MSO205A PRACTICE PROBLEMS SET 9

Question 1. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space and let $A, B \in \mathcal{F}$. Define RVs $X, Y : \Omega \to \mathbb{R}$ by

$$X(\omega) = 1_A(\omega) = \begin{cases} 1, \text{ if } \omega \in A, \\ 0, \text{ otherwise} \end{cases}, \quad Y(\omega) = 1_B(\omega), \forall \omega \in \Omega.$$

Show that RVs X, Y are independent if and only if events A, B are independent.

Question 2. Let $X = (X_1, X_2, \cdots, X_p)$ be a discrete random vector with joint DF F_X, joint p.m.f. f_X and support S_X. Let f_{X_j} denote the marginal p.m.f. of X_j. If $X_1, X_2, \cdots, X_p$ are independent, then show that

$$f_{X_1, X_2, \cdots, X_p}(x_1, x_2, \cdots, x_p) = \prod_{j=1}^{p} f_{X_j}(x_j), \forall x_1, x_2, \cdots, x_p \in \mathbb{R}.$$

Question 3. Let $X = (X_1, X_2)$ be a discrete random vector with joint p.m.f.

$$f_X(x_1, x_2) = \begin{cases} \alpha(2x_1 + x_2), \text{ if } x_1, x_2 \in \{1, 2\}, \\ 0, \text{ otherwise} \end{cases}$$

for some constant $\alpha \in \mathbb{R}$. Find the value of α and identify the marginal p.m.f.s of X_1 and X_2. Are X_1, X_2 independent? If not independent, find the conditional p.m.f. of X_2 given $X_1 = x_1 \in \{1, 2\}$.