

# MSO205A PRACTICE PROBLEMS SET 4

Question 1. Which of the following functions are distribution functions?

$$\begin{aligned}
 (1) \quad F_1(x) &:= \begin{cases} 0, & \text{if } x \leq 1, \\ 1, & \text{if } x > 1. \end{cases} \\
 (2) \quad F_2(x) &:= \begin{cases} 1, & \text{if } x < 1, \\ 0, & \text{if } x \geq 1. \end{cases} \\
 (3) \quad F_3(x) &:= \begin{cases} 0, & \text{if } x < 1, \\ 1, & \text{if } x \geq 1. \end{cases} \\
 (4) \quad F_4(x) &:= \begin{cases} 0, & \text{if } x < 0, \\ \frac{1}{4} + \frac{x}{2}, & \text{if } 0 \leq x \leq 1, \\ \frac{1}{2} + \frac{x}{4}, & \text{if } 1 < x < 2, \\ 1, & \text{if } x \geq 2. \end{cases} .
 \end{aligned}$$

Question 2. Let  $X$  be an RV defined on a probability space  $(\Omega, \mathcal{F}, \mathbb{P})$  with law  $\mathbb{P}_X$  and DF  $F_X$ . Consider the set  $D := \{x \in \mathbb{R} : F_X \text{ is discontinuous at } x\}$ . Show that it is either finite or countably infinite. (Hint: for each  $n = 1, 2, \dots$ , consider the set  $D_n := \{x \in \mathbb{R} : F_X(x+) - F_X(x-) > \frac{1}{n}\} = \{x \in \mathbb{R} : F_X(x) - F_X(x-) > \frac{1}{n}\} = \{x \in \mathbb{R} : \mathbb{P}(X = x) > \frac{1}{n}\}$ . Then  $D = \cup_n D_n$ . What can you say about  $D_n$ ?)