MTH 517: Time Series Analysis
Mid Semester Examination
Full Marks 60

[1] (a) Let $\{X_t\}$ be a covariance stationary ARMA process given by

$$X_t = \phi_1 X_{t-1} + \varepsilon_t - (\phi_1 + \phi_2)\varepsilon_{t-1} + \phi_1\phi_2\varepsilon_{t-2},$$

$\varepsilon_t \sim WN(0,\sigma^2)$. Find the smallest integer k, if any, such that $\gamma_X(h) = 0, \forall |h| > k$.

(b) Let $X_t = U_t + iV_t$ be a complex valued stationary process with $\{U_t\}$ and $\{V_t\}$ real valued stationary processes. **Prove or disprove** "$\gamma_X(h) = \gamma_X(-h); \forall h$".

10 (5+5) Marks

[2] Let $\{\varepsilon_t\}$ be an i.i.d. sequence of $N(0,1)$ random variables. Define a new time series $Y_t = -\frac{3}{2} + \varepsilon_t^2 + \frac{1}{2}\varepsilon_{t-1}^2$. **Prove or disprove** the following statements

(a) $\{Y_t\}$ is covariance stationary.

(b) $\{Y_t\}$ is such that $\sum_{j=0}^{\infty}\left(-\frac{1}{2}\right)^j Y_{t-j} = \eta_t$, where $\{\eta_t\}$ is a white noise sequence.

(c) $\{Y_t\}$ is a Gaussian time series.

(3+4+3) 10 Marks

[3] Let $\{X_t\}$ be a time series given by $X_t = \mu + \varepsilon_t + \phi\varepsilon_{t-1}$, $\varepsilon_t \sim WN(0,\sigma^2)$. Consider $\delta_1 = \frac{2X_2 + X_4}{3}$, $\delta_2 = \frac{X_3 + X_4 + X_5}{3}$ and $\delta_3 = \frac{X_1 + X_2}{2}$ as estimators of μ. Find the value (s) of ϕ, if any, such that

(i) $Var(\delta_1) \geq Var(\delta_2)$,

(ii) $Var(\delta_1) \geq Var(\delta_3)$

10 Marks

[4] Let $\{Z_t\}$ be a covariance stationary AR(1) process given by $Z_t = \theta Z_{t-1} + \varepsilon_t$, $\varepsilon_t \sim WN(0,\sigma^2)$ and $X_t = Z_t + bZ_{t-1}$. Verify whether $\{X_t\}$ is a covariance stationary ARMA process of appropriate order. Further, find the relationship between θ and b such that $\{X_t\}$ is a white noise process.

8 Marks

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[5] Suppose $\{X_t\}$ is a covariance stationary ARMA(1,1) process given by $X_t = \phi X_{t-1} + \varepsilon_t + \theta \varepsilon_{t-1}$, $\varepsilon_t \sim WN(0, \sigma^2)$, $|\phi| < 1$ and $|\theta| < 1$. Find

$$\lim_{N \to \infty} E\left(X_t - \varepsilon_t - (\theta + \phi) \sum_{j=1}^{N} (-\theta)^{j-1} X_{t-j} \right)^2 .$$

12 Marks

[6] Let $\{X_t\}$ be an AR(1) process $X_t = \frac{1}{2} X_{t-1} + \varepsilon_t$, $\varepsilon_t \sim WN(0, \sigma^2)$ and $\{Y_t\}$ be MA(1) process $Y_t = \eta_t + \eta_{t-1}$, $\eta_t \sim WN(0, \sigma^2)$. Furthermore, $\{\varepsilon_t\}$ and $\{\eta_t\}$ are independent. Express the ACGF of $U_t = X_t - X_{t-1} + Y_t - Y_{t-2}$ in terms of ACGFs of $\{\varepsilon_t\}$ and $\{\eta_t\}$.

10 Marks

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