Roll No: 12143

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MTH517/517A: Time Series Analysis
Quiz #2; Full Marks-20

(1) Consider the following bi-variate time series process $\underset{\sim}{Z}_t = (X_t, Y_t)^T$, where

$$X_t = 0.5\varepsilon_t + 0.5$$
$$Y_t = 0.5\varepsilon_{t-1} + 0.5\varepsilon_{t-2}$$

$\varepsilon_t \sim WN(0, \sigma^2)$.

(a) Find $Cov(\underset{\sim}{Z}_t, \underset{\sim}{Z}_{t+2})$.

(b) Verify whether $\underset{\sim}{Z}_t$ is covariance stationary or not.

(c) Find k, if any, such that $Cov(\underset{\sim}{Z}_t, \underset{\sim}{Z}_{t+h})$ is a null matrix $\forall |h| \geq k$.

(d) Show that $\underset{\sim}{Z}_t$ can be written as $\underset{\sim}{Z}_t = \underset{\sim}{\mu} + \Theta_1 \underset{\sim}{\eta}_t + \Theta_2 \underset{\sim}{\eta}_{t-1}$; $\underset{\sim}{\eta}_t = (\varepsilon_t, \varepsilon_{t-1})'$ and $\underset{\sim}{\mu}$, Θ_1, Θ_2 are non-random vector and matrices of appropriate orders.

(e) Using (d) above, or otherwise, verify whether $\underset{\sim}{Z}_t$ is a vector MA process.

(2) Consider the 2-variate Vector ARMA(1,1) process $\underset{\sim}{X}_t = \underset{\sim}{\varepsilon}_t - \frac{1}{4}\underset{\sim}{X}_{t-1} + \underset{\sim}{\varepsilon}_{t-1}$; $\underset{\sim}{\varepsilon}_t \sim VWN(\underset{\sim}{0}, \Sigma)$, $\Sigma > 0$. Prove or disprove

(a) $\{X_t\}$ is covariance stationary, (b) $\{X_t\}$ is invertible, (c) $\{X_t\}$ is causal.

Q.1 $\quad \underset{\sim}{Z}_t = \begin{pmatrix} X_t \\ Y_t \end{pmatrix} \qquad \varepsilon_t \sim WN(0, \sigma^2)$

$cov(\underset{\sim}{Z}_t, \underset{\sim}{Z}_{t+2}) = cov((X_t, Y_t)^T, (X_{t+2}, Y_{t+2})^T)$