MSO205A Mid Semester Examination (September 21, 2022)

Duration: 8:00am - 10:00am **Maximum Marks: 30**

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1. Section 1

Question 1: $(1 + (2 + 2) + 2 + 1 + 2$ marks) It is known that the function $F : \mathbb{R} \to \mathbb{R}$, defined by

$$F(x) = \begin{cases} 0, \text{ if } x < 0, \\ \alpha x + \frac{1}{8}, \text{ if } 0 \leq x \leq 1, \\ \frac{x}{6} + \frac{1}{12}, \text{ if } 1 < x < 2, \\ \beta x - \frac{1}{2}, \text{ if } 2 \leq x \leq 3, \\ 1, \text{ otherwise} \end{cases}$$

is a distribution function (DF) for some $\alpha, \beta \in \mathbb{R}$. Are α, β unique? $\boxed{\text{Yes/No}}$ (underline the correct answer). A possible value of (α, β) for which F is a DF is $\boxed{(\alpha, \beta) = \quad}$. For this (α, β), let γ denote the number of discontinuity points of F. Then, $\boxed{8\alpha + 4\beta + 2\gamma = \quad}$ and the corresponding random variable X is $\boxed{\text{discrete/ continuous / neither discrete nor continuous}}$ (underline the correct answer). Here, $\boxed{\mathbb{P}(X > \frac{3}{2} \mid X \leq \frac{5}{2}) = \quad}$

Question 2: (2 marks) Consider the following two functions $f, g : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) := \begin{cases} \frac{2}{3^x}, & \text{if } x \in \{1, 2, 3, \cdots\}, \\ 0, & \text{otherwise} \end{cases} \qquad g(x) := \frac{4}{\pi} \frac{1}{4^2 + x^2}, \quad \forall x \in \mathbb{R}.$$

Choose the correct option below, by putting a tick (✓) to get credit.

(i) f is a p.m.f. and g is a p.d.f.
(ii) g is a p.d.f., but f is not a p.m.f.
(iii) f is a p.m.f., but g is not a p.d.f.
(iv) f is not a p.m.f. and g is not a p.d.f.

2. Section 2: Descriptive type

Instructions: Answers to these questions should be in the answer-script.

Question 3: Answers to all the sub-parts should be kept together.

(i) ($1 + 2$ marks) Let X be a discrete random variable with the property that $\mathbb{P}(-1 < X \leq 2) = 1$. Is X^2 also a discrete random variable? What can you say about the support of X^2?

(ii) ($1 + 2 + 1$ marks) A person plans to apply to 10 companies for a job. All the companies accept applications only through online mode, but 2 of these companies also require an additional supporting document to be sent by post. An application is considered to be complete once it reaches the intended company along with the supporting document, if any. Assume that online applications always reach the intended company, but the probability that a document sent by post reaches the destination is $\frac{1}{2}$. Further, assume that sending documents by post to different destinations are independent. Let Y denote the number of completed applications once the person sends the applications through online mode and documents by post. Identify the support of the discrete random variable Y and compute the p.m.f. of Y. Also compute $2\mathbb{E}Y^2 - \mathbb{E}Y$.

(iii) (2 marks) Suppose that all the cards numbered 2, 3, 4, 5 and 6 have been removed from a standard deck of cards and that three cards have been drawn at random from the remaining thirty-two cards. Given that at least two cards are from the diamond suite, what is the probability that all cards are from the diamond suite? You may write your answer as a fraction. (Note: binomial coefficients in the answer will not be acceptable.)

Question 4: Answers to all the sub-parts should be kept together.

(i) ($4 + 2$ marks) Consider a continuous random variable X with the p.d.f.

$$f_X(x) = \begin{cases} 2\exp(-2x), & \text{if } x > 0, \\ 0, & \text{otherwise.} \end{cases}$$

Compute the distribution function of $Y = |X - 1|$. Is Y a continuous random variable with a p.d.f.? Justify your answer.

(ii) (3 marks) Let Z be a continuous random variable with p.d.f. f_Z such that $\mathbb{P}(Z \leq 0) = 0$ and $\mathbb{E}Z < \infty$. What can you say about $\lim_{n\to\infty} n\mathbb{P}(Z > n)$? Justify your answer. (Note: if you simply compute the limit for specific random variables Z, then no marks will be provided.)

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1. Section 1

Question 1: $(1 + (2 + 2) + 2 + 1 + 2$ marks) It is known that the function $F : \mathbb{R} \to \mathbb{R}$, defined by

$$F(x) = \begin{cases} 0, \text{ if } x < 0, \\ \beta x + \frac{1}{8}, \text{ if } 0 \leq x \leq 1, \\ \frac{x}{6} + \frac{1}{12}, \text{ if } 1 < x < 2, \\ \alpha x - \frac{1}{2}, \text{ if } 2 \leq x \leq 3, \\ 1, \text{ otherwise} \end{cases}$$

is a distribution function (DF) for some $\alpha, \beta \in \mathbb{R}$. Are α, β unique? $\boxed{\text{Yes/No}}$ (underline the correct answer). A possible value of (α, β) for which F is a DF is $\boxed{(\alpha, \beta) = \quad}$. For this (α, β), let γ denote the number of discontinuity points of F. Then, $\boxed{8\alpha + 4\beta + 2\gamma = \quad}$ and the corresponding random variable X is $\boxed{\text{discrete/ continuous / neither discrete nor continuous}}$ (underline the correct answer). Here, $\boxed{\mathbb{P}(X > \frac{3}{2} \mid X \leq \frac{5}{2}) = \quad}$

Question 2: (2 marks) Consider the following two functions $f, g : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) := \begin{cases} \frac{2}{3^x}, \text{ if } x \in \{1, 2, 3, \cdots\}, \\ 0, \text{ otherwise} \end{cases} \qquad g(x) := \frac{4^2}{\pi} \frac{1}{4^2 + x^2}, \quad \forall x \in \mathbb{R}.$$

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2. Section 2: Descriptive type

Instructions: Answers to these questions should be in the answer-script.

Question 3: Answers to all the sub-parts should be kept together.

(i) (1 + 2 marks) Let X be a discrete random variable with the property that $\mathbb{P}(-1 < X \leq 2) = 1$. Is X^2 also a discrete random variable? What can you say about the support of X^2?

(ii) (1 + 2 + 1 marks) A person plans to apply to 10 companies for a job. All the companies accept applications only through online mode, but 2 of these companies also require an additional supporting document to be sent by post. An application is considered to be complete once it reaches the intended company along with the supporting document, if any. Assume that online applications always reach the intended company, but the probability that a document sent by post reaches the destination is $\frac{1}{2}$. Further, assume that sending documents by post to different destinations are independent. Let Y denote the number of completed applications once the person sends the applications through online mode and documents by post. Identify the support of the discrete random variable Y and compute the p.m.f. of Y. Also compute $2\mathbb{E}Y^2 - \mathbb{E}Y$.

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