

# MSO 201A : Problem Set 4

## Answers

(1) 6

(2)  $\frac{1-2p+2p^2}{p(1-p)}$

(3) (a)  $E(X)$  does not exist

(b)  $E(X)$  does not exist

(c)  $E(X)$  does not exist

(4) (a)  $E(X) = \frac{a}{a+1}$  ;  $V(X) = \frac{a}{a+2} - \left(\frac{a}{a+1}\right)^2$

slightly others

(5)  $E(X) = a\sqrt{\frac{1}{2}+1} + \mu$

$V(X) = \left(a^2\sqrt{\frac{1}{2}+1} + 2a\mu\sqrt{\frac{1}{2}+1} + \mu^2\right) - \left(a\sqrt{\frac{1}{2}+1} + \mu\right)^2$

(6)  $(0.5)^{1/3}$

(7)  $\int_0^\infty (1-F(x)) dx = \int_0^\infty \int_x^\infty f(y) dy dx = \dots = E(X)$

(8)  $E(\underset{\substack{\uparrow \\ \text{score}}}{z}) = 0 \times \frac{1}{3} + 2 \times \frac{1}{6} + 3 \times \frac{1}{6} + 4 \times \frac{1}{3}$

(9) (a)  $M_X(t) = (q + pe^t)^n$  ;  $E(X) = np$  ,  $V(X) = npq$

slightly (b)

(c)  $M_X(t) = (1 - \beta t)^{-\alpha}$   $t < \frac{1}{\beta}$

$E(X) = \alpha\beta$  ;  $V(X) = \alpha\beta^2$

(10)

p.m.f

$x = x$	-5	4	5	25
$P(X=x)$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{5}{24}$

→ d.f (step f<sup>n</sup>)

(11) Markov's inequality

(12) Chebyshev's inequality

(13) By Chebyshev's inequality

$$P(|X| \geq \epsilon) \leq \frac{1}{4\epsilon^2}$$

$$\text{Also } P(|X| \geq \epsilon) = \begin{cases} \frac{1}{4} & 0 < \epsilon \leq 1 \\ 0 & \epsilon > 1 \end{cases}$$

For  $\epsilon = 1$ , bound is attained.

(14) Condition is  $p > \frac{1}{2}$

$$(15) \text{ reqd prob} = \binom{4}{4} \left(\frac{2}{3}\right)^5 + \binom{5}{4} \left(\frac{2}{3}\right)^5 \cdot \left(\frac{1}{3}\right)^1 + \dots$$

$$(16) \text{ reqd prob} = \binom{2N-K}{N} \left(\frac{1}{2}\right)^{2N-K}$$

$$K = 0, 1, \dots, N$$