Introduction to Statistical Time Series

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Introduction to Statistical Time Series

Second Edition

WAYNE A. FULLER

Iowa State University

A Wiley-Interscience Publication

JOHN WILEY & SONS, INC.

New York • Chichester • Brisbane • Toronto • Singapore

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Library of Congress Cataloging in Publication Data:

Fuller, Wayne A.
Introduction to statistical time series / Wayne A. Fuller. -- 2nd ed.
p. cm. -- (Wiley series in probability and statistics)
"A Wiley-Interscience publication."
Includes bibliographical references and index.
ISBN 0-471-55239-9 (cloth : alk. paper)
1. Time-series analysis. 2. Regression analysis. I. Title II. Series.
QA280.F84 1996
519.5'5--dc20 95-14875

10 9 8 7 6 5 4 3

To Evelyn

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Preface to the First Edition

This textbook was developed from a course in time series given at Iowa State University. The classes were composed primarily of graduate students in economics and statistics. Prerequisites for the course were an introductory graduate course in the theory of statistics and a course in linear regression analysis. Since the students entering the course had varied backgrounds, chapters containing elementary results in Fourier analysis and large sample statistics, as well as a section on difference equations, were included in the presentation.

The theorem-proof format was followed because it offered a convenient method of organizing the material. No attempt was made to present the most general results available. Instead, the objective was to give results with practical content whose proofs were generally consistent with the prerequisites. Since many of the statistics students had completed advanced courses, a few theorems were presented at a level of mathematical sophistication beyond the prerequisites. Homework requiring application of the statistical methods was an integral part of the course.

By emphasizing the relationship of the techniques to regression analysis and using data sets of moderate size, most of the homework problems can be worked with any of a number of statistical packages. One such package is SAS (Statistical Analysis System, available through the Institute of Statistics, North Carolina State University). SAS contains a segment for periodogram computations that is particularly suited to this text. The system also contains a segment for regression with time series errors compatible with the presentation in Chapter 9. Another package is available from International Mathematical and Statistical Library, Inc.; this package has a chapter on time series programs.

There is some flexibility in the order in which the material can be covered. For example, the major portions of Chapters 1, 2, 5, 6, 8, and 9 can be treated in that order with little difficulty. Portions of the later chapters deal with spectral matters, but these are not central to the development of those chapters. The discussion of multivariate time series is positioned in separate sections so that it may be introduced at any point.

I thank A. R. Gallant for the proofs of several theorems and for the repair of others: J. J. Goebel for a careful reading of the manuscript that led to numerous substantive improvements and the removal of uncounted mistakes; and D. A.

Dickey, M. Hidiroglou, R. J. Klemm, and G. H. K. Wang for computing examples and for proofreading. G. E. Battese, R. L. Carter, K. R. Crouse, J. D. Cryer, D. P. Hasza, J. D. Jobson, B. Macpherson, J. Mellon, D. A. Pierce and K. N. Wolter also read portions of the manuscript. I also thank my colleagues, R. Groeneveld, D. Isaacson, and O. Kempthorne, for useful comments and discussions. I am indebted to a seminar conducted by Marc Nerlove at Stanford University for the organization of some of the material on Fourier analysis and spectral theory. A portion of the research was supported by joint statistical agreements with the U.S. Bureau of the Census.

I thank Margaret Nichols for the repeated typings required to bring the manuscript to final form and Avonelle Jacobson for transforming much of the original illegible draft into typescript.

WAYNE A. FULLER

Ames, Iowa
February 1976

Preface to the Second Edition

Considerable development in statistical time series has occurred since the first edition was published in 1976. Notable areas of activity include nonstationary models, nonlinear estimation, multivariate models, state space representations and empirical model identification. The second edition attempts to incorporate new results and to respond to recent emphases while retaining the basic format of the first edition.

With the exception of new sections on the Wold decomposition, partial autocorrelation, long memory processes, and the Kalman filter, Chapters one through four are essentially unchanged from the first edition. Chapter 5 has been enlarged, with additional material on central limit theorems for martingale differences, an expanded treatment of nonlinear estimation, a section on estimated generalized least squares, and a section on the roots of polynomials. Chapter 6 and Chapter 8 have been revised using the asymptotic theory of Chapter 5. Also, the discussion of estimation methods has been modified to reflect advances in computing. Chapter 9 has been revised and the material on the estimation of regression equations has been expanded.

The material on nonstationary autoregressive models is now in a separate chapter, Chapter 10. New tests for unit roots in univariate processes and in vector processes have been added.

As with the first edition, the material is arranged in sections so that there is considerable flexibility to the order in which topics can be covered.

I thank David Dickey and Heon Jin Park for constructing the tables of Chapter 10. I thank Anthony An, Rohit Deo, David Hasza, N. K. Nagaraj, Sastry Pantula, Heon Jin Park, Savas Papadopoulos, Sahadeb Sarkar, Dongwan Shin, and George H. K. Wang for many useful suggestions. I am particularly indebted to Sastry Pantula who assisted with the material of Chapters 5, 8, 9, and 10 and made substantial contributions to other parts of the manuscript, including proofs of several results. Sahadeb Sarkar contributed to the material on nonlinear estimation of Chapter 5, Todd Sanger contributed to the discussion of estimated generalized least squares, Yasuo Amemiya contributed to the section on roots of polynomials, Rohit Deo contributed to the material on long memory processes, Sastry Pantula, Sahadeb Sarkar and Dongwan Shin contributed to the material on the limiting distribution of estimators for autoregressive moving averages, and Heon Jin Park contributed to the sections on unit root autoregressive processes. I thank Abdoulaye Adam, Jay Breidt, Rohit Deo, Kevin Dodd, Savas Papadopoulos, and Anindya Roy for computing examples. I thank SAS Institute, Cary, NC, for providing computing support to Heon Jin Park for the construction of tables for unit root tests. The research for the second edition was partly supported by joint statistical agreements with the U.S. Bureau of the Census.

I thank Judy Shafer for the extensive word processing required during preparation of the second edition.

WAYNE A. FULLER

Ames, Iowa
November 1995

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