Lecture 27

Introduction to state-space models

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Motivation

- The model arose in the space tracking setting.
- Here the state equation defines the motion equations for the position or state of a spacecraft with
  - location X_t
  - the data Y_t reflect information that can be observed from a tracking device such as velocity and azimuth.
- Although introduced as a method primarily for use in aerospace-related research, the model has been applied to modeling data from economics, medicine, and the soil sciences.

Things to cover

- We will cover the concepts of
  - prediction
  - filtering
  - smoothing

  state space models and include their derivations.
- We explain how to perform MLE using various techniques, and include methods for handling missing data.

Two principles of state-space models

1. There is a hidden or latent process X_t called the state process. The state process is assumed to be a Markov process; this means that the future $\{X_s; s > t\}$, and past $\{X_s; s < t\}$, are independent conditional on X_t.

$$\begin{aligned} & \pi(X_1, \ldots, X_{t-1}, X_{t+1}, \ldots, X_{t+s} | X_t) \\ = \; & \pi(X_1, \ldots, X_{t+s}) / \pi(X_t) \\ = \; & \pi(X_1, \ldots, X_t) \pi(X_{t+1}, \ldots, X_{t+s} | X_1, \ldots, X_t) / \pi(X_t) \\ = \; & \pi(X_1, \ldots, X_{t-1} | X_t) \pi(X_{t+1}, \ldots, X_{t+s} | X_t) \end{aligned}$$

2. The observations, Y_t are independent given the states X_t.

Diagram

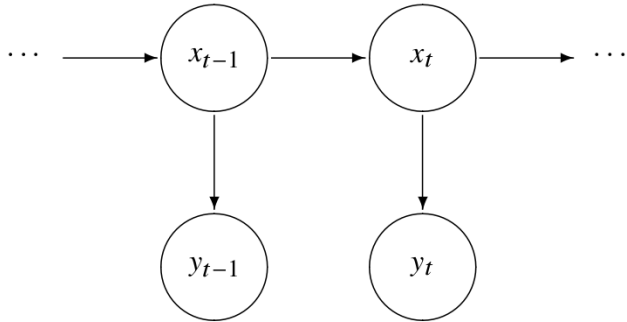

Fig. 6.1. *Diagram of a state space model.*

Why responses are dependent?

$$
\begin{aligned}
& \pi(Y_1, \ldots, Y_T) \\
= & \int \ldots \int \pi(Y_1, \ldots, Y_T, X_1, \ldots, X_T) dX_1 \ldots dX_T \\
= & \int \ldots \int \pi(Y_1, \ldots, Y_T | X_1, \ldots, X_T) \pi(X_1, \ldots, X_T) dX_1 \ldots dX_T \\
= & \int \ldots \int \left\{ \prod_{t=1}^{T} \pi(Y_t | X_t) \right\} \pi(X_1, \ldots, X_T) dX_1 \ldots dX_T \\
= & \int \ldots \int \left\{ \prod_{t=1}^{T} \pi(Y_t | X_t) \right\} \{ \pi(X_1) \pi(X_2 | X_1) \ldots \pi(X_T | X_{T-1}) \} \, dX_1 \ldots dX_T \\
\neq & \prod_{t=1}^{T} \pi(Y_t)
\end{aligned}
$$

Thank you!