MSO205A PRACTICE PROBLEMS SET 12

Question 1. Let X_1, X_2, X_3 be a random sample from $Bernoulli(p)$ distribution, for some $p \in (0,1)$. Find the p.m.f. of $X_{(2)}$.

Question 2. Let $X_1, \cdots, X_n$ be a random sample from $Uniform(0,1)$ distribution. Identify the distribution of $X_{(r)}$ for $r = 1, \cdots, n$.

Question 3. Let $X_1, \cdots, X_n$ be a random sample from a distribution given by a p.d.f. f. Find the joint p.d.f. of $(X_{(r)}, X_{(s)})$ of $1 \leq r < s \leq n$.

Question 4. Let $Y \sim N_p(b, K)$. Then for any $c \in \mathbb{R}^n$ and a $n \times p$ real matrix B, consider the n dimensional random vector $Z = c + BY$. Show that $Z \sim N_n(c + Bb, BKB^t)$.

Question 5. Let $Y \sim N_p(b, K)$ with K being invertible. Then show that $\sum_{j=1}^{p} \lambda_j (Y_j - b_j) = 0$ for some scalars $\lambda_1, \lambda_2, \cdots, \lambda_p$ if and only if $\lambda_1 = \lambda_2 = \cdots = \lambda_p = 0$.

Question 6. Let $c := \sum_{m=1}^{\infty} m^{-3} < \infty$. Then the function $f : \mathbb{R} \to [0,1]$ given by

$$f(x) = \begin{cases} \frac{1}{c} x^{-3}, \text{ if } x \in \{1, 2, \cdots\} \\ 0, \text{ otherwise} \end{cases}$$

is a p.m.f.. Let X be a discrete RV with this p.m.f. and consider the following sequence of RVs $\{X_n\}_n$ defined by

$$X_n = \begin{cases} X, \text{ if } X \leq n, \\ 0, \text{ otherwise} \end{cases}, \forall n.$$

Show that the sequence of RVs $\{X_n\}_n$ converges in first mean to X, but not in the second mean.