MSO 201A : Problem Set #1

Answers

(1) $P(A) = 2 \sum_{i=2}^{5} \frac{1}{2^i}$

$P(B) = 2 \sum_{i=1}^{\infty} \frac{1}{2^{2i}}$

$P(A \cap B) = 2 \times \frac{1}{2^2} + 2 \frac{1}{2^4}$

$P(A^c B) = 2 \sum_{i=3}^{\infty} \frac{1}{2^{2i}}$

(2) (i) AP $\rightarrow \dfrac{49 \times 50}{\binom{100}{3}}$

(ii) GP $\rightarrow \dfrac{53 + 52}{\binom{100}{3}}$

(3)

$$\left(\frac{1}{3}\right)^4 + \left(\binom{4}{3}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right) \times \frac{1}{3}\right)$$

$$+ \left(\binom{5}{3}\left(\frac{1}{3}\right)^3\left(\frac{1}{3}\right)^2 \times \frac{1}{3} + \binom{5}{3}\binom{2}{1}\left(\frac{1}{3}\right)^3\left(\frac{1}{3} \times \frac{1}{3}\right) \times \frac{1}{3}\right)$$

$$+ \left(\binom{6}{3}\binom{3}{1}\left(\frac{1}{3}\right)^3 \frac{1}{3}\left(\frac{1}{3}\right)^2\left(\frac{1}{3}\right)\right)$$

(4) $1 - (1 - 2x)^2$

(5) $\dfrac{365 \times 364 \times \cdots \times (365 - (n-1))}{365^n}$

$= \left(1 - \frac{1}{365}\right)\left(1 - \frac{2}{365}\right) \cdots \left(1 - \frac{n-1}{365}\right)$

(6) $\frac{6}{27}$

(7) $$\frac{2! \times (n-r-1)! \times r! \times \binom{n-2}{r}}{n!}$$

(8)
(a) $$\frac{n(n-1)^{r-1}}{n^r}$$

(b) $$\frac{n(n-1)\cdots(n-r+1)}{n^r}$$

second part $$\frac{\binom{n}{N}\binom{n-N}{N}\binom{n-2N}{N}\cdots\binom{n-(r-1)N}{N}}{\binom{n}{N}^r}$$

(9) $$\left(m^2 - \frac{4}{9}m^2\right) \Big/ m^2$$

(10) $$\left(1 - \frac{r^3}{R^3}\right)^n$$

(11) $$\frac{\cancel{(N-2)}\binom{N-3}{r-1}}{\binom{N-1}{r-1}}$$

(12) $\frac{2}{6}$

(13) $$\frac{5+8+6+9+4+8+2+1}{216}$$

(14) (i) $\mathcal{F}_1$ is not a σ-field

(ii) $\mathcal{F}_2$ is not a σ-field

(iii) $\mathcal{F}_3$ is a σ-field

(15) $\mathcal{F}_1 \cap \mathcal{F}_2$ is a σ-field

Counter example

$$\Omega = \{1, 2, 3\}$$

$\mathcal{F}_1 = \{\phi, \Omega, \{1\}, \{2,3\}\} \leftarrow \sigma$-field

$\mathcal{F}_2 = \{\phi, \Omega, \{2\}, \{1,3\}\} \leftarrow \sigma$-field

$\mathcal{F}_1 \cup \mathcal{F}_2$ is not a σ-field.

(16) $\mathcal{F}_A$ is a σ-field (verify the 3 conditions)