MSO202 - INTRODUCTION TO CPMPLEX ANALYSIS
IIT KANPUR - 2023–2024

Instructions: Course materials and calculators are not allowed.

Assignment 1

The exercises labeled as **(T)** will be covered during the tutorials.

(1) **(T)** Let z, w be two complex numbers. Show that

(a)

$$|z+w|^2 = |z|^2 + |w|^2 + 2\Re(z\overline{w}) \qquad (b).$$

(b) **Law of cosine** Let ΔABC be a triangle with $\angle ACB = \theta$. Prove that $|a|^2 + |b|^2 - 2ab\cos\theta = c^2$, where a, b, c are sides opposite to angles A, B, C respectively.

(c) $|z+w| = |z| + |w|$ if and only if $zw = 0$ of $z = kw$ for some real number k.

(2) Suppose that z_1 and z_2 are complex numbers, with $z_1 z_2$ real and non-zero. Show that there exists a real number r such that $z_1 = r z_2$.

(3) Express following in form of $x + iy$, with $x, y \in \mathbb{R}$.

(a)

$$(a)\left(\frac{1+i}{\sqrt{2}}\right)^{2023}. \quad (b)\left(1+\sqrt{3}i\right)^{1001}. \quad (c)(1-i)^{31}.$$

(4) **(T)** Consider the $n-1$ diagonals of a regular n-gon inscribed in a unit circle obtained by connecting one vertex with all the others. Show that the product of their lengths is n.

(5) Sketch the following sets and determine which ones of these are domains:

$$(a)|z-4| \geqslant |z|. \quad (b)|arg(z+i)| \leqslant \pi/4 \quad (c)|\Im z| < |\Re z|. \quad (d)|z+ia| < |z-a|.$$

(6) **(T)** Let z, w be two complex numbers such that $\overline{z}w \neq 1$. Prove that

$$\left|\frac{w-z}{1-\overline{w}z}\right| < 1 \text{ if } |z| < 1 \text{ and } |w| < 1,$$

and also that

$$\left|\frac{w-z}{1-\overline{w}z}\right| = 1 \text{ if } |z| = 1 \text{ or } |w| = 1,$$

(7) Show that for

$$f(z) = \frac{[(1-i)z + (1+i)\overline{z}]^2}{|z|^2}$$

$\lim_{x\to 0}\lim_{y\to 0} f(x,y) = \lim_{y\to 0}\lim_{x\to 0} f(x,y)$ but limit $\lim_{z\to 0} f(z)$ does not exists.

(8) **T** Show that

$$\frac{R^3 - 4R}{R^2 + R + 1} \leqslant \left| \frac{z^3 + 4z}{z^2 + z + 1} \right| \leqslant \frac{R^3 + 4R}{(R-1)^2},$$

for $|Z| = R > 1$.