3 Actividades de simulación

3.1 Actividad 2.1

Graficar la evolución de la temperatura $\theta(t)$ en base a la ecuación linealizada, considerando

- temperatura inicial 10^{o}
- temperatura ambiente nula
- tensión de entrada constante $e(t) = e_0$

3.2 Actividad 2.2

1. Graficar y explicar los siguientes cambios en $e(t)$, con $\theta_{amb} = cte$:
   1. $e(t) = 1$
   2. $e(t) = 2$
   3. $e(t) = 10$
   4. $e(t) = 0$
2. ¿Qué efecto tiene θ_{amb} sobre la acción de linealización? Mostrarlo con una gráfica.

sen θ

θ^2

2.1 Modelo no lineal

Ecuación diferencial no-lineal:

$$\dot{\theta}(t) = -\frac{1}{RC}\theta + \frac{1}{RC}\theta_{amb} + \frac{1}{rC}e^2(t) \quad g(t)$$

Convirtiendo al modelo de estados:

$$\frac{d}{dt}[\theta(t)] = \underbrace{\left[-\frac{1}{RC}\right]}_{A}[\theta(t)] + \underbrace{\left[\frac{1}{RC} \quad \frac{1}{rC}\right]}_{B}\begin{bmatrix}\theta_{amb}(t)\\ e^2(t)\end{bmatrix}$$

NOTA: esta ecuación corresponde a un sistema no-lineal con respecto a $e(t)$, pero lineal con respecto a $g(t) = e^2(t)$, por eso es posible definir las matrices A y B.

$$y(t) = \theta(t)$$

2.2 Modelo linealizado

Ecuación diferencial linealizada:

e_0

$$\dot{\theta}(t) = -\frac{1}{RC}\theta + \cancel{\frac{1}{RC}\theta_{amb}} + \cancel{\frac{2e_0}{rC}(e(t) - e_0)} + \frac{e_0^2}{rC}$$

Convirtiendo al modelo de estados:

$$\frac{d}{dt}[\theta(t)] = \left[-\frac{1}{RC}\right][\theta(t)] + \left[\frac{1}{RC} \quad \frac{2e_0}{rC} \quad -\frac{e_0^2}{rC}\right]\begin{bmatrix}\theta_{amb}(t)\\ e(t)\\ 1\end{bmatrix}$$

$$y(t) = \theta(t)$$

2.1

$\theta_0 = 10^{\circ}$

$\theta_{amb} = 0$

$$\dot{\theta} = -\frac{1}{Rc}\theta + \frac{e_0^2}{rc}$$

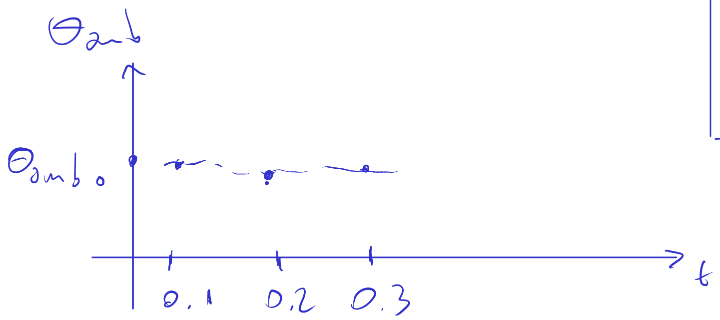
Hecho, clase pasada

2.2 " $\dot{\vec{x}} = A\vec{x} + B\vec{u}$ "

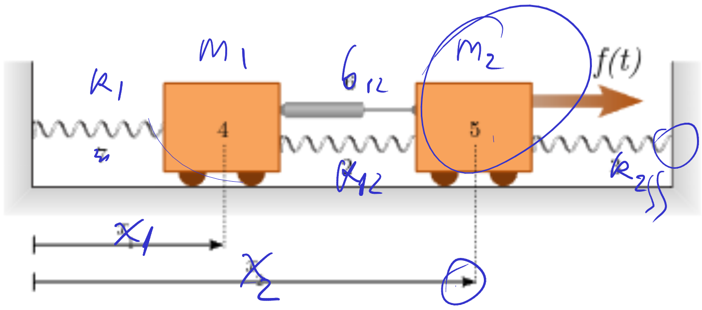
$\vec{y} = C\vec{x} + D\vec{\mu}$

$$y = [1]\,\theta + [0]\,\mu$$

$$\mu = \begin{bmatrix}\theta_{amb}\\ e\\ 1\end{bmatrix}$$

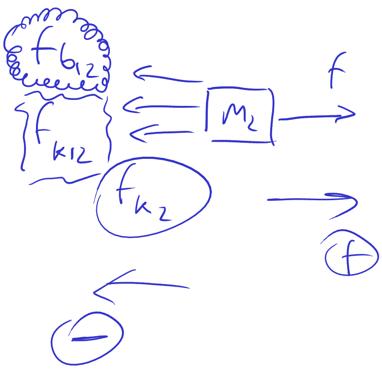


θ_{amb}



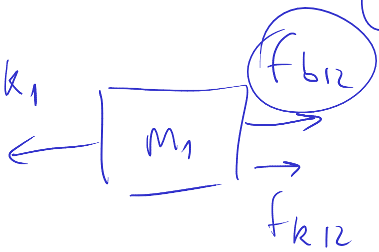
$x_1(t)$

$x_2(t)$



(1) $$f - k_2 x_2 - \overbrace{k_{12}(x_2 - x_1)}^{f_{k12}} - \overbrace{b_{12}(\dot{x}_2 - \dot{x}_1)}^{f_{b12}} = m_2 \ddot{x}_2$$

(2) $$+ b_{12}(\dot{x}_2 - \dot{x}_1) + k_{12}(x_2 - x_1) - k_1 x_1 = m_1 \ddot{x}_1$$



(1) $$m_2 \ddot{x}_2 = f - k_2 x_2 - k_{12}(x_2 - x_1) - b_{12}(\dot{x}_2 - \dot{x}_1)$$

(2) $$m_1 \ddot{x}_1 = b_{12}(\dot{x}_2 - \dot{x}_1) + k_{12}(x_2 - x_1) - k_1 x_1$$

Algoritmo para convertir a ME

(A) Elegir variables de estados

$$z_1 = x_1$$
$$z_2 = \dot{x}_1$$
$$z_3 = x_2$$
$$z_4 = \dot{x}_2$$

$$\dot{v} = Av + Bu$$

B) Derivar variables de estados

$$z_1 = x_1 \longrightarrow \dot{z}_1 = \dot{x}_1$$
$$z_2 = \dot{x}_1 \longrightarrow \dot{z}_2 = \ddot{x}_1$$
$$z_3 = x_2 \longrightarrow \dot{z}_3 = \dot{x}_2$$
$$z_4 = \dot{x}_2 \longrightarrow \dot{z}_4 = \ddot{x}_2$$

C) Reescribir usando var. est. no derivados

$$z_1 = x_1 \longrightarrow \dot{z}_1 = \dot{x}_1 \longrightarrow \dot{z}_1 = z_2$$
$$\boxed{z_2 = \dot{x}_1} \longrightarrow \dot{z}_2 = \ddot{x}_1 \longrightarrow \dot{z}_2 = \ldots \text{magia} \ldots$$
$$z_3 = x_2 \longrightarrow \dot{z}_3 = \dot{x}_2 \longrightarrow \dot{z}_3 = z_4$$
$$\boxed{z_4 = \dot{x}_2} \longrightarrow \dot{z}_4 = \ddot{x}_2 \longrightarrow \dot{z}_4 = \ldots \text{magia} \ldots$$

D) Desarrollar la magia

$$\dot{z}_2 = \ddot{x}_1 = -\frac{k_{12}+k_1}{m_1}\overset{z_1}{x_1} - \frac{k_{12}}{m_1}\overset{z_3}{x_2} - \frac{b_{12}}{m_1}\overset{z_2}{\dot{x}_1} - \frac{b_{12}}{m_1}\overset{z_4}{\dot{x}_2}$$

$$\dot{z}_4 = \ddot{x}_2 = \frac{k_{12}}{m_{\cancel{x}\,2}}\overset{z_1}{x_1} - \frac{k_2+k_{12}}{m_2}\overset{z_3}{x_2} + \frac{b_{12}}{m_2}\overset{z_2}{\dot{x}_1} - \frac{b_{12}}{m_2}\overset{z_4}{\dot{x}_2} + \frac{1}{m_2} f$$

E) Reemplazar por var. est. (mirando paso A)

$$\dot{z}_1 = \cancel{z_2}\ 0 z_1 + 1 z_2 + 0 z_3 + 0 z_4 + 0 f$$
$$\dot{z}_2 = -\frac{k_{12}+k_1}{m_1} z_1 - \frac{b_{12}}{m_1} z_2 - \frac{k_{12}}{m_1} z_3 - \frac{b_{12}}{m_1} z_4 + 0 f$$
$$\dot{z}_3 = z_4 \quad + 0 f$$
$$\dot{z}_4 = \frac{k_{12}}{m_2} z_1 + \frac{b_{12}}{m_2} z_2 - \frac{k_{12}+k_2}{m_2} z_3 - \frac{b_{12}}{m_2} z_4 + \frac{1}{m_2} f$$

F Escribir ME

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{bmatrix} = \frac{d}{dt}\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_{12}+k_1}{m_1} & -\frac{b_{12}}{m_1} & \frac{k_{12}}{m_1} & \frac{b_{12}}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k_{12}}{m_2} & \frac{b_{12}}{m_2} & -\frac{k_2+k_{12}}{m_2} & -\frac{b_{12}}{m_2} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/m_2 \end{bmatrix} f$$

4×1 $\quad$ $4 \times 4 \times 1$ $\quad$ 4×1 $\quad$ 1×1

1.1 Actividad 3.1

Graficar respuesta temporal de la posición de cada carro, considerando $f(t) = \sin t$