# Problem Set #2

(1)

(a) $\Omega = \{0, 1, 2, \dots\}$

Any event A is a collection of pts from $\Omega$

$$P(A) = \sum_{x \in A} \frac{e^{-\lambda} \lambda^x}{x!} \qquad \lambda > 0$$

$$\Rightarrow \frac{e^{-\lambda} \lambda^x}{x!} > 0 \quad \forall x \in A \subseteq \Omega$$

$$P(A) \geq 0$$

$$P(\Omega) = \sum_{x \in \Omega} \frac{e^{-\lambda} \lambda^x}{x!} = e^{-\lambda} \sum_{x=0}^{\infty} \frac{\lambda^x}{x!} = 1$$

Let $A_1, A_2, \dots$ be disjoint $A_i \cap A_j = \phi \ \forall i \neq j$

$$P\left(\bigcup_1^{\infty} A_i\right) = \sum_{x \in \bigcup_1^{\infty} A_i} P(\{x\}) = \sum_{x \in \bigcup_1^{\infty} A_i} \frac{e^{-\lambda} \lambda^x}{x!} \qquad A_i\text{s disjoint}$$

$$= \sum_{i=1}^{\infty} \sum_{x \in A_i} \frac{e^{-\lambda} \lambda^x}{x!} = \sum_{i=1}^{\infty} P(A_i)$$

$\Rightarrow P(.)$ is a prob measure

(b) similar to (a)

(c) Take $A_i = \{i\} \quad i = 1, 2, \dots$

$\bigcup_1^{\infty} A_i = \{1, 2, \dots\}$ infinite # of elements.

$$P\left(\bigcup_1^{\infty} A_i\right) = 0 \neq \sum_1^{\infty} P(A_i) = \sum_1^{\infty} 1$$

Also $P(\Omega) = 0$ ($\Omega$ has infinite no. of elements)

$P(.)$ is not prob measure

(2) Second part

$$P(E) = \sum_3^{\infty} \frac{e^{-\lambda} \lambda^x}{x!} = 1 - e^{-\lambda}\left(1 + \lambda + \frac{\lambda^2}{2}\right)$$

$$P(F) = \sum_1^{2} \frac{e^{-\lambda} \lambda^x}{x!} = \dots$$

$$P(E \cup F) = \sum_1^{\infty} \frac{e^{-\lambda} \lambda^x}{x!} = 1 - e^{-\lambda}$$

Sly others.

(2) $\Omega = \mathbb{R}$

(a) $P(I) = \int_I \frac{1}{2} e^{-|x|} dx \geq 0 \quad \forall I$

$P(\Omega) = \frac{1}{2} \int_{-\infty}^{\infty} e^{-|x|} dx = \frac{1}{2}\left( \int_{-\infty}^{0} e^{x} dx + \int_{0}^{\infty} e^{-x} dx \right) = 1$

$I_1 \cap I_2 = \phi$ : $P(I_1 \cup I_2) = \frac{1}{2}\left[ \int_{I_1} + \int_{I_2} \right] = P(I_1) + P.I_2$

extend ... P is prob measure

(b) Similar to (a)

(c) $P(\Omega) = P(\mathbb{R}) = 0 \neq 1$ P is not prob measure.

(3) P(exactly one of A or B)

$= P((A \cap B^c) \cup (A^c \cap B))$

$= P(A) + P(B) - 2P(A \cap B)$ — on simplification.

(4) $P(AB) - P(A)P(B) = P(A)P(B^c) - P(AB^c)$. — 1st eqn.

[using $P(A) = P(AB) + P(AB^c)$.]

2nd & 3rd eqns can be proved in a similar way.

(5) For $n = 2$

$P(A_1 \cup A_2) = P(A_1 \cup (A_1^c A_2))$

$= P(A_1) + P(A_1^c A_2)$

$= P(A_1) + [P(A_2) - P(A_1 A_2)]$ — true for $n = 2$

Proof by induction

Assume that it is true for $n = m$, then

$P(\bigcup_1^{m+1} A_k) = P((\bigcup_1^m A_k) \cup A_{m+1})$

$= P(\bigcup^m A_k) + P(A_{m+1}) - P((\bigcup^m A_k) \cap A_{m+1})$

$$\text{r.h.s} = \Big[ \sum_{1}^{n} P(A_k) - \sum_{k_1<k_2} P(A_{k_1} \cap A_{k_2}) + \sum_{k_1<k_2<k_3} P(A_{k_1} A_{k_2} A_{k_3}) -$$

$$\cdots + (-1)^{m-1} P\Big(\bigcap_{1}^{m} A_k\Big) \Big]$$

$$+ P(A_{m+1}) - P\Big(\bigcup_{1}^{m} (A_k A_{m+1})\Big) \quad \text{— (1)}$$

$$P\Big(\bigcup_{1}^{m} A_k A_{m+1}\Big) = \sum_{1}^{m} P(A_k A_{m+1}) - \sum_{k_1<k_2} P\big[(A_{k_1} A_{m+1}) \cap (A_{k_2} A_{m+1})\big]$$

$$+ \sum_{k_1<k_2<k_3} P\big(A_{k_1} A_{m+1} \cap A_{k_2} A_{m+1} \cap A_{k_3} A_{m+1}\big) - \cdots$$

$$\cdots + (-1)^{m-1} P\Big(\bigcap_{1}^{m} A_k A_{m+1}\Big) \quad \text{— (2)}$$

Using (2) in (1) gives

$$P\Big(\bigcup_{1}^{m+1} A_k\Big) = \sum_{1}^{m+1} P(A_k) - \sum_{k_1<k_2}^{m+1} P(A_{k_1} A_{k_2}) + \sum_{k_1<k_2<k_3} P(A_{k_1} A_{k_2} A_{k_3})$$

$$- \cdots + (-1)^{m} P\Big(\bigcap_{1}^{m+1} A_k\Big).$$

(6) $\Omega = \{0, 1, 2, \ldots\}$. $P(\{j\}) = c\,\dfrac{2^j}{j!}$ ; $j = 0, 1, 2, \ldots$

(a) $$1 = P(\Omega) = \sum_{j=0}^{\infty} P(\{j\}) = \sum_{j=0}^{\infty} c\,\frac{2^j}{j!} = c\,e^2$$

$$\Rightarrow c = e^{-2}$$

(b) $$P(A) = \sum_{j=2}^{4} e^{-2}\frac{2^j}{j!} \; ; \; P(B) = \sum_{3}^{\infty} e^{-2}\frac{2^j}{j!}$$

$$P(C) = \sum_{0}^{\infty} e^{-2}\frac{2^{2j+1}}{(2j+1)!}$$

$$P(B \cap C) = P(\{3\}) + P(\{5\}) + \cdots = \cdots$$

Other probs can be computed in a similar manner.

(7) Favorite models of dinosaurs numbered 1, 2, 3 (say)

Define events

$A_i$ = model # i not found in 6 packets.

$i = 1, 2, 3$

reqd prob $= P(A_1^c \cap A_2^c \cap A_3^c)$

$= 1 - P(A_1^c A_2^c A_3^c)^c$

$= 1 - P(A_1 \cup A_2 \cup A_3)$.

$= 1 - [P(A_1) + P(A_2) + P(A_3) - P(A_1 A_2) - P(A_1 A_3)$
$- P(A_2 A_3) + P(A_1 A_2 A_3)]$ — (1)

Note that

$$\left.\begin{aligned} P(A_i) &= \left(\frac{4}{5}\right)^6 \quad \forall i \\ P(A_i A_j) &= \left(\frac{3}{5}\right)^6 \quad \forall i \neq j \\ P(A_1 A_2 A_3) &= \left(\frac{2}{5}\right)^6 \end{aligned}\right\} - (2)$$

Use (2) in (1) to get the desired prob.

(8) $A_i$ : match at position $i$

P(at least one match)

$= P(A_1 \cup A_2 \cup \ldots \cup A_n)$

$= \sum_1^n P(A_i) - \sum_{i<j} P(A_i A_j) + \cdots + (-1)^{n-1} P(\bigcap_1^n A_i)$

With

$P(A_{i_1} \cap A_{i_2} \cap \ldots \cap A_{i_r}) = \frac{(n-r)!}{n!}$; $1 \le i_1 < i_2 < \ldots < i_r \le n$
$r = 1(1)n$

$\Rightarrow$ reqd prob $= 1 - \frac{1}{2!} + \frac{1}{3!} + \cdots + (-1)^{n-1} \frac{1}{n!}$

(9)

(a) $A_i$ : event that $i^{th}$ bill goes to $i^{th}$ envelope $(i = 1(1)n)$

Reqd prob

$$P\left(\bigcap_{i=1}^{n} A_i^c\right) = P\left(\bigcup A_i\right)^c$$

$$= 1 - P\left(\bigcup A_i\right)$$

$$= 1 - \left[\sum P(A_i) - \sum_{i<j} P(A_i A_j) + \cdots + (-1)^{n-1} P(A_1 \ldots A_n)\right]$$

$$= 1 - \underbrace{\sum P(A_i)}_{Q_1} + \underbrace{\sum_{i<j} P(A_i A_j)}_{Q_2} + \cdots + \underbrace{(-1)^n P(A_1 \ldots A_n)}_{Q_n}$$

In $Q_i \rightarrow \binom{n}{i}$ terms each equal to $\frac{(n-i)!}{n!} \left(= \frac{1}{(n)_i}\right)$

$$\Rightarrow P\left(\bigcap A_i^c\right) = 1 - \binom{n}{1}\frac{1}{(n)_1} + \binom{n}{2}\frac{1}{(n)_2} - \cdots + (-1)^n \binom{n}{n}\frac{1}{(n)_n}$$

$$= 1 - \frac{n!}{1!\,(n-1)!} \cdot \frac{(n-1)!}{n!} + \frac{n!}{2!\,(n-2)!} \cdot \frac{(n-2)!}{n!} - \cdots + (-1)^n \cdot \frac{1}{n!}$$

$$= 1 - \frac{1}{1!} + \frac{1}{2!} - \cdots + (-1)^n \frac{1}{n!} = \sum_{i=0}^{n} (-1)^i \frac{1}{i!}$$

(b) $B_i$ : $i^{th}$ envelope get $i^{th}$ bill & $i^{th}$ cheque

reqd prob $P\left(\bigcap B_i^c\right) = 1 - P\left(\bigcup B_i\right)$

$$= 1 - \underbrace{\sum P(B_i)}_{R_1} + \underbrace{\sum_{i<j} P(B_i B_j)}_{R_2} - \cdots + \underbrace{(-1)^n P(B_1 \ldots B_n)}_{R_n}$$

In $R_i \rightarrow \binom{n}{i}$ term each equal to $\left(\frac{(n-i)!}{n!} \cdot \frac{(n-i)!}{n!}\right)$.

$\Rightarrow$ $\cancel{P(\cap B_1}$

$$P\left(\bigcap_i B_i^c\right) = 1 - P\left(\bigcup_i B_i\right)$$

$$= 1 - \sum_i P(B_i) + \sum_{i<j} P(B_i B_j) - \cdots + (-1)^n P(B_1 B_2 \cdots B_n)$$

$$= 1 - R_1 + R_2 - R_3 + \cdots + (-1)^n R_n$$

$$R_i = \binom{n}{i} \frac{(n-i)!}{n!} \times \frac{(n-i)!}{n!}$$

$$= \frac{\cancel{n!}}{i!\,\cancel{(n-i)!}} \cdot \frac{(n-i)!}{n!} \cdot \frac{\cancel{(n-i)!}}{\cancel{n!}}$$

$$= \frac{1}{i!} \cdot \frac{1}{(n)_i} \qquad \Big|\ (n)_i = \frac{n!}{(n-i)!}$$

$$\Rightarrow P\left(\bigcap_i B_i^c\right) = \sum_{i=0}^{n} (-1)^i \frac{1}{i!\,(n)_i}$$

(10)

(i) $P(A \cup B | C) = \dfrac{P((A \cup B) \cap C)}{P(C)} = \dfrac{P(AC \cap BC)}{P(C)}$

$= P(A|C) + P(B|C) - P(AB|C)$

(ii) $P(A^c|C) = \dfrac{P(A^c C)}{P(C)} = \dfrac{P(C) - P(AC)}{P(C)} = 1 - P(A|C)$

(11)

(a) true — $P(A|B) + P(A^c|B) = \dfrac{P(AB)}{P(B)} + \dfrac{P(A^c B)}{P(B)} = \dfrac{P(B)}{P(B)} = 1$

(b) $P(A|B) = \dfrac{P(AB)}{P(B)}$ ; $P(A|B^c) = \dfrac{P(AB^c)}{P(B^c)} = \dfrac{P(A) - P(AB)}{1 - P(B)}$

Take $A \subset B$, $P(A) > 0$, $P(B - A) > 0$

$\boxed{0 < P(A), P(B) < 1}$ condition required missing in the question

$P(A|B) + P(A|B^c) = \dfrac{P(AB)}{P(B)} + \dfrac{P(AB^c)}{P(B^c)}$

$= \dfrac{P(A)}{P(B)} < 1$ False.

(c).

Take $A \subset B$, i.e. $B^c \subset A^c$

$P(A|B) + P(A^c|B^c)$.

$= \dfrac{P(AB)}{P(B)} + \dfrac{P(A^c B^c)}{P(B^c)}$.

$= \dfrac{P(A)}{P(B)} + \dfrac{P(B^c)}{P(B^c)} > 1$ false.

$P(A) > 0$.

(12)

(a). $P(A|B) = \frac{1}{4} \Rightarrow P(AB) \neq 0$ False

(i.e $AB \neq \phi$)

(b). $A \subset B \Rightarrow P(AB) = P(A)$

given $P(B|A) = \dfrac{P(AB)}{P(A)} = \dfrac{1}{2}$

$\Rightarrow P(AB) \neq P(A)$ False.

(c) $P(A^c \mid B^c) = \dfrac{P(A^c B^c)}{P(B^c)} = \dfrac{1 - P(A) - P(B) + P(AB)}{1 - P(B)}$ $\quad -(*)$

$P(B|A) = 1/2$

$P(A) = \frac{1}{4}$, $\Rightarrow P(AB) = \frac{1}{8}$

& $P(A|B) = \dfrac{P(AB)}{P(B)} = \dfrac{1/8}{P(B)} = \dfrac{1}{4} \Rightarrow P(B) = \dfrac{1}{2}$

$(*) \Rightarrow P(A^c \mid B^c) = \dfrac{1 - \frac{1}{4} - \frac{1}{2} + \frac{1}{8}}{1/2} = \dfrac{3}{4}$

13 (a): P(exactly 3 white balls, out of 4).

$= \binom{4}{3} \left(\frac{1}{2}\right)^3 \cdot \frac{1}{2} = \ldots$

(b) A : first ball placed is white

$P(A) = \frac{1}{2}$

B : urn contains exactly 3 white balls

$P(B|A) = \dfrac{P(AB)}{P(A)} = \dfrac{\frac{1}{2} \cdot \binom{3}{2} \left(\frac{1}{2}\right)^3}{1/2} = \dfrac{3}{8}$

$= \binom{3}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)$

(14) D : A occurs before B

$P(D) = p_A + p_A p_c + p_A p_c^2 + \ldots$

$= \dfrac{p_A}{1 - p_c} = \dfrac{p_A}{p_A + p_B}$

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$A_i$ : component $i$ functions

$P(A_1 \cup A_2 \cdots \cup A_n)$

$= 1 - P(\cup A_i)^c$

$= 1 - P(\cap A_i^c)$

P(system functions) $= 1 - P(\cap A_i^c)$

$= 1 - \prod_{i=1}^{n} (1 - p_i)$

(16) $A_i$ : Question i is among the 90 questions that the student can answer correctly

reqd prob $P(A_1 A_2 A_3) = P(A_1) P(A_2|A_1) P(A_3|A_1 A_2)$

$$= \frac{90}{100} \cdot \frac{89}{99} \cdot \frac{88}{98} \text{ --}$$

(17)

Appl$^n$ Bayes Th$^m$

$$\text{reqd prob} = \frac{\overset{P(H|F)}{\frac{1}{2}} \times \overset{P(F)}{\frac{2}{3}}}{\frac{1}{2} \times \frac{2}{3} + \underset{P(H|B)}{\frac{2}{3}} \times \underset{P(B)}{\frac{1}{3}}} = \text{--}$$

$P(F|H)$

(18) $A^+, B^+, C^+, D^+$ – events that A, B, C, D passes the paper with the (+) sign

Bayes Th$^m$.

$$\text{reqd prob} = P(A^+|D^+) = \frac{P(A_+)\, P(D_+|A_+)}{P(D^+)}$$

$$P(D^+|A^+) = \left(\tfrac{1}{3}\right)^3 + \binom{3}{2}\left(\tfrac{2}{3}\right)^2 \cdot \tfrac{1}{3} = \frac{13}{27}$$

Also $P(D^+) = P(D^+|A^+)\, P(A^+) + P(D^+|A_+^c)\, P(A_+^c)$

$P(D^+|A_+^c) = P(D \text{ passes with } + \mid A \text{ passes } -)$

$$= \binom{3}{1} \frac{2}{3} \left(\tfrac{1}{3}\right)^2 + \binom{3}{3} \left(\tfrac{2}{3}\right)^3 \left(\tfrac{1}{3}\right)^0 = \frac{14}{27}$$

$$P(D^+) = \frac{13}{27} \cdot \frac{1}{3} + \frac{14}{27} \cdot \frac{2}{3} = \frac{41}{81}$$

$$\Rightarrow P(A^+|D^+) = \frac{13/27 \cdot 1/3}{41/81} = \frac{13}{41}$$

(19)

$$P\left(\text{Silver coin in other} \mid \text{Gold coin in one drawer}\right).$$

Bayes th$^{m}$

$$= \frac{\frac{1}{3}\times\frac{1}{2}}{\frac{1}{3}\times 1+\frac{1}{3}\times\frac{1}{2}+\frac{1}{3}\times 0} = \frac{1}{3}.$$

(20) (a) $P(c_1\, c_2\, c_3\, c_4) = \prod_1^4 P(c_i) = ---$

(b) $P(c_1^c \cap c_2^c \cap c_3^c \cap c_4^c) = \prod_1^4 P(c_i^c)$

$= ---.$

[explain why it is so
$c_1, c_2, c_2, c_4$ indep
$\Rightarrow c_1^c, c_2^c, c_3^c, c_4^c$ are also indep.]

(c) $P(c_1\, c_2^c\, c_3^c\, c_4^c) + P(c_1^c\, c_2\, c_3^c\, c_4^c)$

$+ P(c_1^c\, c_2^c\, c_3\, c_4^c) + P(c_1^c\, c_2^c\, c_3^c\, c_4).$

$= P(c_1) \prod_2^4 P(c_i^3) + ---.$

(d). $P(\text{at least one hits}) = 1 - P(\text{no one hits})$

$= 1 - P(c_1^c\, c_2^c\, c_3^c\, c_4^c).$

$= ---$

(21) $P\left(\bigcap_1^n A_i^c\right) = \prod_1^n P(A_i^c)$

$= \prod_1^n (1-P(A_i)) \le \prod_1^n \exp(-P(A_i))$

$\left[\begin{array}{l} 0<x<1 \\ 1-x<e^{-x} \end{array}\right]$

i.e. $P\left(\bigcap_1^n A_i^c\right) \le \exp\left(-\sum_1^n P(A_i)\right).$

(22) $\Omega = \{1, 2, 3, 4\}$ $\mathcal{F}$ : power set

$P(\{i\}) = \frac{1}{4}$ $i = 1, 2, 3, 4$

$A = \{1, 4\}$, $B = \{2, 4\}$, $C = \{3, 4\}$.

$P(A) = P(B) = P(C) = \frac{1}{2}$

$P(AB) = P(AC) = P(BC) = \frac{1}{4}$ ; $P(ABC) = \frac{1}{4}$

$\Rightarrow$ $P(AB) = P(A)\ P(B)$, $P(AC) = P(A)\ P(C)$

& $P(BC) = P(B)\ P(C)$.

i.e $A, B, C$ are ~~m~~ pairwise indep

but $P(ABC) = \frac{1}{4} \neq P(A) . P(B) P(C) = \frac{1}{8}$

$\Rightarrow$ $A, B, C$ not mutually indep.

(23) Counter example

In prev prob setup take

~~A = {A~~ $A = \{1, 2\}$, $B = \{3, 4\}$, $C = \{1\}$.

$\frac{P(AC)}{P(C)} = \frac{1/4}{1/4} = 1$

$\frac{P(AB)}{P(B)} = 0 = P(A \mid B) < P(A) = \frac{1}{2}$

$0 = P(B \mid C) < P(B) = \frac{1}{2}$ but $P(A \mid C) > P(A) = \frac{1}{2}$

$\Rightarrow$ C does not carry negative information about A.

(24) $A_i$ : $i$ girls are in the list

$i = 0, 1, 2, 3$

$B$ : 1st student is girl

$C$ : 2nd student is boy :- to obtain $P(C|B)$

$$P(B) = P(A_1)P(B|A_1) + P(A_2)P(B|A_2) + P(A_3)P(B|A_3)$$

$$= \frac{\binom{3}{1}\binom{5}{3}}{\binom{8}{4}} \times \frac{1}{4} + \frac{\binom{3}{2}\binom{5}{2}}{\binom{8}{4}} \times \frac{2}{4} + \frac{\binom{3}{3}\binom{5}{1}}{\binom{8}{4}} \times \frac{3}{4}$$

$$P(B) = \frac{105}{4 \times \binom{8}{4}} \qquad \cancel{3} \times 5 \times 2 \quad \frac{\cancel{105}}{7}$$

$$P(C|B) = \frac{P(CB)}{P(B)} = \frac{P\left(C \cap \left(\bigcup_{1}^{3} A_i B\right)\right)}{P(B)} = \frac{P\left(\bigcup_{1}^{3} C A_i B\right)}{P(B)}$$

$$= \sum_{i=1}^{3} \frac{P(CA_iB)}{P(B)} = \sum_{1}^{3} P(C|A_iB)\frac{P(A_iB)}{P(B)}$$

$$= \sum_{1}^{3} P(C|A_iB)\,P(A_i|B)$$

$$= \overset{3B}{1} \times P(A_1|B) + \overset{2B\,1G}{\frac{2}{3}} \times P(A_2|B) + \overset{1B\,2G}{\frac{1}{3}} \times P(A_3|B)$$

$$\left[\begin{array}{l} P(A_1|B) = \dfrac{P(A_1)P(B|A_1)}{P(B)} = \dfrac{2}{7} \\ P(A_2|B) = \dfrac{P(A_2)P(B|A_2)}{P(B)} = \dfrac{4}{7} \quad \& \; P(A_3|B) = \dfrac{1}{7} \end{array}\right]$$

$$\Rightarrow P(C|B) = 1 \times \frac{2}{7} + \frac{2}{3} \times \frac{4}{7} + \frac{1}{3} \times \frac{1}{7}$$

$$= - - -$$

(25) A & B are in series

C & D in parallel

(a) P( the system works)

$= P(A \cap B \cap (C \cup D))$

$= P(A \cap B)\ P(C \cup D)$

$= P(A)\ P(B)\ \big(P(C) + P(D) - P(C)\ P(D)\big)$

$= 0.9 \times 0.9\ (0.8 + 0.8 - 0.8 \times 0.8)$

$= \ldots$

(b) P( C is not working | system is working)

$$= \frac{P(\text{C is not working} \cap \text{system is working})}{P(\text{system is working})}$$

$$= \frac{P(A \cap B \cap C^c \cap D)}{P(\text{system is working})}$$ ← from (a)

$$= \frac{P(A)\ P(B)\ P(C^c)\ P(D)}{P(\text{system is working})}$$

(26) $A_i$ : event that a fly survives $i$th application

$i = 1, 2, 3, 4.$

Note that $A_4 \subset A_3 \subset A_2 \subset A_1$

$\Rightarrow A_4 = A_1 \cap A_2 \cap A_3 \cap A_4$

(a) req. prob = P(a fly survives 4 applications)

$= P(A_1 A_2 A_3 A_4)$

$= P(A_4)$.

$= P(A_1)\, P(A_2 | A_1)\, P(A_3 | A_1 A_2)\, P(A_4 | A_1 A_2 A_3)$

$= (1 - 0.8)\ (1 - 0.4)\ (1 - 0.2)\ (1 - 0.1)$

(from the given conditions)

$= 0.2 \times 0.6 \times 0.8 \times 0.9$

(b) $P(A_4 | A_1) = \dfrac{P(A_4 \cap A_1)}{P(A_1)} = \dfrac{P(A_4))}{P(A_1)}$

$= 0.6 \times 0.8 \times 0.9.$

(27) $B_i$ : event that $i$ of the paintings are forgeries

$i = 0(1)5$

$P(B_0) = 0.76$, $P(B_1) = 0.09$, $P(B_2) = 0.02$, $P(B_3) = 0.01$

$P(B_4) = 0.02$ & $P(B_5) = 0.1$ ( given cond$^n$s)

A : event that the painting sent for authentication turns out to be a forgery.

reqd prob $= P(B_5 | A) = \dfrac{P(B_5)\, P(A|B_5)}{\sum_{i=0}^{5} P(B_i)\, P(A|B_i)}$

↑ Bayes th$^m$

$$P(A) = \sum_{i=0}^{5} P(B_i) P(A|B_i)$$

$$= 0.76 \times 0 + 0.09 \times \frac{1}{5} + 0.02 \times \frac{2}{5} + 0.01 \times \frac{3}{5} + 0.02 \times \frac{4}{5} + 0.10 \times 1$$

$= \ldots$

$$P(B_5|A) = \frac{0.10 \times 1}{P(A)} = \ldots$$