Circle Compactification and T-Duality

String Theory Exam

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Motivation

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Superstring lives in 10d.

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We live in 4d.

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What to do with those extra-dimensions?

What Really is Spacetime?

Kaluza-Klein Compactification

Kaluza-Klein Compactification: Main Results

Compactification of a scalar field on $\mathcal{M}_D \to \mathcal{M}_{D-1} \times S^1$:

Kaluza-Klein Masses

$$m_s^2 = M^2 + \frac{s^2}{R^2}.$$

Negligible for

$$E \ll \frac{1}{R}.$$

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Compactification of Einstein gravity on $\mathcal{M}_D \to \mathcal{M}_{D-1} \times S^1$:

Result

- Decomposition $SO(1, D-1) \to SO(1, D-2)$;
- Gauge symmetry;
- $\mathrm{Vol}(S^1) = \int_0^{2\pi R} \mathrm{d}y \sqrt{G_{yy}^{(0)}} = e^{-(D-3)\alpha_{D-1}\phi} \cdot 2\pi R.$

Superstring

Superstring: the Setting

Closed Critical String in 10d

- Critical setting $\mathcal{M}_{10}$. Closed string;
- Bosonic X^μ and fermionic ψ^μ fields;

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- Lightcone gauge: $X^\mu \to (X^\pm, X^i)$, $\psi^\mu \to (\psi^\pm, \psi^i)$;
- $X^i = X^i_L(\tau + \sigma) + X^i_R(\tau, \sigma)$, $\psi^i = \psi^i_L(\tau + \sigma) + \psi^i_R(\tau - \sigma)$;

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- $X^i = X^i_L(\tau+\sigma) + X^i_R(\tau,\sigma)$, $\psi^i = \psi^i_L(\tau+\sigma) + \psi^i_R(\tau-\sigma)$;
- Mode expansion: $\psi^i_{L/R}(\xi^\pm) = \sqrt{\frac{2\pi}{l}} \sum_{r\in\mathbb{Z}+\phi} \overset{(\sim)}{b}{}^i_r e^{-\frac{2\pi i}{l} r\xi^\pm}$;

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- NS sector: $\phi = 1/2$, $\overset{(\sim)}{a} = 1/2$;
- R sector: $\phi = 0$, $\overset{(\sim)}{a} = 0$;

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- NS sector: $\phi = 1/2$, $\overset{(\sim)}{a} = 1/2$;
- R sector: $\phi = 0$, $\overset{(\sim)}{a} = 0$;
- Level-matching condition: $M_L^2 = M_R^2$;
- Mass-shell condition: $M^2_{L/R} = \frac{2}{\alpha'}\left(N_\perp - \overset{(\sim)}{a}\right)$.

Superstring: the Sectors

sector	G-parity	state	little group rep.	$\alpha' M_R^2/2$	statistics
NS	$-$	$\left\|0\right\rangle_{NS}$	$SO(9): \mathbf{1}$	$-1/2$	boson
NS	$+$	$b^i_{-1/2}\left\|0\right\rangle_{NS}$	$SO(8): \mathbf{8_v}$	0	boson
R	$+$	$\left\|0\right\rangle_R, B^+_{a_1}B^+_{a_2}\left\|0\right\rangle_R,$ $B^+_1B^+_2B^+_3B^+_4\left\|0\right\rangle_R$	$SO(8): \mathbf{8_s}$	0	fermion
R	$-$	$B^+_{a_1}\left\|0\right\rangle_R,$ $B^+_{a_1}B^+_{a_2}B^+_{a_3}\left\|0\right\rangle_R$	$SO(8): \mathbf{8_c}$	0	fermion

Superstring: Type II Theories

Type IIA			Type IIB		
sector	fields	$SO(8)$	sector	fields	$SO(8)$
$(\mathsf{NS}_+, \mathsf{NS}_+)$	Φ, $B_{[\mu\nu]}$, $G_{(\mu\nu)}$	$\mathbf{8_v} \otimes \mathbf{8_v}$	$(\mathsf{NS}_+, \mathsf{NS}_+)$	Φ, $B_{[\mu\nu]}$, $G_{(\mu\nu)}$	$\mathbf{8_v} \otimes \mathbf{8_v}$
$(\mathsf{R}_+, \mathsf{R}_-)$	C_1, C_3	$\mathbf{8_s} \otimes \mathbf{8_c}$	$(\mathsf{R}_+, \mathsf{R}_+)$	C_0, C_2, C_4^+	$\mathbf{8_s} \otimes \mathbf{8_s}$
$(\mathsf{NS}_+, \mathsf{R}_-)$	$\tilde{\lambda}_a$, $\tilde{\psi}_a^\mu$	$\mathbf{8_v} \otimes \mathbf{8_c}$	$(\mathsf{NS}_+, \mathsf{R}_+)$	$\lambda_a^{(1)}$, $\psi_a^{(1)\mu}$	$\mathbf{8_v} \otimes \mathbf{8_s}$
$(\mathsf{R}_+, \mathsf{NS}_+)$	λ_a, ψ_a^μ	$\mathbf{8_s} \otimes \mathbf{8_v}$	$(\mathsf{R}_+, \mathsf{NS}_+)$	$\lambda_a^{(2)}$, $\psi_a^{(2)\mu}$	$\mathbf{8_s} \otimes \mathbf{8_v}$

Type II String: Compactification

Type II String: Field Theoretic Compactification

Type IIA					
sector	$SO(8)$	10d fields	$SO(7)$	$SO(7)$ irrep	9d fields
$(\mathrm{NS}_+,\mathrm{NS}_+)$	$\mathbf{8_v}\otimes\mathbf{8_v}$	$\Phi, B_{[\hat\mu\hat\nu]}, G_{(\hat\mu\hat\nu)}$	$\mathbf{7}\otimes\mathbf{7}$	$\mathbf{1}\oplus\mathbf{21}\oplus\mathbf{27}$	$\phi, B_{[\mu\nu]}, G_{(\mu\nu)}$
			$(\mathbf{7}\otimes\mathbf{1})\oplus(\mathbf{1}\otimes\mathbf{7})$	$\mathbf{7}\oplus\mathbf{7}$	$G_{\mu 9}, B_{\mu 9}$
			$\mathbf{1}\otimes\mathbf{1}$	$\mathbf{1}$	G_{99}
$(\mathrm{R}_+,\mathrm{R}_-)$	$\mathbf{8_s}\otimes\mathbf{8_c}$	$C_{\hat\mu}, C_{[\hat\mu\hat\nu\hat\rho]}$	$\mathbf{8}\otimes\mathbf{8}$	$\mathbf{1}\oplus\mathbf{7}\oplus\mathbf{21}\oplus\mathbf{35}$	$A_9, A_\mu, C_{9\mu\nu}, C_{\mu\nu\rho}$
$(\mathrm{NS}_+,\mathrm{R}_-)$	$\mathbf{8_v}\otimes\mathbf{8_c}$	$\tilde\lambda_a, \tilde\psi_a^{\hat\mu}$	$\mathbf{7}\otimes\mathbf{8}$	$\mathbf{8}\oplus\mathbf{48}$	$\tilde\psi_a^\mu, \tilde\psi_a^9$
			$\mathbf{1}\otimes\mathbf{8}$	$\mathbf{8}$	$\tilde\lambda_a$
$(\mathrm{R}_+,\mathrm{NS}_+)$	$\mathbf{8_s}\otimes\mathbf{8_v}$	$\lambda_a, \psi_a^{\hat\mu}$	$\mathbf{8}\otimes\mathbf{7}$	$\mathbf{8}\oplus\mathbf{48}$	ψ_a^μ, ψ_a^9
			$\mathbf{8}\otimes\mathbf{1}$	$\mathbf{8}$	λ_a
Type IIB					
sector	$SO(8)$	10d fields	$SO(7)$	$SO(7)$ irrep	9d fields
$(\mathrm{NS}_+,\mathrm{NS}_+)$	$\mathbf{8_v}\otimes\mathbf{8_v}$	$\Phi, B_{[\hat\mu\hat\nu]}, G_{(\hat\mu\hat\nu)}$	$\mathbf{7}\otimes\mathbf{7}$	$\mathbf{1}\oplus\mathbf{21}\oplus\mathbf{27}$	$\phi, B_{[\mu\nu]}, G_{(\mu\nu)}$
			$(\mathbf{7}\otimes\mathbf{1})\oplus(\mathbf{1}\otimes\mathbf{7})$	$\mathbf{7}\oplus\mathbf{7}$	$G_{\mu 9}, B_{\mu 9}$
			$\mathbf{1}\otimes\mathbf{1}$	$\mathbf{1}$	G_{99}
$(\mathrm{R}_+,\mathrm{R}_+)$	$\mathbf{8_s}\otimes\mathbf{8_s}$	$C_0, C_{[\hat\mu\hat\nu]}, C^+_{[\hat\mu\hat\nu\hat\rho\hat\sigma]}$	$\mathbf{8}\otimes\mathbf{8}$	$\mathbf{1}\oplus\mathbf{7}\oplus\mathbf{21}\oplus\mathbf{35}$	$a, C_{\mu 9}, C_{\mu\nu}, C_{\mu\nu\rho 9}$
$(\mathrm{NS}_+,\mathrm{R}_+)$	$\mathbf{8_v}\otimes\mathbf{8_s}$	$\lambda_a^{(1)}, \psi_a^{(1)\hat\mu}$	$\mathbf{7}\otimes\mathbf{8}$	$\mathbf{8}\oplus\mathbf{48}$	$\psi_a^{(1)9}, \psi_a^{(1)\mu}$
			$\mathbf{1}\otimes\mathbf{8}$	$\mathbf{8}$	$\lambda_a^{(1)}$
$(\mathrm{R}_+,\mathrm{NS}_+)$	$\mathbf{8_s}\otimes\mathbf{8_v}$	$\lambda_a^{(2)}, \psi_a^{(2)\hat\mu}$	$\mathbf{8}\otimes\mathbf{7}$	$\mathbf{8}\oplus\mathbf{48}$	$\psi_a^{(2)9}, \psi_a^{(2)\mu}$
			$\mathbf{8}\otimes\mathbf{1}$	$\mathbf{8}$	$\lambda_a^{(2)}$

Type II String: Field Theoretic Compactification

Type IIA					
sector	$SO(8)$	10d fields	$SO(7)$	$SO(7)$ irrep	9d fields
$(\mathrm{NS}_+,\mathrm{NS}_+)$	$\mathbf{8_v}\otimes\mathbf{8_v}$	$\Phi, B_{[\hat\mu\hat\nu]}, G_{(\hat\mu\hat\nu)}$	$\mathbf{7}\otimes\mathbf{7}$	$\mathbf{1}\oplus\mathbf{21}\oplus\mathbf{27}$	$\phi, B_{[\mu\nu]}, G_{(\mu\nu)}$
			$(\mathbf{7}\otimes\mathbf{1})\oplus(\mathbf{1}\otimes\mathbf{7})$	$\mathbf{7}\oplus\mathbf{7}$	$G_{\mu 9}, B_{\mu 9}$
			$\mathbf{1}\otimes\mathbf{1}$	$\mathbf{1}$	G_{99}
$(\mathrm{R}_+,\mathrm{R}_-)$	$\mathbf{8_s}\otimes\mathbf{8_c}$	$C_{\hat\mu}, C_{[\hat\mu\hat\nu\hat\rho]}$	$\mathbf{8}\otimes\mathbf{8}$	$\mathbf{1}\oplus\mathbf{7}\oplus\mathbf{21}\oplus\mathbf{35}$	$A_9, A_\mu, C_{9\mu\nu}, C_{\mu\nu\rho}$
$(\mathrm{NS}_+,\mathrm{R}_-)$	$\mathbf{8_v}\otimes\mathbf{8_c}$	$\tilde\lambda_a, \tilde\psi_a^{\hat\mu}$	$\mathbf{7}\otimes\mathbf{8}$	$\mathbf{8}\oplus\mathbf{48}$	$\tilde\psi_a^\mu, \tilde\psi_a^9$
			$\mathbf{1}\otimes\mathbf{8}$	$\mathbf{8}$	$\tilde\lambda_a$
$(\mathrm{R}_+,\mathrm{NS}_+)$	$\mathbf{8_s}\otimes\mathbf{8_v}$	$\lambda_a, \psi_a^{\hat\mu}$	$\mathbf{8}\otimes\mathbf{7}$	$\mathbf{8}\oplus\mathbf{48}$	ψ_a^μ, ψ_a^9
			$\mathbf{8}\otimes\mathbf{1}$	$\mathbf{8}$	λ_a
Type IIB					
sector	$SO(8)$	10d fields	$SO(7)$	$SO(7)$ irrep	9d fields
$(\mathrm{NS}_+,\mathrm{NS}_+)$	$\mathbf{8_v}\otimes\mathbf{8_v}$	$\Phi, B_{[\hat\mu\hat\nu]}, G_{(\hat\mu\hat\nu)}$	$\mathbf{7}\otimes\mathbf{7}$	$\mathbf{1}\oplus\mathbf{21}\oplus\mathbf{27}$	$\phi, B_{[\mu\nu]}, G_{(\mu\nu)}$
			$(\mathbf{7}\otimes\mathbf{1})\oplus(\mathbf{1}\otimes\mathbf{7})$	$\mathbf{7}\oplus\mathbf{7}$	$G_{\mu 9}, B_{\mu 9}$
			$\mathbf{1}\otimes\mathbf{1}$	$\mathbf{1}$	G_{99}
$(\mathrm{R}_+,\mathrm{R}_+)$	$\mathbf{8_s}\otimes\mathbf{8_s}$	$C_0, C_{[\hat\mu\hat\nu]}, C^+_{[\hat\mu\hat\nu\hat\rho\hat\sigma]}$	$\mathbf{8}\otimes\mathbf{8}$	$\mathbf{1}\oplus\mathbf{7}\oplus\mathbf{21}\oplus\mathbf{35}$	$a, C_{\mu 9}, C_{\mu\nu}, C_{\mu\nu\rho 9}$
$(\mathrm{NS}_+,\mathrm{R}_+)$	$\mathbf{8_v}\otimes\mathbf{8_s}$	$\lambda_a^{(1)}, \psi_a^{(1)\hat\mu}$	$\mathbf{7}\otimes\mathbf{8}$	$\mathbf{8}\oplus\mathbf{48}$	$\psi_a^{(1)9}, \psi_a^{(1)\mu}$
			$\mathbf{1}\otimes\mathbf{8}$	$\mathbf{8}$	$\lambda_a^{(1)}$
$(\mathrm{R}_+,\mathrm{NS}_+)$	$\mathbf{8_s}\otimes\mathbf{8_v}$	$\lambda_a^{(2)}, \psi_a^{(2)\hat\mu}$	$\mathbf{8}\otimes\mathbf{7}$	$\mathbf{8}\oplus\mathbf{48}$	$\psi_a^{(2)9}, \psi_a^{(2)\mu}$
			$\mathbf{8}\otimes\mathbf{1}$	$\mathbf{8}$	$\lambda_a^{(2)}$

sector	$SO(8)$	10d fields
$(\mathsf{NS}_+,\mathsf{NS}_+)$	$\mathbf{8_v} \otimes \mathbf{8_v}$	$\Phi, B_{[\hat{\mu}\hat{\nu}]}, G_{(\hat{\mu}\hat{\nu})}$
$(\mathsf{R}_+,\mathsf{R}_-)$	$\mathbf{8_s} \otimes \mathbf{8_c}$	$C_{\hat{\mu}}, C_{[\hat{\mu}\hat{\nu}\hat{\rho}]}$
$(\mathsf{NS}_+,\mathsf{R}_-)$	$\mathbf{8_v} \otimes \mathbf{8_c}$	$\tilde{\lambda}_a, \tilde{\psi}_a^{\hat{\mu}}$
$(\mathsf{R}_+,\mathsf{NS}_+)$	$\mathbf{8_s} \otimes \mathbf{8_v}$	$\lambda_a, \psi_a^{\hat{\mu}}$

Type II String: Field Theoretic Compactification

Type IIA					
sector	$SO(8)$	10d fields	$SO(7)$	$SO(7)$ irrep	9d fields
$(\mathrm{NS}_+,\mathrm{NS}_+)$	$\mathbf{8_v}\otimes\mathbf{8_v}$	$\Phi, B_{[\hat\mu\hat\nu]}, G_{(\hat\mu\hat\nu)}$	$\mathbf{7}\otimes\mathbf{7}$	$\mathbf{1}\oplus\mathbf{21}\oplus\mathbf{27}$	$\phi, B_{[\mu\nu]}, G_{(\mu\nu)}$
			$(\mathbf{7}\otimes\mathbf{1})\oplus(\mathbf{1}\otimes\mathbf{7})$	$\mathbf{7}\oplus\mathbf{7}$	$G_{\mu 9}, B_{\mu 9}$
			$\mathbf{1}\otimes\mathbf{1}$	$\mathbf{1}$	G_{99}
$(\mathrm{R}_+,\mathrm{R}_-)$	$\mathbf{8_s}\otimes\mathbf{8_c}$	$C_{\hat\mu}, C_{[\hat\mu\hat\nu\hat\rho]}$	$\mathbf{8}\otimes\mathbf{8}$	$\mathbf{1}\oplus\mathbf{7}\oplus\mathbf{21}\oplus\mathbf{35}$	$A_9, A_\mu, C_{9\mu\nu}, C_{\mu\nu\rho}$
$(\mathrm{NS}_+,\mathrm{R}_-)$	$\mathbf{8_v}\otimes\mathbf{8_c}$	$\tilde\lambda_a, \tilde\psi_a^{\hat\mu}$	$\mathbf{7}\otimes\mathbf{8}$	$\mathbf{8}\oplus\mathbf{48}$	$\tilde\psi_a^\mu, \tilde\psi_a^9$
			$\mathbf{1}\otimes\mathbf{8}$	$\mathbf{8}$	$\tilde\lambda_a$
$(\mathrm{R}_+,\mathrm{NS}_+)$	$\mathbf{8_s}\otimes\mathbf{8_v}$	$\lambda_a, \psi_a^{\hat\mu}$	$\mathbf{8}\otimes\mathbf{7}$	$\mathbf{8}\oplus\mathbf{48}$	ψ_a^μ, ψ_a^9
			$\mathbf{8}\otimes\mathbf{1}$	$\mathbf{8}$	λ_a
Type IIB					
sector	$SO(8)$	10d fields	$SO(7)$	$SO(7)$ irrep	9d fields
$(\mathrm{NS}_+,\mathrm{NS}_+)$	$\mathbf{8_v}\otimes\mathbf{8_v}$	$\Phi, B_{[\hat\mu\hat\nu]}, G_{(\hat\mu\hat\nu)}$	$\mathbf{7}\otimes\mathbf{7}$	$\mathbf{1}\oplus\mathbf{21}\oplus\mathbf{27}$	$\phi, B_{[\mu\nu]}, G_{(\mu\nu)}$
			$(\mathbf{7}\otimes\mathbf{1})\oplus(\mathbf{1}\otimes\mathbf{7})$	$\mathbf{7}\oplus\mathbf{7}$	$G_{\mu 9}, B_{\mu 9}$
			$\mathbf{1}\otimes\mathbf{1}$	$\mathbf{1}$	G_{99}
$(\mathrm{R}_+,\mathrm{R}_+)$	$\mathbf{8_s}\otimes\mathbf{8_s}$	$C_0, C_{[\hat\mu\hat\nu]}, C^+_{[\hat\mu\hat\nu\hat\rho\hat\sigma]}$	$\mathbf{8}\otimes\mathbf{8}$	$\mathbf{1}\oplus\mathbf{7}\oplus\mathbf{21}\oplus\mathbf{35}$	$a, C_{\mu 9}, C_{\mu\nu}, C_{\mu\nu\rho 9}$
$(\mathrm{NS}_+,\mathrm{R}_+)$	$\mathbf{8_v}\otimes\mathbf{8_s}$	$\lambda_a^{(1)}, \psi_a^{(1)\hat\mu}$	$\mathbf{7}\otimes\mathbf{8}$	$\mathbf{8}\oplus\mathbf{48}$	$\psi_a^{(1)9}, \psi_a^{(1)\mu}$
			$\mathbf{1}\otimes\mathbf{8}$	$\mathbf{8}$	$\lambda_a^{(1)}$
$(\mathrm{R}_+,\mathrm{NS}_+)$	$\mathbf{8_s}\otimes\mathbf{8_v}$	$\lambda_a^{(2)}, \psi_a^{(2)\hat\mu}$	$\mathbf{8}\otimes\mathbf{7}$	$\mathbf{8}\oplus\mathbf{48}$	$\psi_a^{(2)9}, \psi_a^{(2)\mu}$
			$\mathbf{8}\otimes\mathbf{1}$	$\mathbf{8}$	$\lambda_a^{(2)}$

Type II String: Field Theoretic Compactification

Type IIA					
sector	$SO(8)$	10d fields	$SO(7)$	$SO(7)$ irrep	9d fields
$(\mathrm{NS}_+,\mathrm{NS}_+)$	$\mathbf{8_v}\otimes\mathbf{8_v}$	$\Phi, B_{[\hat\mu\hat\nu]}, G_{(\hat\mu\hat\nu)}$	$\mathbf{7}\otimes\mathbf{7}$	$\mathbf{1}\oplus\mathbf{21}\oplus\mathbf{27}$	$\phi, B_{[\mu\nu]}, G_{(\mu\nu)}$
			$(\mathbf{7}\otimes\mathbf{1})\oplus(\mathbf{1}\otimes\mathbf{7})$	$\mathbf{7}\oplus\mathbf{7}$	$G_{\mu 9}, B_{\mu 9}$
			$\mathbf{1}\otimes\mathbf{1}$	$\mathbf{1}$	G_{99}
$(\mathrm{R}_+,\mathrm{R}_-)$	$\mathbf{8_s}\otimes\mathbf{8_c}$	$C_{\hat\mu}, C_{[\hat\mu\hat\nu\hat\rho]}$	$\mathbf{8}\otimes\mathbf{8}$	$\mathbf{1}\oplus\mathbf{7}\oplus\mathbf{21}\oplus\mathbf{35}$	$A_9, A_\mu, C_{9\mu\nu}, C_{\mu\nu\rho}$
$(\mathrm{NS}_+,\mathrm{R}_-)$	$\mathbf{8_v}\otimes\mathbf{8_c}$	$\tilde\lambda_a, \tilde\psi_a^{\hat\mu}$	$\mathbf{7}\otimes\mathbf{8}$	$\mathbf{8}\oplus\mathbf{48}$	$\tilde\psi_a^\mu, \tilde\psi_a^9$
			$\mathbf{1}\otimes\mathbf{8}$	$\mathbf{8}$	$\tilde\lambda_a$
$(\mathrm{R}_+,\mathrm{NS}_+)$	$\mathbf{8_s}\otimes\mathbf{8_v}$	$\lambda_a, \psi_a^{\hat\mu}$	$\mathbf{8}\otimes\mathbf{7}$	$\mathbf{8}\oplus\mathbf{48}$	ψ_a^μ, ψ_a^9
			$\mathbf{8}\otimes\mathbf{1}$	$\mathbf{8}$	λ_a
Type IIB					
sector	$SO(8)$	10d fields	$SO(7)$	$SO(7)$ irrep	9d fields
$(\mathrm{NS}_+,\mathrm{NS}_+)$	$\mathbf{8_v}\otimes\mathbf{8_v}$	$\Phi, B_{[\hat\mu\hat\nu]}, G_{(\hat\mu\hat\nu)}$	$\mathbf{7}\otimes\mathbf{7}$	$\mathbf{1}\oplus\mathbf{21}\oplus\mathbf{27}$	$\phi, B_{[\mu\nu]}, G_{(\mu\nu)}$
			$(\mathbf{7}\otimes\mathbf{1})\oplus(\mathbf{1}\otimes\mathbf{7})$	$\mathbf{7}\oplus\mathbf{7}$	$G_{\mu 9}, B_{\mu 9}$
			$\mathbf{1}\otimes\mathbf{1}$	$\mathbf{1}$	G_{99}
$(\mathrm{R}_+,\mathrm{R}_+)$	$\mathbf{8_s}\otimes\mathbf{8_s}$	$C_0, C_{[\hat\mu\hat\nu]}, C^+_{[\hat\mu\hat\nu\hat\rho\hat\sigma]}$	$\mathbf{8}\otimes\mathbf{8}$	$\mathbf{1}\oplus\mathbf{7}\oplus\mathbf{21}\oplus\mathbf{35}$	$a, C_{\mu 9}, C_{\mu\nu}, C_{\mu\nu\rho 9}$
$(\mathrm{NS}_+,\mathrm{R}_+)$	$\mathbf{8_v}\otimes\mathbf{8_s}$	$\lambda_a^{(1)}, \psi_a^{(1)\hat\mu}$	$\mathbf{7}\otimes\mathbf{8}$	$\mathbf{8}\oplus\mathbf{48}$	$\psi_a^{(1)9}, \psi_a^{(1)\mu}$
			$\mathbf{1}\otimes\mathbf{8}$	$\mathbf{8}$	$\lambda_a^{(1)}$
$(\mathrm{R}_+,\mathrm{NS}_+)$	$\mathbf{8_s}\otimes\mathbf{8_v}$	$\lambda_a^{(2)}, \psi_a^{(2)\hat\mu}$	$\mathbf{8}\otimes\mathbf{7}$	$\mathbf{8}\oplus\mathbf{48}$	$\psi_a^{(2)9}, \psi_a^{(2)\mu}$
			$\mathbf{8}\otimes\mathbf{1}$	$\mathbf{8}$	$\lambda_a^{(2)}$

Type IIA

$SO(7)$	$SO(7)$ irrep	9d fields
$\mathbf{7} \otimes \mathbf{7}$	$\mathbf{1} \oplus \mathbf{21} \oplus \mathbf{27}$	$\phi, B_{[\mu\nu]}, G_{(\mu\nu)}$
$(\mathbf{7} \otimes \mathbf{1}) \oplus (\mathbf{1} \otimes \mathbf{7})$	$\mathbf{7} \oplus \mathbf{7}$	$G_{\mu 9}, B_{\mu 9}$
$\mathbf{1} \otimes \mathbf{1}$	$\mathbf{1}$	G_{99}
$\mathbf{8} \otimes \mathbf{8}$	$\mathbf{1} \oplus \mathbf{7} \oplus \mathbf{21} \oplus \mathbf{35}$	$A_9, A_\mu, C_{9\mu\nu}, C_{\mu\nu\rho}$
$\mathbf{7} \otimes \mathbf{8}$	$\mathbf{8} \oplus \mathbf{48}$	$\tilde{\psi}^\mu_a, \tilde{\psi}^9_a$
$\mathbf{1} \otimes \mathbf{8}$	$\mathbf{8}$	$\tilde{\lambda}_a$
$\mathbf{8} \otimes \mathbf{7}$	$\mathbf{8} \oplus \mathbf{48}$	ψ^μ_a, ψ^9_a
$\mathbf{8} \otimes \mathbf{1}$	$\mathbf{8}$	λ_a

Type II String: Field Theoretic Compactification

Type IIA					
sector	$SO(8)$	10d fields	$SO(7)$	$SO(7)$ irrep	9d fields
$(\mathrm{NS}_+,\mathrm{NS}_+)$	$\mathbf{8_v}\otimes\mathbf{8_v}$	$\Phi, B_{[\hat\mu\hat\nu]}, G_{(\hat\mu\hat\nu)}$	$\mathbf{7}\otimes\mathbf{7}$	$\mathbf{1}\oplus\mathbf{21}\oplus\mathbf{27}$	$\phi, B_{[\mu\nu]}, G_{(\mu\nu)}$
			$(\mathbf{7}\otimes\mathbf{1})\oplus(\mathbf{1}\otimes\mathbf{7})$	$\mathbf{7}\oplus\mathbf{7}$	$G_{\mu 9}, B_{\mu 9}$
			$\mathbf{1}\otimes\mathbf{1}$	$\mathbf{1}$	G_{99}
$(\mathrm{R}_+,\mathrm{R}_-)$	$\mathbf{8_s}\otimes\mathbf{8_c}$	$C_{\hat\mu}, C_{[\hat\mu\hat\nu\hat\rho]}$	$\mathbf{8}\otimes\mathbf{8}$	$\mathbf{1}\oplus\mathbf{7}\oplus\mathbf{21}\oplus\mathbf{35}$	$A_9, A_\mu, C_{9\mu\nu}, C_{\mu\nu\rho}$
$(\mathrm{NS}_+,\mathrm{R}_-)$	$\mathbf{8_v}\otimes\mathbf{8_c}$	$\tilde\lambda_a, \tilde\psi_a^{\hat\mu}$	$\mathbf{7}\otimes\mathbf{8}$	$\mathbf{8}\oplus\mathbf{48}$	$\tilde\psi_a^\mu, \tilde\psi_a^9$
			$\mathbf{1}\otimes\mathbf{8}$	$\mathbf{8}$	$\tilde\lambda_a$
$(\mathrm{R}_+,\mathrm{NS}_+)$	$\mathbf{8_s}\otimes\mathbf{8_v}$	$\lambda_a, \psi_a^{\hat\mu}$	$\mathbf{8}\otimes\mathbf{7}$	$\mathbf{8}\oplus\mathbf{48}$	ψ_a^μ, ψ_a^9
			$\mathbf{8}\otimes\mathbf{1}$	$\mathbf{8}$	λ_a
Type IIB					
sector	$SO(8)$	10d fields	$SO(7)$	$SO(7)$ irrep	9d fields
$(\mathrm{NS}_+,\mathrm{NS}_+)$	$\mathbf{8_v}\otimes\mathbf{8_v}$	$\Phi, B_{[\hat\mu\hat\nu]}, G_{(\hat\mu\hat\nu)}$	$\mathbf{7}\otimes\mathbf{7}$	$\mathbf{1}\oplus\mathbf{21}\oplus\mathbf{27}$	$\phi, B_{[\mu\nu]}, G_{(\mu\nu)}$
			$(\mathbf{7}\otimes\mathbf{1})\oplus(\mathbf{1}\otimes\mathbf{7})$	$\mathbf{7}\oplus\mathbf{7}$	$G_{\mu 9}, B_{\mu 9}$
			$\mathbf{1}\otimes\mathbf{1}$	$\mathbf{1}$	G_{99}
$(\mathrm{R}_+,\mathrm{R}_+)$	$\mathbf{8_s}\otimes\mathbf{8_s}$	$C_0, C_{[\hat\mu\hat\nu]}, C^+_{[\hat\mu\hat\nu\hat\rho\hat\sigma]}$	$\mathbf{8}\otimes\mathbf{8}$	$\mathbf{1}\oplus\mathbf{7}\oplus\mathbf{21}\oplus\mathbf{35}$	$a, C_{\mu 9}, C_{\mu\nu}, C_{\mu\nu\rho 9}$
$(\mathrm{NS}_+,\mathrm{R}_+)$	$\mathbf{8_v}\otimes\mathbf{8_s}$	$\lambda_a^{(1)}, \psi_a^{(1)\hat\mu}$	$\mathbf{7}\otimes\mathbf{8}$	$\mathbf{8}\oplus\mathbf{48}$	$\psi_a^{(1)9}, \psi_a^{(1)\mu}$
			$\mathbf{1}\otimes\mathbf{8}$	$\mathbf{8}$	$\lambda_a^{(1)}$
$(\mathrm{R}_+,\mathrm{NS}_+)$	$\mathbf{8_s}\otimes\mathbf{8_v}$	$\lambda_a^{(2)}, \psi_a^{(2)\hat\mu}$	$\mathbf{8}\otimes\mathbf{7}$	$\mathbf{8}\oplus\mathbf{48}$	$\psi_a^{(2)9}, \psi_a^{(2)\mu}$
			$\mathbf{8}\otimes\mathbf{1}$	$\mathbf{8}$	$\lambda_a^{(2)}$

Type II String: Field Theoretic Compactification

Type IIA					
sector	$SO(8)$	10d fields	$SO(7)$	$SO(7)$ irrep	9d fields
$(\mathrm{NS}_+,\mathrm{NS}_+)$	$\mathbf{8_v}\otimes\mathbf{8_v}$	$\Phi, B_{[\hat\mu\hat\nu]}, G_{(\hat\mu\hat\nu)}$	$\mathbf{7}\otimes\mathbf{7}$	$\mathbf{1}\oplus\mathbf{21}\oplus\mathbf{27}$	$\phi, B_{[\mu\nu]}, G_{(\mu\nu)}$
			$(\mathbf{7}\otimes\mathbf{1})\oplus(\mathbf{1}\otimes\mathbf{7})$	$\mathbf{7}\oplus\mathbf{7}$	$G_{\mu 9}, B_{\mu 9}$
			$\mathbf{1}\otimes\mathbf{1}$	$\mathbf{1}$	G_{99}
$(\mathrm{R}_+,\mathrm{R}_-)$	$\mathbf{8_s}\otimes\mathbf{8_c}$	$C_{\hat\mu}, C_{[\hat\mu\hat\nu\hat\rho]}$	$\mathbf{8}\otimes\mathbf{8}$	$\mathbf{1}\oplus\mathbf{7}\oplus\mathbf{21}\oplus\mathbf{35}$	$A_9, A_\mu, C_{9\mu\nu}, C_{\mu\nu\rho}$
$(\mathrm{NS}_+,\mathrm{R}_-)$	$\mathbf{8_v}\otimes\mathbf{8_c}$	$\tilde\lambda_a, \tilde\psi_a^{\hat\mu}$	$\mathbf{7}\otimes\mathbf{8}$	$\mathbf{8}\oplus\mathbf{48}$	$\tilde\psi_a^\mu, \tilde\psi_a^9$
			$\mathbf{1}\otimes\mathbf{8}$	$\mathbf{8}$	$\tilde\lambda_a$
$(\mathrm{R}_+,\mathrm{NS}_+)$	$\mathbf{8_s}\otimes\mathbf{8_v}$	$\lambda_a, \psi_a^{\hat\mu}$	$\mathbf{8}\otimes\mathbf{7}$	$\mathbf{8}\oplus\mathbf{48}$	ψ_a^μ, ψ_a^9
			$\mathbf{8}\otimes\mathbf{1}$	$\mathbf{8}$	λ_a
Type IIB					
sector	$SO(8)$	10d fields	$SO(7)$	$SO(7)$ irrep	9d fields
$(\mathrm{NS}_+,\mathrm{NS}_+)$	$\mathbf{8_v}\otimes\mathbf{8_v}$	$\Phi, B_{[\hat\mu\hat\nu]}, G_{(\hat\mu\hat\nu)}$	$\mathbf{7}\otimes\mathbf{7}$	$\mathbf{1}\oplus\mathbf{21}\oplus\mathbf{27}$	$\phi, B_{[\mu\nu]}, G_{(\mu\nu)}$
			$(\mathbf{7}\otimes\mathbf{1})\oplus(\mathbf{1}\otimes\mathbf{7})$	$\mathbf{7}\oplus\mathbf{7}$	$G_{\mu 9}, B_{\mu 9}$
			$\mathbf{1}\otimes\mathbf{1}$	$\mathbf{1}$	G_{99}
$(\mathrm{R}_+,\mathrm{R}_+)$	$\mathbf{8_s}\otimes\mathbf{8_s}$	$C_0, C_{[\hat\mu\hat\nu]}, C^+_{[\hat\mu\hat\nu\hat\rho\hat\sigma]}$	$\mathbf{8}\otimes\mathbf{8}$	$\mathbf{1}\oplus\mathbf{7}\oplus\mathbf{21}\oplus\mathbf{35}$	$a, C_{\mu 9}, C_{\mu\nu}, C_{\mu\nu\rho 9}$
$(\mathrm{NS}_+,\mathrm{R}_+)$	$\mathbf{8_v}\otimes\mathbf{8_s}$	$\lambda_a^{(1)}, \psi_a^{(1)\hat\mu}$	$\mathbf{7}\otimes\mathbf{8}$	$\mathbf{8}\oplus\mathbf{48}$	$\psi_a^{(1)9}, \psi_a^{(1)\mu}$
			$\mathbf{1}\otimes\mathbf{8}$	$\mathbf{8}$	$\lambda_a^{(1)}$
$(\mathrm{R}_+,\mathrm{NS}_+)$	$\mathbf{8_s}\otimes\mathbf{8_v}$	$\lambda_a^{(2)}, \psi_a^{(2)\hat\mu}$	$\mathbf{8}\otimes\mathbf{7}$	$\mathbf{8}\oplus\mathbf{48}$	$\psi_a^{(2)9}, \psi_a^{(2)\mu}$
			$\mathbf{8}\otimes\mathbf{1}$	$\mathbf{8}$	$\lambda_a^{(2)}$

sector	$SO(8)$	10d fields
$(\mathsf{NS}_+,\mathsf{NS}_+)$	$\mathbf{8_v} \otimes \mathbf{8_v}$	$\Phi, B_{[\hat{\mu}\hat{\nu}]}, G_{(\hat{\mu}\hat{\nu})}$
$(\mathsf{R}_+,\mathsf{R}_+)$	$\mathbf{8_s} \otimes \mathbf{8_s}$	$C_0, C_{[\hat{\mu}\hat{\nu}]}, C^+_{[\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}]}$
$(\mathsf{NS}_+,\mathsf{R}_+)$	$\mathbf{8_v} \otimes \mathbf{8_s}$	$\lambda^{(1)}_a, \psi^{(1)\hat{\mu}}_a$
$(\mathsf{R}_+,\mathsf{NS}_+)$	$\mathbf{8_s} \otimes \mathbf{8_v}$	$\lambda^{(2)}_a, \psi^{(2)\hat{\mu}}_a$

Type II String: Field Theoretic Compactification

Type IIA					
sector	$SO(8)$	10d fields	$SO(7)$	$SO(7)$ irrep	9d fields
$(\mathrm{NS}_+,\mathrm{NS}_+)$	$\mathbf{8_v}\otimes\mathbf{8_v}$	$\Phi, B_{[\hat\mu\hat\nu]}, G_{(\hat\mu\hat\nu)}$	$\mathbf{7}\otimes\mathbf{7}$	$\mathbf{1}\oplus\mathbf{21}\oplus\mathbf{27}$	$\phi, B_{[\mu\nu]}, G_{(\mu\nu)}$
			$(\mathbf{7}\otimes\mathbf{1})\oplus(\mathbf{1}\otimes\mathbf{7})$	$\mathbf{7}\oplus\mathbf{7}$	$G_{\mu 9}, B_{\mu 9}$
			$\mathbf{1}\otimes\mathbf{1}$	$\mathbf{1}$	G_{99}
$(\mathrm{R}_+,\mathrm{R}_-)$	$\mathbf{8_s}\otimes\mathbf{8_c}$	$C_{\hat\mu}, C_{[\hat\mu\hat\nu\hat\rho]}$	$\mathbf{8}\otimes\mathbf{8}$	$\mathbf{1}\oplus\mathbf{7}\oplus\mathbf{21}\oplus\mathbf{35}$	$A_9, A_\mu, C_{9\mu\nu}, C_{\mu\nu\rho}$
$(\mathrm{NS}_+,\mathrm{R}_-)$	$\mathbf{8_v}\otimes\mathbf{8_c}$	$\tilde\lambda_a, \tilde\psi_a^{\hat\mu}$	$\mathbf{7}\otimes\mathbf{8}$	$\mathbf{8}\oplus\mathbf{48}$	$\tilde\psi_a^\mu, \tilde\psi_a^9$
			$\mathbf{1}\otimes\mathbf{8}$	$\mathbf{8}$	$\tilde\lambda_a$
$(\mathrm{R}_+,\mathrm{NS}_+)$	$\mathbf{8_s}\otimes\mathbf{8_v}$	$\lambda_a, \psi_a^{\hat\mu}$	$\mathbf{8}\otimes\mathbf{7}$	$\mathbf{8}\oplus\mathbf{48}$	ψ_a^μ, ψ_a^9
			$\mathbf{8}\otimes\mathbf{1}$	$\mathbf{8}$	λ_a
Type IIB					
sector	$SO(8)$	10d fields	$SO(7)$	$SO(7)$ irrep	9d fields
$(\mathrm{NS}_+,\mathrm{NS}_+)$	$\mathbf{8_v}\otimes\mathbf{8_v}$	$\Phi, B_{[\hat\mu\hat\nu]}, G_{(\hat\mu\hat\nu)}$	$\mathbf{7}\otimes\mathbf{7}$	$\mathbf{1}\oplus\mathbf{21}\oplus\mathbf{27}$	$\phi, B_{[\mu\nu]}, G_{(\mu\nu)}$
			$(\mathbf{7}\otimes\mathbf{1})\oplus(\mathbf{1}\otimes\mathbf{7})$	$\mathbf{7}\oplus\mathbf{7}$	$G_{\mu 9}, B_{\mu 9}$
			$\mathbf{1}\otimes\mathbf{1}$	$\mathbf{1}$	G_{99}
$(\mathrm{R}_+,\mathrm{R}_+)$	$\mathbf{8_s}\otimes\mathbf{8_s}$	$C_0, C_{[\hat\mu\hat\nu]}, C^+_{[\hat\mu\hat\nu\hat\rho\hat\sigma]}$	$\mathbf{8}\otimes\mathbf{8}$	$\mathbf{1}\oplus\mathbf{7}\oplus\mathbf{21}\oplus\mathbf{35}$	$a, C_{\mu 9}, C_{\mu\nu}, C_{\mu\nu\rho 9}$
$(\mathrm{NS}_+,\mathrm{R}_+)$	$\mathbf{8_v}\otimes\mathbf{8_s}$	$\lambda_a^{(1)}, \psi_a^{(1)\hat\mu}$	$\mathbf{7}\otimes\mathbf{8}$	$\mathbf{8}\oplus\mathbf{48}$	$\psi_a^{(1)9}, \psi_a^{(1)\mu}$
			$\mathbf{1}\otimes\mathbf{8}$	$\mathbf{8}$	$\lambda_a^{(1)}$
$(\mathrm{R}_+,\mathrm{NS}_+)$	$\mathbf{8_s}\otimes\mathbf{8_v}$	$\lambda_a^{(2)}, \psi_a^{(2)\hat\mu}$	$\mathbf{8}\otimes\mathbf{7}$	$\mathbf{8}\oplus\mathbf{48}$	$\psi_a^{(2)9}, \psi_a^{(2)\mu}$
			$\mathbf{8}\otimes\mathbf{1}$	$\mathbf{8}$	$\lambda_a^{(2)}$

Type II String: Field Theoretic Compactification

Type IIA					
sector	$SO(8)$	10d fields	$SO(7)$	$SO(7)$ irrep	9d fields
$(\mathrm{NS}_+,\mathrm{NS}_+)$	$\mathbf{8_v}\otimes\mathbf{8_v}$	$\Phi, B_{[\hat\mu\hat\nu]}, G_{(\hat\mu\hat\nu)}$	$\mathbf{7}\otimes\mathbf{7}$	$\mathbf{1}\oplus\mathbf{21}\oplus\mathbf{27}$	$\phi, B_{[\mu\nu]}, G_{(\mu\nu)}$
			$(\mathbf{7}\otimes\mathbf{1})\oplus(\mathbf{1}\otimes\mathbf{7})$	$\mathbf{7}\oplus\mathbf{7}$	$G_{\mu 9}, B_{\mu 9}$
			$\mathbf{1}\otimes\mathbf{1}$	$\mathbf{1}$	G_{99}
$(\mathrm{R}_+,\mathrm{R}_-)$	$\mathbf{8_s}\otimes\mathbf{8_c}$	$C_{\hat\mu}, C_{[\hat\mu\hat\nu\hat\rho]}$	$\mathbf{8}\otimes\mathbf{8}$	$\mathbf{1}\oplus\mathbf{7}\oplus\mathbf{21}\oplus\mathbf{35}$	$A_9, A_\mu, C_{9\mu\nu}, C_{\mu\nu\rho}$
$(\mathrm{NS}_+,\mathrm{R}_-)$	$\mathbf{8_v}\otimes\mathbf{8_c}$	$\tilde\lambda_a, \tilde\psi_a^{\hat\mu}$	$\mathbf{7}\otimes\mathbf{8}$	$\mathbf{8}\oplus\mathbf{48}$	$\tilde\psi_a^\mu, \tilde\psi_a^9$
			$\mathbf{1}\otimes\mathbf{8}$	$\mathbf{8}$	$\tilde\lambda_a$
$(\mathrm{R}_+,\mathrm{NS}_+)$	$\mathbf{8_s}\otimes\mathbf{8_v}$	$\lambda_a, \psi_a^{\hat\mu}$	$\mathbf{8}\otimes\mathbf{7}$	$\mathbf{8}\oplus\mathbf{48}$	ψ_a^μ, ψ_a^9
			$\mathbf{8}\otimes\mathbf{1}$	$\mathbf{8}$	λ_a
Type IIB					
sector	$SO(8)$	10d fields	$SO(7)$	$SO(7)$ irrep	9d fields
$(\mathrm{NS}_+,\mathrm{NS}_+)$	$\mathbf{8_v}\otimes\mathbf{8_v}$	$\Phi, B_{[\hat\mu\hat\nu]}, G_{(\hat\mu\hat\nu)}$	$\mathbf{7}\otimes\mathbf{7}$	$\mathbf{1}\oplus\mathbf{21}\oplus\mathbf{27}$	$\phi, B_{[\mu\nu]}, G_{(\mu\nu)}$
			$(\mathbf{7}\otimes\mathbf{1})\oplus(\mathbf{1}\otimes\mathbf{7})$	$\mathbf{7}\oplus\mathbf{7}$	$G_{\mu 9}, B_{\mu 9}$
			$\mathbf{1}\otimes\mathbf{1}$	$\mathbf{1}$	G_{99}
$(\mathrm{R}_+,\mathrm{R}_+)$	$\mathbf{8_s}\otimes\mathbf{8_s}$	$C_0, C_{[\hat\mu\hat\nu]}, C^+_{[\hat\mu\hat\nu\hat\rho\hat\sigma]}$	$\mathbf{8}\otimes\mathbf{8}$	$\mathbf{1}\oplus\mathbf{7}\oplus\mathbf{21}\oplus\mathbf{35}$	$a, C_{\mu 9}, C_{\mu\nu}, C_{\mu\nu\rho 9}$
$(\mathrm{NS}_+,\mathrm{R}_+)$	$\mathbf{8_v}\otimes\mathbf{8_s}$	$\lambda_a^{(1)}, \psi_a^{(1)\hat\mu}$	$\mathbf{7}\otimes\mathbf{8}$	$\mathbf{8}\oplus\mathbf{48}$	$\psi_a^{(1)9}, \psi_a^{(1)\mu}$
			$\mathbf{1}\otimes\mathbf{8}$	$\mathbf{8}$	$\lambda_a^{(1)}$
$(\mathrm{R}_+,\mathrm{NS}_+)$	$\mathbf{8_s}\otimes\mathbf{8_v}$	$\lambda_a^{(2)}, \psi_a^{(2)\hat\mu}$	$\mathbf{8}\otimes\mathbf{7}$	$\mathbf{8}\oplus\mathbf{48}$	$\psi_a^{(2)9}, \psi_a^{(2)\mu}$
			$\mathbf{8}\otimes\mathbf{1}$	$\mathbf{8}$	$\lambda_a^{(2)}$

Type IIB

$SO(7)$	$SO(7)$ irrep	9d fields
$\mathbf{7} \otimes \mathbf{7}$	$\mathbf{1} \oplus \mathbf{21} \oplus \mathbf{27}$	$\phi, B_{[\mu\nu]}, G_{(\mu\nu)}$
$(\mathbf{7} \otimes \mathbf{1}) \oplus (\mathbf{1} \otimes \mathbf{7})$	$\mathbf{7} \oplus \mathbf{7}$	$G_{\mu 9}, B_{\mu 9}$
$\mathbf{1} \otimes \mathbf{1}$	$\mathbf{1}$	G_{99}
$\mathbf{8} \otimes \mathbf{8}$	$\mathbf{1} \oplus \mathbf{7} \oplus \mathbf{21} \oplus \mathbf{35}$	$a, C_{\mu 9}, C_{\mu\nu}, C_{\mu\nu\rho 9}$
$\mathbf{7} \otimes \mathbf{8}$	$\mathbf{8} \oplus \mathbf{48}$	$\psi_a^{(1)9}, \psi_a^{(1)\mu}$
$\mathbf{1} \otimes \mathbf{8}$	$\mathbf{8}$	$\lambda_a^{(1)}$
$\mathbf{8} \otimes \mathbf{7}$	$\mathbf{8} \oplus \mathbf{48}$	$\psi_a^{(2)9}, \psi_a^{(2)\mu}$
$\mathbf{8} \otimes \mathbf{1}$	$\mathbf{8}$	$\lambda_a^{(2)}$

Type II String: Field Theoretic Compactification

Type IIA					
sector	$SO(8)$	10d fields	$SO(7)$	$SO(7)$ irrep	9d fields
$(\mathrm{NS}_+,\mathrm{NS}_+)$	$\mathbf{8_v}\otimes\mathbf{8_v}$	$\Phi, B_{[\hat\mu\hat\nu]}, G_{(\hat\mu\hat\nu)}$	$\mathbf{7}\otimes\mathbf{7}$	$\mathbf{1}\oplus\mathbf{21}\oplus\mathbf{27}$	$\phi, B_{[\mu\nu]}, G_{(\mu\nu)}$
			$(\mathbf{7}\otimes\mathbf{1})\oplus(\mathbf{1}\otimes\mathbf{7})$	$\mathbf{7}\oplus\mathbf{7}$	$G_{\mu 9}, B_{\mu 9}$
			$\mathbf{1}\otimes\mathbf{1}$	$\mathbf{1}$	G_{99}
$(\mathrm{R}_+,\mathrm{R}_-)$	$\mathbf{8_s}\otimes\mathbf{8_c}$	$C_{\hat\mu}, C_{[\hat\mu\hat\nu\hat\rho]}$	$\mathbf{8}\otimes\mathbf{8}$	$\mathbf{1}\oplus\mathbf{7}\oplus\mathbf{21}\oplus\mathbf{35}$	$A_9, A_\mu, C_{9\mu\nu}, C_{\mu\nu\rho}$
$(\mathrm{NS}_+,\mathrm{R}_-)$	$\mathbf{8_v}\otimes\mathbf{8_c}$	$\tilde\lambda_a, \tilde\psi_a^{\hat\mu}$	$\mathbf{7}\otimes\mathbf{8}$	$\mathbf{8}\oplus\mathbf{48}$	$\tilde\psi_a^\mu, \tilde\psi_a^9$
			$\mathbf{1}\otimes\mathbf{8}$	$\mathbf{8}$	$\tilde\lambda_a$
$(\mathrm{R}_+,\mathrm{NS}_+)$	$\mathbf{8_s}\otimes\mathbf{8_v}$	$\lambda_a, \psi_a^{\hat\mu}$	$\mathbf{8}\otimes\mathbf{7}$	$\mathbf{8}\oplus\mathbf{48}$	ψ_a^μ, ψ_a^9
			$\mathbf{8}\otimes\mathbf{1}$	$\mathbf{8}$	λ_a
Type IIB					
sector	$SO(8)$	10d fields	$SO(7)$	$SO(7)$ irrep	9d fields
$(\mathrm{NS}_+,\mathrm{NS}_+)$	$\mathbf{8_v}\otimes\mathbf{8_v}$	$\Phi, B_{[\hat\mu\hat\nu]}, G_{(\hat\mu\hat\nu)}$	$\mathbf{7}\otimes\mathbf{7}$	$\mathbf{1}\oplus\mathbf{21}\oplus\mathbf{27}$	$\phi, B_{[\mu\nu]}, G_{(\mu\nu)}$
			$(\mathbf{7}\otimes\mathbf{1})\oplus(\mathbf{1}\otimes\mathbf{7})$	$\mathbf{7}\oplus\mathbf{7}$	$G_{\mu 9}, B_{\mu 9}$
			$\mathbf{1}\otimes\mathbf{1}$	$\mathbf{1}$	G_{99}
$(\mathrm{R}_+,\mathrm{R}_+)$	$\mathbf{8_s}\otimes\mathbf{8_s}$	$C_0, C_{[\hat\mu\hat\nu]}, C^+_{[\hat\mu\hat\nu\hat\rho\hat\sigma]}$	$\mathbf{8}\otimes\mathbf{8}$	$\mathbf{1}\oplus\mathbf{7}\oplus\mathbf{21}\oplus\mathbf{35}$	$a, C_{\mu 9}, C_{\mu\nu}, C_{\mu\nu\rho 9}$
$(\mathrm{NS}_+,\mathrm{R}_+)$	$\mathbf{8_v}\otimes\mathbf{8_s}$	$\lambda_a^{(1)}, \psi_a^{(1)\hat\mu}$	$\mathbf{7}\otimes\mathbf{8}$	$\mathbf{8}\oplus\mathbf{48}$	$\psi_a^{(1)9}, \psi_a^{(1)\mu}$
			$\mathbf{1}\otimes\mathbf{8}$	$\mathbf{8}$	$\lambda_a^{(1)}$
$(\mathrm{R}_+,\mathrm{NS}_+)$	$\mathbf{8_s}\otimes\mathbf{8_v}$	$\lambda_a^{(2)}, \psi_a^{(2)\hat\mu}$	$\mathbf{8}\otimes\mathbf{7}$	$\mathbf{8}\oplus\mathbf{48}$	$\psi_a^{(2)9}, \psi_a^{(2)\mu}$
			$\mathbf{8}\otimes\mathbf{1}$	$\mathbf{8}$	$\lambda_a^{(2)}$

Is This a Coincidence?

Is This a Coincidence?
is This a Feature of the Massless Spectrum?

Type II String: Compactification on S^1

Setting and Main Results

Lightcone quantization:

- $\mathcal{M}_{10} \to \mathcal{M}_9 \times S^1, X^9 \simeq X^9 + 2\pi R$;
- $X^i(\tau, \sigma + l) = X^i(\tau, \sigma)$;

Type II String: Compactification on S^1

Setting and Main Results

Lightcone quantization:

- $\mathcal{M}_{10} \to \mathcal{M}_9 \times S^1, X^9 \simeq X^9 + 2\pi R$;
- $X^i(\tau, \sigma + l) = X^i(\tau, \sigma)$;
- $X^9(\tau, \sigma + l) = X^9(\tau, \sigma) + 2\pi R\omega$;
- $p_9 = s/R$, $s \in \mathbb{Z}$.

Type II String: Compactification on S^1

Setting and Main Results

Lightcone quantization:

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Mode decomposition:

- $X^9_{L/R}(\xi^\pm) = \frac{x^9}{2} + \frac{\alpha'\pi}{l} p_{L/R}\xi^\pm + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \frac{\overset{(\sim)}{\alpha^9_n}}{n} e^{-\frac{2\pi i}{l} n\xi^\pm}$;
- $p_L = \left(\frac{s}{R} + \frac{\omega R}{\alpha'}\right), p_R = \left(\frac{s}{R} - \frac{\omega R}{\alpha'}\right)$;

Setting and Main Results

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- $\mathcal{M}_{10} \to \mathcal{M}_9 \times S^1, X^9 \simeq X^9 + 2\pi R$;
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Mode decomposition:

- $X^9_{L/R}(\xi^\pm) = \frac{x^9}{2} + \frac{\alpha'\pi}{l} p_{L/R}\xi^\pm + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \frac{\overset{(\sim)}{\alpha^9_n}}{n} e^{-\frac{2\pi i}{l} n\xi^\pm}$;
- $p_L = \left(\frac{s}{R} + \frac{\omega R}{\alpha'}\right), p_R = \left(\frac{s}{R} - \frac{\omega R}{\alpha'}\right)$;
- $M_L^2 = \frac{p_L^2}{2} + \frac{2}{\alpha'}\left(\tilde{N}_\perp - \tilde{a}\right), \quad M_R^2 = \frac{p_R^2}{2} + \frac{2}{\alpha'}\left(N_\perp - a\right)$;
- $M_L^2 = M_R^2$, $M^2 = M_L^2 + M_R^2$.

T-Duality: Introduction

Mass-Shell Condition

$$M^2 = \frac{s^2}{R^2} + \frac{\omega^2 R^2}{\alpha'^2} + \frac{2}{\alpha'}(\tilde{N}_\perp + N_\perp - a - \tilde{a}),$$

$$p_L = \left(\frac{s}{R} + \frac{\omega R}{\alpha'}\right), p_R = \left(\frac{s}{R} - \frac{\omega R}{\alpha'}\right).$$

T-Duality

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$$M^2 = \frac{s^2}{R^2} + \frac{\omega^2 R^2}{\alpha'^2} + \frac{2}{\alpha'}(\tilde{N}_\perp + N_\perp - a - \tilde{a}),$$

$$p_L = \left(\frac{s}{R} + \frac{\omega R}{\alpha'}\right), p_R = \left(\frac{s}{R} - \frac{\omega R}{\alpha'}\right).$$

From Spectrum

$$R \to R' = \alpha'/R,$$

$$(s, \omega) \to (s', \omega') = (\omega, s).$$

T-Duality

Mass-Shell Condition

$$M^2 = \frac{s^2}{R^2} + \frac{\omega^2 R^2}{\alpha'^2} + \frac{2}{\alpha'}(\tilde{N}_\perp + N_\perp - a - \tilde{a}),$$
$$p_L = \left(\frac{s}{R} + \frac{\omega R}{\alpha'}\right), p_R = \left(\frac{s}{R} - \frac{\omega R}{\alpha'}\right).$$

From Spectrum

$$R \to R' = \alpha'/R,$$
$$(s, \omega) \to (s', \omega') = (\omega, s).$$

From Full Theory

$$p_L \to p_L, p_R \to -p_R,$$
$$X_L^9 \to X_L^9, X_R^9 \to -X_R^9.$$

T-Duality

Mass-Shell Condition

$$M^2 = \frac{s^2}{R^2} + \frac{\omega^2 R^2}{\alpha'^2} + \frac{2}{\alpha'}(\tilde{N}_\perp + N_\perp - a - \tilde{a}),$$
$$p_L = \left(\frac{s}{R} + \frac{\omega R}{\alpha'}\right), p_R = \left(\frac{s}{R} - \frac{\omega R}{\alpha'}\right).$$

From Spectrum

$$R \to R' = \alpha'/R,$$
$$(s, \omega) \to (s', \omega') = (\omega, s).$$

From Full Theory

$$p_L \to p_L, p_R \to -p_R,$$
$$X^9_L \to X^9_L, X^9_R \to -X^9_R.$$

from Worldsheet Supersymmetry

$$\psi^9_L \to \psi^9_L, \ \psi^9_R \to -\psi^9_R.$$

T-Duality: Consequence

T-Duality

Transformation:

$$X^9(\tau,\sigma) = X^9_L(\xi^+) + X^9_R(\xi^-) \to X'^9(\tau,\sigma) = X^9_L(\xi^+) - X^9_R(\xi^-),$$
$$\psi^9(\tau,\sigma) = \psi^9_L(\xi^+) + \psi^9_R(\xi^-) \to \psi'^9(\tau,\sigma) = \psi^9_L(\xi^+) - \psi^9_R(\xi^-).$$

T-Duality: Consequence

T-Duality

Transformation:

$$X^9(\tau,\sigma) = X^9_L(\xi^+) + X^9_R(\xi^-) \to X'^9(\tau,\sigma) = X^9_L(\xi^+) - X^9_R(\xi^-),$$
$$\psi^9(\tau,\sigma) = \psi^9_L(\xi^+) + \psi^9_R(\xi^-) \to \psi'^9(\tau,\sigma) = \psi^9_L(\xi^+) - \psi^9_R(\xi^-).$$

On the modes:

$$\tilde{b}^9_r \to \tilde{b}'^9_r = \tilde{b}^9_r, \quad b^9_r \to b'^9_r = -b^9_r,$$
$$B^\pm_4 = \frac{1}{\sqrt{2}}\left(b^8_0 \pm i b^9_0\right) \to \frac{1}{\sqrt{2}}\left(b^8_0 \mp i b^9_0\right) = B^\mp_4.$$

T-Duality: Consequence

T-Duality

Transformation:

$$X^9(\tau,\sigma) = X^9_L(\xi^+) + X^9_R(\xi^-) \to X'^9(\tau,\sigma) = X^9_L(\xi^+) - X^9_R(\xi^-),$$
$$\psi^9(\tau,\sigma) = \psi^9_L(\xi^+) + \psi^9_R(\xi^-) \to \psi'^9(\tau,\sigma) = \psi^9_L(\xi^+) - \psi^9_R(\xi^-).$$

On the modes:

$$\tilde{b}^9_r \to \tilde{b}'^9_r = \tilde{b}^9_r, \quad b^9_r \to b'^9_r = -b^9_r,$$
$$B^\pm_4 = \frac{1}{\sqrt{2}}\left(b^8_0 \pm i b^9_0\right) \to \frac{1}{\sqrt{2}}\left(b^8_0 \mp i b^9_0\right) = B^\mp_4.$$

On the sectors:

$$(\mathsf{R}_+, \mathsf{R}_\pm) \to (\mathsf{R}_+, \mathsf{R}_\mp),$$
$$(\mathsf{NS}_+, \mathsf{R}_\pm) \to (\mathsf{NS}_+, \mathsf{R}_\mp).$$

T-Duality: Conclusion

Type IIB on S^1 with R $\cong$ Type IIA on S'^1 with $R' = \dfrac{\alpha'}{R}$.

T-Duality: Conclusion

$$X^9(\tau,\sigma) = X_L^9(\xi^+) + X_R^9(\xi^-),$$
$$X'^9(\tau,\sigma) = X_L^9(\xi^+) - X_R^9(\xi^-).$$

Then, What Really is Spacetime?