# Lab session 2

## Sampling different time series using R

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# Question 1

- Draw a realization of length $T = 100$ from a time series $W_t \overset{IID}{\sim} \mathrm{Normal}(0, 1)$ using `R`.

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```
rnorm(n=100, mean = 0, sd = 1) # Option 1

rnorm(100) # Option 2
```

# Question 2

- Draw a realization of length $T = 100$ from a time series $W_t \stackrel{IID}{\sim} \mathrm{Normal}(0, 10^2)$ using `R`.

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```
rnorm(n=100, mean = 0, sd = 10) # Option 1

10 * rnorm(100) # Option 2
```

# Question 3

- Draw a realization of length $T = 100$ from a trivariate time series $\boldsymbol{W}_t \overset{IID}{\sim} \mathrm{MVN}(\mathbf{0}, \boldsymbol{\Sigma})$ using R. Here $\boldsymbol{\Sigma} = \begin{pmatrix} 1 & 0.75 & 0.75 \\ 0.75 & 1 & 0.75 \\ 0.75 & 0.75 & 1 \end{pmatrix}$.

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```
library(mvtnorm)
Sigma <- matrix(0.75, 3, 3) + 0.25 * diag(3)
rmvnorm(n = 100, mean = rep(0, 3), sigma = Sigma)
```

# Question 4

- Let us consider the moving average process obtained with replacing $W_t$ by an average of its current value and its immediate neighbors in the past and future, i.e.,

$$V_t = \frac{1}{3}[W_{t-1} + W_t + W_{t+1}].$$

  Draw a realization of length $T = 100$ from the time series $V_t, t = 1, 2, \ldots$.

# Question 4

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Draw a realization of length $T = 100$ from the time series $V_t, t = 1, 2, \ldots$.

```
# Option 1
W <- rnorm(n=100, mean = 0, sd = 1)
V <- filter(W, sides = 2, filter = rep(1 / 3, 3))

# Option 2
V <- c(NA, W[-(1:2)] + W[-c(1, 100)] + W[-(99:100)], NA) / 3
```

# Question 5

- Let us consider the moving average process obtained with replacing $W_t$ by an average of its current value and its two immediate neighbors in the past and one immediate neighbor in the future, i.e.,

$$V_t = \frac{1}{4}[W_{t-2} + W_{t-1} + W_t + W_{t+1}].$$

Draw a realization of length $T = 100$ from the time series $V_t, t = 1, 2, \ldots$.

# Question 5

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Draw a realization of length $T = 100$ from the time series $V_t, t = 1, 2, \ldots$.

```
V <- c(NA, NA, W[-(1:3)] + W[-c(1:2, 100)] +
        W[-c(1, 99:100)] + W[-(98:100)], NA) / 4

# WRONG SOLUTION
filter(W, sides = 2, filter = c(1, 1, 1, 1, 0) / 4)
```

# Question 6

- Suppose we consider $W_t$'s as input and calculate the output using the second-order equation

$$X_t = X_{t-1} - 0.9X_{t-2} + W_t$$

successively for $t = 1, 2, \ldots, 100$. Assume $X_{-1} = 0, X_0 = 0$. Draw a realization of length $T = 100$ from the time series $X_t, t = 1, 2, \ldots$.

# Question 6

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successively for $t = 1, 2, \ldots, 100$. Assume $X_{-1} = 0, X_0 = 0$. Draw a realization of length $T = 100$ from the time series $X_t, t = 1, 2, \ldots$.

```
filter(W, filter = c(1, -.9), method = "recursive") # Option 1

X.minus1 <- 0
X0 <- 0
X <- rep(NA, 100)
X[1] <- X0 - 0.9 * X.minus1 + W[1]
X[2] <- X[1] - 0.9 * X0 + W[2]
for(i in 3:100){X[i] <- X[i-1] - 0.9 * X[i-2] + W[i]} # Option 2
```

# Question 7

- A model for analyzing trend such as seen in the global temperature data is the random walk with drift model given by

$$X_t = \delta + X_{t-1} + W_t$$

for $t = 1, 2, \ldots$, with initial condition $X_0 = 0$, and where $W_t$ is white noise. Draw a realization of length $T = 100$ from the time series $X_t, t = 1, 2, \ldots$.

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```
X <- cumsum(W)
delta <- 0.2
W.delta <- w + delta
X.delta <- cumsum(W.delta)
```

# Question 8

- Consider the model
$$X_t = 2\cos\left(2\pi\frac{t+15}{50}\right) + W_t$$
for $t = 1, 2, \ldots$. Draw a realization of length $T = 100$ from the time series $X_t, t = 1, 2, \ldots$ where $W_t$ are white Gaussian noise with standard deviation 1 and 5 respectively.

# Question 8

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for $t = 1, 2, \ldots$. Draw a realization of length $T = 100$ from the time series $X_t, t = 1, 2, \ldots$ where $W_t$ are white Gaussian noise with standard deviation 1 and 5 respectively.

```
cs <- 2 * cos(2 * pi * 1:100 / 50 + 0.6 * pi)
X <- cs + rnorm(100)
X <- cs + 5 * rnorm(100)
```

Thank you!