ACF and PACF Analysis of Time Series Models

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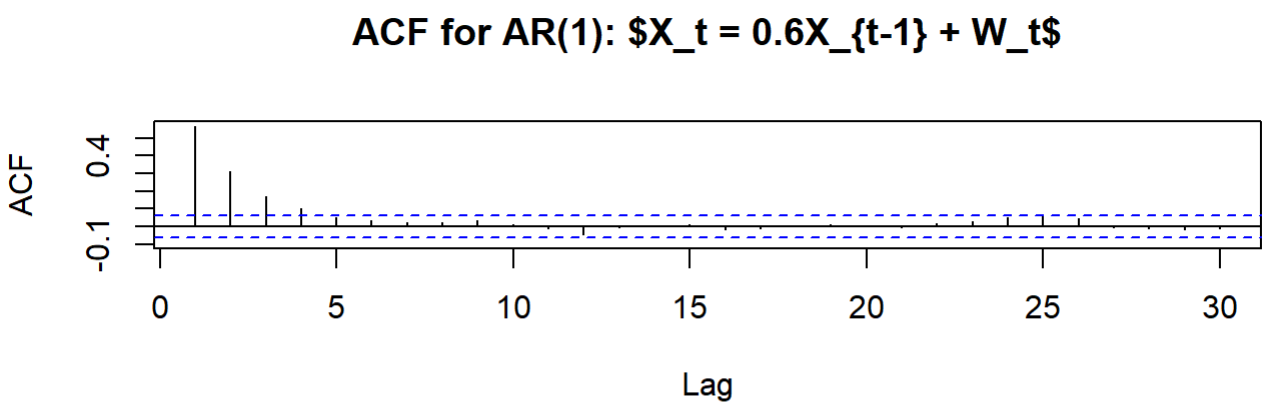
Simulation and ACF/PACF Analysis for AR(1), MA(1), and ARMA(1,1) Models

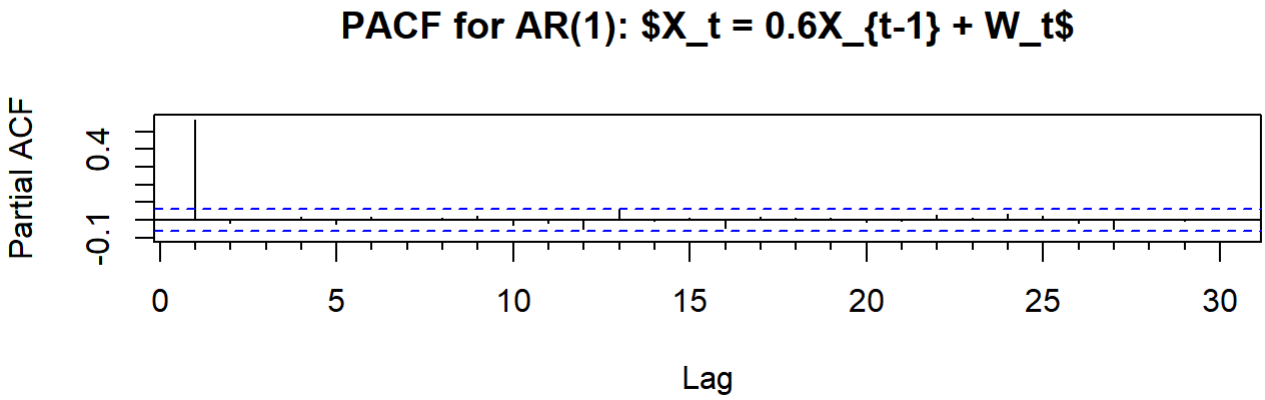
```
# Setting parameters
phi <- 0.6
theta <- 0.9
n <- 1000

# Generating data for each model
set.seed(123) # For reproducibility
data_ar1 <- arima.sim(model = list(ar = phi), n = n)
data_ma1 <- arima.sim(model = list(ma = theta), n = n)
data_arma11 <- arima.sim(model = list(ar = phi, ma = theta), n = n)
```

Plotting ACF and PACF for AR(1) Model

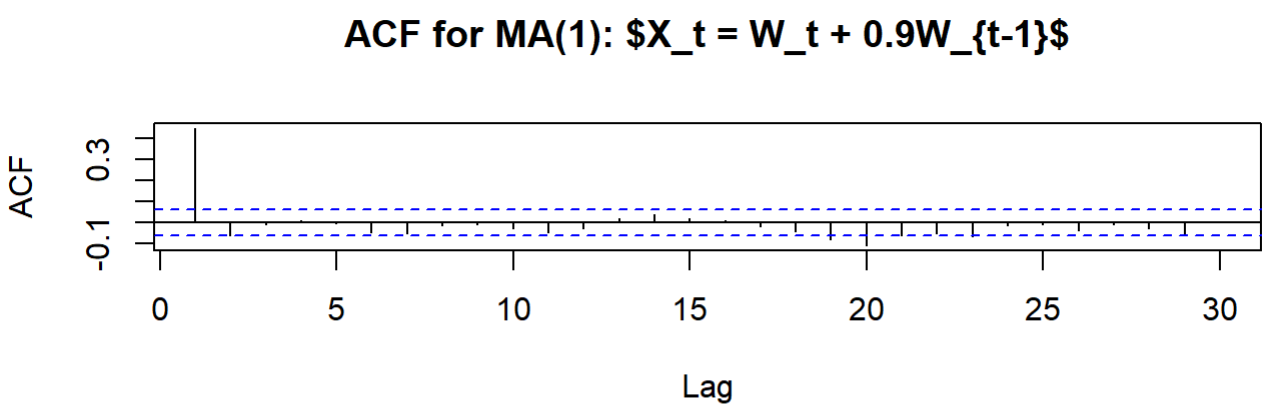
```
# ACF and PACF for AR(1)
par(mfrow=c(2,1)) # Setting plot area for two plots
Acf(data_ar1, main="ACF for AR(1): $X_t = 0.6X_{t-1} + W_t$")
Pacf(data_ar1, main="PACF for AR(1): $X_t = 0.6X_{t-1} + W_t$")
```

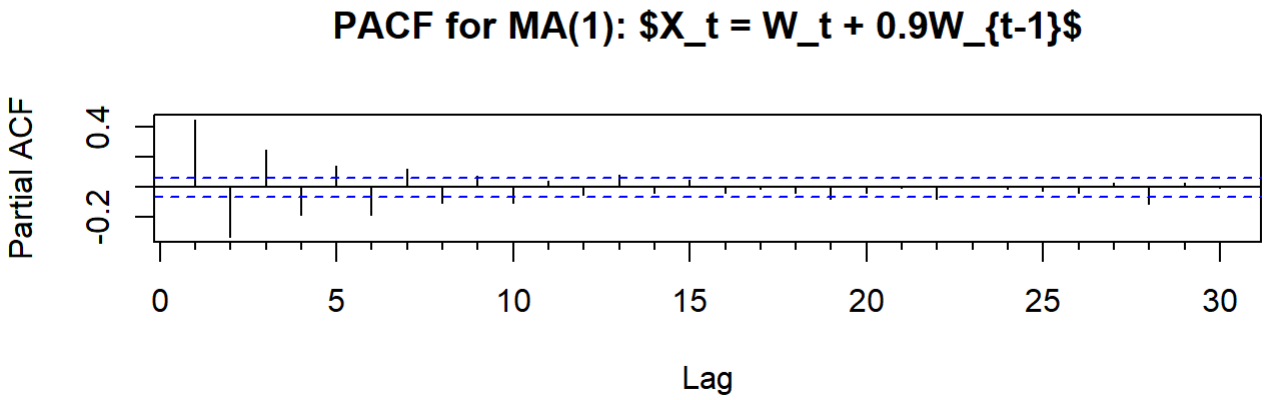




Plotting ACF and PACF for MA(1) Model

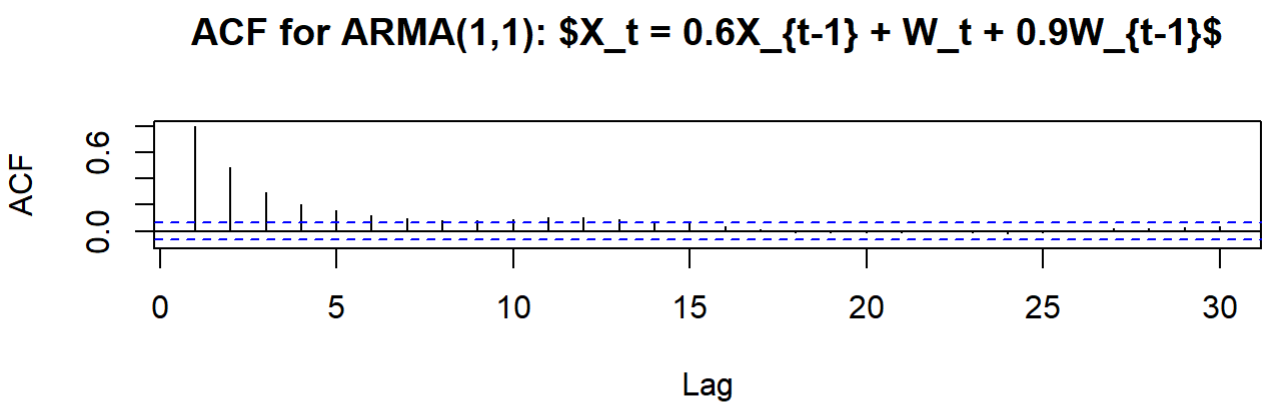
```
# ACF and PACF for MA(1)
par(mfrow=c(2,1)) # Setting plot area for two plots
Acf(data_ma1, main="ACF for MA(1): $X_t = W_t + 0.9W_{t-1}$")
Pacf(data_ma1, main="PACF for MA(1): $X_t = W_t + 0.9W_{t-1}$")
```

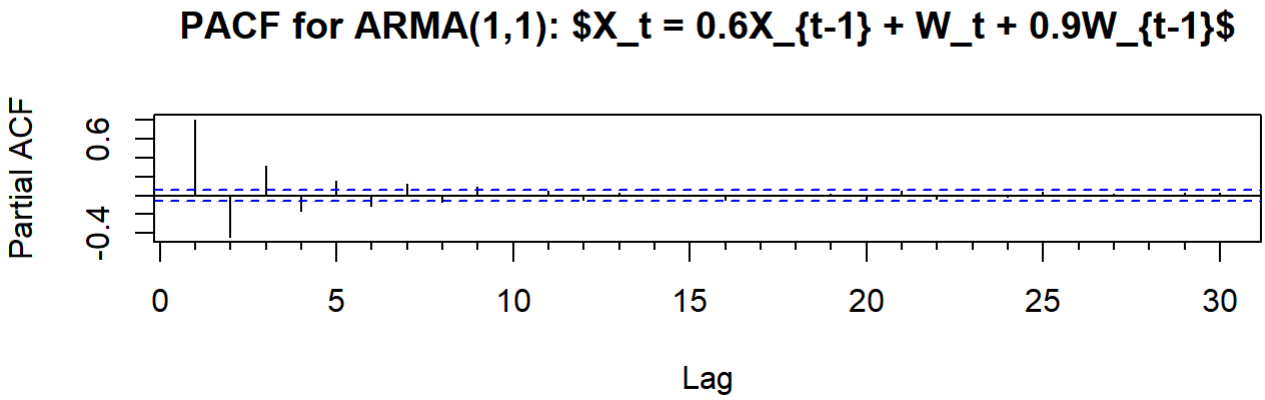




Plotting ACF and PACF for ARMA(1,1) Model

```
# ACF and PACF for ARMA(1,1)
par(mfrow=c(2,1)) # Setting plot area for two plots
Acf(data_arma11, main="ACF for ARMA(1,1): $X_t = 0.6X_{t-1} + W_t + 0.9W_{t-1}$")
Pacf(data_arma11, main="PACF for ARMA(1,1): $X_t = 0.6X_{t-1} + W_t + 0.9W_{t-1}$")
```





Commentary on ACF and PACF Patterns

- **AR(1) Model**: The ACF shows exponential decay, which is typical for AR models, indicating the presence of one significant lag. The PACF cuts off sharply after one lag, supporting the AR(1) structure.
- **MA(1) Model**: The ACF for the MA(1) model typically cuts off after the first lag, reflecting the memory of the moving average process. The PACF, conversely, decays slowly, indicating the effect of the MA component on the series.
- **ARMA(1,1) Model**: For the ARMA(1,1) model, both the ACF and PACF exhibit more complex patterns, showing a mixture of decay and cutoffs. This complexity reflects the combined autoregressive and moving average behaviors.

These plots provide visual diagnostics to confirm the theoretical expectations of how AR, MA, and combined ARMA processes behave in terms of autocorrelation and partial autocorrelation.