MSO 202A : Complex Variables
Quiz, 23rd August 2022

Total Marks: 30 **Time: 6:10 pm - 7 pm**

- Answer all questions.
- Write each step clearly.

1. Prove or disprove the following statements. Explain your answer with complete details. Here $\mathbb{D} = \{z| \ |z| < 1\}$ is the unit disc.

   (a) If $f(z) = u + \imath v : \mathbb{D} \to \mathbb{C}$ is a function such that $f^2(z)$ is analytic, then $f(z)$ itself is analytic.

   [5]

   (b) If $f(z) = u + \imath v : \mathbb{D} \to \mathbb{C}$ is a function such that u, v has continuous partial derivatives on $\mathbb{D}$ and $f^2(z)$ is analytic, then $f(z)$ itself is analytic.

   [5]

2. We know that $\lim\limits_{x\to 0} \frac{\sin x}{x} = 1$ and $\lim\limits_{x\to 0} x\sin(1/x) = 0$, where $x \in \mathbb{R}$. Using the definitions of limits only, determine whether the following limits exist or not. If it exist, find its value.

$$(a) \quad \lim_{x\to 0} \frac{\sin z}{z}; \quad (b) \quad \lim_{z\to 0} z\sin(1/z), \quad z \in \mathbb{C}.$$

   [5+5]

3. (a) Use ML-ineuality to show that

$$\left| \int_\gamma \frac{e^z dz}{z^2+1} \right| \leq e^2 \frac{8\pi}{3},$$

   where γ is the circle $|z| = 2$ travelled twice anticlockwise.

   [6]

   (b) Evaluate $\int_\gamma e^{z^2} dz$, where $\gamma(t) = t(1-t)e^t + i\cos(2\pi t^3), \quad t \in [0, 1]$. Explain your answer clearly.

   [4]