

MSO205A PRACTICE PROBLEMS SET 5

Question 1. Is it possible that the following function $f : \mathbb{R} \rightarrow \mathbb{R}$ is a p.m.f. of an RV? If yes, also compute the corresponding DF. Take f as follows: for some constant $c \in \mathbb{R}$,

$$f(x) = \begin{cases} \frac{c}{(2x-1)(2x+1)}, & \text{if } x \in \{1, 2, 3, \dots\}, \\ 0, & \text{otherwise.} \end{cases}$$

Question 2. Let X be a continuous RV with p.d.f. $f_X : \mathbb{R} \rightarrow \mathbb{R}$ given by

$$f(x) = \begin{cases} -\frac{4}{3}x, & \text{if } x \in [-1, 0), \\ \frac{x^2}{8}, & \text{if } x \in [0, 2], \\ 0, & \text{otherwise.} \end{cases}$$

Write down the DF F_X . Compute $\mathbb{P}(X > 0)$, $\mathbb{P}(X \leq 1)$ and $\mathbb{P}(X > 0 \mid X \leq 1)$.

Question 3. Suppose that an RV X has the DF F_X given by any of the following functions. In each case, check if X is discrete or continuous. If so, also find the corresponding p.m.f./p.d.f. (as appropriate) of X .

(1)

$$F_X(x) := \begin{cases} 0, & \text{if } x < 0, \\ \frac{x}{4}, & \text{if } 0 \leq x < 1, \\ \frac{x}{3}, & \text{if } 1 \leq x < 2, \\ \frac{3x}{8}, & \text{if } 2 \leq x < \frac{5}{2}, \\ 1, & \text{if } x \geq \frac{5}{2}. \end{cases}$$

(2)

$$F_X(x) = \begin{cases} 0, & \text{if } x < 0, \\ \frac{1}{2}, & \text{if } 0 \leq x < 2, \\ \frac{3}{4}, & \text{if } 2 \leq x < 3, \\ 1, & \text{if } x \geq 3. \end{cases}$$

(3)

$$F_X(x) := \begin{cases} 0, & \text{if } x < 0, \\ \frac{x^2}{2}, & \text{if } 0 \leq x < 1, \\ \frac{x}{2}, & \text{if } 1 \leq x < 2, \\ 1, & \text{if } x \geq 2. \end{cases}$$