Department of Mathematics and Statistics
Indian Institute of Technology Kanpur
MSO202A/MSO202 Assignment 6
Introduction To Complex Analysis

The problems marked **(T)** *need an explicit discussion in the tutorial class. Other problems are for enhanced practice.*

1. Expand each of the following functions in Laurent series in the neighbourhood of the indicated points z_0 and, in each case, determine the largest domain where the resulting Laurent series converges:
   $(i)\ \frac{1}{z(1-z)}$, $z_0 = 0, 1$ and ∞ **(T)** $(ii)\ z^2 e^{1/z}$, $z_0 = 0$ and ∞ $(iii)\ \frac{1}{z^2+1}$, $z_0 = -i, \infty$.

2. For each of the following functions, determine the nature of its isolated singularities by considering the relevant Laurent series
   (T) $(i)\ \frac{1-\cosh z}{z^3}$ $(ii)\ \frac{\sin z}{z}$ $(iii)\ e^z$ (iv) $1 + 2z + 7z^3 + 3z^7$.

3. For the following functions, determine the residues at each of their isolated singularities in the extended complex plane:
   $(i)\ \frac{1}{z^3 - z^5}$ $(ii)\ \frac{z^{2n}}{(1+z)^n}$ **(T)** $(iii)\ e^z\, e^{1/z}$.

4. Find residues of the following functions at all its poles:
   $(i)\ \cot z$ $(ii)\ \frac{z}{z^n - 1}$ **(T)** $(iii)\ \frac{z(z^3+5)}{(z-1)^3}$.

5. Evaluate
   $(i)\ \int_{|z|=2} \tan z\, dz$ $(ii)\ \int_{|z|=2} \frac{1}{\sin 2z} dz$ **(T)** $(iii)\ \int_{|z|=8} \frac{e^{z/3}}{\sinh z} dz$
   The integration in each case being in anticlockwise direction.

6. **(T)** Show that the functions that are analytic in the whole complex plane and have a non-essential isolated singularity at ∞ are polynomials.

7. Evaluate the following integrals using Cauchy Residue Theorem:
   (i) $\int_0^{2\pi} \frac{1+\sin\theta}{3+\cos\theta} d\theta$ **(T)**(ii) $\int_0^{2\pi} \cos^{2n}\theta\, d\theta$

8. Use Cauchy Residue Theorem to evaluate (i) $\int_0^{\infty} \frac{\cos x}{(x^2+1)^2} dx$ **(T)**(ii) $\int_0^{\infty} \frac{\sin^2 x}{1+x^2} dx$

9. **(T)**Evaluate $\int_{-\infty}^{\infty} \frac{1}{(x-1)(x^2+4)} dx$ by indenting the singularity on real axis.