Q85 Given ARMA model

$$X_t = 0.80 X_{t-1} \cancel{4} - 0.15 X_{t-2} + W_t - 0.3 W_{t-1}$$

$$X_t - 0.8 X_{t-1} + 0.15 X_{t-2} = W_t - 0.3 W_{t-1}$$

$$\cancel{X_t} \; (1 - 0.8B + 0.15B^2) X_t = (1 - 0.3B) W_t$$

$$(1 - 0.3B - 0.5B + 0.15B^2) X_t = (1 - 0.3B) W_t$$

$$(1 - 0.3B)(1 - 0.5B) X_t = (1 - 0.3B) W_t$$

Now AR polynomial $\phi(z) = (1 - 0.3z)(1 - 0.5z)$ and MA polymomial $\theta(z) = (1 - 0.3z)$ has the common root So there is parameter redundancy So the model is

$$(1 - 0.5B) X_t = W_t$$

$$X_t = 0.5 X_{t-1} + W_t$$

which is AR(1) model.

Also If I consider the root of the AR polynomial

$\phi(z) = 1 - 0.5z$

$z = 2$ and $|z| > 1$

So the process is causal.

Also $X_t = 0.5 X_{t-1} + w_t$

$$w_t = X_t - 0.5 X_{t-1}$$

$$= \sum_{j=0}^{\infty} \pi_j X_{t-j}$$

$\pi_0 = 1$

$\pi_1 = -0.5$

$\pi_t = 0 \quad \forall t \geq 2$

So the process is inveshible also.