Indian Institute of Technology Kanpur

Department of Mathematics and Statistics

Time Series Analysis (MTH442)

Assignment 4, Due date: October 30, 2024, Wednesday

Answers should be provided neatly. In case the handwriting is unreadable, the instructor and the teaching assistants hold the right to give a zero score. In case of cheating, all students involved will get zero, irrespective of who copied from whom. Write on a paper using some dark ink and scan using a good scanner to obtain a clearly readable PDF file. For the coding related question, use R markdown and generate the PDF output with echo = TRUE mode (so that the codes are also visible along with the outputs). Finally, join the two PDFs and submit the single PDF file only. The final file should be named as RollNo_Lastname_Firstname.pdf. If the file is not submitted in this nomenclature format, marks will be zero. No request should be made in that case.

1. Verify that the IMA(1,1) model $X_t = X_{t-1} + W_t - \lambda W_{t-1}$ with $|\lambda| < 1$ and $X_0 = 0$ can be inverted and written as $X_t = \sum_{j=0}^{\infty}(1-\lambda)\lambda^{j-1}X_{t-j} + W_t$ as an approximation for large t. Here assume that $X_t = 0$ for $t < 0$. (1 point)

2. For the ARIMA(1, 1, 0) model with drift, $(1 - \phi B)(1 - B)X_t = \delta + W_t$, let $Y_t = \nabla X_t$.
(a) Noting that Y_t is AR(1), show that, for $j \geq 1$,
$$Y^T_{T+j} = \delta[1 + \phi + \ldots + \phi^{j-1}] + \phi^j Y_T.$$
(b) Use part (a) to show that, for $m = 1, 2, \ldots,$
$$X^T_{T+m} = X_T + \frac{\delta}{1-\phi}\left[m - \frac{\phi(1-\phi^m)}{1-\phi}\right] + (X_T - X_{T-1})\frac{\phi(1-\phi^m)}{1-\phi}.$$
(c) For large T, the mean-squared prediction error can be approximated (in some cases exact, like here) by $P^T_{T+m} = \sigma^2_W \sum_{j=0}^{m-1} \psi_j^{*2}$ where ψ^*_j is the coefficient of z^j in $\psi^*(z) = \frac{\theta(z)}{\phi(z)(1-z)^d}$. Use this to find P^T_{T+m} by first showing that $\psi^*_0 = 1$, $\psi^*_1 = (1+\phi)$, and $\psi^*_j - (1+\phi)\psi^*_{j-1} + \phi\psi^*_{j-2} = 0$ for $j \geq 2$, in which case $\psi^*_j = (1-\phi)^{-1}(1-\phi^{j+1})$ for $j \geq 1$. (1+1+1=3 points)

3. We discussed quarterly United States Gross National Product (GNP) data analysis in the lab. We said that AR(1) and MA(2) are two possible models for the differenced log GNP data. We discussed the model diagnostics for MA(2) model but also pointed out that AR(1) is preferable. Show the model diagnostics for AR(1) model. Repeat the diagnostics for ARMA(1, 2) model and compare the results. You should properly report your findings using English sentences. Just running the codes and reporting the `R` outputs is not enough. (1+1=2 points)

4. Fit a seasonal ARIMA model of your choice to the unemployment data in `unemp` from the `R` package `astsa`. Use the estimated model to forecast the next 12 months. You should properly report your findings using English sentences. Just running the codes and reporting the `R` outputs is not enough. (1+1=2 points)

5. Fit an appropriate seasonal ARIMA model to the log-transformed Johnson and Johnson earnings series (`jj` from the `R` package `astsa`) discussed in Lecture 2. Use the estimated model to forecast the next 4 quarters. You should properly report your findings using English sentences. Just running the codes and reporting the `R` outputs is not enough. (1+1 = 2 points)