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MTH517/517A: Time Series Analysis
Quiz #1; Full Marks-20

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Let $\{X_t\}$ be a time series given by $X_t = \alpha + \beta t + S_t + Y_t$; where, α and β are real constants, S_t is a seasonal component with period 6 and $Y_t = \varepsilon_t - \varepsilon_{t-1}$; $\{\varepsilon_t\}$ is a sequence of independent and identically distributed $N(0, \sigma^2)$ random variables.

Prove or disprove the following statements:

- (a) $\text{Cov}(X_t, X_{t+h}) = 0; \forall |h| \geq 2$
- (b) $\{X_t\}$ is covariance stationary
- (c) $\{X_t\}$ is a Gaussian time series
- (d) $\{\nabla X_t\}$ is covariance stationary
- (e) $\text{Cov}(\nabla_6 X_t, \nabla_6 X_{t+h}) = 0; h = \pm 2, \pm 3, \pm 4$
- (f) $\{\nabla_6 X_t\}$ is covariance stationary
- (g) $\{\nabla_6 X_t\}$ is strict stationary
- (h) $\{\nabla_6^2 X_t\}$ is an MA process

$$X_t = \alpha + \beta t + S_t + Y_t$$
$$= S_t = S_{t+6}$$

$$Y_t = \varepsilon_t - \varepsilon_{t-1}$$
$$\varepsilon_t \sim N(0, \sigma^2)$$