Q3 b

Introduction

In this analysis, we fit a moving average model (MA(1)) to the differenced logarithm of the glacial varve series data. This analysis aims to show that the differenced series of `Y_t` follows an MA(1) process. We will apply the model to glacial varve data, available in the `astsa` package as `varve`, to assess its fit and interpret the results.

Mathematical Derivation

Model Setup

Given a univariate state-space model:

$$Y_t = X_t + V_t$$

$$X_t = X_{t-1} + W_t$$

where $W_t \sim \mathcal{N}(0, \sigma_W^2)$ and $V_t \sim \mathcal{N}(0, \sigma_V^2)$ are Gaussian white noise processes, and both are independent.

Differencing the Observed Series

The observed data Y_t can be transformed into a stationary series by differencing:

$$\nabla Y_t = Y_t - Y_{t-1} = (X_t + V_t) - (X_{t-1} + V_{t-1})$$

Expanding, we get:

$$\nabla Y_t = (X_t - X_{t-1}) + (V_t - V_{t-1}) = W_t + \nabla V_t$$

Since W_t is white noise and $\nabla V_t = V_t - V_{t-1}$ is a first-order moving average process, ∇Y_t follows an MA(1) model:

$$\nabla Y_t = W_t + \theta V_{t-1}$$

Interpretation of Parameters

The MA(1) model is defined by:

$$\nabla Y_t = W_t + \theta W_{t-1}$$

where: - θ is the MA(1) coefficient. - W_t is white noise with variance σ_W^2.

Our goal is to fit this MA(1) model to the differenced series and interpret the results.

Data Preparation

We start by loading and transforming the glacial varve series data.

```r
data(varve, package = "astsa")
log_varve <- log(varve)
diff_log_varve <- diff(log_varve)
```

Model Fitting

We fit an MA(1) model to the differenced log-transformed series.

```r
# Fit an MA(1) model to the differenced series
model <- Arima(diff_log_varve, order = c(0, 0, 1))
summary(model)
```

```
## Series: diff_log_varve
## ARIMA(0,0,1) with non-zero mean
##
## Coefficients:
##           ma1     mean
##       -0.7710  -0.0013
## s.e.   0.0341   0.0044
##
## sigma^2 = 0.236:  log likelihood = -440.68
## AIC=887.36   AICc=887.39   BIC=900.71
##
## Training set error measures:
##                       ME      RMSE       MAE  MPE MAPE      MASE      ACF1
## Training set 0.0004021077 0.4850623 0.3823767 -Inf  Inf 0.4963925 0.1200001
```

Observations

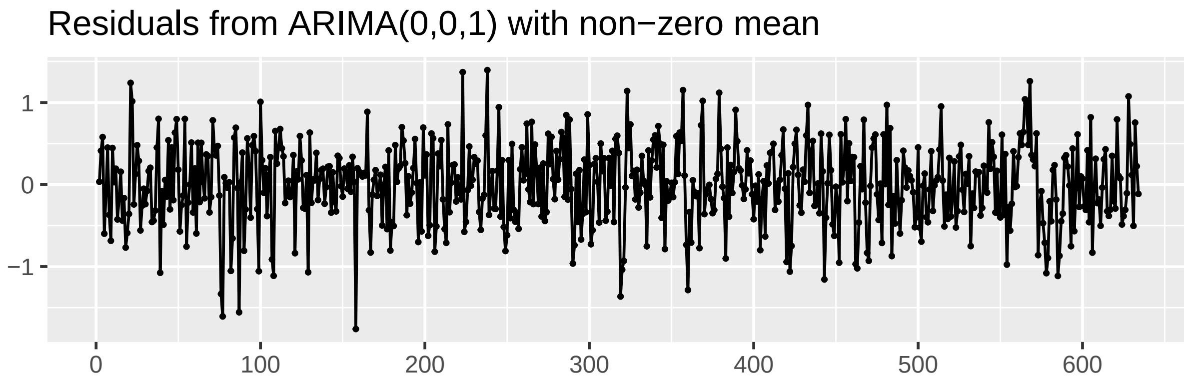
From the model output, we obtain: - The estimated MA(1) coefficient θ. - The variance σ^2 of the residuals. - Information criteria (AIC, BIC) that assess the model fit.

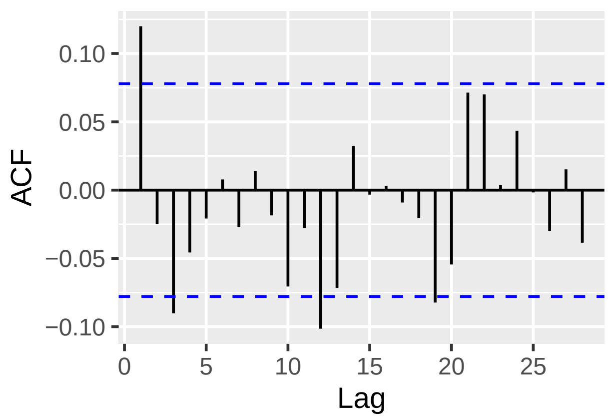
These values provide insights into how well an MA(1) process fits the differenced series.

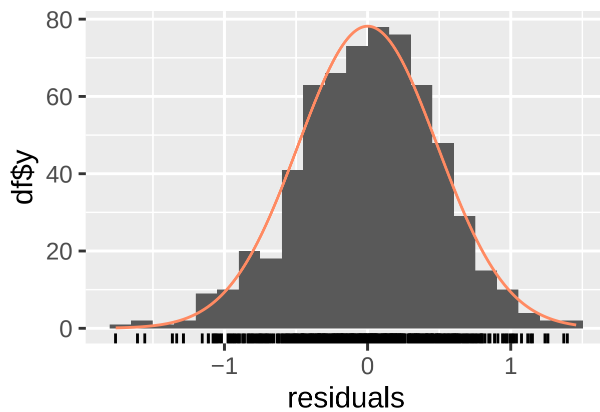
Diagnostic Checks

After fitting the model, we examine the residuals to ensure they resemble white noise, as required by the MA(1) assumption.

```r
# Plot residuals and perform diagnostic checks
checkresiduals(model)
```








```
##
##  Ljung-Box test
##
## data:  Residuals from ARIMA(0,0,1) with non-zero mean
## Q* = 20.441, df = 9, p-value = 0.01538
##
## Model df: 1.   Total lags used: 10
```

Interpretation of Diagnostics

The diagnostic plot includes:

1. **Residual Time Series Plot**: Checks if residuals appear randomly distributed around zero.
2. **ACF Plot of Residuals**: Residual autocorrelations should lie within the confidence bounds, indicating no significant correlation.
3. **Residual Density Plot**: Ensures normality of residuals.
4. **Ljung-Box Test**: A formal test for residual autocorrelation. A high p-value supports the null hypothesis of no autocorrelation.

If these diagnostics are satisfactory, it suggests the MA(1) model is appropriate for the data.

Summary of Findings

The fitted MA(1) model and diagnostics indicate:

- **Model Fit**: The MA(1) model seems appropriate for the differenced log-transformed varve series.
- **Coefficient Interpretation**: The estimated MA(1) parameter indicates the extent of correlation between Y_t and Y_{t-1} after differencing.
- **Residual Analysis**: Diagnostic checks confirm that residuals approximate white noise, validating the model's suitability.

In conclusion, an MA(1) model provides a reasonable fit to the differenced series, and this model can help understand the underlying process in the glacial varve data. "'