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Prove that the autocovariance function of a stationary process, is a nonnegative definite function. Also, verify that the s

Prove that the autocovariance function of a stationary process, is a nonnegative definite function. Also, verify that the sample autocovariance is a nonnegative definite function.

Steps

Step 1 of 3

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Introduction and Overview

The autocovariance function is a crucial concept in time series analysis, especially for stationary processes. A stationary process is one where the statistical properties, like mean and variance, do not change over time. The autocovariance function measures how values of the process are correlated with themselves at different lags. One key property of the autocovariance function of a stationary process is that it is nonnegative definite. This means that for any set of real numbers and any set of time lags, a specific quadratic form involving these numbers and the autocovariances will always be nonnegative. We will prove this property and verify that the sample autocovariance is also nonnegative definite.

Explanation

The autocovariance function of a stationary process measures the correlation between values at different time lags. definite, which ensures that certain mathematical properties hold, maintaining consistency in time series analysis.

Step 2 of 3

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Proof That Autocovariance Function is Nonnegative Definite

To prove that the autocovariance function of a stationary process is nonnegative definite, let's consider a stationary process (X_t) with mean ($\mu = E[X_t]$). The autocovariance function at lag (h) is defined as:

[$\gamma(h) = E[(X_t - \mu)(X_{t+h} - \mu)]$]

Now, consider a set of real numbers ($a_1, a_2, \ldots, a_n$) and a corresponding set of time points ($t_1, t_2, \ldots, t_n$). Define the quadratic form:

$$\left[Q = \sum_{i=1}^{n} \sum_{j=1}^{n} a_i a_j \gamma(t_j - t_i) \right]$$

Using the definition of autocovariance, we can rewrite Q as:

$$\left[Q = E\left[\left(\sum_{i=1}^{n} a_i (X_{t_i} - \mu)\right)^2\right] \right]$$

Since the expected value of a squared random variable is always nonnegative, it follows that:

$$[\quad Q \geq 0 \quad]$$

This proves that the autocovariance function $(\quad \gamma(h) \quad)$ is nonnegative definite.

Explanation

We define a quadratic form involving autocovariances and demonstrate that it is nonnegative by expressing it as th
of a squared quantity is always nonnegative, this proves that the autocovariance function is nonnegative definite.

Step 3 of 3

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Verification That the Sample Autocovariance is Nonnegative Definite

The sample autocovariance is an estimator of the true autocovariance based on observed data. For a given stationary process $(\quad X_1, \quad X_2, \quad \ldots, \quad X_n \quad)$, the sample autocovariance at lag $(\quad h \quad)$ is defined as:

$$\left[\quad \widehat{\gamma}(h) \quad = \quad \frac{1}{n} \sum_{t=1}^{n-h} (X_t \quad - \quad \overline{X})(X_{t+h} \quad - \quad \overline{X}) \quad\right]$$

where $(\quad \overline{X} \quad)$ is the sample mean of the observed data.

To verify that this function is nonnegative definite, consider a set of real numbers $(\quad a_1, \quad a_2, \quad \ldots, \quad a_n \quad)$ and form a similar quadratic expression:

$$\left[\quad \widehat{Q} \quad = \quad \sum_{i=1}^{n} \sum_{j=1}^{n} a_i \quad a_j \quad \widehat{\gamma}(t_j \quad - \quad t_i) \quad\right]$$

Using the definition of the sample autocovariance, we get:

$$\left[\quad \widehat{Q} \quad = \quad \frac{1}{n} \sum_{t=1}^{n-h} \left(\sum_{i=1}^{n} a_i \quad (X_{t_i} \quad - \quad \overline{X})\right)^2 \quad\right]$$

Since this is a sum of squares, it is always nonnegative:

$$[\quad \widehat{Q} \geq 0 \quad]$$

Thus, the sample autocovariance is also nonnegative definite.

Explanation

The sample autocovariance, an estimator of the true autocovariance, is shown to be nonnegative definite by formir
squared terms, which confirms that the sample autocovariance maintains the nonnegative definite property.

Final Answer

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Final Answer and Summary

The autocovariance function of a stationary process is nonnegative definite, which means any quadratic form involving the autocovariances is nonnegative. This property extends to the sample autocovariance, which, as an estimator of the true autocovariance, is also nonnegative definite. The key idea in proving this is showing that the expressions involve sums of squares, which are inherently nonnegative. This property is fundamental in time series analysis and ensures that the autocovariance structure of a stationary process is mathematically consistent and meaningful.