MSO-203M Partial Differential Equations
PDE

The main aim of this course is to study basic important PDEs

1. Transport Eqn. → 3
2. Laplace Eqn. → 4
3. Wave Eqn. → 5
4. Heat Equations. → 6

Reduction to Cannonical forms

Two basic tools that will be required for us

1: Sturm-Liouville Eigen Value problems. → 2nd we
2: Fourier Series → 1-weeks

Fourier "Series"

$$\sum_{n=1}^{\infty} a_n \to (C) / (d)$$

$\{a_n\}_{n \geq 1}$ sequence of real numbers

$$S_n = \sum_{i=1}^{n} a_i$$

$\{S_n\}$

Applications of "Fourier" Series

$$\sum_{n=1}^{\infty} \frac{1}{(2n+1)^2} \longrightarrow (\mathbb{C})$$

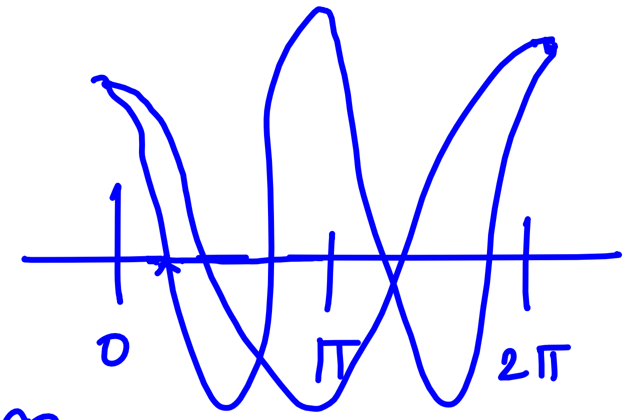
1. We will be able find 'sum' of several series (may not all).

2. PDE (study).

Idea!!

$f_n(x) = \cos(nx)$ $\qquad n \in \mathbb{Z}^+ \cup \{0\}$

$g_n(x) = \sin(nx)$



Good!

$$f(x) = \sum_{n=0}^{\infty} a_n \cos(nx) + b_n \sin(nx) \longrightarrow \circledast$$

$a_n, b_n \in \mathbb{R}$

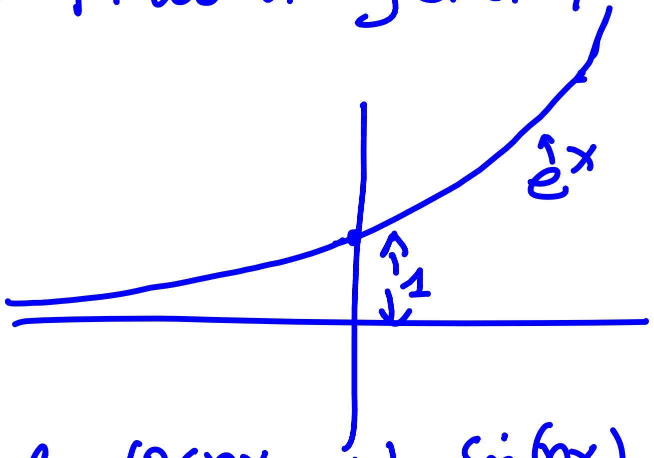
$x = x_0$

$x = x_1$ whenever $\circledast$ is true by plugging in different points we will start finding sum of different series.

$$\to \quad f(x) = \sum_{n=0}^{\infty} a_n \cos(nx) + b_n \sin(nx) \quad \to (*)$$

Example (*) not true in general

$f(x) = e^x$



cannot write

$$e^x = \sum_{n=0}^{\infty} a_n \cos nx + b_n \sin(nx), \quad x \in \mathbb{R}$$

Plug $x = 0$, $x = 2\pi$, and see if the Equality holds. to get contradiction $\perp$

— x —

We need to study the concept of 'Periodic functions'

Defn $f : I \to \mathbb{R}$ $\quad I \subseteq \mathbb{R}$

is said to be periodic, if $\exists\, p > 0$, such that $\boxed{f(x+p) = f(x),\ \forall x \in I}$

and 'p' – is called a period of 'f'.

Ex:

$$f(x) = \sin x, \quad \cos x$$

$p = 2\pi$ is period of both of them.

$$\underset{\uparrow p}{\sin(x + 2\pi)} = \sin x$$

→ **OBSERVATION**

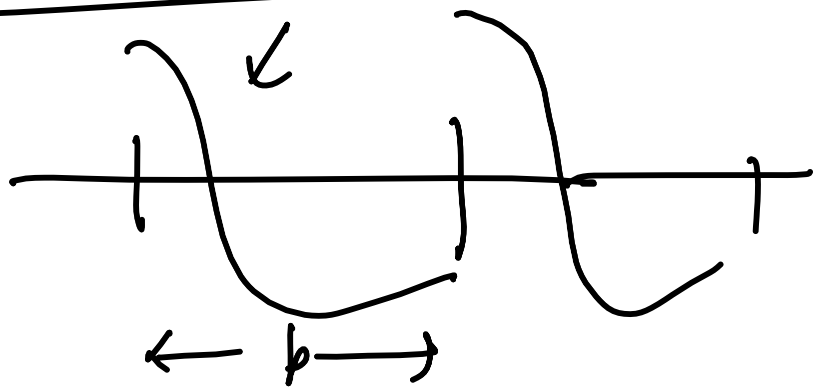
If 'p' is a period of 'f' then '2p', '3p', ... are all periods of 'f'.

$$f(x + 2p) = f(\underbrace{(x+p)}_{\text{new pt}} + p) = f(x+p) = f(x)$$

Fundamental Period

The smallest period 'p' of function 'f' is called the fundamental period of 'f'.

→ Fundamental period of $\sin x$ is 2π.



$f(x) = e^x$ is not periodic

$$\left\{ \begin{array}{l} e^{x+p} = e^x, \ \forall x \in \mathbb{R} \\ \Rightarrow p = 0 \end{array} \right.$$

2π is the fundamental period of $\sin x$

$\rightarrow \exists\, p > 0,\ p < 2\pi$

$\sin(x+p) = \sin x \quad \forall x \in \mathbb{R}$

$p = 2\pi, 4\pi, 6\pi$

"Verify"

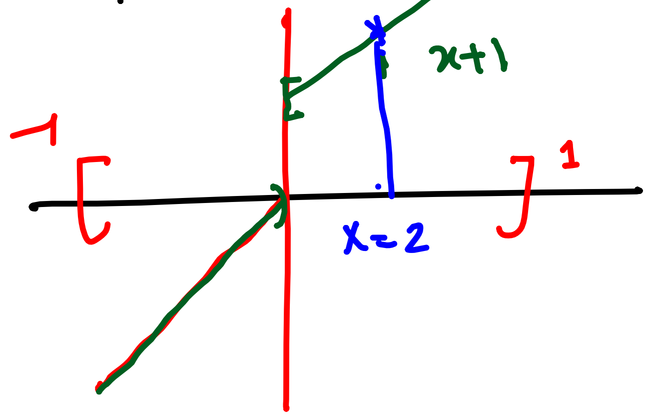
$\rightarrow$ smallest '2π'

Piece wise Continous function

Defn: A function $f: (a,b) \to \mathbb{R}$ is said to be piecewise continuous if it is continous except finitely many points.

Ex $f: [-1, 1] \to \mathbb{R}$, $f(x) = \begin{cases} x, & x < 0 \\ x+1, & x \geq 0 \end{cases}$

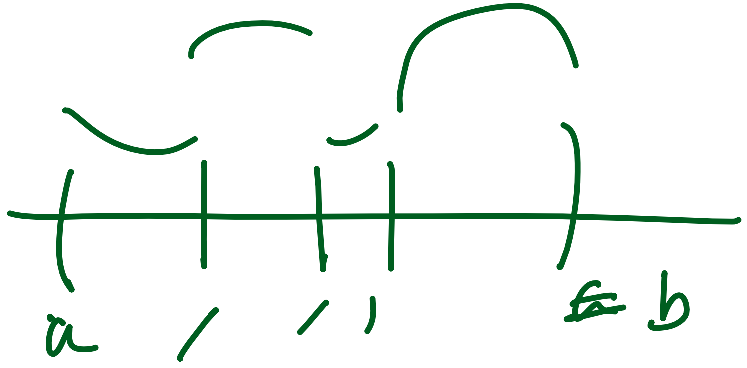
$f(2+) = 3 = f(2-)$



RHL

$f(0+) = 1$

$f(0-) = 0$



— x —

$f: I \to \mathbb{R}$

Left hand Limit (LHL)

Right " (RHL)

$\lim_{\substack{h \to 0 \\ \to h > 0}} f(x+h) := f(x+)$ (RHL)

$x+h > x$

LHD (Left hand derivatives)

(Right " ")

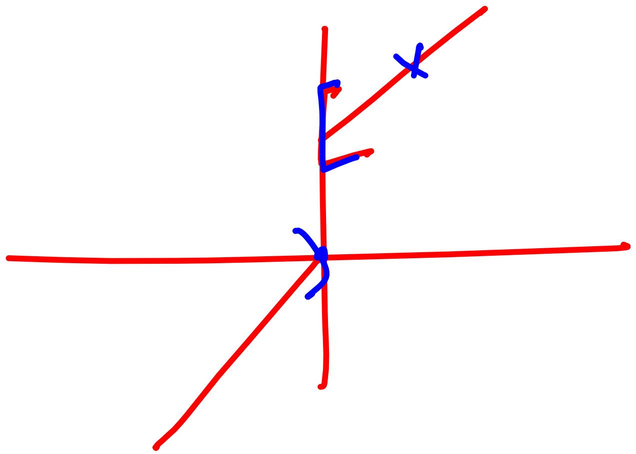
$$f'(x-) := \lim_{\substack{h \to 0 \\ h < 0}} \frac{f(x+h) - f(x)}{h} := \lim_{h \to 0-} \frac{f(x+h) - f(x)}{h}$$

Similarly RHD

$$f'(x+) = \lim_{\substack{h \to 0 \\ h > 0}} \frac{f(x+h) - f(x)}{h}.$$

$$f(x) = \begin{cases} x & x \in [-1, 0) \\ x+1 & x \in [0, 1] \end{cases}$$

closed.



LHD

$$\lim_{\substack{h \to 0 \\ h < 0}} \frac{f(h) - f(0)}{h}$$

$$= \lim_{\substack{h \to 0 \\ h < 0}} \frac{h - 1}{h}$$

does not Exist

RHD

$$\lim_{\substack{h \to 0 \\ h > 0}} \frac{f(h) - f(0)}{h}$$

$$= \lim_{\substack{h \to 0 \\ h > 0}} \frac{(h+1) - 1}{h} = \lim_{\substack{h \to 0 \\ h > 0}} 1$$

$$= 1$$

— x —