y > 1 \end{cases} \end{aligned}$$

for $0 \leq y \leq 1$

$$P(-\sqrt{y} \leq X \leq \sqrt{y}) = P(0 \leq X \leq \sqrt{y}) = F_X(\sqrt{y}) = \sqrt{y}$$

$$\Rightarrow F_Y(y) = \begin{cases} 0, & y < 0 \\ \sqrt{y}, & 0 \leq y \leq 1 \\ 1, & y > 1 \end{cases}$$

$$\text{p.d.f. of } Y : f_Y(y) = \begin{cases} \frac{1}{2\sqrt{y}} & 0 \leq y \leq 1 \\ 0 & \text{o/w.} \end{cases}$$

$$(c) \quad Y = 2X + 3 \rightarrow (3, 5)$$

$$\begin{aligned} \text{d.f. of } Y: F_Y(y) &= P(2X + 3 \leq y) \\ &= P\left(X \leq \frac{y-3}{2}\right) = \begin{cases} 0, & y < 3 \\ \frac{y-3}{2}, & 3 \leq y \leq 5 \\ 1, & y > 5 \end{cases} \end{aligned}$$

$$\text{p.d.f. } f_Y(y) = \begin{cases} \frac{1}{2}, & 3 \leq y \leq 5 \\ 0, & \text{o/w} \end{cases} \Rightarrow Y \sim U(3, 5)$$

$$(d) \quad Y = -\lambda \log X \rightarrow (0, \infty)$$

$$\begin{aligned} F_Y(y) &= P(Y \leq y) = P(-\lambda \log X \leq y) = P(X > e^{-y/\lambda}) \\ &= 1 - P(X \leq e^{-y/\lambda}) \end{aligned}$$

$$\text{i.e. } F_Y(y) = \begin{cases} 0 & y < 0 \\ 1 - e^{-y/\lambda} & y \geq 0 \end{cases}$$

$$\text{p.d.f. of } Y: f_Y(y) = \begin{cases} \frac{1}{\lambda} e^{-y/\lambda}, & y \geq 0 \\ 0, & \text{o/w} \end{cases}$$

$$\text{i.e. } Y \sim \text{Exp}(\lambda) \text{ (scale } \lambda)$$

$$(2) \quad X \sim U(0, \theta)$$

$$Y = \min(X, \theta/2) \rightarrow (0, \theta/2) \leftarrow \text{range of } Y$$

$$\begin{aligned} F_Y(y) &= P(Y \leq y) = P(\min(X, \theta/2) \leq y) \\ &= 1 - P(\min(X, \theta/2) > y) \\ &= 1 - P(X > y \cap \theta/2 > y) \end{aligned}$$

$$\text{Now } P(X > y, \theta/2 > y) = 1 \quad \text{if } y < 0 \\ = 0 \quad \text{if } y \geq \theta/2$$

$$\text{for } 0 \leq y < \theta/2; \quad P(X > y, \theta/2 > y) = P(X > y) \\ = \frac{1}{\theta} \int_y^{\theta} dx = \frac{\theta - y}{\theta}$$

$$\Rightarrow F_Y(y) = \begin{cases} 0, & y < 0 \\ y/\theta, & 0 \leq y < \theta/2 \\ 1, & y \geq \theta/2 \end{cases}$$

Note: $F_Y(y)$ has a jump discontinuity at $\frac{\theta}{2}$

$$(3) \quad f_X(x) = \begin{cases} \frac{1}{2}, & -\frac{1}{2} \leq x \leq \frac{3}{2} \\ 0, & \text{o/w} \end{cases} \quad X \sim U(-\frac{1}{2}, \frac{3}{2})$$

$$Y = X^2 \rightarrow Y \in (0, 9/4)$$

$$F_Y(y) = P(X^2 \leq y) = P(-\sqrt{y} \leq X \leq \sqrt{y})$$

$$\text{For } y < 0; \quad F_Y(y) = 0$$

$$\& \quad y > \frac{9}{4}; \quad F_Y(y) = 1$$

$$\text{For, } 0 \leq y \leq \frac{1}{4}; \quad F_Y(y) = \int_{-\sqrt{y}}^{\sqrt{y}} \frac{1}{2} dx = \sqrt{y}$$

$$\text{for, } \frac{1}{4} < y \leq \frac{9}{4}; \quad F_Y(y) = \int_{-\sqrt{y}}^{-1/2} 0 \cdot dx + \int_{-1/2}^{\sqrt{y}} \frac{1}{2} dx = \frac{1}{2} \left(\sqrt{y} + \frac{1}{2} \right) \\ = \left(\frac{1}{4} + \frac{\sqrt{y}}{2} \right)$$

$$\Rightarrow F_Y(y) = \begin{cases} 0 & y < 0 \\ \sqrt{y} & 0 \leq y \leq \frac{1}{4} \\ \frac{1}{2}(\sqrt{y} + \frac{1}{2}), & \frac{1}{4} \leq y \leq \frac{9}{4} \\ 1, & y \geq \frac{9}{4} \end{cases}$$

$$\text{p.d.f. } f_Y(y) = \begin{cases} \frac{1}{2\sqrt{y}}, & 0 \leq y \leq \frac{1}{4} \\ \frac{1}{4\sqrt{y}}, & \frac{1}{4} < y \leq \frac{9}{4} \end{cases}$$

$$(4) \quad f_X(x) = \begin{cases} k \frac{x^{p-1}}{(1+x)^{p+q}}, & x > 0 \\ 0 & \text{o/w} \end{cases}$$

$$Y = \frac{1}{1+X} \Rightarrow X = \frac{1-Y}{Y} = \bar{g}^{-1}(Y); \quad Y \in (0,1)$$

$$J = \frac{dx}{dy} = -\frac{1}{y^2}$$

$$f_Y(y) = \begin{cases} f_X(\bar{g}^{-1}(y)) |J|, & 0 \leq y \leq 1 \\ 0 & \text{o/w} \end{cases}$$

$$= \begin{cases} k \cdot \left(\frac{1-y}{y}\right)^{p-1} \left(\frac{1}{y^2}\right)^{p+q} \cdot \frac{1}{y^2}, & 0 \leq y \leq 1 \\ 0, & \text{o/w} \end{cases}$$

$$\text{i.e. } f_Y(y) = \begin{cases} k y^{q-1} (1-y)^{p-1}, & 0 \leq y \leq 1 \\ 0 & \text{o/w.} \end{cases}$$

$$K = (\text{Beta}(q, p))^{-1}$$

$$\Rightarrow Y \sim \text{Beta}(q, p)$$

$$(5) f_X(x) = \begin{cases} K x^{\beta-1} e^{-\alpha x^{\beta}}, & x > 0 \\ 0, & \text{o/w} \end{cases}$$

$$Y = X^{\beta} \quad J = \frac{dx}{dy} = \frac{1}{\beta} y^{1/\beta-1} \quad (x = y^{1/\beta} = g^{-1}(y))$$

$$\frac{y > 0}{f_Y(y)} = \begin{cases} f_X(g^{-1}(y)) |J|, & y > 0 \\ 0, & \text{o/w} \end{cases}$$

$$= \begin{cases} K \cdot (y^{1/\beta})^{\beta-1} e^{-\alpha y} \left(\frac{1}{\beta} \cdot y^{1/\beta-1} \right), & y > 0 \\ 0 & \text{o/w} \end{cases}$$

$$= \begin{cases} K y^{1-1/\beta} e^{-\alpha y} \frac{1}{\beta} \cdot y^{1/\beta-1}, & y > 0 \\ 0 & \text{o/w} \end{cases}$$

$$= \begin{cases} K \frac{e^{-\alpha y}}{\beta}, & y > 0 \\ 0, & \text{o/w} \end{cases}$$

$$K = \alpha \beta \Rightarrow Y \sim \text{Exp}\left(\frac{1}{\alpha}\right).$$

$$(6) \quad f_v(v) = \begin{cases} K v^2 e^{-\beta v^2}, & v > 0 \\ 0, & \text{o/w} \end{cases}$$

$$E = \frac{1}{2} m v^2 \quad ; \quad v^2 = \frac{2E}{m}$$

$$\frac{\partial E}{\partial v} = m v$$

$$J = \frac{\partial v}{\partial E} = \frac{1}{\sqrt{2me}}$$

$$f_E(E) = \begin{cases} K \cdot \left(\frac{2E}{m}\right) e^{-\beta\left(\frac{2E}{m}\right)} \cdot \frac{1}{\sqrt{2me}} ; & E > 0 \\ 0 & \text{o/w} \end{cases}$$

$$= \begin{cases} c \cdot e^{1/2} \exp\left(-\frac{2\beta}{m} E\right), & E > 0 \\ 0, & \text{o/w} \end{cases}$$

$$c \cdot \int_0^{\infty} e^{1/2} \exp\left(-\frac{2\beta}{m} E\right) dE = 1$$

$$\therefore c \cdot \frac{\Gamma_{3/2}}{\left(\frac{2\beta}{m}\right)^{3/2}} = 1 \Rightarrow c = \frac{\left(\frac{2\beta}{m}\right)^{3/2}}{\Gamma_{3/2}}$$

$$\Rightarrow E \sim \text{Gamma}(\dots)$$

$$(7) \quad f_X(x) = \begin{cases} \frac{3}{8}(x+1)^2, & -1 < x < 1 \\ 0, & \text{o/w.} \end{cases}$$

$$Y = 1 - X^2, \quad Y \in (0, 1)$$

$$x^2 = 1 - y \Rightarrow x = \pm \sqrt{1 - y}$$

$$x \in (-1, 0) \rightarrow x = -\sqrt{1 - y} = g_1^{-1}(y) \rightarrow \left| \frac{dx}{dy} \right| = \frac{1}{2\sqrt{1 - y}}$$

$$x \in (0, 1) \rightarrow x = \sqrt{1 - y} = g_2^{-1}(y) \rightarrow \left| \frac{dx}{dy} \right| = \frac{1}{2\sqrt{1 - y}}$$

$$\begin{aligned} f_Y(y) &= \overset{(-1, 0)}{\downarrow} f_X(g_1^{-1}(y)) \left| \frac{dx}{dy} \right| + \overset{(0, 1)}{\downarrow} f_X(g_2^{-1}(y)) \left| \frac{dx}{dy} \right| \cdot 0 < y < 1 \\ &= \frac{3}{8} (1 - \sqrt{1 - y})^2 \cdot \frac{1}{2\sqrt{1 - y}} + \frac{3}{8} (1 + \sqrt{1 - y})^2 \cdot \frac{1}{2\sqrt{1 - y}} \\ &= \frac{3}{16\sqrt{1 - y}} \left((1 - \sqrt{1 - y})^2 + (1 + \sqrt{1 - y})^2 \right) \\ &= \frac{3}{16\sqrt{1 - y}} \left(2(1 + (1 - y)) \right) \end{aligned}$$

$$\therefore f_Y(y) = \begin{cases} \frac{3}{8} \left((1 - y)^{-1/2} + (1 - y)^{1/2} \right), & 0 < y < 1 \\ 0, & \text{o/w.} \end{cases}$$

$$(8) \quad Y = \min(X, 1 - X) \rightarrow \text{range of } Y \text{ is } (0, \frac{1}{2})$$

$$\begin{aligned} F_Y(y) &= P(Y \leq y) = P(\min(X, 1 - X) \leq y) = 1 - P(\min(X, 1 - X) > y) \\ &= 1 - P(X > y, 1 - X > y) \\ &= 1 - P(X > y, 1 - y > X) \\ &= 1 - P(y < X < 1 - y) \end{aligned}$$

$$P(y < X < 1-y) = \begin{cases} 1 & y \leq 0 \\ \int_y^{1-y} dx & \text{if } 0 < y < \frac{1}{2} \\ 0 & y \geq \frac{1}{2} \end{cases}$$

$$\Rightarrow F_Y(y) = \begin{cases} 0 & y \leq 0 \\ 2y & 0 < y < \frac{1}{2} \\ 1 & y \geq \frac{1}{2} \end{cases} \Rightarrow \text{p.d.f. } f_Y(y) = \begin{cases} 2, & 0 < y < \frac{1}{2} \\ 0, & \text{o/w} \end{cases}$$

transform. $Z = \frac{1-Y}{Y} = \frac{1}{Y} - 1 \rightarrow \text{range of } Z \text{ is } (1, \infty)$

$$F_Z(z) = P(Z \leq z) -$$

If $z \leq 1$, then $F_Z(z) = 0$

If $z > 1$, then $P(Z \leq z) = P\left(\frac{1}{Y} - 1 \leq z\right) = P\left(\frac{1}{Y} \leq z+1\right)$
 $= P\left(Y \geq \frac{1}{z+1}\right) = 1 - P\left(Y < \frac{1}{z+1}\right)$
 $= 1 - \frac{2}{z+1} \quad (\text{using d.f. of } Y)$

$$\Rightarrow F_Z(z) = \begin{cases} 0, & \text{if } z \leq 1 \\ 1 - \frac{2}{z+1}, & \text{if } z > 1 \end{cases}$$

$$\Rightarrow \text{p.d.f. of } Z \text{ is } f_Z(z) = \begin{cases} \frac{2}{(z+1)^2}, & z > 1 \\ 0 & \text{o/w} \end{cases}$$

$$(9) X \sim N(\mu, \sigma^2)$$

$$Y = 2X - 6; \quad Y \in (-\infty, \infty)$$

$$\begin{aligned} F_Y(y) &= P(Y \leq y) = P(2X - 6 \leq y) \\ &= P\left(X \leq \frac{y+6}{2}\right) \\ &= P\left(\frac{X-\mu}{\sigma} \leq \left(\frac{y+6}{2} - \mu\right)/\sigma\right) \\ &= \Phi\left(\frac{y+6-2\mu}{2\sigma}\right) \end{aligned}$$

$$\begin{aligned} f_Y(y) &= \phi\left(\frac{y+6-2\mu}{2\sigma}\right) \cdot \frac{1}{2\sigma} \quad y \in (-\infty, \infty) \\ &= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{y-(2\mu-6)}{2\sigma}\right)^2} \cdot \frac{1}{2\sigma} \\ &= \frac{1}{\sqrt{2\pi}(2\sigma)} \exp\left(-\frac{1}{2(4\sigma^2)}(y - (2\mu-6))^2\right) \end{aligned}$$

$$\Rightarrow Y \sim N(2\mu-6, 4\sigma^2).$$

$$(10) X \sim f_X(x)$$

$$Z = -\log F(X); \quad Z \in (0, \infty)$$

$$Z = -\log F(X) \Rightarrow X = F^{-1}(e^{-Z})$$

$$\left| \frac{\partial z}{\partial x} \right| = \left(\frac{f(x)}{F(x)} \right) \Rightarrow |J| = \frac{F(x)}{f(x)}$$

$$\begin{aligned} \text{p.d.f. of } Z &= f(F^{-1}(e^{-Z})) \cdot \frac{F(F^{-1}(e^{-Z}))}{f(F^{-1}(e^{-Z}))} \\ &= F(F^{-1}(e^{-Z})) = e^{-Z}; \end{aligned}$$

$$\Rightarrow f_Z(z) = \begin{cases} e^{-z}, & z > 0 \\ 0, & \text{o.w.} \end{cases}$$

$$(11) \quad f_X(x) = \begin{cases} 6x(1-x), & 0 \leq x \leq 1 \\ 0 & \text{o/w} \end{cases}$$

$$F_X(x) = P(X \leq x) = \begin{cases} 0, & x < 0 \\ \int_0^x (6y - 6y^2) dy, & 0 \leq x \leq 1 \\ 1, & x > 1 \end{cases}$$

$$= \begin{cases} 0, & x < 0 \\ x^2(3-2x), & 0 \leq x \leq 1 \\ 1, & x > 1 \end{cases}$$

$$F_X(x) = x^2(3-2x)$$

If $X \sim F_X(x)$ distⁿ fⁿ, then $Y = F(X) \sim U(0,1)$

$$\left[\begin{array}{l} \uparrow \\ \text{Cont r.v.} \end{array} \begin{array}{l} X \sim f_X(x) \text{ p.d.f. \& d.f. } F \\ Y = F(X) \rightarrow y \in (0,1) \\ x = F^{-1}(y) \end{array} \right]$$

$$\frac{dy}{dx} = f(x) \quad \left| \frac{dx}{dy} \right| = \left| \frac{1}{f(x)} \right|$$

$$\Rightarrow \text{p.d.f. of } Y : f_{Y^{-1}(y)} = \begin{cases} \frac{f_X(F^{-1}(y))}{f_X(F^{-1}(y))} = 1 & \text{if } 0 < y < 1 \\ 0 & \text{o/w} \end{cases}$$

$$\Rightarrow Y \sim U(0,1)$$

\Rightarrow For the given distⁿ $X^2(3-2x) = F(x) \sim U(0,1)$

(12) $X \sim \text{Double exponential}$

$$f_X(x) = \frac{1}{2} e^{-|x|}; \quad -\infty < x < \infty$$

$$Y = |X| \quad \text{range of } Y : (0, \infty)$$

$$x \in (-\infty, 0) \rightarrow x = -y \rightarrow \left| \frac{dx}{dy} \right| = 1$$

$$x \in (0, \infty) \rightarrow x = y \rightarrow \left| \frac{dx}{dy} \right| = 1$$

$$\Rightarrow f_Y(y) = f_X(g_1^{-1}(y)) \left| J_1 \right| + f_X(g_2^{-1}(y)) \left| J_2 \right|$$

\uparrow
 $x \in (-\infty, 0)$

$$\therefore f_Y(y) = \begin{cases} \frac{1}{2} e^{-y} + \frac{1}{2} e^{-y}, & 0 < y < \infty \\ 0, & \text{o/w} \end{cases}$$

$$\therefore \text{p.d.f. of } Y : f_Y(y) = \begin{cases} e^{-y}, & 0 < y < \infty \\ 0, & \text{o/w} \end{cases}$$

(13)

$$X_i = 0, 1, 2, 3 \quad \text{for } i = 1, 2, 3$$

$$N = 1, 2, 3$$

Possible configurations with 3 boxes and 3 balls

B_1	B_2	B_3
3	0	0
0	3	0
0	0	3
2	1	0
2	0	1
1	2	0
0	2	1
1	0	2
0	1	2
1	1	1

→

→ each with

$$\text{prob} = \frac{1}{\binom{3+3-1}{3}} = \frac{1}{10}$$

$N \backslash$	X_1	X_2	X_3
1	3	0	0
1	0	3	0
1	0	0	3
2	2	1	0
2	2	0	1
2	1	2	0
2	0	2	1
2	1	0	2
2	0	1	2
3	1	1	1

jt p.m.f. of (N, X_1)

$N \backslash X_1$	0	1	2	3
1	$\frac{2}{10}$	0	0	$\frac{1}{10}$
2	$\frac{2}{10}$	$\frac{2}{10}$	$\frac{2}{10}$	0
3	0	$\frac{1}{10}$	0	0
	$\frac{4}{10}$	$\frac{3}{10}$	$\frac{2}{10}$	$\frac{1}{10}$

Marginal of X_1

jt. p.m.f. of (X_1, X_2)

$X_1 \backslash X_2$	0	1	2	3
0	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$
1	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	0
2	$\frac{1}{10}$	$\frac{1}{10}$	0	0
3	$\frac{1}{10}$	0	0	0
	$\frac{4}{10}$	$\frac{3}{10}$	$\frac{2}{10}$	$\frac{1}{10}$

Marg of X_2

$$(14) \sum_{(x,y)} p(x,y) = c \sum_{(x,y)} (xy) = 1$$

$$\Rightarrow c [(1,1) + (2,1) + (2,2) + (3,1)] = 1$$

$$\Rightarrow c = \frac{1}{10}$$

jt p.m.f.

X \ Y	1	2	
1	1/10	0	1/10
2	2/10	4/10	6/10
3	3/10	0	3/10
	6/10	4/10	1

marginal of X

marginal of Y

Conditional p.m.f. of X given Y=2, $\frac{p(x,2)}{p_Y(2)} = 1 \quad \text{if } x=2$
 $= 0 \quad \text{if } x=1, 3$

(15) jt p.m.f.

X \ Y	1	2	
1	3/18	5/18	8/18
2	4/18	6/18	10/18
	7/18	11/18	1

marg of X

marg of Y

(b) $P(X=1, Y=1) = \frac{3}{18} \neq P(X=1) \cdot P(Y=1) = \frac{8}{18} \cdot \frac{7}{18}$
 $\Rightarrow X \& Y$ not ind.

(c) $P(X < Y) = P(X=1, Y=2) = \frac{5}{18}$

$$P(X+Y > 2) = P(X=1, Y=2) + P(X=2, Y=1) + P(X=2, Y=2)$$

$$= \frac{15}{18}$$

(d) marg of X: $p_X(x) = P(X=x) = \frac{x+3}{9} \quad ; \quad x=1, 2$

$$p_{Y|X=x} = \frac{\frac{1}{18}(x+2y)}{\frac{1}{18}(2x+6)} = \frac{x+2y}{2x+6} \quad ; \quad y=1, 2$$

(16) j.t. p.m.f. of X_1, X_2, X_3

$$p_{X_1, X_2, X_3}(x_1, x_2, x_3) = \frac{\binom{13}{x_1} \binom{13}{x_2} \binom{13}{x_3} \binom{13}{5-x_1-x_2-x_3}}{\binom{52}{5}}; \quad x_i \geq 0 \text{ and } \sum_{i=1}^3 x_i \leq 5$$

$$p_{X_i}(x) = \frac{\binom{13}{x} \binom{39}{5-x}}{\binom{52}{5}} \quad x = 0, 1, 2, 3, 4, 5$$

$$p_{X_1}(x_1) \neq p_{X_2}(x_2) \neq p_{X_3}(x_3) \neq p_{X_1, X_2, X_3}(x_1, x_2, x_3)$$

$$\Rightarrow (X_1, X_2, X_3) \text{ are not indep.}$$

(17). X_1 : # of white balls

X_2 : # of black balls.

3 W, 2 B, 1 R = 7

$$p_{X_1, X_2}(x_1, x_2) = \frac{3!}{x_1! x_2! (3-x_1-x_2)!} \left(\frac{3}{8}\right)^{x_1} \left(\frac{2}{8}\right)^{x_2} \left(\frac{3}{8}\right)^{3-x_1-x_2};$$

$$(X_1, X_2) \sim \text{Mult}(3, \frac{3}{8}, \frac{2}{8}) \quad x_i \geq 0, x_1 + x_2 \leq 3$$

$$p_{X_1}(x_1) = \binom{3}{x_1} \left(\frac{3}{8}\right)^{x_1} \left(\frac{5}{8}\right)^{3-x_1}; \quad x_1 = 0, 1, 2, 3$$

$$p_{X_2}(x_2) = \binom{3}{x_2} \left(\frac{2}{8}\right)^{x_2} \left(\frac{6}{8}\right)^{3-x_2}; \quad x_2 = 0, 1, 2, 3$$

$$\therefore X_1 \sim B(3, \frac{3}{8}); \quad X_2 \sim B(3, \frac{2}{8})$$

$$p_{X_1}(x_1) p_{X_2}(x_2) \neq p_{X_1, X_2}(x_1, x_2)$$

$$\Rightarrow X_1 \& X_2 \text{ are not indep.}$$

(18) From the j.t. p.m.f. of X_1, X_2

$$P(X_1=0, X_2=0) = P(X_1=0, X_2=1) = P(X_1=1, X_2=0) = P(X_1=1, X_2=1) = \frac{1}{4}$$

$$\text{Further } (X_1, X_2) \stackrel{d}{=} (X_1, X_3) \stackrel{d}{=} (X_2, X_3)$$

$$\& P(X_i=0) = \frac{1}{2} = P(X_i=1); \quad i=1, 2, 3$$

$$\Rightarrow X_1, X_2, X_3 \text{ are pairwise indep.}$$

$$\text{But } P(X_1=0, X_2=0, X_3=0) = \frac{1}{4} \neq P(X_1=0) P(X_2=0) P(X_3=0) = \frac{1}{8}$$

$$\Rightarrow X_1, X_2, X_3 \text{ are not indep.}$$