Indian Institute of Technology Kanpur

Department of Mathematics and Statistics

Time Series Analysis (MTH442)

Assignment 2, Due date: September 27, 2024, Friday

Answers should be provided neatly. In case the handwriting is unreadable, the instructor and the teaching assistants hold the right to give a zero score. In case of cheating, all students involved will get zero, irrespective of who copied from whom. Write on a paper using some dark ink and scan using a good scanner to obtain a clearly readable PDF file. For the coding related question, use R markdown and generate the PDF output with echo = TRUE mode (so that the codes are also visible along with the outputs). Finally, join the two PDFs and submit the single PDF file only. The final file should be named as RollNo_Lastname_Firstname.pdf. If the file is not submitted in this nomenclature format, marks will be zero. No request should be made in that case.

1. For an MA(1) model, show that the absolute value of the lag-1 ACF is bounded above by 0.5. (1 point)

2. Let $\{W_t; t = 0, 1, \ldots\}$ be a white noise process with variance σ_W^2 and let $|\phi| < 1$ be a constant. Consider the process $X_0 = W_0$, and $X_t = \phi X_{t-1} + W_t,\ t = 1, 2, \ldots$. $(0.5 \times 8 = 4$ points)
   (a) Show that $X_t = \sum_{j=0}^{t} \phi^j W_{t-j}$ for any $t = 0, 1, \ldots$.
   (b) Find the $E(X_t)$.
   (c) Find the $\mathrm{Var}(X_t)$.
   (d) Show that, for $h \geq 0$, derive $\mathrm{Cov}(X_{t+h}, X_t)$.
   (e) Is X_t stationary? Comment based on the conditions of weak stationarity.
   (f) Is X_t "asymptotically stationary", i.e., comment on the conditions of weak stationarity in the limiting sense as $t \uparrow \infty$?
   (g) Comment on how you could use these results to simulate n observations of a stationary Gaussian AR(1) model from simulated IID $N(0, 1)$ values.
   (h) Now suppose $X_0 = W_0/\sqrt{1 - \phi^2}$. Is this process X_t stationary?

3. For an AR(2) model $(1 - \phi_1 B - \phi_2 B^2)X_t = W_t$, derive the conditions on ϕ_1 and ϕ_2 for causality of X_t. (1 point)

4. For the AR(2) models given by $X_t = 0.25X_{t-2} + W_t$ and $X_t = -0.9X_{t-2} + W_t$, find the roots of the autoregressive polynomials, and then plot their ACFs, $\rho(h)$. $(0.5 \times 2 = 1$ point)

5. For the ARMA model $X_t = 0.80X_{t-1} - 0.15X_{t-2} + W_t - 0.30W_{t-1}$ Identify if there is any parameter redundancy, and determine whether they are causal and/or invertible. (1 point)

6. Plot the theoretical ACFs (all derived in the class) of the three series AR(1)=ARMA(1,0) $X_t = \phi X_{t-1} + W_t$, MA(1)=ARMA(0,1) $X_t = W_t + \theta W_{t-1}$, and ARMA(1,1) $X_t = \phi X_{t-1} + W_t + \theta W_{t-1}$ on the same graph for $\phi = 0.6$, $\theta = 0.9$, and comment on the diagnostic capabilities of the ACF in this case. (1 point)

7. Generate $n = 1000$ observations from each of the models in the previous question and comment on the important patterns in the ACF and PACF plots. (1 point)