

# MTH210 (2024): Quiz 1

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Roll. No. ..... Solutions .....

Show all mathematical details and steps. Marks will not be given unless all work is shown.

**Q:** Consider a discrete target distribution with pmf described by the table below:

$\Pr(X = 0)$	$\Pr(X = 2)$	$\Pr(X = 3)$
1/5	1/5	3/5

Our goal is to sample from the above distribution using Accept-Reject algorithm. Consider two different proposal distributions: Geometric(.5) and Poisson(1). The pmf of Geometric( $p$ ) distribution and Poisson( $\lambda$ ) distribution respectively are

$$q_1(y) = (1-p)^y p \quad y = 0, 1, 2, \dots \quad \text{and} \quad q_2(y) = \frac{\lambda^y e^{-\lambda}}{y!} \quad y = 0, 1, 2, \dots$$

Further, the following R output may be useful (or maybe it won't be)

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> dpois(c(0,1,2,3), lambda = 1)
[1] 0.36787944 0.36787944 0.18393972 0.06131324

> dgeom(c(0,1,2,3), prob = .5)
[1] 0.5000 0.2500 0.1250 0.0625
```

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(6) (a) Which of the two proposals is better and why?

(4) (b) Write down the full AR algorithm using the better proposal. (-1) if notation not defined

(a) Let  $p_x$  denote the pmf of  $X$  at  $x$ :  $p_x = \Pr(X=x)$ . The proposal we choose will be the one with the smaller value of "c"

Geometric:  $\max_{x=0,2,3} \frac{p_x}{q_1(x)} = \max \left\{ \frac{1}{5} \cdot \frac{1}{0.5}, \frac{1}{5} \cdot \frac{1}{0.25}, \frac{3}{5} \cdot \frac{1}{0.0625} \right\}$

$$= \max \left\{ \frac{1}{5} \cdot \frac{1}{0.5}, \frac{1}{5} \cdot \frac{1}{0.25}, \frac{1}{5} \cdot \frac{1}{0.0205} \right\}$$

$$= \frac{1}{5} \cdot \frac{1}{0.0205} \therefore c_1$$

Poisson:  $\max_{x=0,2,3} \frac{p_x}{q_2(x)} = \max \left\{ \frac{1}{5} \cdot \frac{1}{0.3679}, \frac{1}{5} \cdot \frac{1}{0.1839}, \frac{3}{5} \cdot \frac{1}{0.0613} \right\}$

$$= \frac{1}{5} \cdot \frac{1}{0.1839} \therefore c_2$$

(-1) or (-2) if calculation mistake.

Since  $c_1 < c_2$ , we will choose Geometric( $\frac{1}{2}$ ) as our proposal as a smaller value of "c" means a more computationally efficient algorithm

(-5) if both are not calculated

(-3) if no reason is given

(b) Below is the AR algorithm using Geometric(0.5) proposal:

- ① Draw  $U \sim U(0,1)$   
(-1) if independently is not mentioned
- ② Independently, draw  $Y=y \sim \text{Geometric}\left(\frac{1}{2}\right)$  (-1) if proposal not specified.
- ③ If  $U \leq \frac{p_y}{c_1 \left(\frac{1}{2}\right)^y \frac{1}{2}}$ , return  $X=y$ .  
→ (-1) if not solved.
- ④ Else go to ①.  
(-2) or (-3) if algorithm is incorrect