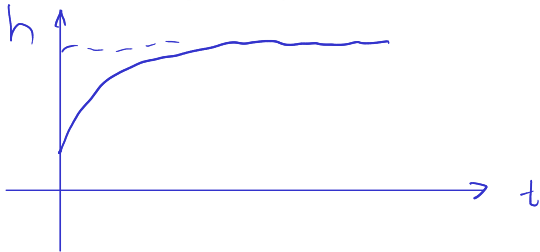




$$\dot{h}(t) = -\frac{k}{C}h(t) + \frac{1}{C}q_i(t) \rightarrow \text{ec. dif.} \rightarrow \text{simular } (2)$$

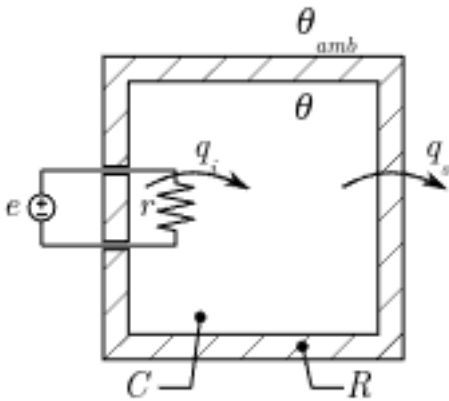
$$h(t) = \frac{U_0}{k} + \left(h_0 - \frac{U_0}{k}\right)e^{-(k/C)t} \rightarrow \text{graficar } (1) \checkmark$$



valor final! $h(t \rightarrow \infty) = \left[\frac{U_0}{k} \right]$

¿tiempo de simulación? $\frac{k}{C} = \frac{1}{\tau} \rightarrow \tau = \frac{C}{k}$

Clase 2



$$(1) q_i - q_s = C \frac{d\theta}{dt} \quad [\theta] = ^\circ\text{C}$$

$$[q_i] = \text{W (watt)} = [q_s]$$

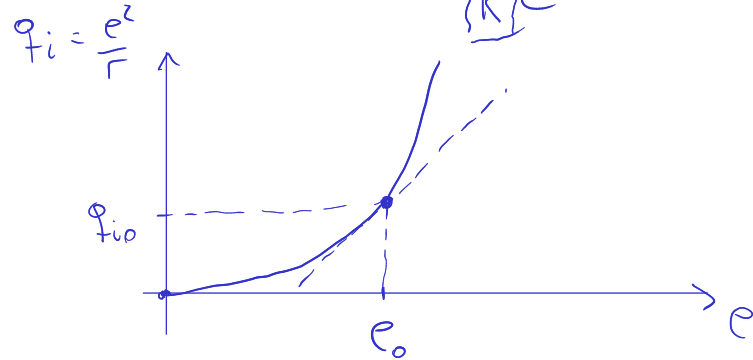
$$(2) q_i(t) = \frac{\{e(t)\}^2}{r}$$

$$(3) q_s = \frac{\theta - \theta_{amb}}{R}$$

$$C \frac{d\theta}{dt} = - \frac{\theta - \theta_{amb}}{R} + \frac{e^2}{r}$$



$$\dot{\Theta}(t) = -\frac{1}{RC} \Theta(t) + \frac{1}{RC} \Theta_{amb}(t) + \frac{1}{C} \left(\frac{e^2}{r} \right)$$



$$q_{i-lin} - q_{i0} = \left[\frac{2e_0}{r} \right] (e - e_0)$$

$$q_{i-lin} = \left[\frac{2e_0}{r} (e - e_0) + \frac{e_0^2}{r} \right]$$

Cerca del punto de op. $q_i \approx q_{i-lin}$

$$\dot{\Theta}(t) = -\frac{1}{RC} \Theta(t) + \frac{1}{RC} \Theta_{amb}(t) + \frac{1}{rC} e^2(t)$$

$$\dot{\Theta}(t) \approx -\frac{1}{RC} \Theta(t) + \frac{1}{RC} \Theta_{amb} + \frac{1}{C} \left[\frac{2e_0}{r} (e - e_0) + \frac{e_0^2}{r} \right]$$

3 Actividades de simulación

3.1 Actividad 2.1

Grafiar la evolución de la temperatura $\theta(t)$ en base a la ecuación linealizada, considerando

- temperatura inicial 10°
- temperatura ambiente nula
- tensión de entrada constante $e(t) = e_0$

$$R=1 \quad r=1 \quad C=1$$

$$e_0=1 \quad \Theta_0=10$$

$$\dot{\Theta}(t) = \left[\frac{-1}{RC} \right] \Theta(t) + \left[\frac{e_0^2}{rC} \right]$$

$$\tau = \frac{1}{1/RC} = RC = 1$$

$$t_s = 5\tau$$

settling $\leftarrow 98\% \text{ Vf.}$



Modelo de Estados

Sist. de ecs. dif \rightarrow Ec matricial

$$\dot{\Theta}(t) = \underbrace{\frac{-1}{RC}}_{\text{matrices}} \Theta(t) + \underbrace{\frac{1}{RC}}_{\text{matrices}} \Theta_{amb} + \underbrace{\frac{1}{C} \left[\frac{2e_0}{r} (e - e_0) + \frac{e_0^2}{r} \right]}_{\text{matrices}} \underbrace{\frac{2e_0 e - 2e_0^2}{r} + \frac{e_0^2}{r}}_{\text{matrices}}$$

//

$$\begin{cases} \dot{x}_1 = f(x_1, x_2, x_3) + u_1 \\ \dot{x}_2 = g(x_1, x_2, x_3) + u_2 \\ \dot{x}_3 = h(x_1, x_2, x_3) + u_3 \end{cases}$$

//

$$\dot{X} = A X + B u$$

↑ ↑ ↑
vector vector vector

matrices

$$\dot{\Theta} \approx \frac{-1}{RC} \Theta(t) + \frac{1}{RC} \Theta_{amb}(t) + \frac{2e_0}{rC} e(t) - \frac{e_0^2}{rC}$$

. variables de la ec. dif: 1 $\Theta(t)$ $\rightarrow x$
. señales externas: 2 "y medio" $\begin{cases} \Theta_{amb} \\ e(t) \\ \frac{1}{2} e_0^2 / rC \text{ (linealización)} \end{cases}$ $\rightarrow u$

$$x = \Theta$$

$$u = \begin{bmatrix} \Theta_{amb}(t) \\ e(t) \\ 1 \end{bmatrix}$$



2.1 No lineal

Ecuación diferencial no-lineal:

$$\dot{\theta}(t) = -\frac{1}{RC}\theta + \frac{1}{RC}\theta_{amb} + \frac{1}{rC}e^2(t)$$

Convirtiendo al modelo de estados:

$$\frac{d}{dt} [\theta(t)] = \left[-\frac{1}{RC} \right] [\theta(t)] + \left[\frac{1}{RC} \quad \frac{1}{rC} \right] \begin{bmatrix} \theta_{amb}(t) \\ e^2(t) \end{bmatrix}$$
$$y(t) = \theta(t)$$

2.2 Lineal

Ecuación diferencial linealizada:

$$\dot{\theta}(t) = -\frac{1}{RC}\theta + \frac{1}{RC}\theta_{amb} + \frac{2e_0}{rC}(e(t) - e_0) + \frac{e_0^2}{rC}$$

Convirtiendo al modelo de estados:

$$\frac{d}{dt} [\theta(t)] = \left[-\frac{1}{RC} \right] [\theta(t)] + \left[\frac{1}{RC} \quad \frac{2e_0}{rC} \quad -\frac{e_0^2}{rC} \right] \begin{bmatrix} \theta_{amb}(t) \\ e(t) \\ 1 \end{bmatrix}$$
$$y(t) = \theta(t)$$