MTH 207A: Assignment 1
g–inverse

P 1. Let A and B represent $m \times n$ matrices. Then, for any $r \times m$ matrix C of full column rank (i.e., of rank m) and any $n \times p$ matrix D of full row rank (i.e., of rank n) prove that $CAD = CBD$ implies $A = B$.

P 2. Let B represent an $m \times n$ matrix and G an $n \times m$ matrix. Then, for any $r \times m$ full column rank matrix A and $n \times p$ full row rank matrix C, G is a generalized inverse of ABC if and only if $CGA = H$ for some generalized inverse H of B.

P 3. Let k represents a nonzero scalar. Consider the matrices A, B, and C of dimensions $m \times n$, $m \times p$, and $q \times n$ respectively. Let (G_1, G_2) and $\begin{pmatrix} H_1 \\ H_2 \end{pmatrix}$ are the generalized inverses of (A, B) and $\begin{pmatrix} A \\ C \end{pmatrix}$ respectively. Then show that the generalized inverses of (A, kB) and $\begin{pmatrix} A \\ kC \end{pmatrix}$ are $\begin{pmatrix} G_1 \\ k^{-1}G_2 \end{pmatrix}$ and $\begin{pmatrix} H_1 & k^{-1}H_2 \end{pmatrix}$ respectively.

P 4. Let A be of rank r. Choose any $r \times r$ nonsingular submatrix of A such that

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

where A_{11} is $r \times r$ and nonsingular. Show that

$$A_{22} = A_{21} A_{11}^{-1} A_{12}.$$

P 5. Show that, for any $m \times n$ matrix A, $A^+ = A'$ if and only if $A'A$ is idempotent.

P 6. Consider a matrix $A = \mathbf{u}\mathbf{v}' \neq 0$ where $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$. Then show that $B = c^{-1}A'$ is a MP inverse of A where $c = trace(A'A)$.