MTH 207A: Assignment 3

P 1. Let Y_1, Y_2, Y_3, and Y_4 be four random variables such that $\mathbb{E}(Y_1) = \theta_1 - \theta_3$, $\mathbb{E}(Y_2) = \theta_1 + \theta_2 - \theta_3$, $\mathbb{E}(Y_3) = \theta_1 - \theta_3$, and $\mathbb{E}(Y_4) = \theta_1 - \theta_2 - \theta_3$, where θ_i's are unknown for $i = 1, \ldots, 4$. Then which one is true

1. θ_1 and θ_3 are estimable
2. $\theta_1 + \theta_2$ is estimable
3. $\theta_1 - \theta_3$ is estimable
4. θ_2 are estimable.

P 2. Prove that in a linear regression model $\mathbb{E}(\mathbf{y}) = \mathrm{X}\boldsymbol{\beta}$, if X is of full rank then every $\mathbf{l}'\boldsymbol{\beta}$ is estimable.

P 3. Consider the model $\mathbb{E}(y_1) = 2\beta_1 - \beta_2 - \beta_3$, $\mathbb{E}(y_2) = \beta_2 - \beta_4$, $\mathbb{E}(y_3) = \beta_2 + \beta_3 - 2\beta_4$ with usual assumptions. Describe the estimable functions.

P 4. Consider the model $\mathbb{E}(y_1) = \beta_1 + \beta_2$, $\mathbb{E}(y_2) = \beta_1 - \beta_2$, $\mathbb{E}(y_3) = \beta_1 + 2\beta_2$ with usual assumptions. Obtain the BLUE of $2\beta_1 + \beta_2$ and find its variance.

P 5. Prove that for all the estimable linear functions $\mathbf{l}'\boldsymbol{\beta}$, $\mathbf{l}'\widehat{\boldsymbol{\beta}}$ is invariant to the choice of g–inverse.

P 6. For a full rank regression model, derive the BLUE $\widehat{\boldsymbol{\beta}}$ of $\boldsymbol{\beta}$ using least squares method. Find the expectation and variance of $\widehat{\boldsymbol{\beta}}$.