- Rubric [Unnecessary arguments can further deduct marks !!].

1. Those who attempted to prove Γ_T

[Partial steps where most of the students made mistake].

Given $\gamma(0) > 0$ and $\gamma(h) \longrightarrow 0$ as $h \longrightarrow 0$

- Define, $\underset{\sim}{c}_T = \left(c_{T_1}, \ldots, c_{T_s}\right)^T$ and ,

$$\underset{\sim}{X} = \left(x_1, x_2, \ldots, x_s\right)^T$$

- Eigen decomposition of $\Gamma_T = P \Lambda_s P^T$ with $0 \leq \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_s$

- $$\begin{aligned}
\gamma(0) = \mathrm{Var}\left[\underset{\sim}{X}_T\right] &= \underset{\sim}{c}_T^T P \Lambda P^T \underset{\sim}{c}_T \\
&\geq \lambda_1 \underset{\sim}{c}_T^T P P^T \underset{\sim}{c}_T \\
&= \lambda_1 \underset{\sim}{c}_T^T \underset{\sim}{c}_T \quad \left[\because PP^T = I\right] \\
&= \lambda \sum_{i=1}^{s} c_i^2
\end{aligned}$$

Wrong arguement:

$$\gamma(0) = \operatorname{cov}\left(\underset{\sim}{x}_T, \underset{\sim}{c}_T^{T} \underset{\sim}{x}\right)$$

$$= \operatorname{cov}\left(\underset{\sim}{x}_T, \underset{\sim}{c}_T^{T} \underset{\sim}{x}\right)$$

$$= \sum_{i=1}^{8} \underset{\sim}{c}_T^{T} \operatorname{cov}\left(\underset{\sim}{x}_T, \underset{\sim}{x}\right)$$

$$= \sum_{i=1}^{8} \underset{\sim}{c}_T^{T} \gamma(T-i)$$

$$\leq \sum_{i=1}^{8} |c_{T_i}| \, |\gamma(T-i)|$$

Suppose $|c_{T_i}| \leq M \quad \forall\, i = 1(1)8.$

and $\gamma(h) = \exp\left(-\frac{|h|}{\phi}\right)$

or $\infty \; \frac{1}{h^2}$

Then $0 < \gamma(0) \leq M \sum_{i=1}^{8} \exp\left(-\frac{i}{8}\right) = M^*$

[with stationarity assumption on $\gamma(\cdot)$]

[$\sum_{n=1}^{\infty} \frac{1}{n^2}$, $\sum_{n=1}^{\infty} \exp(-n)$ doesn't converge to zero]

Hence, $\gamma(0) \in (0, M^*]$ (say). This fact doesn't contradict the assumption

① $\gamma(0) > 0$

② $|\gamma(h)| \downarrow 0$ as $h \longrightarrow 0$.

[0 marks]

<u>2nd wrong approach:</u>

- If anyone claims $\gamma(0) > 0$ and each diagonal non-zero with $\gamma(h) \to 0$ as $h \to \infty$ implies smallest eigenvalue λ_1 of Γ_T [must be positive]

[0 marks]

- <u>Hint:</u>
  - Γ_1 is p.d.
  - Show Γ_2 is pd also (by computing eigenvalues)
  - By induction say Γ_K is p.d for
  - Show $\Gamma_{K+1} = \left[\begin{array}{c|c} \Gamma_K & \underset{\sim}{\gamma_K} \\ \hline \underset{\sim}{\gamma_K}^T & \gamma(0) \end{array}\right]$ is p.d by using results of partitioned matrices.

② Correct Answers: (Since no set.seed() is given, the close answers with decimals as well as coding formulas). But MAD, MSE ↓ T & coverage ↑ T.

Summary of ϕ:

T	MSE	MAD	Coverage
50	0·0204	0·1017	0·894.
200	0·00175	0·0313	0·939
500	0·000556	0·0179	0·940

Summary of θ:

T	MSE	MAD	coverage
50	0·0224	0·11544	0·909
200	0·0043	0·05182	0·931
500	0·0016	0·03180	0·949

Summary of σ^2:

T	MSE	MAD	Coverage
50	0.04164	0.16541	NA
200	0.01018	0.08067	NA
500	0.0040	0.05073	NA

- Wrong coverage or MSE or MAD (-0.5 marks for each).

- Should print 3x3 tables for each of θ, σ^2, ϕ. Calculation of MSE, MAD, coverage is awarded $\boxed{\text{zero}}$ marks.

③ Comparison of AR(2) and OLS.

Correct answers:

Parameters	Yule-Walker	OLS
$\hat{\phi}_1$	0.42859	0.43395
$\hat{\phi}_2$	0.44178	0.43758
$\hat{\sigma}^2$	32.3175	32.8405

- Those who didn't reported $\hat{\sigma}^2$ and commented on comparison (-0.5)
- Those who reported wrong YW/OLS (one of them) (-1) and both wrong (-2.5).

4. Complete and detailed derivation (2.5)

- Over-stepping (-1) [No consesation]

5. MSE = 3 × (MSE of optimum predictor)

So, linear regression is not at all good for this problem

[No comment (-0.5) marks].

* (-1) marks from total marks with wrong file name.