- **Disclaimer:**

① Please follow the marks deduction scheme for each questions. Any unnecessary arguments will deduct further negative marking.

② For any doubt in Q-1, 3 talk to Arghya.
Q-2, 4 talk to Sayan.

- **Marking scheme:**

Question-1: ⓐ and ⓑ Incorrect answers with detailed steps will award **Zero** marks. **No** partial marking.

1. ⓐ The state equation:

$$\begin{bmatrix} X_t \\ X_{t-1} \end{bmatrix} = \begin{bmatrix} 0 & \phi \\ 1 & 0 \end{bmatrix} \begin{bmatrix} X_{t-1} \\ X_{t-2} \end{bmatrix} + \begin{bmatrix} W_t \\ 0 \end{bmatrix}$$

The observed equation:

$$Y_t = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} X_t \\ X_{t-1} \end{bmatrix} + V_t$$

ⓑ $\sigma_0^2 = \dfrac{\sigma_w^2}{1-\phi^2}$, $\sigma_1^2 = \dfrac{\sigma_w^2}{1-\phi^2}$

③

(a) ∇Y_t is MA(1) process with

$$\gamma_{\nabla Y_t}(h) = \begin{cases} \sigma_w^2 + 2\sigma_v^2 & , h = 0 \\ -\sigma_v^2 & h = \pm 1 \\ 0 & \text{otherwise} \end{cases}$$

zero marks will be given if the autocorrelation function is missing.

(ii) Reporting summary of coefficients from Arima (∇log_varve, order = c(0,0,1)) object

	Estimate	SE	p. value
ma1	−0.771	0.0341	0.000
constant	−0.0013	0.0044	0.7782

$\hat{\sigma}^2 = 0.2355$ (must)

[1 marks for coeff^n + $\hat{\sigma}^2$

0.5 marks for coeff^{n}s]

- Reporting only ma(1) coeff^n (−0.771) will awarded (0.25) marks.

(ii) Residual analysis (1 marks)

- ACF plot for residuals [lag-1 significance] (0.5 marks)

or

- Ljung-Box test statistic: p-value = $0.015 < 0.05$

(Rejected at 5% level of significance, H_0: no auto correlation is rejected at 5% l.o.s)

- QQ plot / density plot for normality assumption (0.5)

[Plots with no explanation (−0.75) ⊛]

(2) Find $\text{cov}(\varepsilon_s, \varepsilon_t)$ by explaining each term without skipping any remarkable step. Marks deducted if written in just 1-2 line.

(a) show that $\text{cov}(\varepsilon_s, \varepsilon_t) = 0$ for $s \neq t$ (1)

(b) show that $\text{cov}(\varepsilon_s, \varepsilon_t) = P_t^{t-1^2} + \sigma_v^2$ for $s = t$ (1)

(4) (a) Write the expression of ε_t

then write why $\partial_i \varepsilon_t = -A_t \partial_i x_t^{t-1}$ (0.5)

(b) Start with correct expression of x_t^{t-1}

$$x_t^{t-1} = \Phi x_{t-1}^{t-2} + K_{t-1} \varepsilon_{t-1} \quad (0.5)$$

from Shumway, stoffer eq. 6.100 - 6.107

(c) $\Sigma_t = A_t P_t^{t-1} A_t' + R$

Explain why differentiation only with P_t^{t-1} (0.5)

(d) Use expression of K_t as mentioned shumway, stoffer $K_t = \Phi P_t^{t-1} A_t' \Sigma_t^{-1}$ (0.5)

(e) Use the given expression of P_t^{t+1} and end up explaining each 3 terms differentiation. (1)

Note:

(1) 1 Mark deducted if submitted with wrong file name.

(2) Partial marks deducted if there is broad step jump.