

MSO202 - INTRODUCTION TO COMPLEX ANALYSIS
IIT KANPUR - 2023–2024

Instructions: Course materials and calculators are not allowed.

ASSIGNMENT 2

The exercises labeled as **(T)** will be covered during the tutorials.

- (1) **(T)** Which of the following functions $f(z)$ can be defined at $z = 0$ so that they become continuous at $z = 0$:

$$(a) \frac{\Re z}{|z|} \quad (b) \frac{\Re z^2}{|z|^2} \quad (c) \frac{a \Re z}{z} \quad (d) \frac{iz^2}{|z|}.$$

- (2) **(T)** Show that

$$f(z) = \begin{cases} \frac{z^2}{|z|} & \text{if } z \neq 0 \\ 0 & z = 0 \end{cases}$$

is continuous at $z = 0$, first order partial derivatives of its real and imaginary part exist at $z = 0$, but $f(z)$ is not differentiable at $z = 0$.

- (3) Prove that for a fixed $w \in D = \{z \in \mathbb{C} : |z| < 1\}$, the mapping

$$F : z \mapsto \frac{w - z}{1 - \bar{w}z}$$

satisfy the following conditions:

- F maps D to itself and $F : D \rightarrow D$ is holomorphic.
- F interchanges 0 and w , i.e., $F(0) = w$ and $F(w) = 0$.
- $|F(z)| = 1$ if $|z| = 1$.
- F is bijective.

- (4) Write the following functions $f(z)$ in the forms $f(z) = u(x, y) + iv(x, y)$.

$$(a) f(z) = \overline{\exp(z^2)} \quad (b) f(z) = \frac{1}{i - z}.$$

- (5) Which of the following maps are holomorphic?

- (a) $P(x + iy) = x^3 - 3xy^2 - x + i(3x^2y - y^3 - y)$
 (b) $P(x + iy) = x^2 + iy^2$
 (c) **(T)** $P(x + iy) = 2xy + i(y^2 - x^2)$

- (6) Determine if there exist an analytic function with u as real part. Also, find the harmonic conjugate.

- (a) $x^2 - y^2$ (b) $\sinh x \cos y$ (c) $2x(1 - y)$ (d) $x^2 - y^2 - x + y$ (e) $e^x \sin y$
 (f) **(T)** $e^{(x^2 - y^2)} \cos(2xy)$.

- (7) **(T)** Show that the set of natural numbers \mathbb{N} can not be partitioned into finite number of subsets that are in arithmetic progression with distinct common difference.
- (8) **(T)** Prove that
- (a) If $f(z)$ and $\overline{f(z)}$ both are analytic in a domain, then it is a constant function.
 - (b) If $f(z)$ is analytic in a domain D and $f'(z) \equiv 0$ in D , then it is a constant function.
 - (c) If $f(z)$ is analytic in a domain D and $u_x + v_y \equiv 0$ in D , then $f'(z)$ is a constant function.
 - (d) Let $f(z) = u + iv = Re^{i\theta}$ be an analytic function in a domain D . If either of the functions u, v, R, θ is a constant function in D , then f is a constant function.
- (9) Write down C-R equation in polar co-ordinates. Express $f'(z)$ in terms of polar co-ordinates.