Home Work 3 (2023)
MTH 212
Elementary Stochastic Process

1. Prove that in a finite state Markov Chain all the states can not be transient.

2. Consider the following transition probability matrix

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ p & 0 & 0 & 1-p \\ 0 & 1-p & p & 0 \end{bmatrix}.$$

   If $0 < p < 1$, find $\mathbf{P}^n$ and also $\lim_{n\to\infty} \mathbf{P}^n$. How many equivalent classes are there? What are the recurrent states and what are the transient states?

3. Consider the following 4×4, transition probability matrix

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ p & 0 & 1-p & 0 \\ 0 & p & 0 & 1-p \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

   If $0 < p < 1$, find $\mathbf{P}^n$ and also $\lim_{n\to\infty} \mathbf{P}^n$. How many equivalent classes are there? What are the recurrent states and what are the transient states?

4. Consider a two persons zero sum game with the transition probability matrix of Player 1, is as given in the previous question. Let us assume that the game stops as soon as somebody hits 0. Show that the game stops with probability one. Find the expected duration of the game. Find the expected duration of the game when Player 1 hits 0.

5. Find the transition probability matrix $\mathbf{P}$ of the following Markov Chain. Consider a sequence of tosses of a coin with the probability of head p. At time n, after n tosses of the coin, the state of the process is the number of heads in the n tosses minus the number of tails. Find the equivalent classes. Are they recurrent or transient? Find the period of each state.

6. Consider an urn with a total of D tokens; some numbered +1, and some -1. The composition of the urn changes over time as follows. At each turn a token is picked at random, and the sign is changed and put back in the urn. Let X_n denote the number of +1 sign at time n. Show that X_n is a Markov chain, and find the transition probability matrix. Find the equivalent classes. Are they recurrent or transient? Find the period of each state.

7. Show that for a finite transition probability matrix one of the eigenvalue is 1. Find the corresponding eigenvector.