Theoretical ACFs of Time Series Models

Jiyanshu Dhaka

2024-09-27

Question 6: Theoretical ACFs of AR(1), MA(1), and ARMA(1,1)

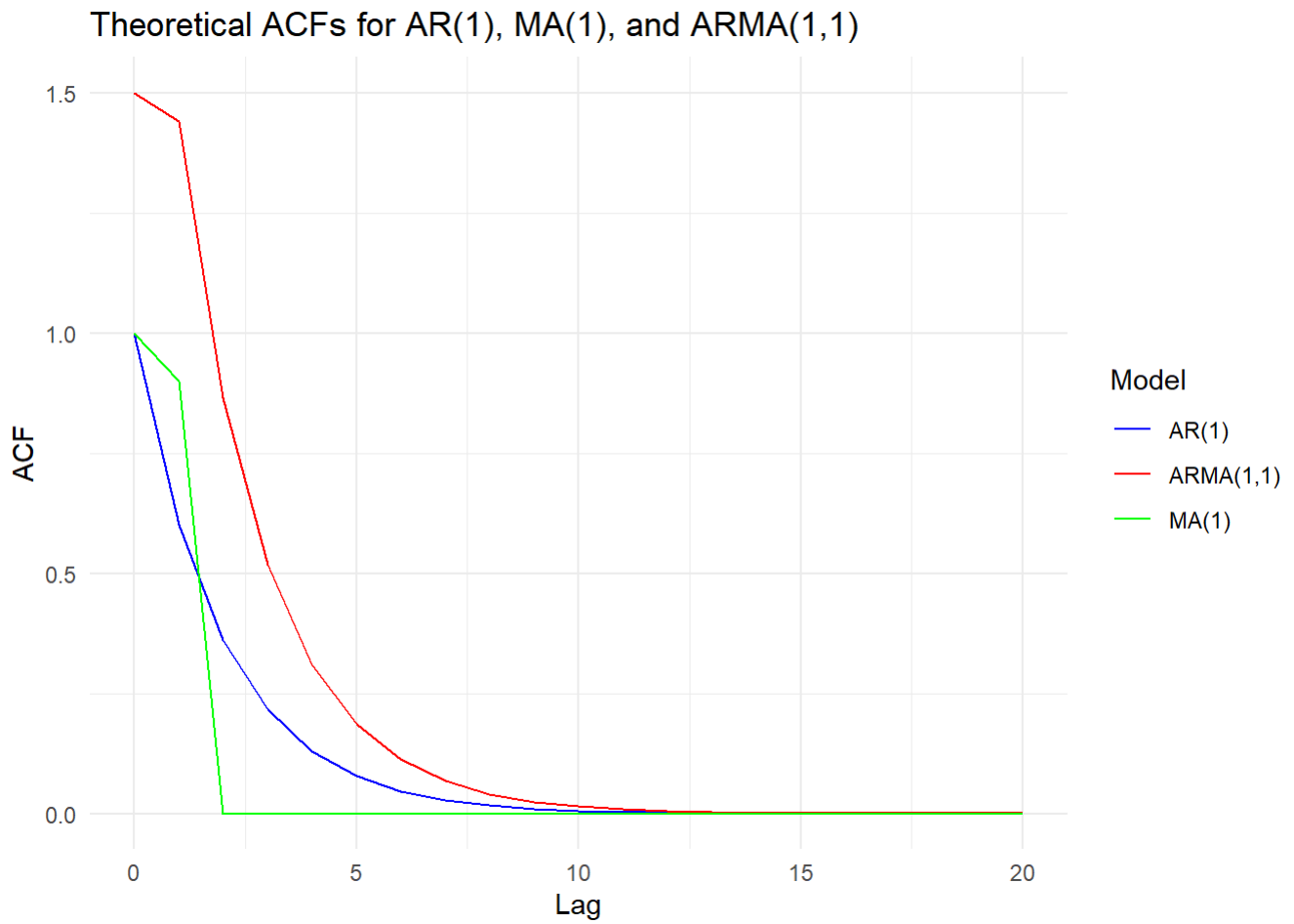
We will plot the theoretical ACFs for three different models: 1. AR(1) = ARMA(1,0): $X_t = 0.6X_{t-1} + W_t$ 2. MA(1) = ARMA(0,1): $X_t = W_t + 0.9W_{t-1}$ 3. ARMA(1,1): $X_t = 0.6X_{t-1} + W_t + 0.9W_{t-1}$

```
# Define lags
lags <- 0:20

# Theoretical ACFs
acf_ar1 <- 0.6^lags
acf_ma1 <- c(1, 0.9, rep(0, 19))
acf_arma11 <- (0.6 + 0.9) * 0.6^lags + 0.9 * c(0, 0.6^(lags[-1]))

# Data frame for plotting
df_acf <- data.frame(lag = rep(lags, 3),
                     acf = c(acf_ar1, acf_ma1, acf_arma11),
                     Model = rep(c("AR(1)", "MA(1)", "ARMA(1,1)"), each = length(lags)))

# Plotting
ggplot(df_acf, aes(x = lag, y = acf, color = Model)) +
  geom_line() +
  labs(title = "Theoretical ACFs for AR(1), MA(1), and ARMA(1,1)",
       x = "Lag",
       y = "ACF") +
  scale_color_manual(values = c("blue", "red", "green")) +
  theme_minimal()
```



Theoretical ACFs of AR(1), MA(1), and ARMA(1,1)

Commentary on Diagnostic Capabilities

The ACF plot illustrates distinct decay patterns and behavior for each model: - **AR(1)**: Exhibits exponential decay in the ACF, which does not cut off, indicating a single autoregressive term. - **MA(1)**: Shows a sharp cut-off after the first lag, typical for a single moving average term. - **ARMA(1,1)**: Combines features of both AR(1) and MA(1) models, showing a more complex decay pattern which does not cut off immediately but dampens more quickly than a pure AR model.

These differences in the ACF can be diagnostically useful in determining the underlying model type when analyzing real time series data, highlighting whether autoregressive or moving average components (or both) are present.