Coding Quiz 2

April 1, 2023

Instructions:

- Drop only your .R file here: https://www.dropbox.com/request/OZbZC9osbgV7WSBLFSz4.

  NO other files should be submitted through this link.

- Accepted format of the file: 111111.R (where '111111' is your roll number).

  Use your full name (e.g., Subhajit Dutta) and IITK email address (e.g., duttas@iitk.ac.in) while submitting your R code.

  If you submit multiple files, I will execute your .R file only.

  Moreover, **grading will be based solely on this file**.

- Output should be printed strictly in the order of the questions given below.
- Total marks: 20
- Time: 30 minutes (6:30pm to 7pm).

  Needless to say, R codes dropped **after 7pm will NOT be graded**.

- **Set the seed to be** 1 **at the beginning of your R code**.
- Only text written below in Red should be printed when the R code is executed.
- Please avoid spamming by NOT uploading incorrect and/or multiple files.

Read the set of instructions given above again before moving to the next page.

1. Consider the following probability density function:

$$f(y;\beta) = \frac{2y}{\beta^2} e^{-\left(\frac{y}{\beta}\right)^2} \text{ for } y > 0.$$

Set the true value (say, β_0) to be 5, and draw n i.i.d. observations $y_1, \ldots, y_n$ from this distribution. Obtain a mathematical expression for the maximum likelihood estimate (MLE) of β.

Write a function using an optimization method available in R to numerically compute the MLE of β.

Now, compute the MLE of β for the sequence of sample sizes 10, 50, 100, 500, 1000 and 5000 using the two methods stated above.

Print the two sequences estimates of β for varying values of n. Interpret the results.

2. Let $y_1, \ldots, y_{100}$ be a random sample from $N(\mu, 1)$. What is the 95% confidence interval for the population mean μ?

Set $\mu_0 = 10$. Let p denote the probability that an additional observation (independent of the random sample generated above) from $N(\mu_0, 1)$ will fall in this 95% confidence interval. Conduct an experiment to compute p with number of repetitions 10, 100, 1000, 10000 and 100000.

(a) Print the estimates of p for increasing number of repetitions (along with their standard errors).

(b) Prepare a plot for the estimates of p (along with their standard errors) against the number of repetitions.

Output marks: $5 + (3 + 2) = 10$

R code marks: $5 + 5 = 10$