

## MSO205A PRACTICE PROBLEMS SET 6

Question 1. Verify that the following distribution function  $F : \mathbb{R} \rightarrow \mathbb{R}$  corresponds to a discrete RV. Take

$$F(x) := \begin{cases} 0, & \text{if } x < 0, \\ \frac{1}{2}, & \text{if } 0 \leq x < 1, \\ 1, & \text{if } x \geq 1. \end{cases}$$

Compute the p.m.f.  $f_X$ .

Question 2. Verify that the following distribution function  $F : \mathbb{R} \rightarrow \mathbb{R}$  corresponds to a continuous RV. Take

$$F(x) := \begin{cases} 0, & \text{if } x < 0, \\ \frac{x}{2}, & \text{if } 0 \leq x < 1, \\ \frac{1}{2}, & \text{if } 1 \leq x < 2, \\ \frac{x-1}{2}, & \text{if } 2 \leq x < 3, \\ 1, & \text{if } x \geq 3. \end{cases}$$

Compute the p.d.f.  $f_X$ .

Question 3. Let  $X$  be a continuous RV with the p.d.f.

$$f_X(x) := \begin{cases} \exp(-x), & \text{if } x > 0, \\ 0, & \text{otherwise.} \end{cases}$$

Find the support of  $X$ .

Question 4. Consider a discrete RV  $X$  with the p.m.f.

$$f_X(x) := \begin{cases} \frac{1}{4} \left(\frac{3}{4}\right)^x, & \text{if } x \in \{0, 1, 2, \dots\}, \\ 0, & \text{otherwise.} \end{cases}$$

Consider the RV  $Y = \frac{X}{X+1}$ .

- (1) First find the DF  $F_Y$  and then compute the p.m.f.  $f_Y$ .
- (2) First find the p.m.f.  $f_Y$  and then compute the DF  $F_Y$ .

Question 5. Consider a continuous RV  $X$  with the p.d.f.

$$f_X(x) := \begin{cases} \frac{1}{2}, & \text{if } x \in (-1, 0), \\ \frac{1}{3}, & \text{if } x \in (0, \frac{3}{2}), \\ 0, & \text{otherwise.} \end{cases}$$

Consider the RV  $Y = X^4$ .

- (1) First find the DF  $F_Y$  and then compute the p.d.f.  $f_Y$ .
- (2) First find the p.d.f.  $f_Y$  and then compute the DF  $F_Y$ .