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MTH552: Quiz #1
Full Marks: 20

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[1] Let $\underline{X} = (X_1, X_2)^T$ be a random vector with $E(\underline{X}) = \underline{0}$ and $Cov(\underline{X}) = \Sigma = \begin{bmatrix} 1 & \sigma \\ \sigma & 1 \end{bmatrix}; \sigma = -0.9$.

(a) Derive the principal components and find the proportion of total variation explained by the first principal component.

(b) If $\underline{Y} = (Y_1, Y_2)^T$ denote the vector of principal components, find $\text{Correlation}(Y_1, X_2)$ and $\text{Correlation}(Y_2, X_2)$.

(c) Find $Cov(\underline{\tilde{X}} - \underline{\tilde{Y}}, \underline{\tilde{X}} + \underline{\tilde{Y}})$.

[2] Let $\underline{x}_1 = (2, 2)'$, $\underline{x}_2 = (4, 2)'$, $\underline{x}_3 = (6, 6)'$ and $\underline{x}_4 = (2, 8)'$ be four observed feature vectors. Find the cluster structures and the corresponding ESS measures for the first 2 steps of Ward's clustering algorithm.

$$\Sigma = \begin{bmatrix} 1 & -0.9 \\ -0.9 & 1 \end{bmatrix}$$

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