MSO 202A: Complex Variables
August-September 2022
Assignment-5

Throughout C_R will denote the circle of radius R around origin, oriented counterclockwise. and $C_1 = C$.

1. **(T)**Find the max value of $|f|$ on $|z| \leq 1$ where $f(z) = z^2 + 3z - 1$. Where is it attained?

2. **Analytic functions satisfies Mean Value Property.** Let f be holomorphic function on a an open set containing the closed disc $|z - z_0| \leq r$. Show that u satisfies :

$$f(z_0) = \frac{1}{2\pi} \int_0^{2\pi} f(z_0 + re^{it})dt.$$

   (Remark: $\frac{1}{2\pi} \int_0^{2\pi} f(z_0 + re^{it})dt$ called the **Mean Value of** f over the circle $|z - z_0| = r$.)

3. **(T)Harmonic functions satisfies Mean Value Property.** Let u be Harmonic function on a an open set Ω containing the closed disc $|z - z_0| \leq r$. Show that u satisfies :

$$u(z_0) = \frac{1}{2\pi} \int_0^{2\pi} u(z_0 + re^{it})dt.$$

4. Find the mean value of the function $u(x, y) = x^2 - y^2 + x$ on the circle $|z - i| = 2$.

5. **(T)**Let f be a nowhere zero, entire function. Prove that there exists an entire function g such that $\exp(g) = f$.

6. Find out all entire functions f such that $f(x) = e^x$ for $x \in \mathbb{R}$.

7. Find out the order of the all zeros for

   (T)(a) $f(z) = z^2(e^{z^2} - 1)$ (b) $z \sin z$. (c) $\sin^3(z)/z$

8. **(T)**Does there exist a holomorphic function f on $\mathbb{D}$ such that $Z_f = \{z \in \mathbb{D} : \ f(z) = 0\}$ is equal to the following set? Justify your answer.

   (a) $S_1 = \{1/n : \ n \in \mathbb{N}\}$. (b) $S_2 = \{1 - 1/n : \ n \in \mathbb{N}\}$. (c) $S_1 = \{z \in \mathbb{D} : \mathrm{Re}(z) = 0\}$ (d) $S_4 = \{z \in \mathbb{D} : -1/2 < y < 1/2\}$.

9. **(T)**Evaluate $\int_\Gamma \frac{1}{z^4} dz$ where Γ is the part of clockwise oriented ellipse $\frac{(x-3)^2}{1} + \frac{y^2}{4} = 1$ lying on the upper half plane $y > 0$.

10. **(T)**Write down the power series expansion $\sum a_n (z - z_0)^n$ of the function $Log(z) = \log_\pi(z)$ around $z_0 = -1 + \iota$.

    What is the radius of convergence of the power series?

    For which values of z, we have $Log(z) = \sum a_n (z - z_0)^n$.?

11. (**T**)Evaluate the following integrals on the square γ, oriented on the counterclockwise direction with sides $x = \pm 2, y = \pm 2$.

   (a) $\int_\gamma \frac{\cos z}{z(z^2+8)} dz$ (b) $\int_\gamma \frac{\cosh z}{z^4} dz$.

12. Find out

$$\int_C \frac{2z^3 + z^2 + 4}{z^4 + 4z^2}; \quad C : |z - 2| = 4$$