Lecture 4

Characterization of a Time Series

Arnab Hazra

Characterization of a Time Series

- A complete description of a time series, observed as a collection of n random variables at arbitrary time points $t_1, t_2, \ldots, t_n$, for any positive integer n, is provided by the joint CDF

$$F_{t_1,t_2,\ldots,t_n}(c_1, c_2, \ldots, c_n) = Pr[X_{t_1} \leq c_1, X_{t_2} \leq c_2, \ldots, X_{t_n} \leq c_n].$$

- Unfortunately, these joint CDFs cannot usually be written easily unless the random variables are jointly normal.
- Although the joint distribution function describes the data completely, it is an unwieldy tool for displaying and analyzing time series data.
- The distribution function must be evaluated as a function of n arguments, so any plotting of the corresponding multivariate densities is virtually impossible.
- The marginal distribution functions and density functions are often informative for examining the marginal behavior of a series.

Mean function

- The mean function is defined as

$$\mu_{xt} = E(X_t) = \int x f_t(x) dx$$

where f_t is the density of X_t. In proper contexts, we will replace μ_{xt} by μ_t.

- Mean Function of a Moving Average Series:

$$\mu_{vt} = \frac{1}{3}\{E[W_{t-1}] + E[W_t] + E[W_{t+1}]\} = 0.$$

- Mean Function of a Random Walk with Drift:

$$\mu_{xt} = \delta t + E[\sum_{j=1}^{t} W_j] = \delta t.$$

Thank you!