

QUIZ-2B
ELEMENTARY STOCHASTIC PROCESS (MTH-212A)

Name (Roll Number)

Time: 20 mins.

Maximum Marks: 15

Minimum Marks: 0

Instructions: Both the questions are of multiple choice. More than one answers might be correct in both the questions. Each correct answer will give you one point and if you tick a wrong answer it will be negative 1. In Question 1, if you can identify all the correct answers without ticking any wrong answer, you will get eight points, and similarly in Question 2, you will get 7 points. No extra sheet will be provided.

1. Suppose X_1, X_2, \dots are independent and identically distributed random variables with $P(X_1 = 0) = P(X_1 = 1) = \frac{1}{2}$. If $Y_n = \max\{X_1, \dots, X_n\}$, and let us denote \mathbf{P} as the transition probability matrix of $\{Y_n\}$. Then which of the statements are correct.

1. $\lim_{n \rightarrow \infty} p_{01}^n = 1$
2. $\lim_{n \rightarrow \infty} p_{11}^n = 1$
3. The period of state $\{0\}$ is 1.
4. The period of state $\{1\}$ is 1.
5. The state $\{1\}$ is reachable from state $\{0\}$.

The correct answers are: [1],[2],[3],[4],[5]

Solutions of 1: $\mathbf{P} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 0 & 1 \end{bmatrix}$, $\mathbf{P}^n = \begin{bmatrix} \frac{1}{2^n} & 1 - \frac{1}{2^n} \\ 0 & 1 \end{bmatrix}$. $\{1\}$ and $\{0\}$ do not communicate with each other, $d(0) = d(1) = 1$.

2. Let $\{X_n\}$ be a Markov Chain with the following transition probability matrix

$$\mathbf{P} = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

Then which of the statements are correct.

1. \mathbf{P}^n depends on n .
2. \mathbf{P}^n does not depend on n .
3. There are two equivalent classes.
4. All the states communicate with each other.
5. The periods of all the states are same.

The correct answers are: [2],[3],[5]

Solution: $\mathbf{P}^n = \mathbf{P}$. If we denote the states as $\{1, 2, 3, 4\}$, then there are two equivalent classes $\{1, 3\}$ and $\{2, 4\}$. All the states have period 1.