MSO205A Quiz 2 Descriptive Question (Qn 4) and Solution

1. Set 1

Question (6 marks): Let X be an RV with the MGF $M_X(t) = \frac{1}{5}\left(e^{-2t} + 2e^{1.5t} + 2e^{3t}\right), \forall t \in \mathbb{R}$. Find the DF of X and compute $\mathbb{P}(X \geq 1 | X \leq 2)$. Justify your answer.

Answer: Considering the structure of the MGF, we look at the p.m.f. (identification of the p.m.f. - part marks: 1.5)

$$f(x) := \begin{cases} \frac{1}{5}, \text{ if } x = -2, \\ \frac{2}{5}, \text{ if } x = 1.5 \text{ or } 3, \\ 0, \text{ otherwise.} \end{cases}$$

The MGF corresponding to the p.m.f. f above agrees with the MGF M_X of X. Since an MGF, if it exists, uniquely determines the distribution, the RV X is a discrete RV with the above p.m.f. f (unique identification of a distribution through MGF - this statement, part marks: 1).

Hence, the DF F_X is as follows: (part marks: 1.5)

$$F_X(x) = \mathbb{P}(X \leq x) = \begin{cases} 0, \text{ if } x < -2, \\ \frac{1}{5}, \text{ if } -2 \leq x < 1.5, \\ \frac{3}{5}, \text{ if } 1.5 \leq x < 3, \\ 1, \text{ if } x \geq 3. \end{cases}$$

Now, (part marks: 2)

$$\mathbb{P}(X \geq 1 | X \leq 2) = \frac{\mathbb{P}(1 \leq X \leq 2)}{\mathbb{P}(X \leq 2)} = \frac{F_X(2) - F_X(1-)}{F_X(2)} = \frac{\frac{3}{5} - \frac{1}{5}}{\frac{3}{5}} = \frac{2}{3}.$$

2. Set 2

Question (6 marks): Let X be an RV with the MGF $M_X(t) = \frac{1}{4}\left(2e^{-t} + e^{1.5t} + e^{3t}\right), \forall t \in \mathbb{R}$. Find the DF of X and compute $\mathbb{P}(X \geq 1 | X \leq 2)$. Justify your answer.

Answer: Considering the structure of the MGF, we look at the p.m.f. (identification of the p.m.f. - part marks: 1.5)

$$f(x) := \begin{cases} \frac{2}{4}, \text{ if } x = -1, \\ \frac{1}{4}, \text{ if } x = 1.5 \text{ or } 3, \\ 0, \text{ otherwise.} \end{cases}$$

The MGF corresponding to the p.m.f. f above agrees with the MGF M_X of X. Since an MGF, if it exists, uniquely determines the distribution, the RV X is a discrete RV with the above p.m.f. f (unique identification of a distribution through MGF - this statement, part marks: 1).

Hence, the DF F_X is as follows: (part marks: 1.5)

$$F_X(x) = \mathbb{P}(X \le x) = \begin{cases} 0, \text{ if } x < -1, \\ \frac{1}{2}, \text{ if } -1 \le x < 1.5, \\ \frac{3}{4}, \text{ if } 1.5 \le x < 3, \\ 1, \text{ if } x \ge 3. \end{cases}$$

Now, (part marks: 2)

$$\mathbb{P}(X \ge 1 | X \le 2) = \frac{\mathbb{P}(1 \le X \le 2)}{\mathbb{P}(X \le 2)} = \frac{F_X(2) - F_X(1-)}{F_X(2)} = \frac{\frac{3}{4} - \frac{1}{2}}{\frac{3}{4}} = \frac{1}{3}.$$

3. Set 3

Question (6 marks): Let X be an RV with the MGF $M_X(t) = \frac{1}{6}\left(e^{-2t} + 2e^{1.5t} + 3e^{3t}\right), \forall t \in \mathbb{R}$. Find the DF of X and compute $\mathbb{P}(X \ge 1 | X \le 2)$. Justify your answer.

Answer: Considering the structure of the MGF, we look at the p.m.f. (identification of the p.m.f. - part marks: 1.5)

$$f(x) := \begin{cases} \frac{1}{6}, \text{ if } x = -2, \\ \frac{2}{6}, \text{ if } x = 1.5, \\ \frac{3}{6}, \text{ if } x = 3, \\ 0, \text{ otherwise.} \end{cases}$$

The MGF corresponding to the p.m.f. f above agrees with the MGF M_X of X. Since an MGF, if it exists, uniquely determines the distribution, the RV X is a discrete RV with the above p.m.f. f (unique identification of a distribution through MGF - this statement, part marks: 1).

Hence, the DF F_X is as follows: (part marks: 1.5)

$$F_X(x) = \mathbb{P}(X \le x) = \begin{cases} 0, \text{ if } x < -2, \\ \frac{1}{6}, \text{ if } -2 \le x < 1.5, \\ \frac{1}{2}, \text{ if } 1.5 \le x < 3, \\ 1, \text{ if } x \ge 3. \end{cases}$$

Now, (part marks: 2)

$$\mathbb{P}(X \ge 1 | X \le 2) = \frac{\mathbb{P}(1 \le X \le 2)}{\mathbb{P}(X \le 2)} = \frac{F_X(2) - F_X(1-)}{F_X(2)} = \frac{\frac{1}{2} - \frac{1}{6}}{\frac{1}{2}} = \frac{2}{3}.$$

4. Set 4

Question (6 marks): Let X be an RV with the MGF $M_X(t) = \frac{1}{5}\left(2e^{-2t} + 2e^{1.5t} + e^{3t}\right), \forall t \in \mathbb{R}$. Find the DF of X and compute $\mathbb{P}(X \ge 1 | X \le 2)$. Justify your answer.

Answer: Considering the structure of the MGF, we look at the p.m.f. (identification of the p.m.f. - part marks: 1.5)

$$f(x) := \begin{cases} \frac{2}{5}, \text{ if } x = -2 \text{ or } 1.5, \\ \frac{1}{5}, \text{ if } x = 3, \\ 0, \text{ otherwise.} \end{cases}$$

The MGF corresponding to the p.m.f. f above agrees with the MGF M_X of X. Since an MGF, if it exists, uniquely determines the distribution, the RV X is a discrete RV with the above p.m.f. f (unique identification of a distribution through MGF - this statement, part marks: 1).
Hence, the DF F_X is as follows: (part marks: 1.5)

$$F_X(x) = \mathbb{P}(X \leq x) = \begin{cases} 0, \text{ if } x < -2, \\ \frac{2}{5}, \text{ if } -2 \leq x < 1.5, \\ \frac{4}{5}, \text{ if } 1.5 \leq x < 3, \\ 1, \text{ if } x \geq 3. \end{cases}$$

Now, (part marks: 2)

$$\mathbb{P}(X \geq 1 | X \leq 2) = \frac{\mathbb{P}(1 \leq X \leq 2)}{\mathbb{P}(X \leq 2)} = \frac{F_X(2) - F_X(1-)}{F_X(2)} = \frac{\frac{4}{5} - \frac{2}{5}}{\frac{4}{5}} = \frac{1}{2}.$$

5. Set 5

Question (6 marks): Let X be an RV with the MGF $M_X(t) = \frac{1}{4}\left(e^{-t} + 2e^{1.5t} + e^{3t}\right), \forall t \in \mathbb{R}$. Find the DF of X and compute $\mathbb{P}(X \geq 1 | X \leq 2)$. Justify your answer.

Answer: Considering the structure of the MGF, we look at the p.m.f. (identification of the p.m.f. - part marks: 1.5)

$$f(x) := \begin{cases} \frac{1}{4}, \text{ if } x = -1 \text{ or } 3, \\ \frac{2}{4}, \text{ if } x = 1.5, \\ 0, \text{ otherwise.} \end{cases}$$

The MGF corresponding to the p.m.f. f above agrees with the MGF M_X of X. Since an MGF, if it exists, uniquely determines the distribution, the RV X is a discrete RV with the above p.m.f. f (unique identification of a distribution through MGF - this statement, part marks: 1).
Hence, the DF F_X is as follows: (part marks: 1.5)

$$F_X(x) = \mathbb{P}(X \leq x) = \begin{cases} 0, \text{ if } x < -1, \\ \frac{1}{4}, \text{ if } -1 \leq x < 1.5, \\ \frac{3}{4}, \text{ if } 1.5 \leq x < 3, \\ 1, \text{ if } x \geq 3. \end{cases}$$

Now, (part marks: 2)

$$\mathbb{P}(X \geq 1 | X \leq 2) = \frac{\mathbb{P}(1 \leq X \leq 2)}{\mathbb{P}(X \leq 2)} = \frac{F_X(2) - F_X(1-)}{F_X(2)} = \frac{\frac{3}{4} - \frac{1}{4}}{\frac{3}{4}} = \frac{2}{3}.$$