MTH 207A: Assignment 4

P 1. Consider the one–way ANOVA model

$$y_{ij} = \mu + \alpha_i + \epsilon_{ij},\ i = 1, \ldots, K,\ j = 1, \ldots, n_i \tag{1}$$

where ϵ_{ij} are i.i.d. $N(0, \sigma^2)$.

(a) Find the residual sum of squares (RSS) for the model (1).

(b) Find the distribution of RSS/σ^2.

P 2. Consider the ANOVA model (1). The regression sum of squares SS_{Reg} is defined as

$$SS_{\text{Reg}} = \sum_{i=1}^{K} \sum_{j=1}^{n_i} (\widehat{y}_{ij} - \bar{y}), \tag{2}$$

where $\widehat{y}_{ij} = \widehat{\mu} + \widehat{\alpha}_i$, $\widehat{\mu}$ and $\widehat{\alpha}_i$ are least squares estimators of μ and α_i, $\bar{y} = \sum_{i=1}^{K} n_i \bar{y}_i / \sum_{i=1}^{K} n_i$ and $\bar{y}_i = \sum_{j=1}^{n_i} y_{ij}$ for $i = 1, \ldots, K$. Find the distribution of SS_{Reg}/σ^2.

Remark: The hypothesis

$$H_0 : \alpha_1 = \cdots = \alpha_K \text{ versus } H_1 : \alpha_i \neq \alpha_j \text{ for at least one pair of } (i, j)$$

can tested using the statistics

$$F = \frac{RSS/df_1}{SS_{\text{Reg}}/df_2} \sim F_{df_1, df_2},$$

where df_1 and df_2 are degrees of freedoms associated to the distributions of RSS and SS_{Reg} respectively.

P 3. Consider the model $y_1 = \theta_1 + \theta_2 + \epsilon_1$, $y_2 = 2\theta_1 + \epsilon_2$, $y_3 = \theta_1 - \theta_2 + \epsilon_3$, where ϵ_i, $i = 1, 2, 3$, are i.i.d. $N(0, \sigma^2)$. Derive the F–statistics for testing $\theta_1 = \theta_2$.

R: Assignment

P 5. Three teaching methods, A, B, C, are to be compared. Each method was ad- ministered to a group of 4 students, and the scores obtained by the students on a test are given below. Carry out an F -test at level of significance 0.01 to decide whether the mean scores under the three methods are significantly different.

Table 1: Data.

Method		
A	B	C
75	82	78
79	93	81
71	86	76
69	88	81

P 6. Consider the model $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \epsilon$ with the usual assumptions. Test the following hypotheses: **(a)** $H_0 : \beta_2 = 0$ versus $H_1 : \beta_2 \neq 0$, **(b)** $H_0 : \beta_3 = 0$ versus $H_1 : \beta_3 \neq 0$ using the data given below.

Table 2: Data.

y	x_1	x_2	y	x_1	x_2
10	21	2.67	13	48	7.20
12	32	3.12	21	81	9.12
6	46	2.11	14	93	3.21
14	91	4.21	11	88	4.87
20	20	6.43	18	46	5.38
5	65	1.76	17	24	8.71
8	26	2.88	27	11	8.11
15	74	6.15			