HOME WORK 5 MTH 212 ELEMENTARY STOCHASTIC PROCESS

- 1. Consider an urn with a total of D tokens; some numbered +1, and some -1. The composition of the urn changes over time as follows. At each turn a token is picked at random, and the sign is changed and put back in the urn. Let X_n denote the number of +1 sign at time n. Show that X_n is a Markov chain, and find the transition probability matrix. Is this an irreducible Markov chain?
- 2. Prove that for a finite Markov chain all states cannot be null recurrent.
- 3. Is it possible that in an infinite Markov Chain all states are null recurrent?
- 4. Consider the following transition probability matrix with the state space $S = \{0, 1, 2, 3, 4\}$

$$\begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
\frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\
0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\
0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}$$

Find $\lim_{n\to\infty} p_{ij}^{(n)}$, for all $i,j\in S$.

5. Consider the following transition probability matrix with the state space $S = \{0, 1, 2, 3, 4, \dots, N\}$

$$\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & \dots & 0 \\
\frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 & & \\
0 & \frac{1}{3} & 0 & \frac{2}{3} & 0 & \dots & 0 \\
0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} & \dots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & \dots & 0 & \frac{1}{3} & 0 & \frac{2}{3} \\
0 & 0 & 0 & 0 & 0 & \dots & 1
\end{bmatrix}$$

Find $\lim_{n\to\infty} p_{ij}^{(n)}$, for all $i, j \in S$.

6. Consider the following transition probability matrix with the state space $S = \{1, 2, 3, 4\}$

$$\left[\begin{array}{ccccc}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\
\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4}
\end{array}\right]$$

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Find $\lim_{n\to\infty} p_{ij}^{(n)}$, for all $i, j \in S$.

7. Consider the following transition probability matrix

$$\mathbf{P} = \begin{bmatrix} 1/2 & 1/2 & 0 & 0 & 0 & 0 & \dots \\ 1/2 & 0 & 1/2 & 0 & 0 & 0 & \dots \\ 1/2 & 0 & 0 & 1/2 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}.$$

Prove that the Markov Chain is an irreducible aperiodic recurrent Markov Chain. Find $\lim p_{00}^{(n)}$.

8. Consider the following transition probability matrix

$$\mathbf{P} = \begin{bmatrix} 2/3 & 1/3 & 0 & 0 & 0 & 0 & \dots \\ 2/3 & 0 & 1/3 & 0 & 0 & 0 & \dots \\ 2/3 & 0 & 0 & 1/3 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}.$$

Prove that the Markov Chain is an irreducible aperiodic recurrent Markov Chain. Find $\lim p_{00}^{(n)}$.

9. Consider the following transition probability matrix

$$\mathbf{P} = \begin{bmatrix} p & 1-p & 0 & 0 & 0 & 0 & \dots \\ p & 0 & 1-p & 0 & 0 & 0 & \dots \\ p & 0 & 0 & 1-p & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}.$$

Here 0 . Find the values of <math>p such that the Markov Chain is an irreducible aperiodic recurrent Markov Chain. Find $\lim p_{00}^{(n)}$.

10. Consider the following transition probability matrix

$$\mathbf{P} = \begin{bmatrix} 1 - p & p & 0 & 0 & 0 & 0 & \dots \\ 1 - p^2 & 0 & p^2 & 0 & 0 & 0 & \dots \\ 1 - p^3 & 0 & 0 & p^3 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}.$$

Here 0 . Find the values of <math>p such that the Markov Chain is an irreducible aperiodic recurrent Markov Chain. Find $\lim p_{00}^{(n)}$.

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