MSO205A Quiz 2 (October 17, 2022)

Duration: 11:00am - 11:40am **Maximum Marks: 15**

Instructions:

1. Write your name and roll number clearly on the designated place. IITK student ID card must be carried in person for verification.
2. You may use books, notebooks, handwritten/photocopied notes of classroom lectures, printouts of supplementary lecture materials and writing instruments during the quiz. Usage of internet or any e-material (including e-books) is prohibited. Electronic communication devices like mobile phones must be switched off and kept in the place designated by the invigilator(s). The invigilators will not be responsible for loss of such a device. If such a device is found on person during the quiz, appropriate action shall be taken. Usage of calculators is not allowed.
3. DO NOT do any rough work on this sheet. If required, do it in your notebook. No additional sheet(s) will be provided and DO NOT attach any additional sheet to this page.
4. Write your answers at the designated places. Any statements written outside the designated place will be taken as rough work and no credit will be provided for such statements. If your answer is not legible, you shall not get credit. You may write your answers as a fraction and include e or π, if required.

Name:

Roll No.:

Question 1. (2 marks) Let A denote the integer formed by the last two digits of your roll number. Let $X \sim Uniform(-1, 105 - A)$. Then $F_X(2) =$ (answers involving the letter/character A will not be accepted)

Question 2. (1.5 + 1.5 marks) Consider the following functions $F_1, F_2 : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$F_1(x,y) := \begin{cases} 1, & \text{if } x + 3y \geq 1 \\ 0, & \text{otherwise.} \end{cases} \quad , \quad F_2(x,y) := \begin{cases} 1, & \text{if } x \geq 4 \text{ and } y \geq -1 \\ 0, & \text{otherwise.} \end{cases}$$

Is F_1 the joint DF of a 2-dimensional random vector? Yes/No (underline the correct answer)

Is F_2 the joint DF of a 2-dimensional random vector? Yes/No (underline the correct answer)

Question 3. (2 + 2 marks) Consider the continuous RV Y given by the p.d.f.

$$f_Y(x) := 2x(1-x)1_{(0,1)}(x) + \frac{2}{3}\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(x-2)^2}{2}\right), \forall x \in \mathbb{R}.$$

Then $\mathbb{E}Y =$ and $\mathbb{E}Y^2 =$

Please turn over

Question 4. (6 marks) Let X be an RV with the MGF $M_X(t) = \frac{1}{4}\left(e^{-t} + 2e^{1.5t} + e^{3t}\right), \forall t \in \mathbb{R}$. Find the DF of X and compute $\mathbb{P}(X \geq 1 | X \leq 2)$. Justify your answer. (Write your answer on this side of the page)

MSO205A Quiz 2 (October 17, 2022)

Duration: 11:00am - 11:40am **Maximum Marks: 15**

Instructions:

1. Write your name and roll number clearly on the designated place. IITK student ID card must be carried in person for verification.
2. You may use books, notebooks, handwritten/photocopied notes of classroom lectures, printouts of supplementary lecture materials and writing instruments during the quiz. Usage of internet or any e-material (including e-books) is prohibited. Electronic communication devices like mobile phones must be switched off and kept in the place designated by the invigilator(s). The invigilators will not be responsible for loss of such a device. If such a device is found on person during the quiz, appropriate action shall be taken. Usage of calculators is not allowed.
3. DO NOT do any rough work on this sheet. If required, do it in your notebook. No additional sheet(s) will be provided and DO NOT attach any additional sheet to this page.
4. Write your answers at the designated places. Any statements written outside the designated place will be taken as rough work and no credit will be provided for such statements. If your answer is not legible, you shall not get credit. You may write your answers as a fraction and include e or π, if required.

Name:

Roll No.:

Question 1. (2 marks) Let A denote the integer formed by the last two digits of your roll number. Let $X \sim Uniform(-1, 105 - A)$. Then $F_X(2) =$ (answers involving the letter/character A will not be accepted)

Question 2. (1.5 + 1.5 marks) Consider the following functions $F_1, F_2 : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$F_1(x,y) := \begin{cases} 1, & \text{if } x \geq 0 \text{ and } y \geq 5 \\ 0, & \text{otherwise.} \end{cases} \quad , \quad F_2(x,y) := \begin{cases} 1, & \text{if } x + 2y \geq 1 \\ 0, & \text{otherwise.} \end{cases}$$

Is F_1 the joint DF of a 2-dimensional random vector? Yes/No (underline the correct answer)

Is F_2 the joint DF of a 2-dimensional random vector? Yes/No (underline the correct answer)

Question 3. (2 + 2 marks) Consider the continuous RV Y given by the p.d.f.

$$f_Y(x) := \frac{1}{5}\exp(-x)1_{(0,\infty)}(x) + \frac{4}{5}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{(x+2)^2}{2}\right), \forall x \in \mathbb{R}.$$

Then $\mathbb{E}Y =$ and $\mathbb{E}Y^2 =$

Please turn over

Question 4. (6 marks) Let X be an RV with the MGF $M_X(t) = \frac{1}{5}\left(2e^{-2t} + 2e^{1.5t} + e^{3t}\right), \forall t \in \mathbb{R}$. Find the DF of X and compute $\mathbb{P}(X \geq 1 | X \leq 2)$. Justify your answer. (Write your answer on this side of the page)

MSO205A Quiz 2 (October 17, 2022)

Duration: 11:00am - 11:40am **Maximum Marks: 15**

Instructions:

1. Write your name and roll number clearly on the designated place. IITK student ID card must be carried in person for verification.
2. You may use books, notebooks, handwritten/photocopied notes of classroom lectures, printouts of supplementary lecture materials and writing instruments during the quiz. Usage of internet or any e-material (including e-books) is prohibited. Electronic communication devices like mobile phones must be switched off and kept in the place designated by the invigilator(s). The invigilators will not be responsible for loss of such a device. If such a device is found on person during the quiz, appropriate action shall be taken. Usage of calculators is not allowed.
3. DO NOT do any rough work on this sheet. If required, do it in your notebook. No additional sheet(s) will be provided and DO NOT attach any additional sheet to this page.
4. Write your answers at the designated places. Any statements written outside the designated place will be taken as rough work and no credit will be provided for such statements. If your answer is not legible, you shall not get credit. You may write your answers as a fraction and include e or π, if required.

Name:

Roll No.:

Question 1. (2 marks) Let A denote the integer formed by the last two digits of your roll number. Let $X \sim Uniform(-1, 105 - A)$. Then $F_X(2) =$ (answers involving the letter/character A will not be accepted)

Question 2. (1.5 + 1.5 marks) Consider the following functions $F_1, F_2 : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$F_1(x,y) := \begin{cases} 1, & \text{if } 3x + y \geq 1 \\ 0, & \text{otherwise.} \end{cases} \quad , \quad F_2(x,y) := \begin{cases} 1, & \text{if } x \geq 3 \text{ and } y \geq 0 \\ 0, & \text{otherwise.} \end{cases}$$

Is F_1 the joint DF of a 2-dimensional random vector? Yes/No (underline the correct answer)

Is F_2 the joint DF of a 2-dimensional random vector? Yes/No (underline the correct answer)

Question 3. (2 + 2 marks) Consider the continuous RV Y given by the p.d.f.

$$f_Y(x) := \frac{1}{3}\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(x+2)^2}{2}\right) + 4x(1-x)1_{(0,1)}(x), \forall x \in \mathbb{R}.$$

Then $\mathbb{E}Y =$ and $\mathbb{E}Y^2 =$

Please turn over

Question 4. (6 marks) Let X be an RV with the MGF $M_X(t) = \frac{1}{6}\left(e^{-2t} + 2e^{1.5t} + 3e^{3t}\right), \forall t \in \mathbb{R}$. Find the DF of X and compute $\mathbb{P}(X \geq 1 | X \leq 2)$. Justify your answer. (Write your answer on this side of the page)

MSO205A Quiz 2 (October 17, 2022)

Duration: 11:00am - 11:40am **Maximum Marks: 15**

Instructions:

1. Write your name and roll number clearly on the designated place. IITK student ID card must be carried in person for verification.
2. You may use books, notebooks, handwritten/photocopied notes of classroom lectures, printouts of supplementary lecture materials and writing instruments during the quiz. Usage of internet or any e-material (including e-books) is prohibited. Electronic communication devices like mobile phones must be switched off and kept in the place designated by the invigilator(s). The invigilators will not be responsible for loss of such a device. If such a device is found on person during the quiz, appropriate action shall be taken. Usage of calculators is not allowed.
3. DO NOT do any rough work on this sheet. If required, do it in your notebook. No additional sheet(s) will be provided and DO NOT attach any additional sheet to this page.
4. Write your answers at the designated places. Any statements written outside the designated place will be taken as rough work and no credit will be provided for such statements. If your answer is not legible, you shall not get credit. You may write your answers as a fraction and include e or π, if required.

Name:

Roll No.:

Question 1. (2 marks) Let A denote the integer formed by the last two digits of your roll number. Let $X \sim Uniform(-1, 105 - A)$. Then $F_X(2) =$ (answers involving the letter/character A will not be accepted)

Question 2. (1.5 + 1.5 marks) Consider the following functions $F_1, F_2 : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$F_1(x,y) := \begin{cases} 1, \text{ if } x \geq -3 \text{ and } y \geq 2 \\ 0, \text{ otherwise.} \end{cases} \quad , \quad F_2(x,y) := \begin{cases} 1, \text{ if } 2x + y \geq 1 \\ 0, \text{ otherwise.} \end{cases}$$

Is F_1 the joint DF of a 2-dimensional random vector? Yes/No (underline the correct answer)

Is F_2 the joint DF of a 2-dimensional random vector? Yes/No (underline the correct answer)

Question 3. (2 + 2 marks) Consider the continuous RV Y given by the p.d.f.

$$f_Y(x) := \frac{1}{5}\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(x+2)^2}{2}\right) + \frac{4}{5}\exp(-x)1_{(0,\infty)}(x), \forall x \in \mathbb{R}.$$

Then $\mathbb{E}Y =$ and $\mathbb{E}Y^2 =$

Please turn over

Question 4. (6 marks) Let X be an RV with the MGF $M_X(t) = \frac{1}{4}\left(2e^{-t} + e^{1.5t} + e^{3t}\right), \forall t \in \mathbb{R}$. Find the DF of X and compute $\mathbb{P}(X \geq 1 | X \leq 2)$. Justify your answer. (Write your answer on this side of the page)

MSO205A Quiz 2 (October 17, 2022)

Duration: 11:00am - 11:40am **Maximum Marks: 15**

Instructions:

1. Write your name and roll number clearly on the designated place. IITK student ID card must be carried in person for verification.
2. You may use books, notebooks, handwritten/photocopied notes of classroom lectures, printouts of supplementary lecture materials and writing instruments during the quiz. Usage of internet or any e-material (including e-books) is prohibited. Electronic communication devices like mobile phones must be switched off and kept in the place designated by the invigilator(s). The invigilators will not be responsible for loss of such a device. If such a device is found on person during the quiz, appropriate action shall be taken. Usage of calculators is not allowed.
3. DO NOT do any rough work on this sheet. If required, do it in your notebook. No additional sheet(s) will be provided and DO NOT attach any additional sheet to this page.
4. Write your answers at the designated places. Any statements written outside the designated place will be taken as rough work and no credit will be provided for such statements. If your answer is not legible, you shall not get credit. You may write your answers as a fraction and include e or π, if required.

Name:

Roll No.:

Question 1. (2 marks) Let A denote the integer formed by the last two digits of your roll number. Let $X \sim Uniform(-1, 105 - A)$. Then $F_X(2) =$ (answers involving the letter/character A will not be accepted)

Question 2. (1.5 + 1.5 marks) Consider the following functions $F_1, F_2 : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$F_1(x,y) := \begin{cases} xy, & \text{if } 0 < y < x < 1 \\ 0, & \text{otherwise.} \end{cases} \quad , \quad F_2(x,y) := \begin{cases} 1, & \text{if } x + y \geq 2 \\ 0, & \text{otherwise.} \end{cases}$$

Is F_1 the joint DF of a 2-dimensional random vector? Yes/No (underline the correct answer)

Is F_2 the joint DF of a 2-dimensional random vector? Yes/No (underline the correct answer)

Question 3. (2 + 2 marks) Consider the continuous RV Y given by the p.d.f.

$$f_Y(x) := \frac{1}{4}\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(x-2)^2}{2}\right) + \frac{3}{4}\frac{1}{2\pi} \exp\left(-\frac{(x-3)^2}{4\pi}\right), \forall x \in \mathbb{R}.$$

Then $\mathbb{E}Y =$ and $\mathbb{E}Y^2 =$

Please turn over

Question 4. (6 marks) Let X be an RV with the MGF $M_X(t) = \frac{1}{5}\left(e^{-2t} + 2e^{1.5t} + 2e^{3t}\right), \forall t \in \mathbb{R}$. Find the DF of X and compute $\mathbb{P}(X \geq 1 | X \leq 2)$. Justify your answer. (Write your answer on this side of the page)