Georgia Institute of Technology

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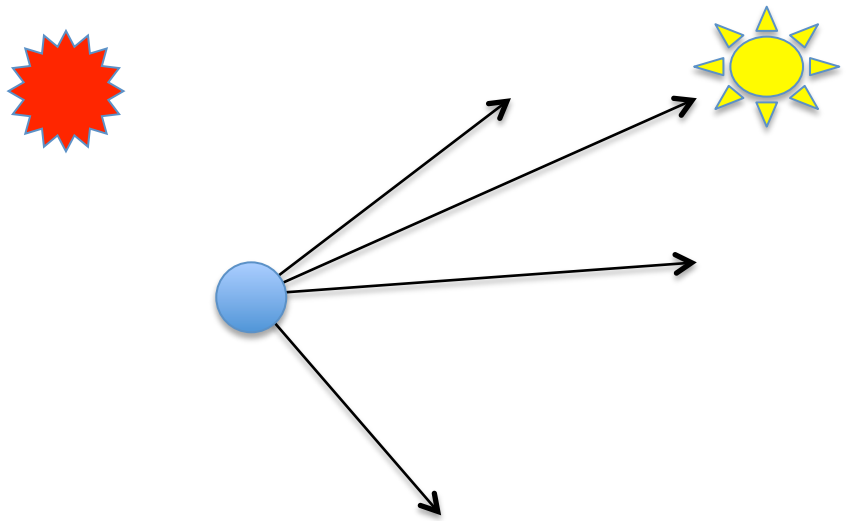
Control of Mobile Robots

Module 6
The Navigation Problem

How make mobile robots move in effective, safe, predictable, and collaborative ways using modern control theory?

Lecture 6.1 – Behaviors Revisited

- Let's return to the navigation problem using behaviors
- Use control theory to describe what is going on



Start Simple: The Model

- For the purpose of "planning", assume that

$$\dot{x} = u, \; x \in \Re^2$$

$$\dot{x} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} u$$

$$\Gamma = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} I & 0 \end{bmatrix}, \; \text{rank}(\Gamma) = 2 = \text{dimension of the system } (n)$$

CC!

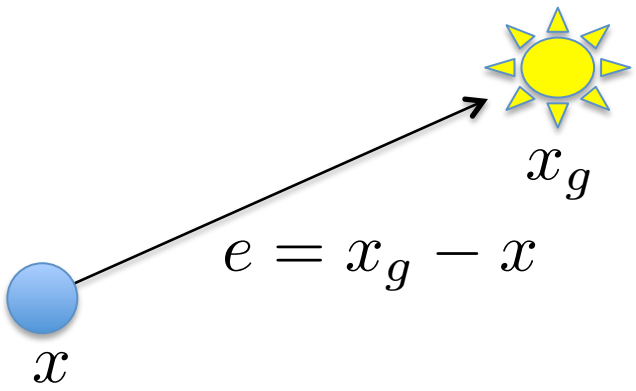
The Dynamic Duo

- The two bread-and-butter behaviors are go-to-goal and avoid-obstacle
  - Go-to-goal: Drives the robot towards the goal
  - Avoid-obstacle: Don't slam into things

} main controllers

- Control design task: pick a desired motion vector and set that equal to the input u

Go-To-Goal



$$u = Ke \;\Rightarrow\; \dot{e} = 0 - \dot{x} = -Ke$$

$\dot{x}_g$

$\dot{x} = u = -Ke$

Asymptotically stable?

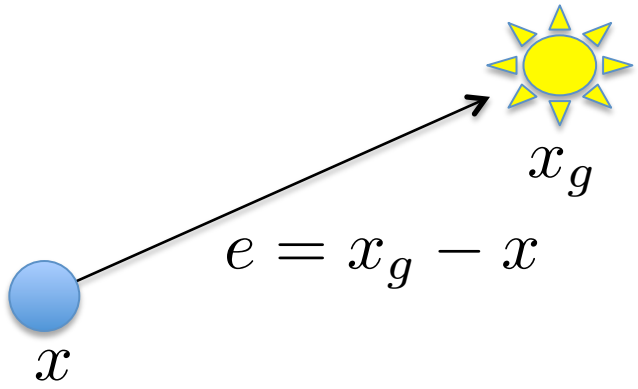
$K > 0$ or $K \succ 0$ $(\text{eig}[K] > 0)$

$e \to 0$

drive to zero

A Concern

- A linear controller means the robot goes faster the further away the goal is.
- Solution, in practice, make the gain K a function of e



$$\dot{e} = -K(\|e\|)e$$

A Concern

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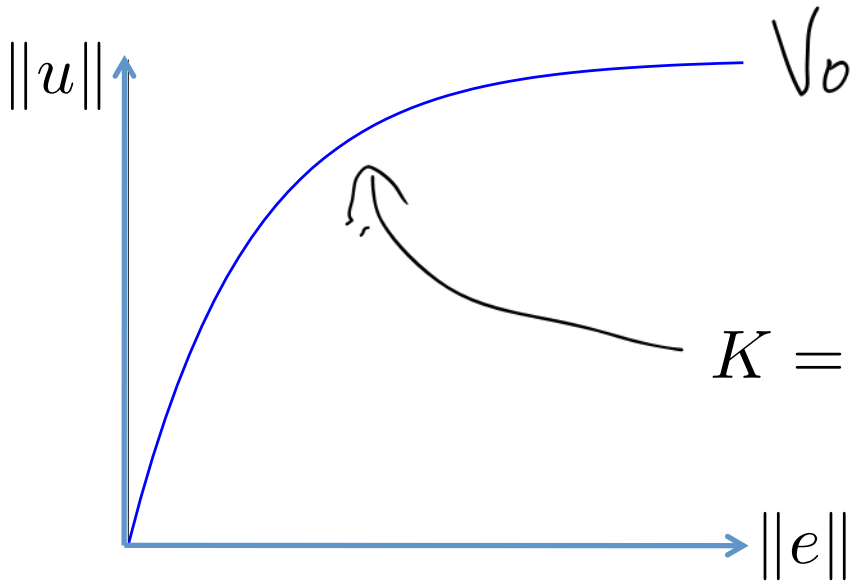
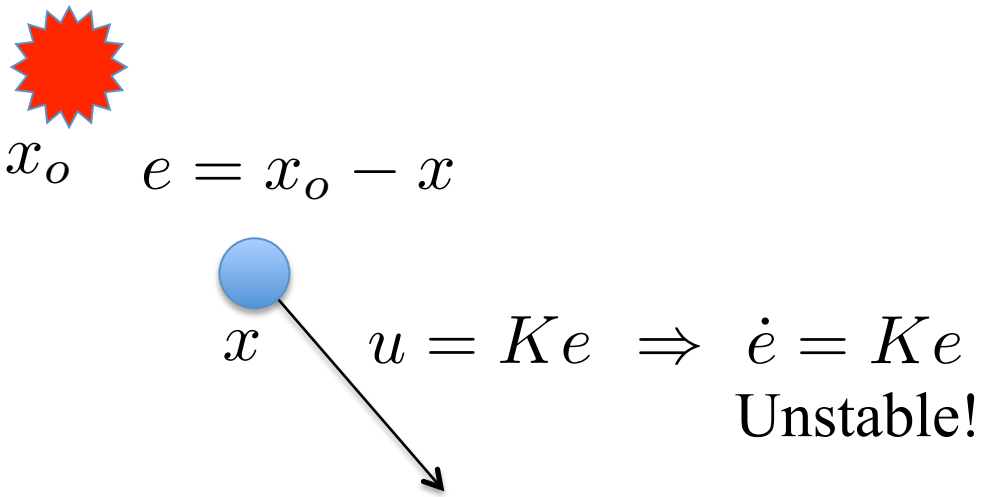


$$\dot{e} = -K(\|e\|)e$$

$$K = \frac{v_0\left(1 - e^{-\alpha\|e\|^2}\right)}{\|e\|}$$

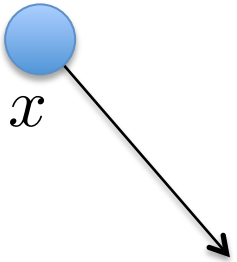
Avoid-Obstacle

- To get the direction, just flip the sign in the go-to-goal behavior



More Concerns

- The robot drives off to infinity?
- It is overly cautious?
- We care less the closer we get?



"Solutions"

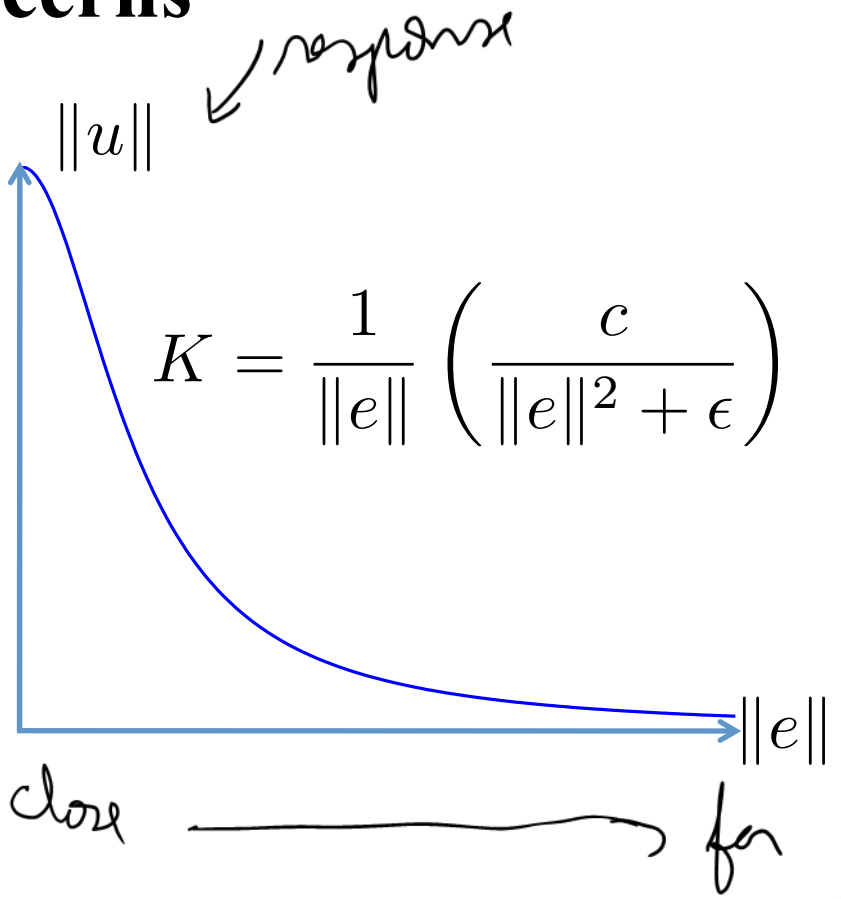
- Make K dependent on e!
- Switch between behaviors!
- Use the induced mode!

More Concerns

- The robot drives off to infinity?
- It is overly cautious?
- We care less the closer we get?

"Solutions"

- Make K dependent on e!
- Switch between behaviors!
- Use the induced mode!

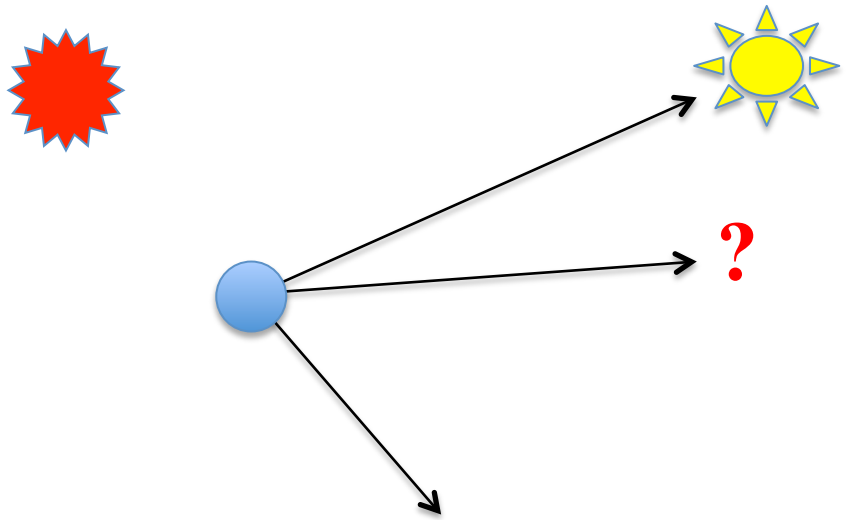


Behavior/Mode Transitions?

- What's missing?
  - Couple actual robot dynamics to the behaviors (next module)
  - Figure out the mode transitions (next lecture)
- But first, let's try it for real!

Lecture 6.2 –Hard Switches vs Blending

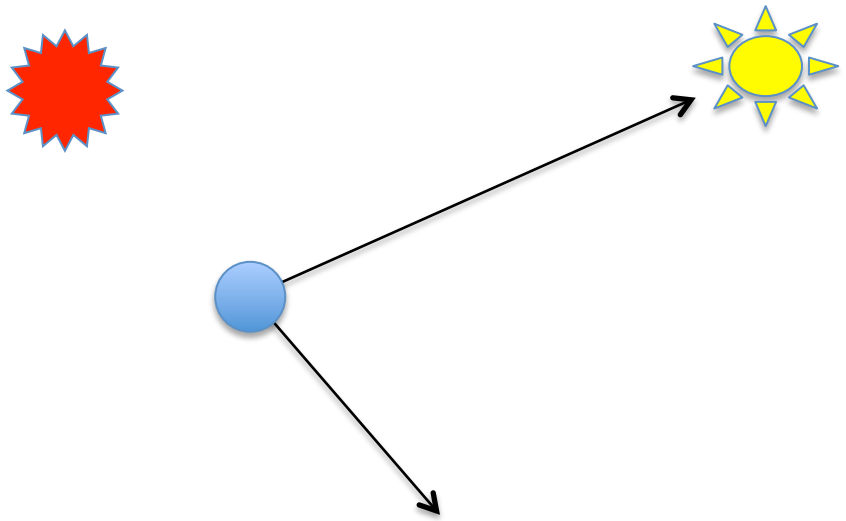
- Given two different behaviors, how should they be combined?



- Two options:
  - Hard Switches
  - Blended Behaviors

Pros and Cons

- Philosophical Question: Can robots chew gum and walk at the same time?



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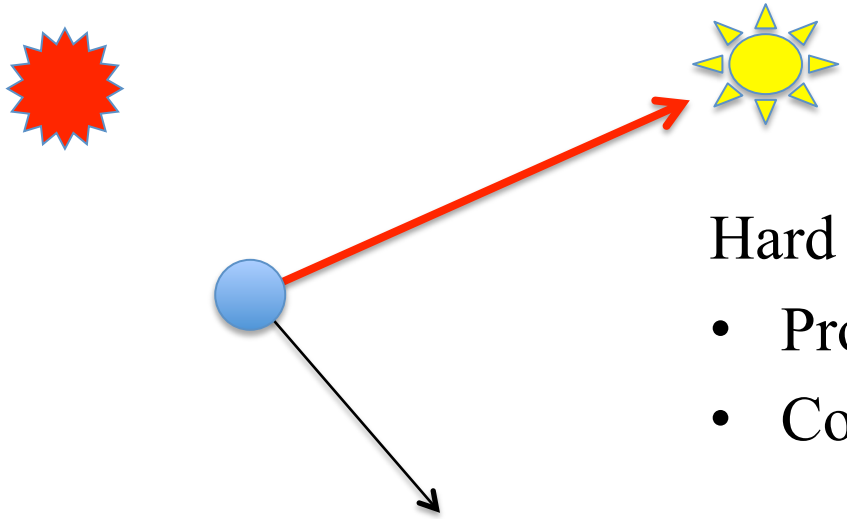
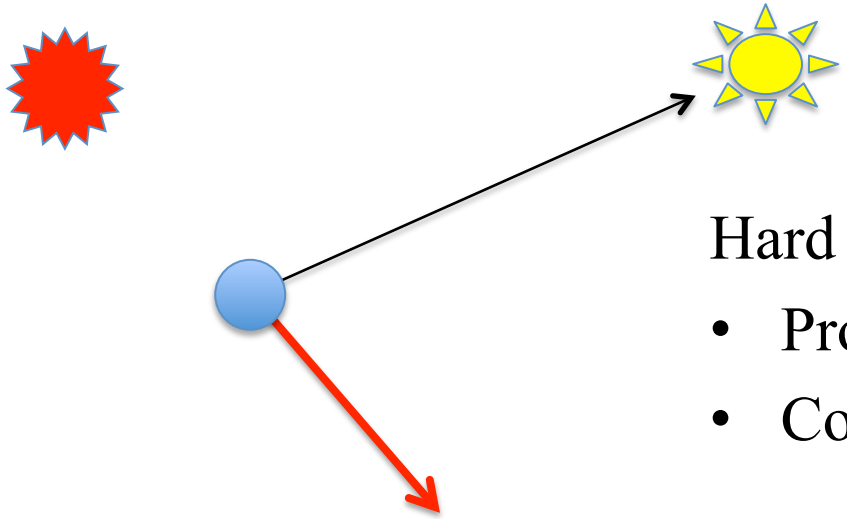


Hard Switches:

- Pro: Performance guarantees
- Con: Bumpy ride, Zeno?

Pros and Cons

- Philosophical Question: Can robots chew gum and walk at the same time?



Hard Switches:

- Pro: Performance guarantees
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Pros and Cons

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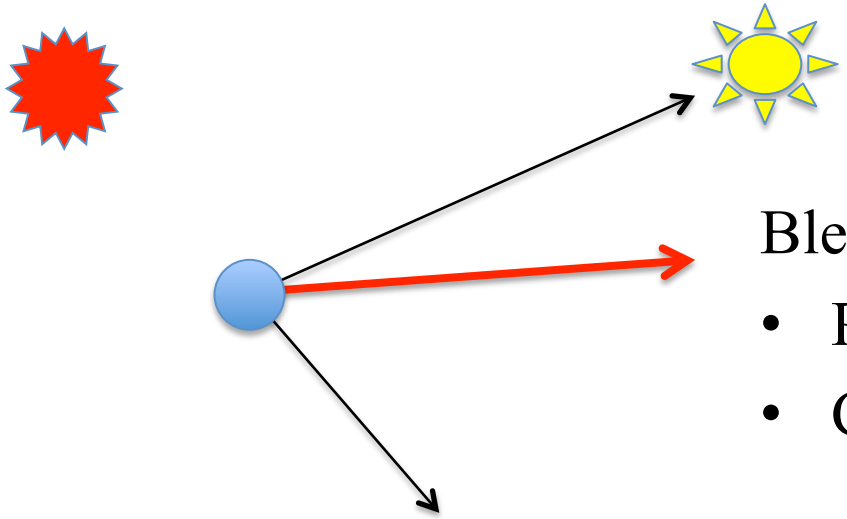
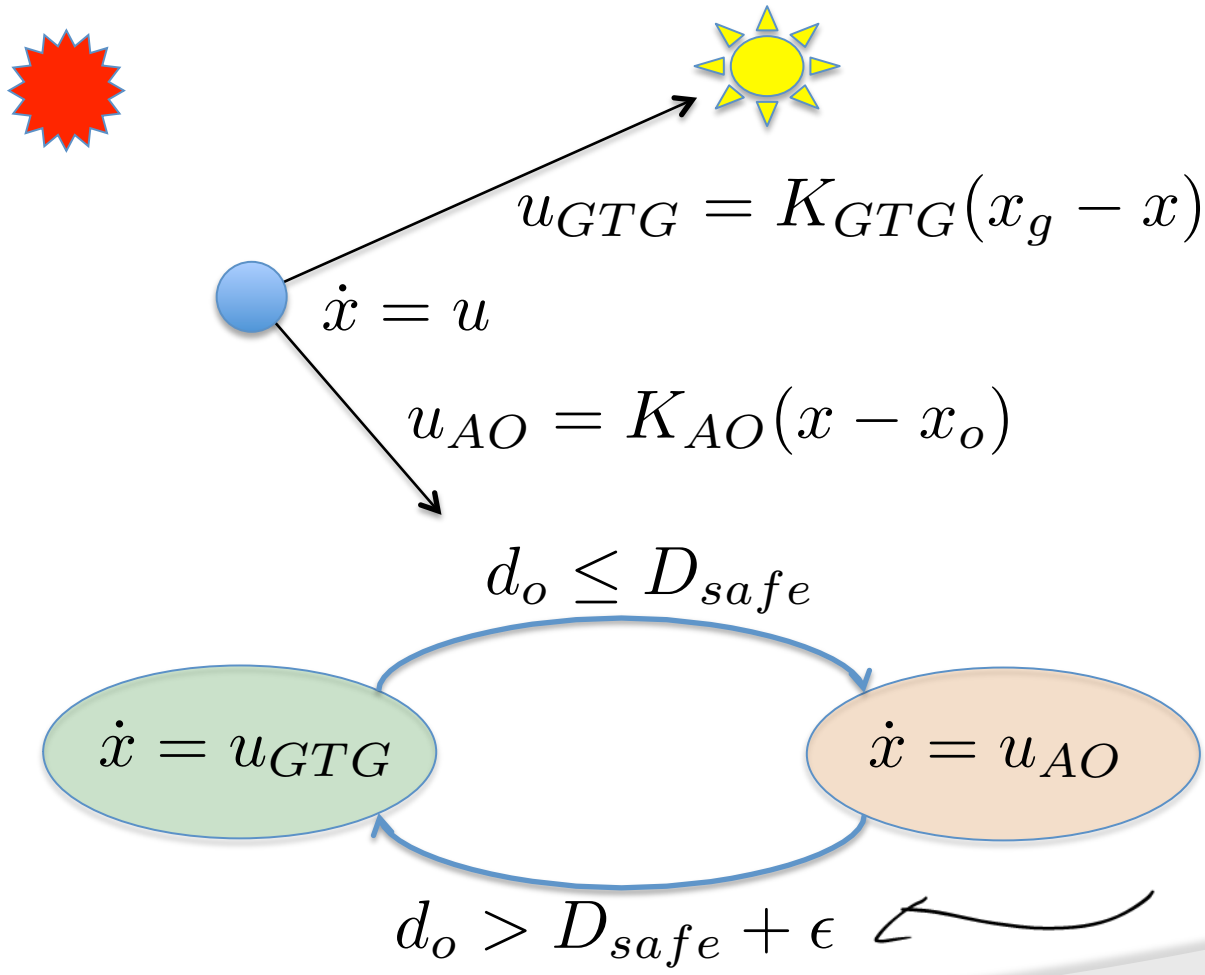


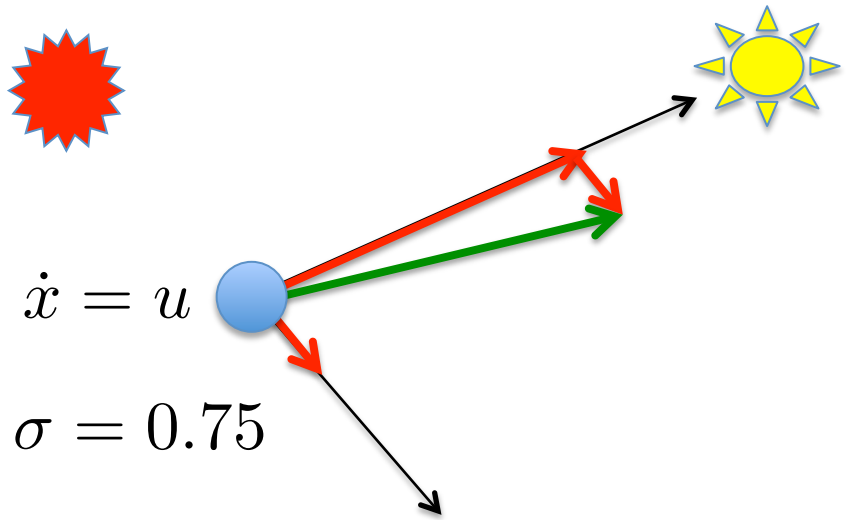
Blending:

- Pro: Smooth ride
- Con: No guarantees

A Switch HA


$u_{GTG} = K_{GTG}(x_g - x)$
$\dot{x} = u$
$u_{AO} = K_{AO}(x - x_o)$
$d_o \leq D_{safe}$
$\dot{x} = u_{GTG}$
$\dot{x} = u_{AO}$
$d_o > D_{safe} + \epsilon$
To avoid switching quickly

Blending



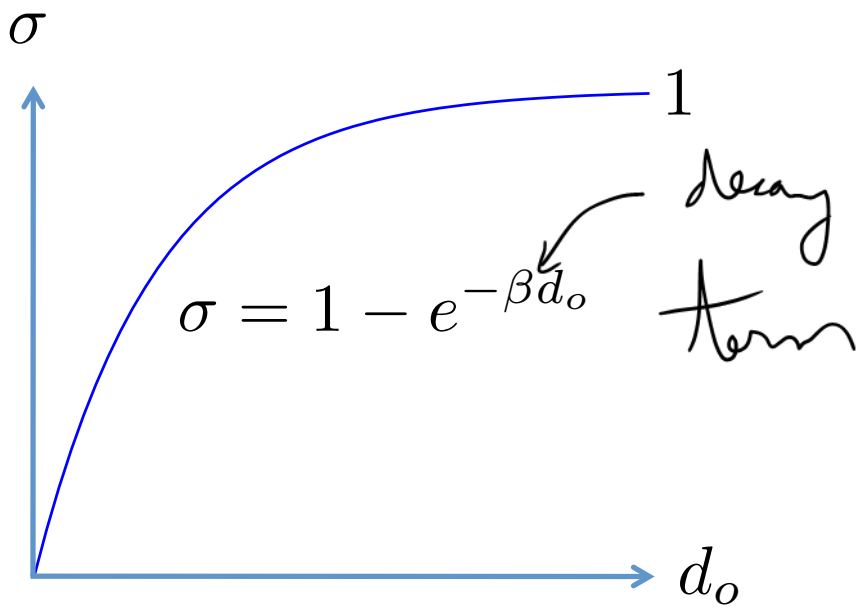
Blending function

$\sigma(d_o) \in [0, 1]$

↳ function on base of the distance

$$\dot{x} = \sigma(d_o)u_{GTG} + (1 - \sigma(d_o))u_{AO}$$

Example



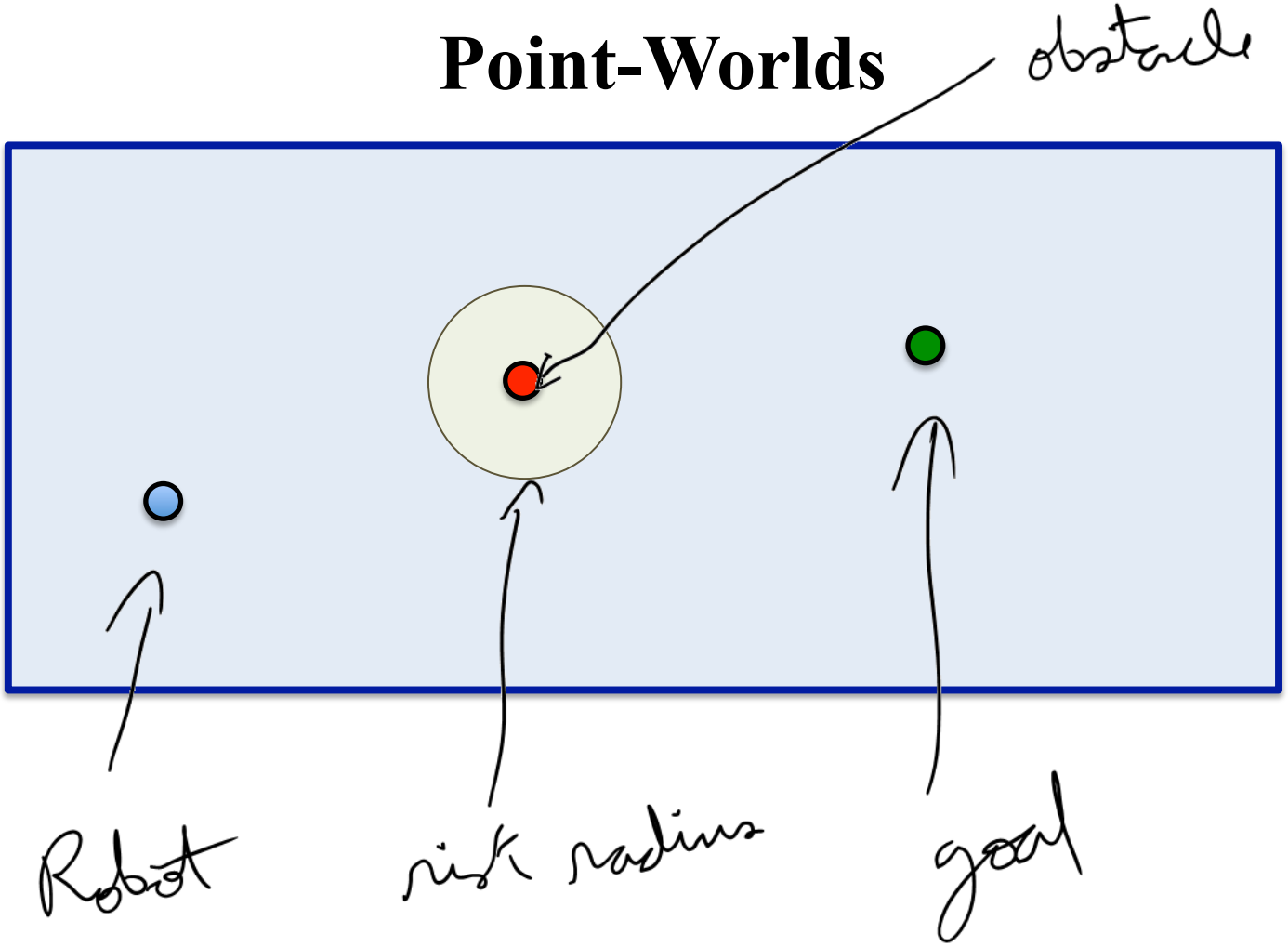
Punchlines

- Two choices – both of them come with pros and cons
- Would like to have our cake and eat it (smooth ride + performance guarantees)
- Enter: the induced mode

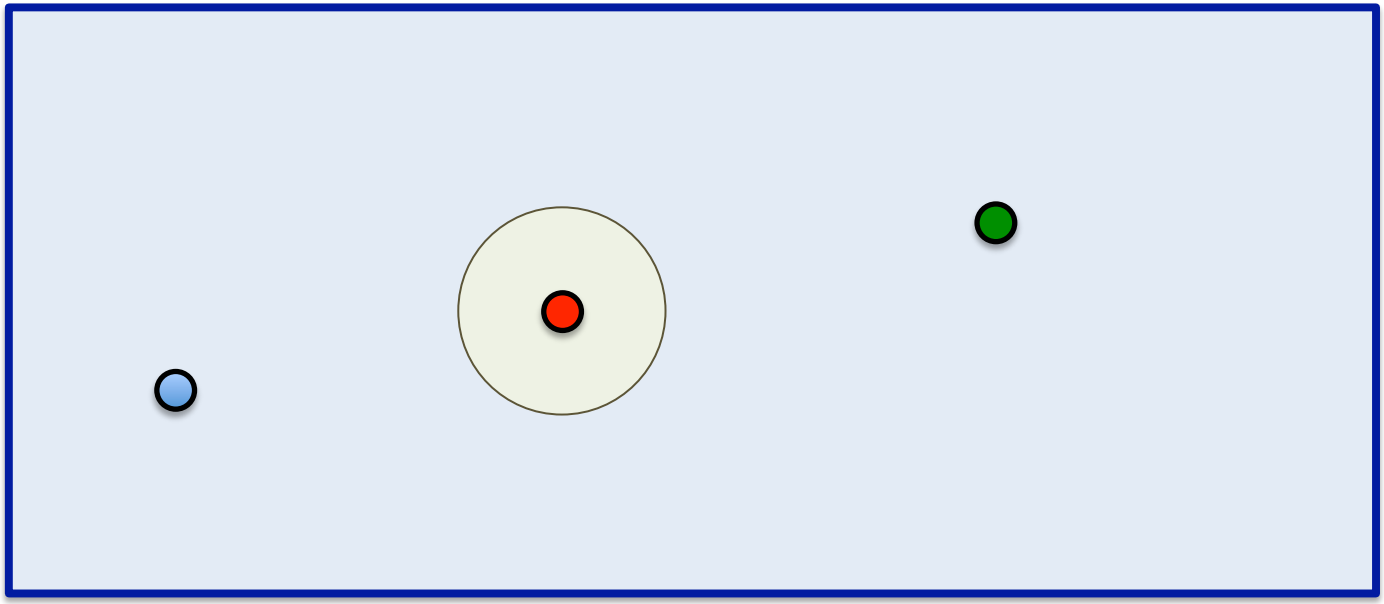
Lecture 6.3 –Convex and Non-Convex Worlds

- Let's see when our two basic behaviors work and when we need more behaviors!
- We will start simple and add complexity
  - Point-worlds
  - Circular obstacles
  - Convex worlds
  - Non-convex obstacles
  - Labyrinths

Point-Worlds

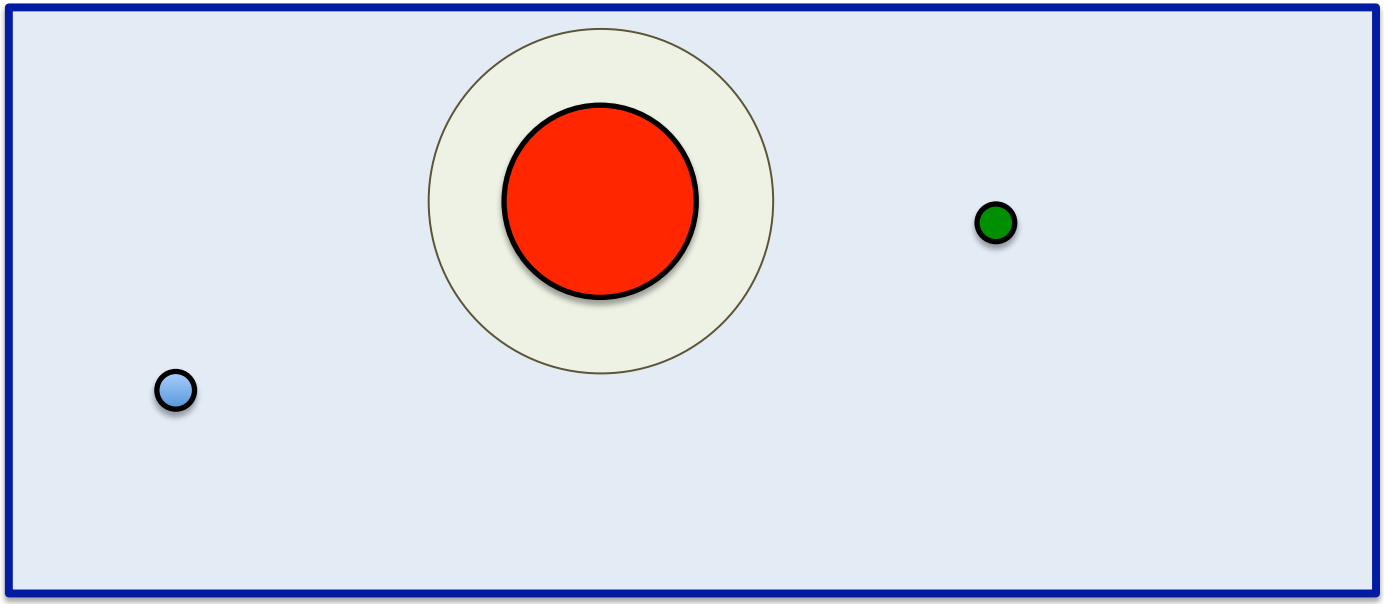


Point-Worlds



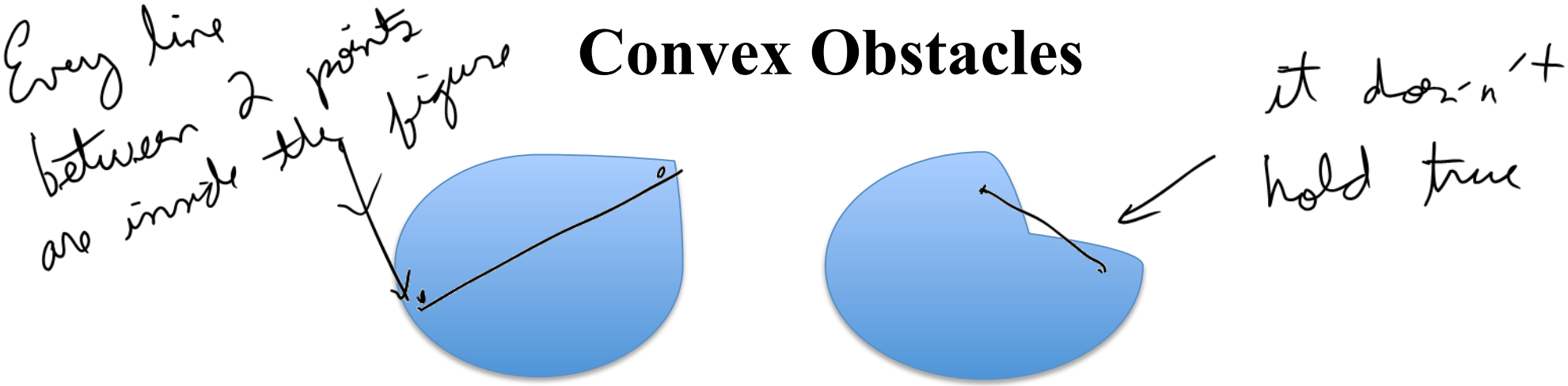
- Here the two behaviors are sufficient unless we are super-unlucky. (Maybe some “mild” Zeno…)
- Solution: Add a little bit of noise – in practice this is never really needed since the world is noisy

Circular Obstacles



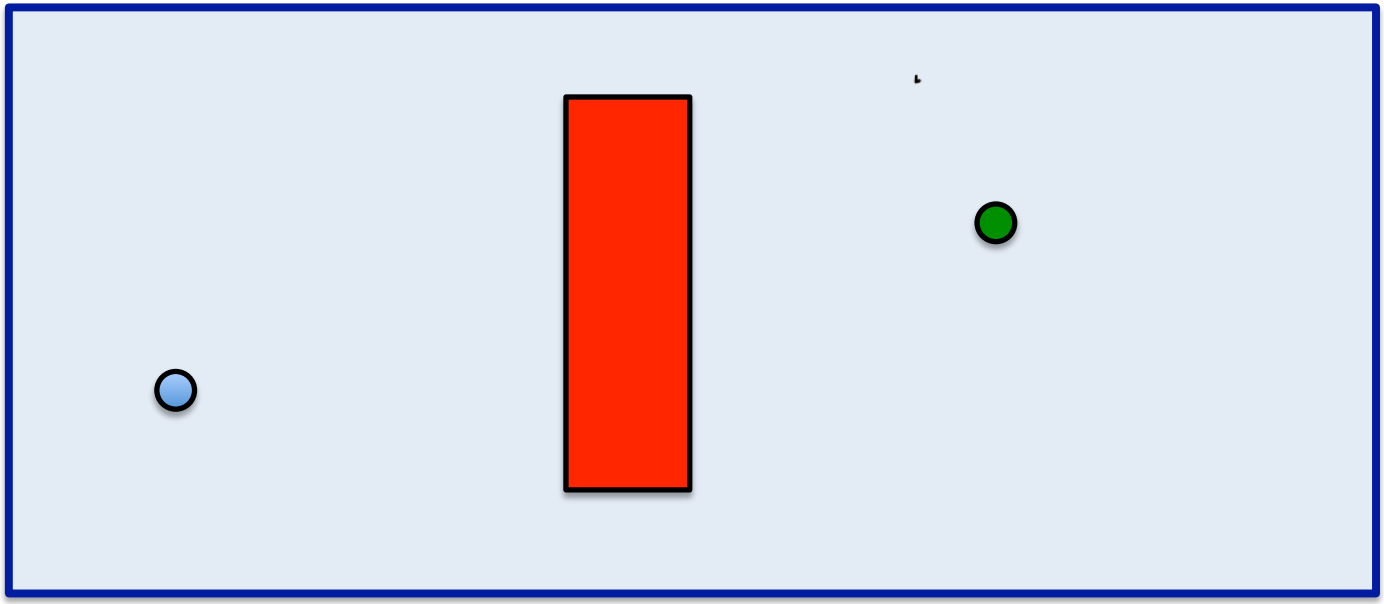
- This is the same as for point-obstacles

Convex Obstacles



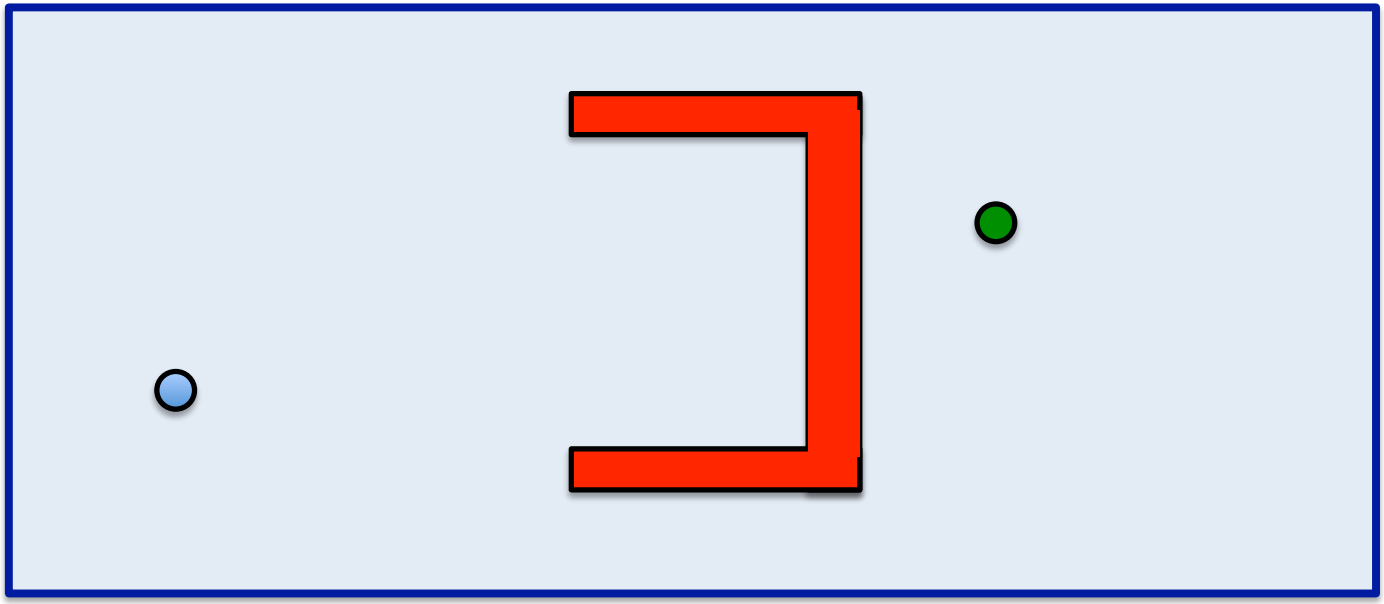
- A set is convex if every line in-between two points in the set lies in the set

Convex Obstacles



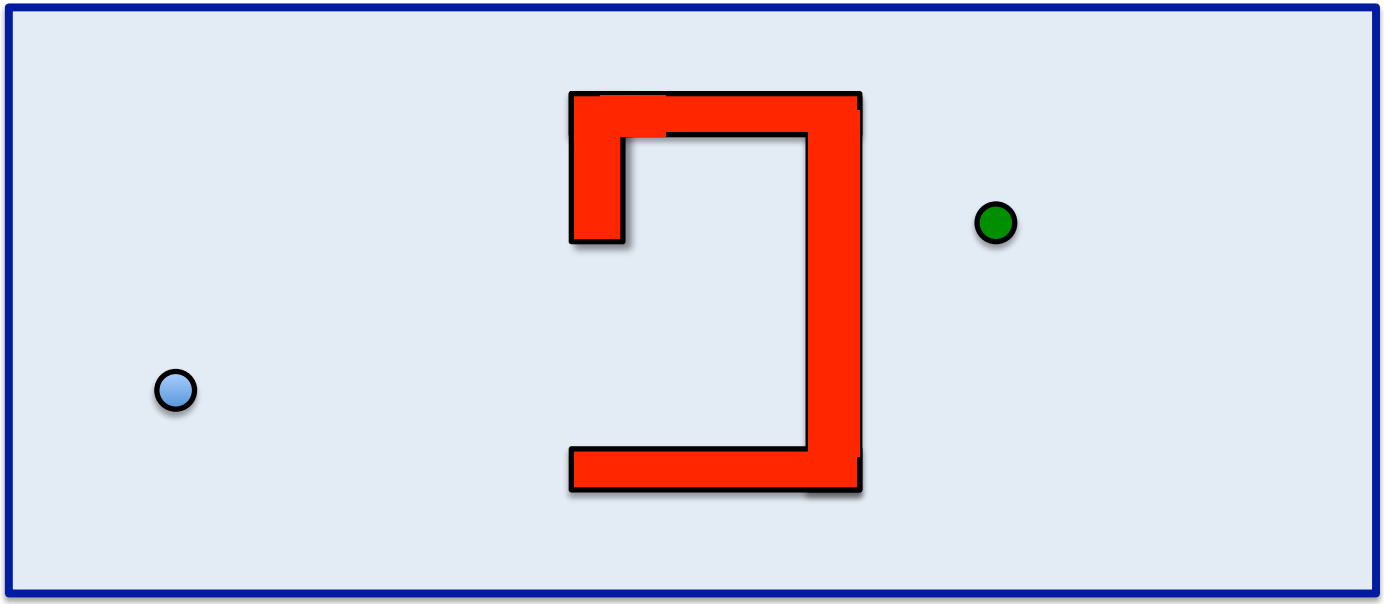
- Even though the obstacles are quite nice-looking, we can get stuck for real with only these two behaviors
- We need some way to keep going along the boundary of the obstacle!

Non-Convex Obstacles



- Even worse!

Labyrinths



- Forget about it!

Next Lecture

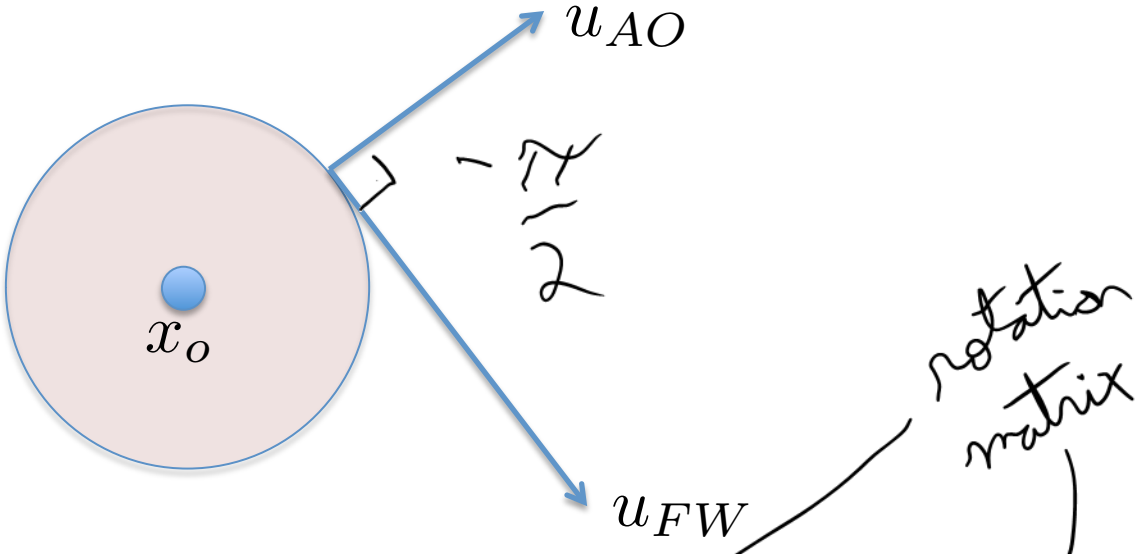
- We really need at least one more behavior to be able to deal with non-circular worlds!

Lecture 6.4 –Wall Following

- We need some way of negotiating complex environments
- A really useful behavior is one that makes the robot follow the boundary of an obstacle/wall
- This will also allow us to introduce the induced mode in a systematic way

How To Follow Walls

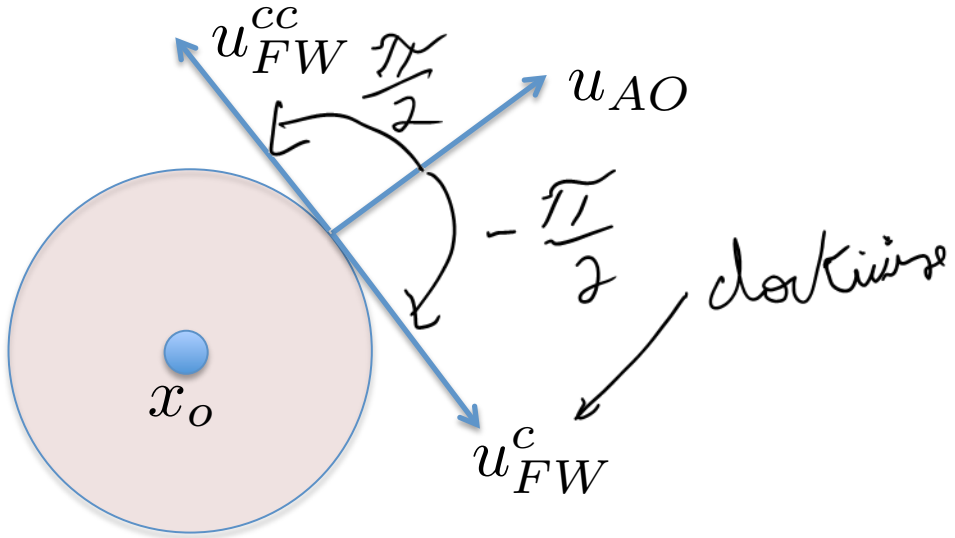
- The follow-wall behavior should maintain a constant distance to the obstacle/wall it is if following



$$u_{FW} = \alpha \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} u_{AO} = \alpha R(-\pi/2) u_{AO}$$

Clockwise or Counter-Clockwise?

- We can clearly move in two different directions along a wall!



Clockwise or Counter-Clockwise?

- We can clearly move in two different directions along a wall!

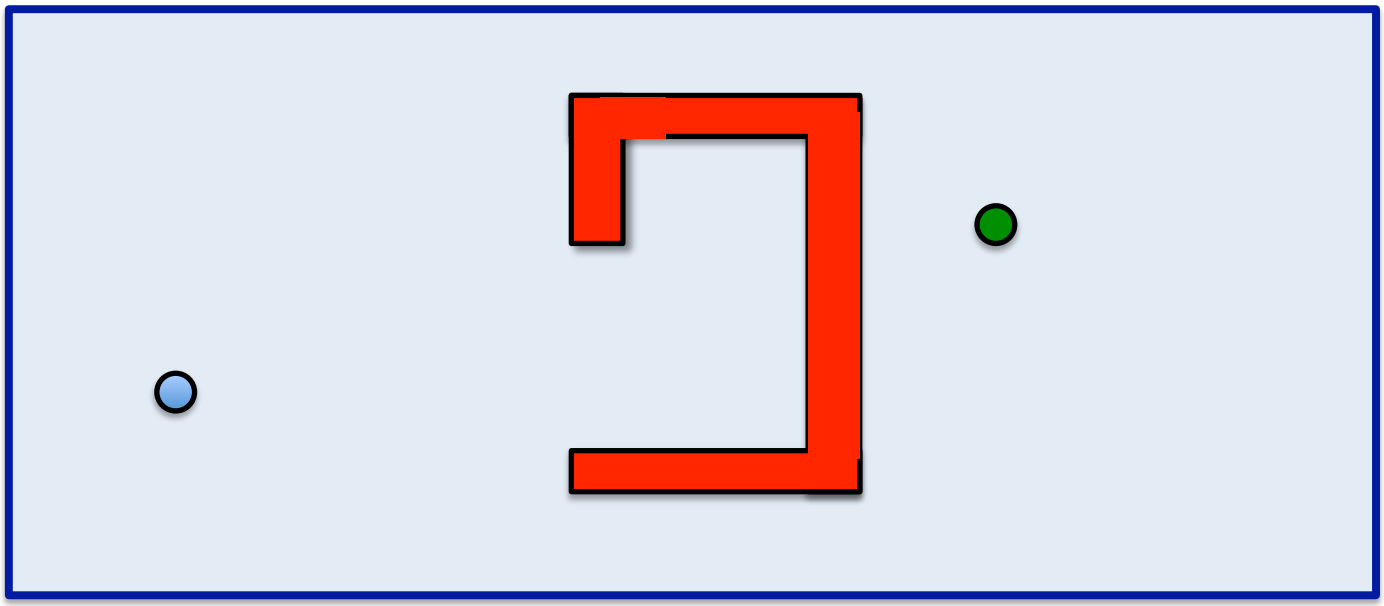
Rotation Matrix: $R(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$

$$u_{FW}^{c} = \alpha R(-\pi/2) u_{AO} = \alpha \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} u_{AO}$$

$$u_{FW}^{cc} = \alpha R(\pi/2) u_{AO} = \alpha \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} u_{AO}$$

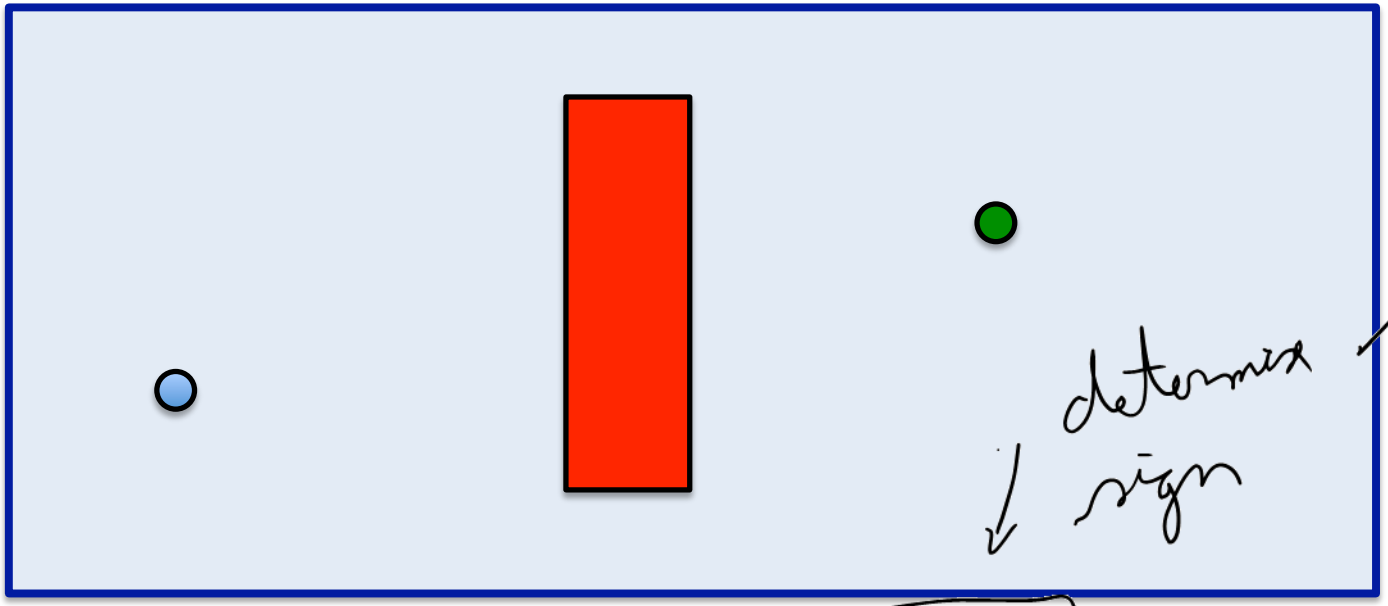
Clockwise or Counter-Clockwise?

- Which direction to choose?



- There is no obvious answer to that!
- Maybe let the goal-to-goal direction determine? (when goal to goal vector is going)

Clockwise or Counter-Clockwise?



$$\langle v, w \rangle = v^T w = \|v\| \|w\| \cos(\angle(v, w))$$

dot product

$$\langle u_{GTG}, u^c_{FW} \rangle > 0 \;\Rightarrow\; u^c_{FW}$$

$$\langle u_{GTG}, u^{cc}_{FW} \rangle > 0 \;\Rightarrow\; u^{cc}_{FW}$$

Issues

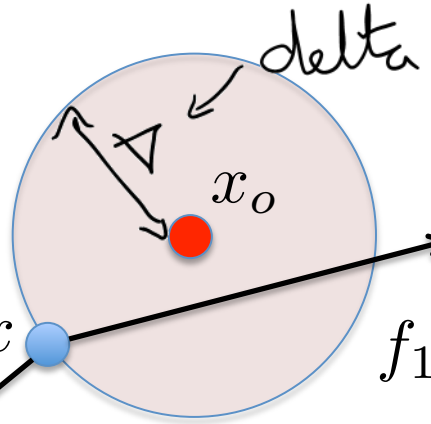
- When release the follow-wall behavior?
- Is there a systematic way to scale the behavior?

$$\alpha R(\pm\pi/2)u_{AO}$$

Lecture 6.5 – The Induced Mode

- We have seen that we need wall-following to solve complex navigation problems
- Last lecture: $\alpha R(\pm\pi/2)u_{AO}$
- This lecture: How can this be connected to the induced mode between go-to-goal and avoid-obstacle in a more systematic manner?

The Setup



$u_{GTG} =$

$f_1 = C_{GTG}(x_g - x)$

u_{AO}

$\|x - x_o\| = \Delta$

$f_2 = C_{AO}(x - x_o)$

$g(x) = \frac{1}{2}(\|x - x_o\|^2 - \Delta^2) = 0$

f_1 when $g > 0$

f_2 when $g < 0$

we added squares for ease on the math

Connecting to the Induced Mode

$$g(x) = \frac{1}{2}(\|x - x_o\|^2 - \Delta^2) = 0$$

$$L_{f_2} g = \frac{dg}{dx} f_1$$

$$f_1 = C_{GTG}(x_g - x)$$

$$f_2 = C_{AO}(x - x_o)$$

The Induced Mode:

$$\dot{x} = \frac{1}{L_{f_2} g - L_{f_1} g}\left(L_{f_2} g f_1 - L_{f_1} g f_2\right)$$

$$\frac{\partial g}{\partial x} = (x - x_o)^T$$

Constant

$$L_{f_2} g = \frac{\partial g}{\partial x} f_2 = (x - x_o)^T C_{AO}(x - x_o) = C_{AO}\|x - x_o\|^2$$

$$L_{f_1} g = \frac{\partial g}{\partial x} f_1 = C_{GTG}(x - x_o)^T (x_g - x)$$

Constant

Connecting to the Induced Mode

$$L_{f_1} g = C_{GTG}(x - x_o)^T (x_g - x)$$

$$L_{f_2} g = C_{AO} \|x - x_o\|^2$$

The Induced Mode:

$$\dot{x} = \frac{1}{L_{f_2} g - L_{f_1} g} \left(L_{f_2} g f_1 - L_{f_1} g f_2 \right)$$

Quite a mess!

plug in is a mess

Recall:

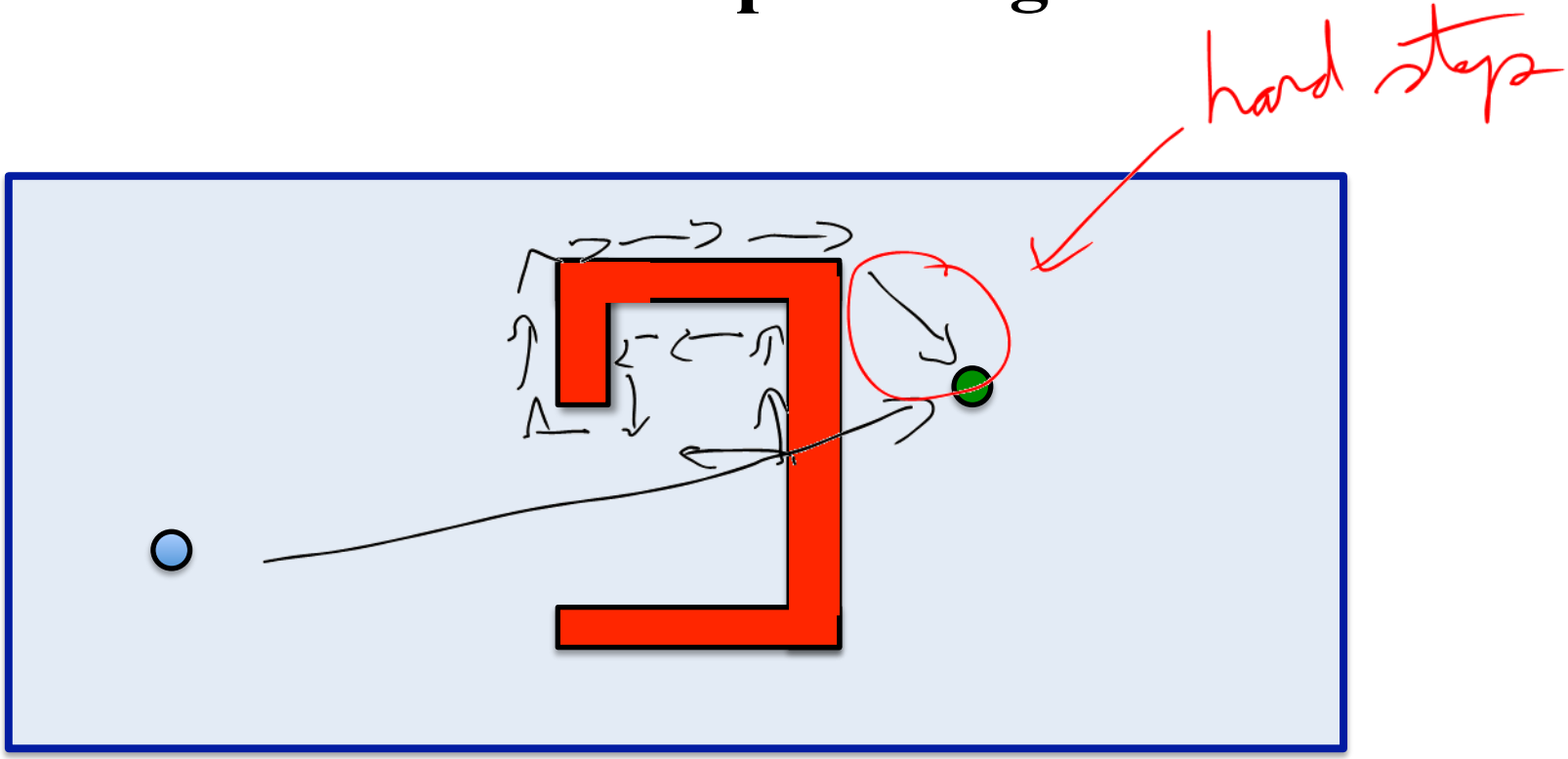
$$u_{FW}^{c} = \alpha R(-\pi/2) u_{AO}$$

$$u_{FW}^{cc} = \alpha R(\pi/2) u_{AO}$$

They are the same!!

$\alpha = 1 \rightarrow$ good initial assumption

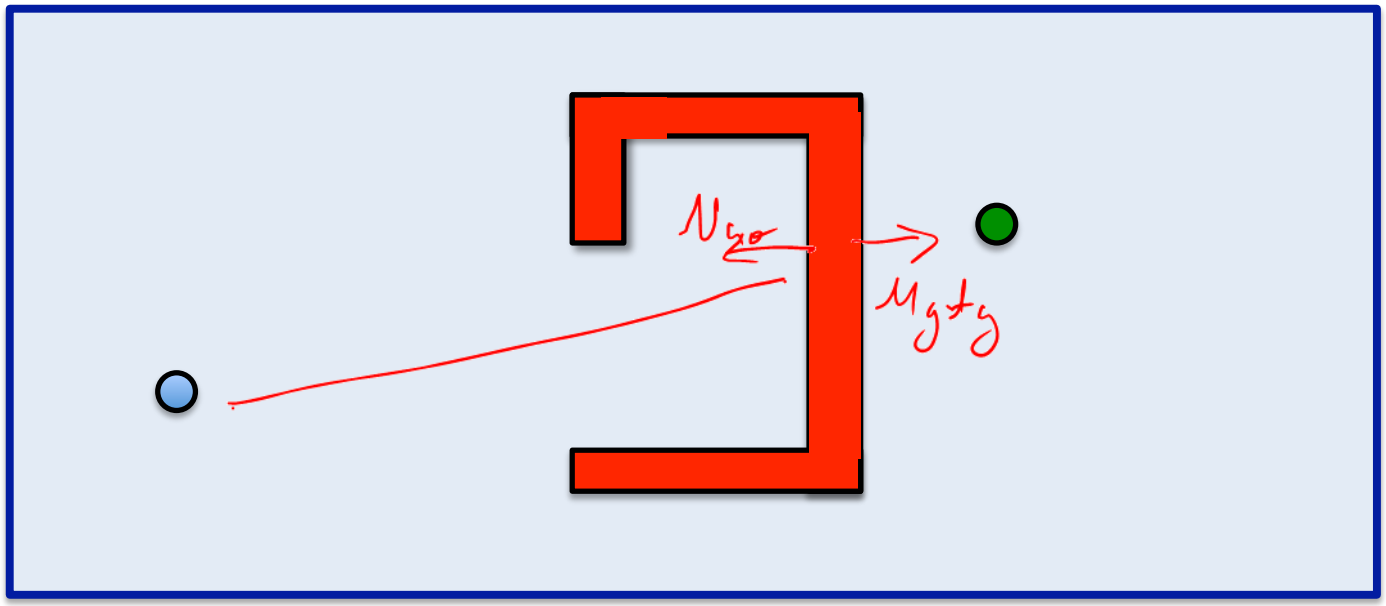
When to Stop Sliding?



- Not so easy!
- Next time….

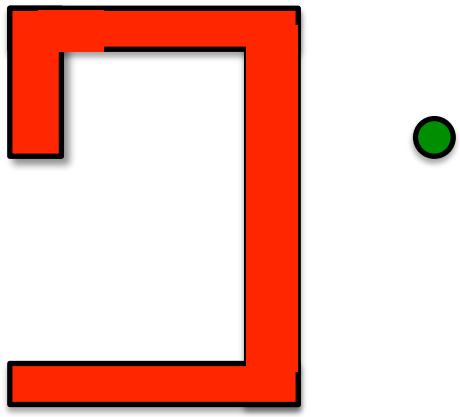
Lecture 6.6 – A Complete Navigation System

- When do we stop following walls?
- Not when we stop sliding!



When To Stop?

- We should stop when enough progress has been made and we have a "clear" shot to the goal!



$\tau =$ time of last switch ← when we started sliding

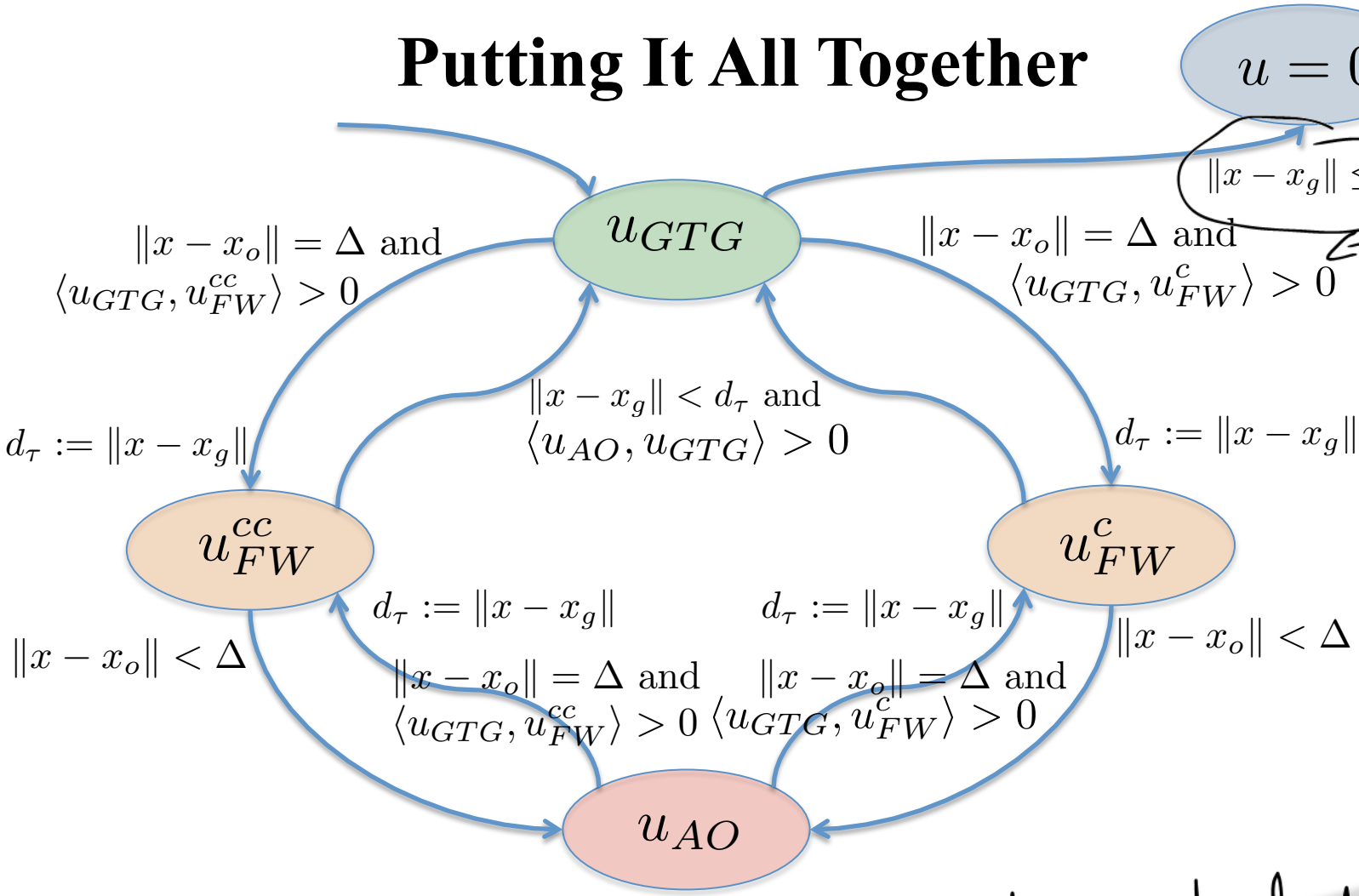
progress: $\|x - x_g\| < \|x(\tau) - x_g\|$

clear shot: $\langle u_{AO}, u_{GTG} \rangle > 0$

$u_{AO}^T u_{GTG} > 0$

We want to be close to when we started sliding

Putting It All Together

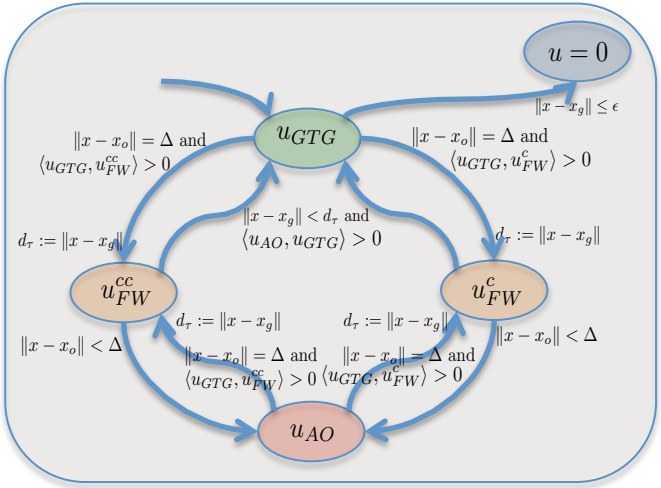


stop condition

guard rest

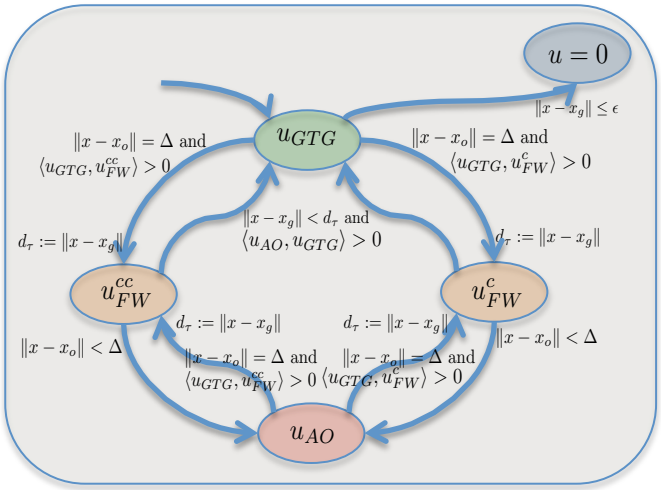
↳ safety control for real world

Does This Work?



- Nope!
- Next time: Practical considerations!

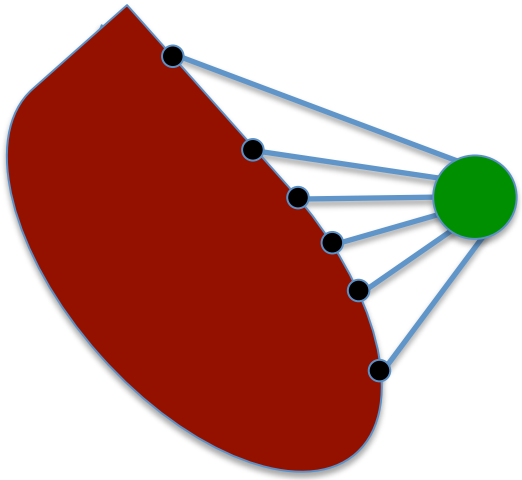
Lecture 6.7 – Practical Considerations



- Non-Point-Obstacles → objects are multi-dimensional
- Fat Guards → due to noise
- Tweak, Tweak, Tweak
  - ↳ all scalar parameters

Obstacles Aren't Points

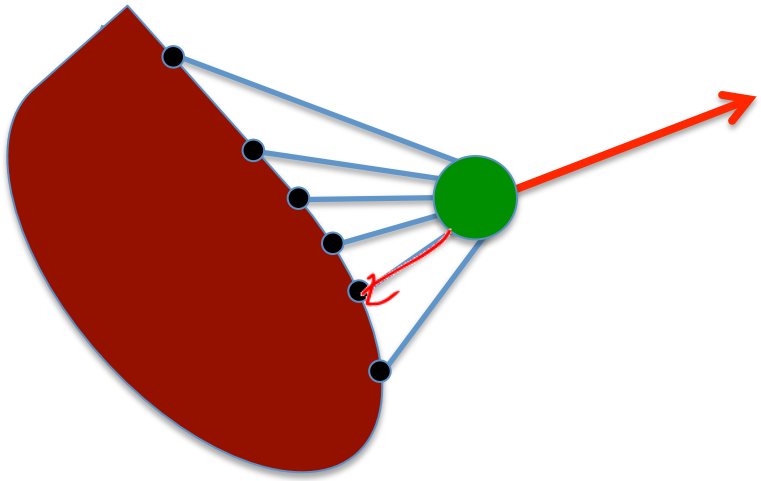
- We have to allow for non-point obstacles!
- But most (all) sensors really return points



How to deal with this situation?

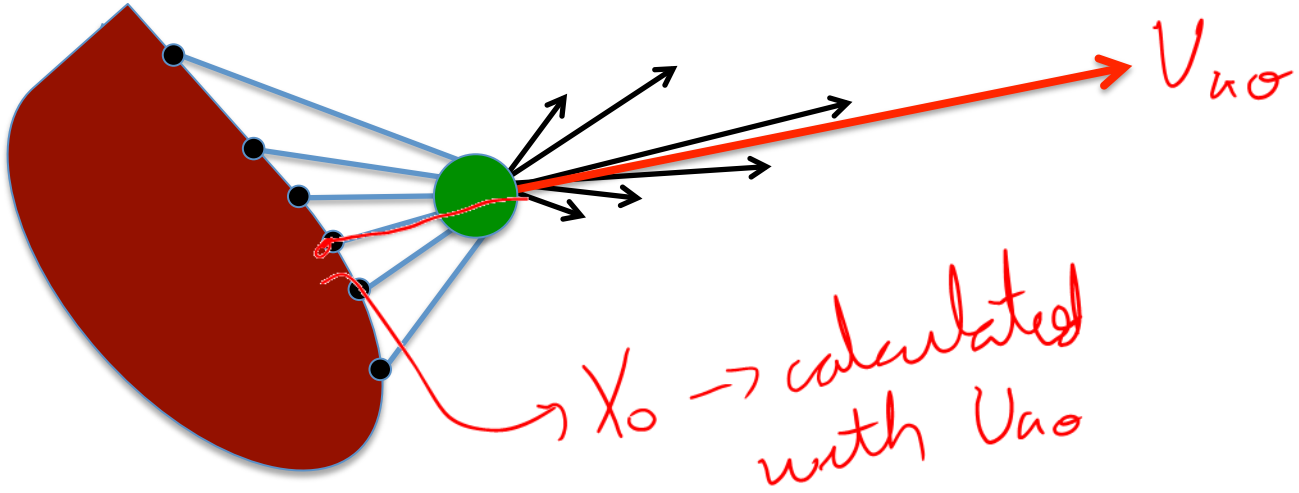
Some Options

1. Go with the closest obstacle point (not bad)



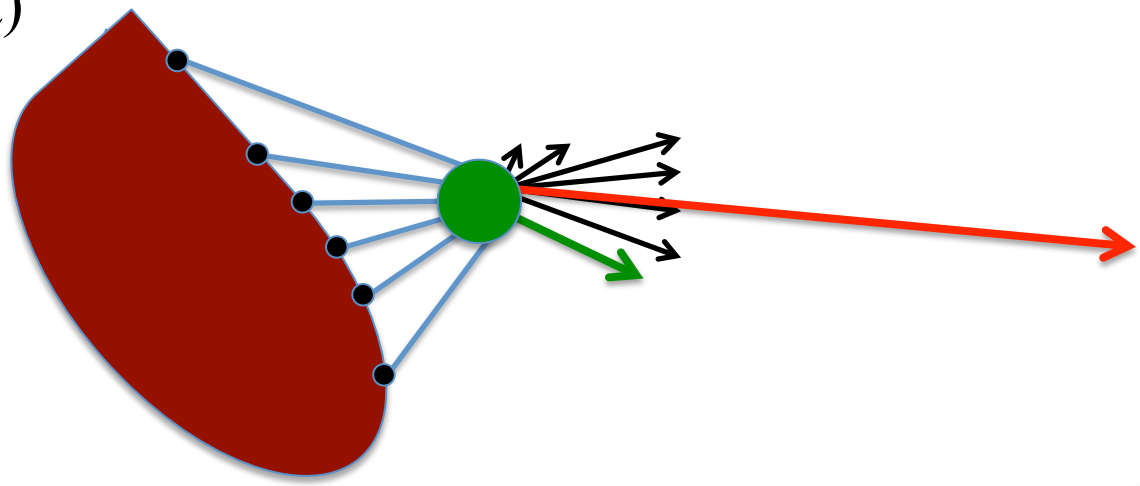
Some Options

1. Go with the closest obstacle point (not bad)
2. Weigh and add the "obstacle vectors" depending on distance (better)



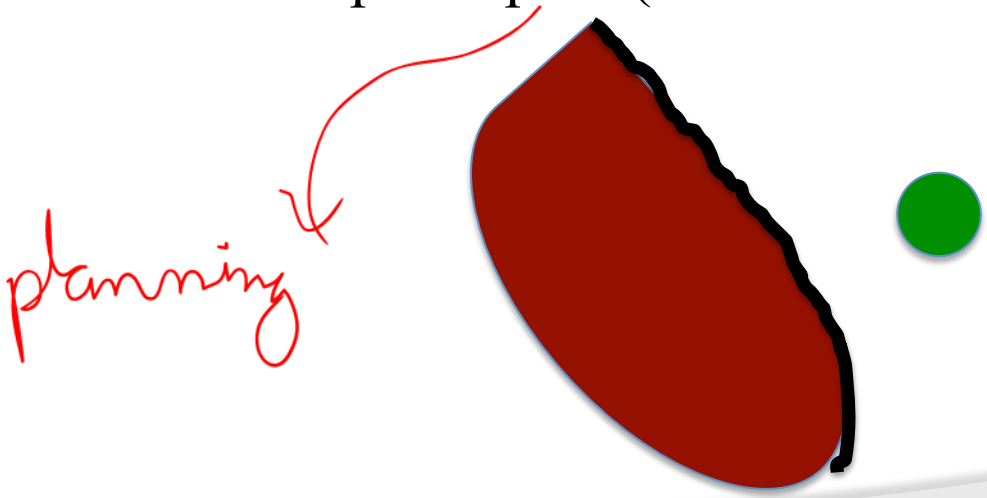
Some Options

1. Go with the closest obstacle point (not bad)
2. Weigh and add the "obstacle vectors" depending on distance (better)
3. Weight and add depending on distance and direction of travel (best)



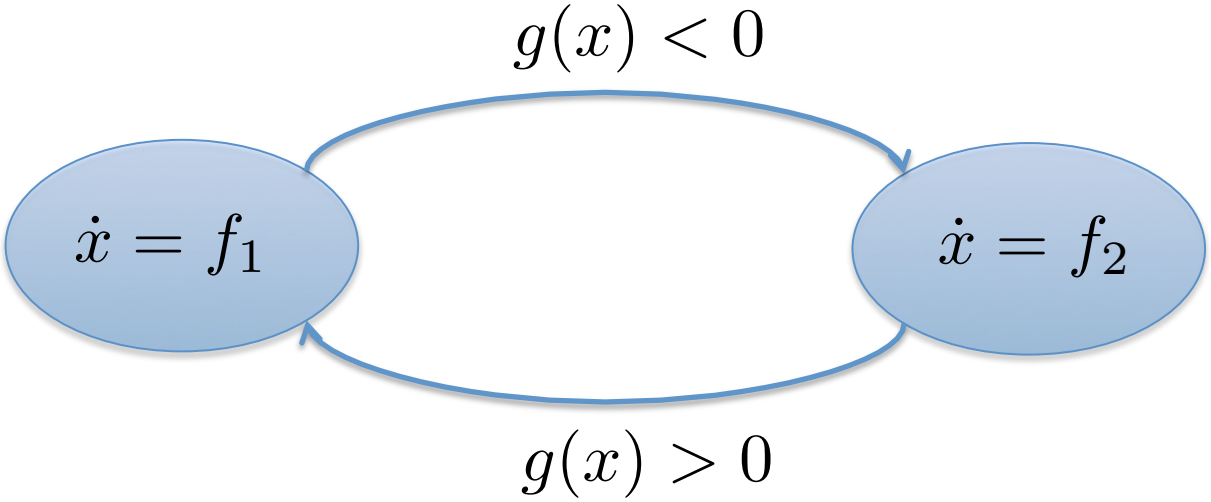
Some Options

1. Go with the closest obstacle point (not bad)
2. Weigh and add the "obstacle vectors" depending on distance (better)
3. Weight and add depending on distance and direction of travel (best)
4. Make a map and plan (most bestest – not in this class)



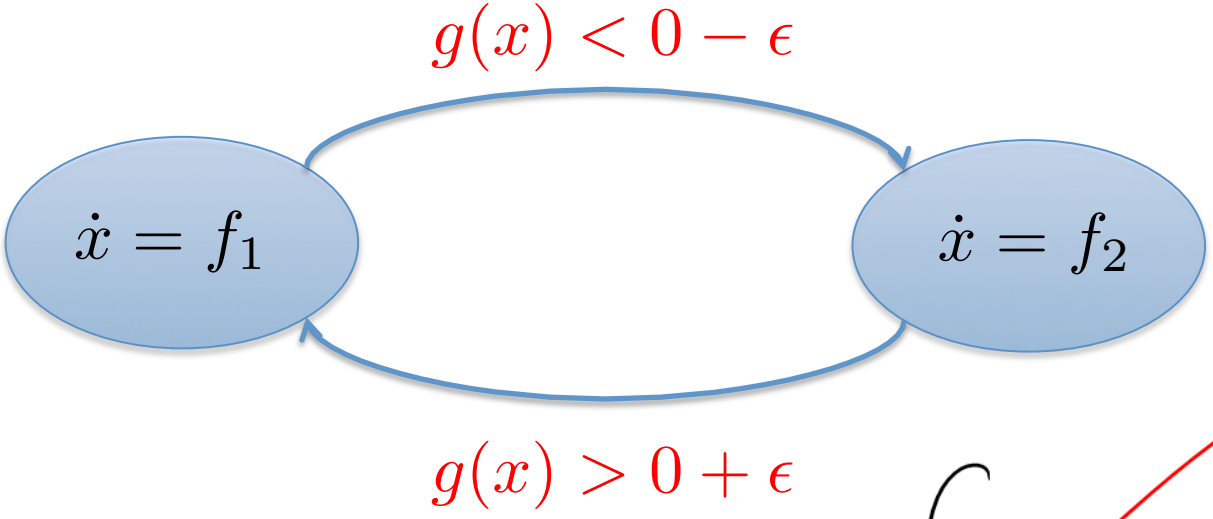
Fat Guards

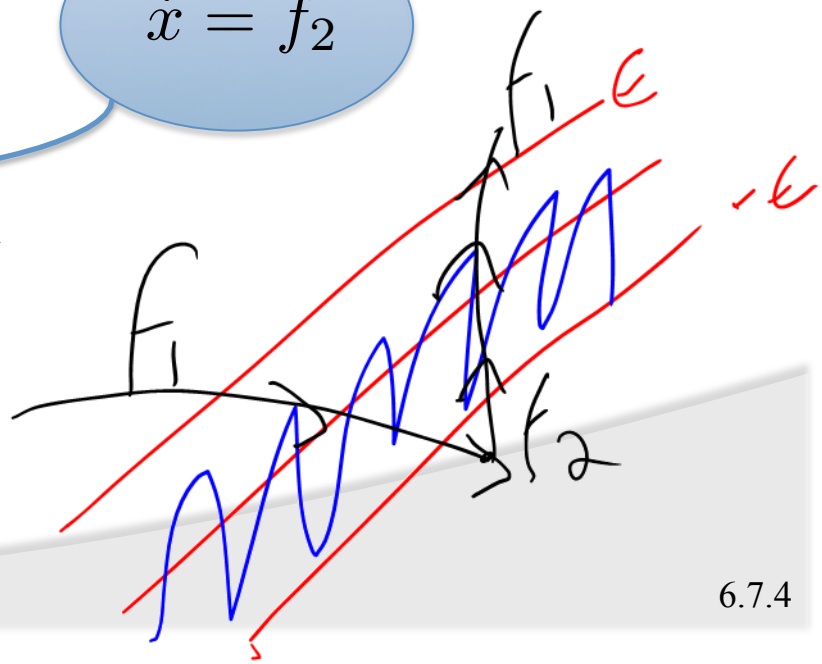
- Since no sensor is perfect, always allow for "fat" guards in the navigation HA



Fat Guards

- Since no sensor is perfect, always allow for "fat" guards in the navigation HA





And The Final Advise Is….

- TWEAK, TWEAK, TWEAK!!!!

Lecture 6.8 –Let's Do It!

- Enough Talk!