MTH 441: Lab 10: Logistic Regression

P. Consider the The Pneumoconiosis data concerning the proportion of coal miners who exhibit symptoms of severe pneumoconiosis and the number of years of exposure.

Table 1: The Pneumoconiosis Data.

Years of Exposure	Severe Cases	Total Miners	Proportions of Severe Cases
5.8	0	98	0
15.0	1	54	0.0185
21.5	3	43	0.0698
27.5	8	48	0.1667
33.5	9	51	0.1765
39.5	8	38	0.2105
46.0	10	28	0.3571
51.5	5	11	0.4545

- (1) Fit a simple logistic regression model and obtain the MLE of the intercept β_0 and the coefficient of Severe Cases β_1 .
- (ii) Find the estimated variance $\widehat{Var}(\widehat{\boldsymbol{\beta}})$ of $\widehat{\boldsymbol{\beta}}$ where $\boldsymbol{\beta} = (\widehat{\beta}_0, \widehat{\beta}_1)^{\mathrm{T}}$ is the vector of MLE of $\boldsymbol{\beta} = (\beta_0, \beta_1)^{\mathrm{T}}$.
- (iii) Find the odds of contracting a severe case of pneumoconiosis for every additional year of exposure.
- (iv) Test the following hypothesis

$$H_0: \beta_1 = 0$$
 versus $H_1: \beta_1 \neq 0$.