Lecture 10

Exploratory data analysis Part 1

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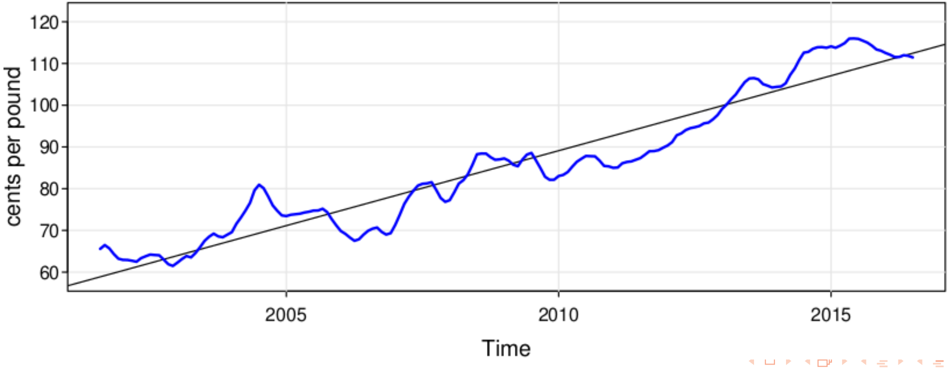
Stationary Time Series (Recap)

- A **weakly stationary** time series is one for which
  - variance of the process is finite at each time point,
  - the mean value function μ_t is constant and does not depend on t,
  - the autocovariance function, $\gamma(s, t)$ depends on s and t only through their difference $|s - t|$.

Too strong as an assumption! (Recap)

- Consider the monthly price (per pound) of a chicken in the US from mid-2001 to mid-2016 (180 months).
- We might fit the model

$$X_t = \beta_0 + \beta_1 z_t + W_t, \quad z_t = 2001\frac{7}{12}, 2001\frac{8}{12}, \ldots, 2016\frac{6}{12}, \; W_t \overset{IID}{\sim} \mathrm{Normal}(0, \sigma_W^2)$$



Trend stationary models (Recap)

- Suppose the observed time series is a realization of the process

$$X_t = \beta_0 + \beta_1 t + Y_t, \;\; t = 1, \dots$$

  where Y_t is stationary.

- In general, suppose $X_t = \mu_t + Y_t, \;\; t = 1, \dots$ where μ_t is a deterministic trend.

- If $\mu_Y = E[Y_t]$, $\mu_{X,t} = E[X_t] = ?$

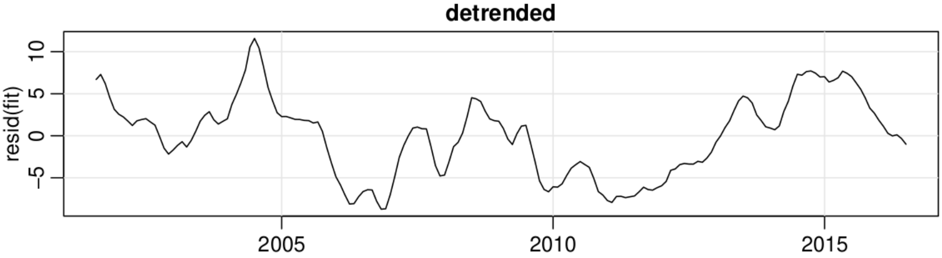
- If $\sigma_Y^2 = \mathrm{Var}(Y_t)$, then $\sigma_{X,t}^2 = \mathrm{Var}[X_t] = ?$

- If $\gamma_Y(h) = \mathrm{Cov}(Y_{t+h}, Y_t)$, then $\gamma_X(h) = ?$

Detrending

- We may write trend stationary type of model as $X_t = \mu_t + Y_t$, where X_t are the observations, μ_t denotes the trend, and Y_t is a stationary process.
- Strong trend will obscure the behavior of the stationary process Y_t.
- Hence, there is some advantage to removing the trend as a first step in an exploratory analysis of such time series.
- The steps involved are to obtain a reasonable estimate of the trend component, say $\hat{\mu}_t$, and then work with the residuals $\hat{Y}_t = X_t - \hat{\mu}_t$.
- In the chicken price example, we estimated the trend using ordinary least squares and found $\hat{\mu}_t = -7131 + 3.59t$ and thus $\hat{Y}_t = X_t + 7131 - 3.59t$.

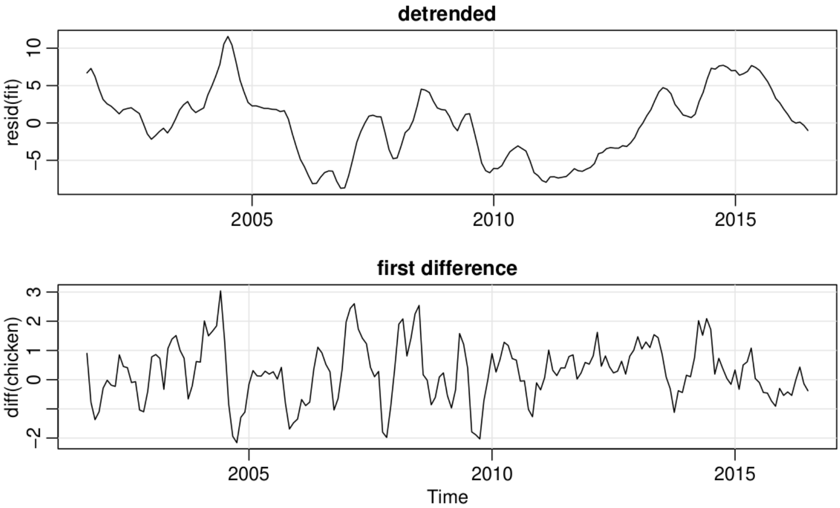
Detrended data



Differencing

- Rather than modeling trend as fixed, we might model trend as a stochastic component as
$$\mu_t = \delta + \mu_{t-1} + W_t.$$
- Then
$$X_t - X_{t-1} = (\mu_t + Y_t) - (\mu_{t-1} + Y_{t-1}) = \delta + W_t + Y_t - Y_{t-1}.$$
- Because Y_t is stationary, $Y_t - Y_{t-1}$ is stationary. Because W_t is white noise, $X_t - X_{t-1}$ is also stationary.
- If $\mu_t = \beta_0 + \beta_1 t$, the differencing leads to a stationary process
$$X_t - X_{t-1} = (\mu_t + Y_t) - (\mu_{t-1} + Y_{t-1}) = \beta_1 + Y_t - Y_{t-1}$$
- Because differencing plays a central role in time series analysis, it receives its own notation
$$\nabla X_t = X_t - X_{t-1}$$

Detrended and differenced chicken price series



ACFs of original, detrended, and differenced chicken price series

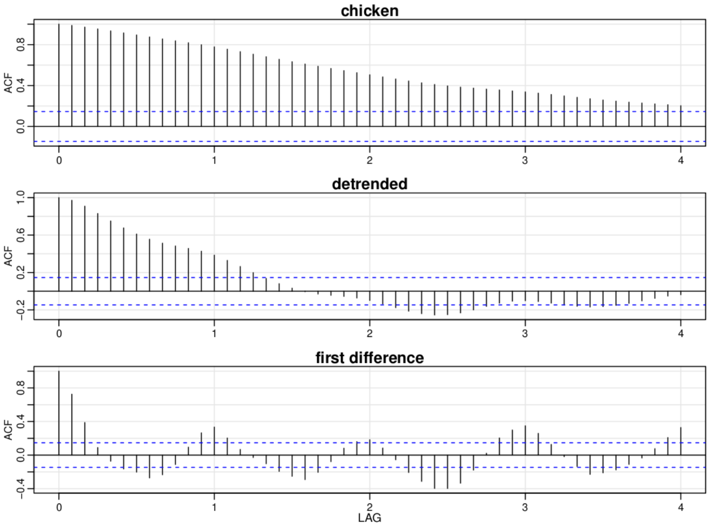


Fig. 2.5. *Sample ACFs of chicken prices (top), and of the detrended (middle) and the differenced (bottom) series. Compare the top plot with the sample ACF of a straight line:* `acf(1:100)`.

Differenced global temperature data and their ACF

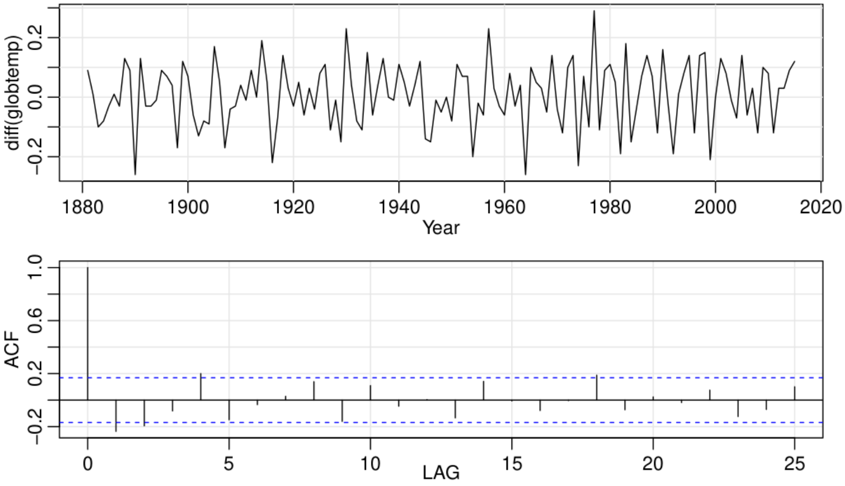


Fig. 2.6. *Differenced global temperature series and its sample ACF.*

Back-shift and forward-shift operator

- We define the backshift operator by $BX_t = X_{t-1}$ and extend it to powers $B^2X_t = B(BX_t) = BX_{t-1} = X_{t-2}$, and so on. Thus, $B^kX_t = X_{t-k}$.

- We define the forward-shift operator by if we require $B^{-1}B = 1$.

- Thus, $X_t = B^{-1}BX_t = B^{-1}X_{t-1}$.

- We can rewrite the differences by $\nabla X_t = X_t - X_{t-1} = X_t - BX_t = (1 - B)X_t$.

Higher order differences

- The difference of the first difference is

$$\nabla(\nabla X_t) = \nabla(X_t - X_{t-1}) = (X_t - X_{t-1}) - (X_{t-1} - X_{t-2}) = X_t - 2X_{t-1} + X_{t-2}.$$

- Using the back-shift operator, the second difference can also be written as

$$\nabla^2 X_t = (1 - B)^2 X_t = (1 - 2B + B^2) X_t = X_t - 2X_{t-1} + X_{t-2}.$$

- Differences of order d are defined as

$$\nabla^d = (1 - B)^d$$

  where we may expand the operator $(1 - B)^d$ algebraically to evaluate for higher integer values of d.

- When $d = 1$, we drop it from the notation.

Thank you!