MSO205A Quiz 3 (November 7, 2022)

Duration: 11:00am - 11:40am **Maximum Marks: 15**

Instructions:

1. Write your name and roll number clearly on the designated place. IITK student ID card must be carried in person for verification.
2. You may use books, notebooks, handwritten/photocopied notes of classroom lectures, printouts of supplementary lecture materials and writing instruments during the quiz. Usage of internet or any e-material (including e-books) is prohibited. Electronic communication devices like mobile phones must be switched off and kept in the place designated by the invigilator(s). The invigilators will not be responsible for loss of such a device. If such a device is found on person during the quiz, appropriate action shall be taken. Usage of calculators is not allowed.
3. DO NOT do any rough work on this sheet. If required, do it in your notebook. No additional sheet(s) will be provided and DO NOT attach any additional sheet to this page.
4. Write your answers at the designated places. Any statements written outside the designated place will be taken as rough work and no credit will be provided for such statements. If your answer is not legible, you shall not get credit. You may write your answers as fractions or roots and include e or π, if required.

Name:

Roll No.:

Question 1. (1.5 + 1 + 1.5 + 1 marks) Consider a continuous random vector (X, Y) with the joint p.d.f.

$$f_{X,Y}(x,y) = \begin{cases} \frac{c}{\pi}, & \text{if } x^2 + y^2 < \frac{1}{2}, \\ 0, & \text{otherwise,} \end{cases}$$

where c is a positive constant. Then $c =$,

$\mathbb{E}X =$ and $Cov(X,Y) =$

Are the RVs X and Y independent? Yes/No (underline the correct answer)

Please turn over

Question 2. (3 marks) Given that $\Phi(0.84) = 0.8, \Phi(0.525) = 0.7, \Phi(0.675) = 0.75$, where Φ denotes the DF of $X \sim N(0,1)$. Let α denote the lower quartile of the distribution of $Y = 1 + 2X$. Then

$2\alpha =$

Question 3. (4 marks) Let X and Y be two non-degenerate discrete RVs defined on the same probability space $(\Omega, \mathcal{F}, \mathbb{P})$ such that the MGF M_X exists on $\mathbb{R}$, $\mathbb{P}(X \geq 0) = 1$ and $\mathbb{E}Y^2 < \infty$. Which of the following statement(s) is/are necessarily true? Put a tick (✓) beside all correct statement(s) to get credit. No partial marking is applicable.

(a) $\exp(\mathbb{E}X) \leq M_X(1)$.

(b) $\left(\mathbb{E}X^3\right)^2 > \mathbb{E}X^6$

(c) $\mathbb{P}(X \geq \alpha) \leq e^{-\lambda\alpha} M_X(\lambda)$ for all $\alpha > 0, \lambda > 0$.

(d) $F_X(0) = 0$

(e) $\mathbb{E}(X+Y)^2$ may not exist.

Question 4. (1.5 + 1.5 marks) Let $X \sim N(1,4), Y \sim N(3,9), Z \sim N(-2,16)$ be independent RVs. Then

$\left(\frac{Y-3}{3}\right)^2 + \left(\frac{Z+2}{4}\right)^2 \sim$ and $2^{\frac{X-1}{Z+2}} \sim$

MSO205A Quiz 3 (November 7, 2022)

Duration: 11:00am - 11:40am **Maximum Marks: 15**

Instructions:

1. Write your name and roll number clearly on the designated place. IITK student ID card must be carried in person for verification.
2. You may use books, notebooks, handwritten/photocopied notes of classroom lectures, printouts of supplementary lecture materials and writing instruments during the quiz. Usage of internet or any e-material (including e-books) is prohibited. Electronic communication devices like mobile phones must be switched off and kept in the place designated by the invigilator(s). The invigilators will not be responsible for loss of such a device. If such a device is found on person during the quiz, appropriate action shall be taken. Usage of calculators is not allowed.
3. DO NOT do any rough work on this sheet. If required, do it in your notebook. No additional sheet(s) will be provided and DO NOT attach any additional sheet to this page.
4. Write your answers at the designated places. Any statements written outside the designated place will be taken as rough work and no credit will be provided for such statements. If your answer is not legible, you shall not get credit. You may write your answers as fractions or roots and include e or π, if required.

Name:

Roll No.:

Question 1. (1.5 + 1 + 1.5 + 1 marks) Consider a continuous random vector (X, Y) with the joint p.d.f.

$$f_{X,Y}(x,y) = \begin{cases} \frac{c}{\pi}, & \text{if } x^2 + y^2 < \frac{1}{3}, \\ 0, & \text{otherwise,} \end{cases}$$

where c is a positive constant. Then $c =$,

$\mathbb{E}X =$ and $Cov(X,Y) =$

Are the RVs X and Y independent? Yes/No (underline the correct answer)

Please turn over

Question 2. (3 marks) Given that $\Phi(0.84) = 0.8, \Phi(0.525) = 0.7, \Phi(0.675) = 0.75$, where Φ denotes the DF of $X \sim N(0,1)$. Let α denote the quantile of order 0.2 of the distribution of $Y = 1 + 3X$. Then $\boxed{2\alpha =}$

Question 3. (4 marks) Let X and Y be two non-degenerate discrete RVs defined on the same probability space $(\Omega, \mathcal{F}, \mathbb{P})$ such that the MGF M_X exists on $\mathbb{R}$, $\mathbb{P}(X \geq 0) = 1$ and $\mathbb{E}Y^2 < \infty$. Which of the following statement(s) is/are necessarily true? Put a tick (✓) beside all correct statement(s) to get credit. No partial marking is applicable.

(a) $\exp(\mathbb{E}X) \leq M_X(1)$.

(b) $\left(\mathbb{E}X^3\right)^2 > \mathbb{E}X^6$

(c) $\mathbb{P}(X \geq \alpha) \leq e^{-\lambda\alpha} M_X(\lambda)$ for all $\alpha > 0, \lambda > 0$.

(d) $F_X(0) = 0$

(e) $\mathbb{E}(X+Y)^2$ may not exist.

Question 4. (1.5 + 1.5 marks) Let $X \sim N(1,4), Y \sim N(3,9), Z \sim N(-2,16)$ be independent RVs and set $W = X + 2Y$. Then $\boxed{\frac{\left(\frac{Y-3}{3}\right)^2 + \left(\frac{Z+2}{4}\right)^2}{2\left(\frac{X-1}{2}\right)^2} \sim}$ and $\boxed{F_W(W) \sim}$ where F_W denotes the DF of W.

MSO205A Quiz 3 (November 7, 2022)

Duration: 11:00am - 11:40am **Maximum Marks: 15**

Instructions:

1. Write your name and roll number clearly on the designated place. IITK student ID card must be carried in person for verification.
2. You may use books, notebooks, handwritten/photocopied notes of classroom lectures, printouts of supplementary lecture materials and writing instruments during the quiz. Usage of internet or any e-material (including e-books) is prohibited. Electronic communication devices like mobile phones must be switched off and kept in the place designated by the invigilator(s). The invigilators will not be responsible for loss of such a device. If such a device is found on person during the quiz, appropriate action shall be taken. Usage of calculators is not allowed.
3. DO NOT do any rough work on this sheet. If required, do it in your notebook. No additional sheet(s) will be provided and DO NOT attach any additional sheet to this page.
4. Write your answers at the designated places. Any statements written outside the designated place will be taken as rough work and no credit will be provided for such statements. If your answer is not legible, you shall not get credit. You may write your answers as fractions or roots and include e or π, if required.

Name:

Roll No.:

Question 1. (1.5 + 1 + 1.5 + 1 marks) Consider a continuous random vector (X, Y) with the joint p.d.f.

$$f_{X,Y}(x,y) = \begin{cases} \frac{c}{\pi}, & \text{if } x^2 + y^2 < \frac{1}{5}, \\ 0, & \text{otherwise,} \end{cases}$$

where c is a positive constant. Then $c =$,

$\mathbb{E}X =$ and $Cov(X,Y) =$

Are the RVs X and Y independent? Yes/No (underline the correct answer)

Please turn over

Question 2. (3 marks) Given that $\Phi(0.84) = 0.8, \Phi(0.525) = 0.7, \Phi(0.675) = 0.75$, where Φ denotes the DF of $X \sim N(0,1)$. Let α denote the lower quartile of the distribution of $Y = -1 + 2X$. Then

$2\alpha =$

Question 3. (4 marks) Let X and Y be two non-degenerate discrete RVs defined on the same probability space $(\Omega, \mathcal{F}, \mathbb{P})$ such that the MGF M_X exists on $\mathbb{R}$, $\mathbb{P}(X \geq 0) = 1$ and $\mathbb{E}Y^2 < \infty$. Which of the following statement(s) is/are necessarily true? Put a tick (✓) beside all correct statement(s) to get credit. No partial marking is applicable.

(a) $\exp(\mathbb{E}X) \leq M_X(1)$.

(b) $\left(\mathbb{E}X^3\right)^2 > \mathbb{E}X^6$

(c) $\mathbb{P}(X \geq \alpha) \leq e^{-\lambda\alpha} M_X(\lambda)$ for all $\alpha > 0, \lambda > 0$.

(d) $F_X(0) = 0$

(e) $\mathbb{E}(X+Y)^2$ may not exist.

Question 4. (1.5 + 1.5 marks) Let $X \sim N(1,4), Y \sim N(3,9), Z \sim N(-2,16)$ be independent RVs and set $W = 2Y - X$. Then $W \sim$. Moreover, $\beta \frac{Z+2}{W-5} \sim t_1$ for $\beta =$

MSO205A Quiz 3 (November 7, 2022)

Duration: 11:00am - 11:40am **Maximum Marks: 15**

Instructions:

1. Write your name and roll number clearly on the designated place. IITK student ID card must be carried in person for verification.
2. You may use books, notebooks, handwritten/photocopied notes of classroom lectures, printouts of supplementary lecture materials and writing instruments during the quiz. Usage of internet or any e-material (including e-books) is prohibited. Electronic communication devices like mobile phones must be switched off and kept in the place designated by the invigilator(s). The invigilators will not be responsible for loss of such a device. If such a device is found on person during the quiz, appropriate action shall be taken. Usage of calculators is not allowed.
3. DO NOT do any rough work on this sheet. If required, do it in your notebook. No additional sheet(s) will be provided and DO NOT attach any additional sheet to this page.
4. Write your answers at the designated places. Any statements written outside the designated place will be taken as rough work and no credit will be provided for such statements. If your answer is not legible, you shall not get credit. You may write your answers as fractions or roots and include e or π, if required.

Name:

Roll No.:

Question 1. (1.5 + 1 + 1.5 + 1 marks) Consider a continuous random vector (X, Y) with the joint p.d.f.

$$f_{X,Y}(x,y) = \begin{cases} \frac{c}{\pi}, & \text{if } x^2 + y^2 < \frac{1}{6}, \\ 0, & \text{otherwise,} \end{cases}$$

where c is a positive constant. Then $c =$,

$\mathbb{E}X =$ and $Cov(X,Y) =$

Are the RVs X and Y independent? Yes/No (underline the correct answer)

Please turn over

Question 2. (3 marks) Given that $\Phi(0.84) = 0.8, \Phi(0.525) = 0.7, \Phi(0.675) = 0.75$, where Φ denotes the DF of $X \sim N(0,1)$. Let α denote the quantile of order 0.3 of the distribution of $Y = -1 + 2X$. Then $\boxed{2\alpha =}$

Question 3. (4 marks) Let X and Y be two non-degenerate discrete RVs defined on the same probability space $(\Omega, \mathcal{F}, \mathbb{P})$ such that the MGF M_X exists on $\mathbb{R}$, $\mathbb{P}(X \geq 0) = 1$ and $\mathbb{E}Y^2 < \infty$. Which of the following statement(s) is/are necessarily true? Put a tick (✓) beside all correct statement(s) to get credit. No partial marking is applicable.

(a) $\exp(\mathbb{E}X) \leq M_X(1)$.

(b) $\left(\mathbb{E}X^3\right)^2 > \mathbb{E}X^6$

(c) $\mathbb{P}(X \geq \alpha) \leq e^{-\lambda\alpha} M_X(\lambda)$ for all $\alpha > 0, \lambda > 0$.

(d) $F_X(0) = 0$

(e) $\mathbb{E}(X+Y)^2$ may not exist.

Question 4. (1.5 + 1.5 marks) Let $X \sim N(1,4), Y \sim N(-2,16)$ be independent RVs. Then, $\boxed{\left(\frac{X-1}{2}\right)^2 + \left(\frac{Y+2}{4}\right)^2 \sim}$ and $\boxed{\frac{1}{2}\frac{Y+2}{X-1} \sim}$