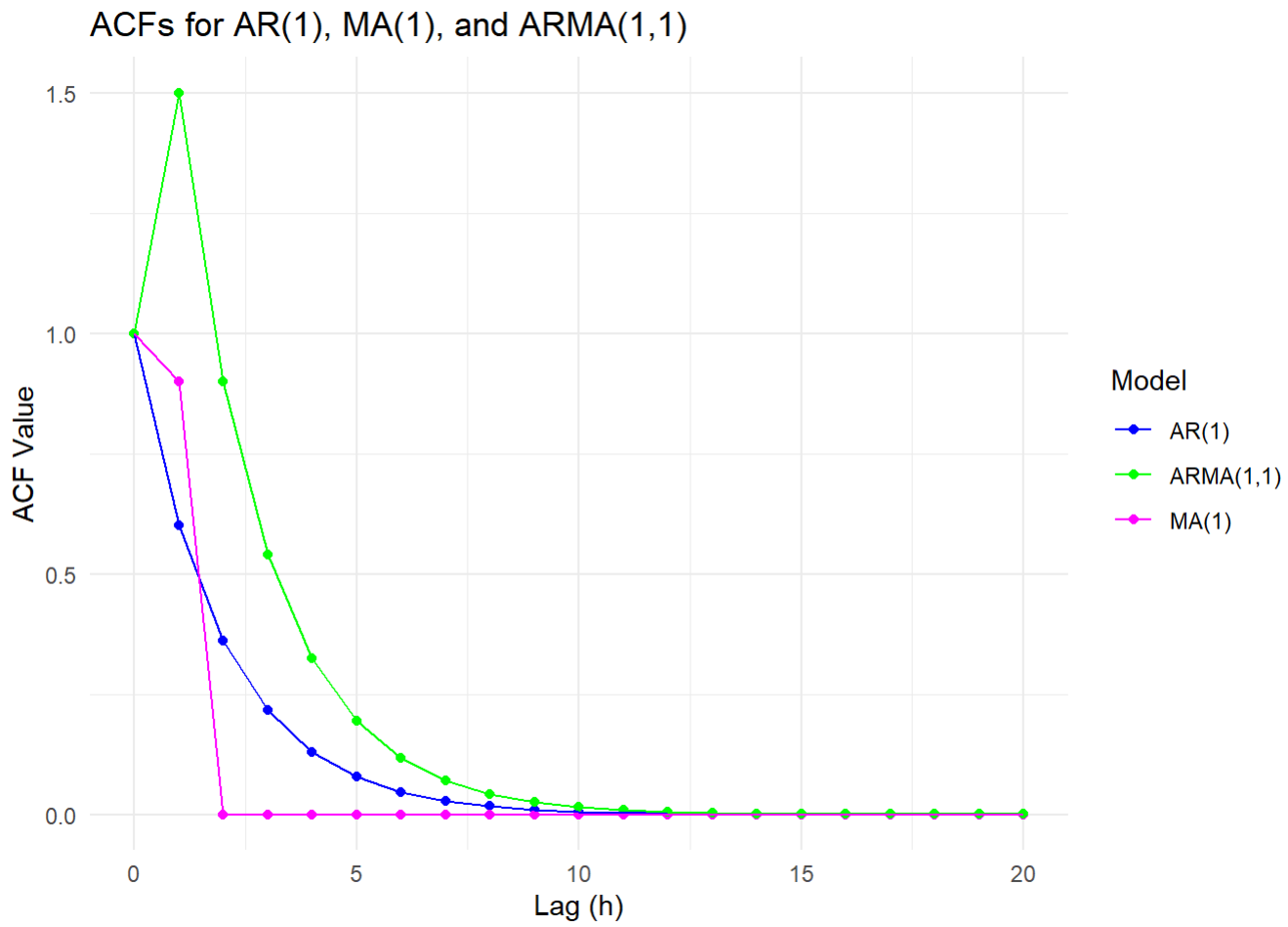
Q6

ACFs with ϕ = 0.6, θ = 0.9:

1. AR(1)= ARMA(1,0): Xt=0.6Xt-1+Wt
2. MA(1)= ARMA(0,1): Xt=Wt+0.9Wt-1
3. ARMA(1,1): Xt=0.6Xt-1+Wt+0.9Wt-1

Plot

```
p<-0.6
t<-0.9

a<-function(h,p){return(p^h)}

m<-function(h,t){
  if(h==0){return(1)}
  else if(h==1){return(t)}
  else{return(0)}
}

x<-function(h,p,t){
  if(h==0){return(1)}
  else if(h==1){return(p+t)}
  else{return(p^h+t*(p^(h-1)))}
}

l<-0:20

v1<-a(l,p)
v2<-sapply(l,m,t)
v3<-sapply(l,x,p,t)

d<-data.frame(Lag=rep(l,3),ACF=c(v1,v2,v3),Model=factor(rep(c("AR(1)","MA(1)","ARMA(1,1)"),ea
ch=length(l))))

ggplot(d,aes(x=Lag,y=ACF,color=Model))+
  geom_point()+
  geom_line()+
  labs(title="ACFs for AR(1), MA(1), and ARMA(1,1)",x="Lag (h)",y="ACF Value")+
  theme_minimal()+
  scale_color_manual(values=c("blue","green","magenta"))
```



Diagnostic Capabilities

patterns in models:

- **AR(1)**: exponential decay in ACF without sharp cut-off, past values affect current value.
- **MA(1)**: sharp cut-off after first lag, so only current and 1 previous noise value has impact.
- **ARMA(1,1)**: ACF has complex decay pattern. It does not cut off quickly and slows down faster than AR model. past values and noise both affect current value.