MTH517: TIME SERIES ANALYSIS

End Semester Examination: Full Marks 100

Date: November 18, 2014

[1] (a) Let $\{X_t\}$ and $\{Y_t\}$ be two time series defined by $X_t = t\,e^{-Y}$ and $Y_t = X_t - X_{t-2}$, where $Y \sim \exp(1)$. Prove or disprove "$\{X_t\}$ and $\{Y_t\}$ are both covariance stationary".

(b) Let $\{\gamma_X(h)\}$ be the autocovariance sequence of an $MA(\infty)$ process, $\{X_t\}$, given by $X_t = \sum_{j=0}^{\infty} \psi_j \varepsilon_{t-j}$, $\sum_j |\psi_j| < \infty$, $\varepsilon_t \sim WN(0,\sigma^2)$. Prove or disprove "$\sum_{h=-\infty}^{\infty} \gamma_X(h) = \sigma^2 \left(\sum_{j=0}^{\infty} \psi_j\right)^2$".

(c) Let $\{X_t\}$ be a covariance stationary time series with mean μ and autocovariance sequence $\{\gamma_X(h)\}$. Prove or disprove "if $\lim_{n\to\infty} Cov(X_n, \bar{X}_n) = 0$ then $\lim_{h\to\infty} \gamma_X(h) = 0$".

(d) Let $\{X_t\}$ be a covariance stationary time series with mean μ and autocovariance sequence $\{\gamma_X(h)\}$ such that $\lim_{n\to\infty} E(\bar{X}_n - \mu)^2 = 0$, find $\lim_{n\to\infty} \left(\sum_{h=0}^{n-1} \frac{\gamma_X(h)}{n} \right)^2$.

20 (5+5+5+5) Marks

[2] Let $\{X_t\}$ be a sequence of independent and identically distributed $N(0,1)$ random variables. Define $Y_t = (-1)^t + X_t + X_{t-1} + X_{t-2}$ and $Z_t = (-1)^t X_t$. Prove or disprove the following statements:

(a) $\{Y_t\}$ is Gaussian and strictly stationary.

(b) $\{Z_t\}$ is Gaussian and strictly stationary.

(c) $\begin{pmatrix} Y_t \\ Z_t \end{pmatrix}$ is covariance stationary bivariate process.

15 (5+5+5) Marks

[3] Let $\{X_t\}$ and $\{Y_t\}$ be two time series given by $X_t = 0.5X_{t-1} + \varepsilon_t$, $Y_t = 0.5Y_{t-1} + \varepsilon_{t-1}$, $\varepsilon_t \sim WN(0,\sigma^2)$.

(a) Find the BLP of X_{t+2} based on Y_t.

(b) Find the mean square prediction error of the BLP obtained in (a).

12 (7+5) Marks

[4] Consider the 2-variate vector process $\{\underset{\sim}{X}_t\}$ given by $\underset{\sim}{X}_t = \Phi\, \underset{\sim}{X}_{t-1} + \underset{\sim}{\varepsilon}_t + \Theta\, \underset{\sim}{\varepsilon}_{t-1}$, $\Phi = \begin{pmatrix} 0.5 & 0 \\ 2 & 0 \end{pmatrix}$,

$\Theta = \begin{pmatrix} 0 & 0.5 \\ 0 & 2 \end{pmatrix}$ and $\underset{\sim}{\varepsilon}_t \sim VWN(\underset{\sim}{0}, \Sigma)$, $\Sigma > 0$.

(a) Find Ψ_j associated with the $VMA(\infty)$ representation $\underset{\sim}{X}_t = \sum_{j=0}^{\infty} \Psi_j \underset{\sim}{\varepsilon}_{t-j}$ of $\{\underset{\sim}{X}_t\}$.

(b) Verify whether $\{\Psi_j\}_{j=0}^{\infty}$ is absolutely summable.

(c) Obtain impulse response of the 2 variables (in $\underset{\sim}{X}_t$) with respect to shocks in the other variable.

15 (6+5+4) Marks

[5] Let $\{X_t\}$ and $\{Y_t\}$ be two independent covariance stationary time series such that $X_t \sim WN\left(0,\sigma_1^2\right)$ and $Y_t = \delta + \varepsilon_t + \varepsilon_{t-1}$, $\varepsilon_t \sim WN\left(0,\sigma_2^2\right)$. Find the spectral density function of $Z_t = X_t Y_t$ and comment on the nature of the time series $\{Z_t\}$.

6 Marks

[6] Let $\{Z_t\}$ be a covariance stationary time series with spectral density function $f_Z(\lambda) = \dfrac{1}{2\pi}$; $-\pi \le \lambda \le \pi$.

$\{X_t\}$ is obtained from $\{Z_t\}$ using a linear filter with coefficients g_{-1}, g_0, g_1 as $X_t = g_{-1}Z_{t+1} + g_0 Z_t + g_1 Z_{t-1}$. The ACVF of $\{X_t\}$ is $\gamma_X(h) = e^{-|h|}$ and spectral density function $f_X(\lambda)$. Let $Y_t = X_{t-2} - X_{t+2}$ and $f_Y(\lambda)$ the spectral density function of $\{Y_t\}$.

(a) Obtain the values of $f_X(0)$ and $f_Y(0)$.

(b) Find the cross covariance $\gamma_{ZY}(2) = Cov\left(Z_t, Y_{t+2}\right)$.

12 (8+4) Marks

[7] The spectral distribution function of a covariance stationary time series is given by

$$F_X(\lambda) = \begin{cases} \dfrac{3(\pi+\lambda)}{2\pi}, & -\pi \le \lambda < -\pi/2 \\[2mm] \dfrac{5\pi+3\lambda}{2\pi}, & -\pi/2 \le \lambda < -\pi/3 \\[2mm] \dfrac{3(2\pi+\lambda)}{2\pi}, & -\pi/3 \le \lambda < \pi/3 \\[2mm] \dfrac{7\pi+3\lambda}{2\pi}, & \pi/3 \le \lambda < \pi/2 \\[2mm] \dfrac{3(3\pi+\lambda)}{2\pi}, & \pi/2 \le \lambda \le \pi \end{cases}$$

$3\pi + \frac{\pi}{2} = \frac{3\times 7\pi}{2\times 2\pi} = \frac{21}{4}$

(a) Find the continuous spectrum and/or discrete spectrum associated with $F_X(.)$.

(b) Using $F_X(.)$, obtain the ACVF sequence.

(c) Identify the time series for which the above is the spectral distribution function.

20 (8+8+4) Marks

Useful Information:

- p.d.f. of $Y \sim \exp(1)$ is $f_Y(y) = \begin{cases} e^{-y}, & y > 0 \\ 0, & o/w. \end{cases}$