```
fun append (xs,ys) =
    if xs=[]
    then ys
    else (hd xs)::append(tl xs,ys)

fun map (f,xs) =
    case xs of
      [] => []
      | x::xs' => (f x)::(map(f,xs'))

val a = map (increment, [4,8,12,16])
val b = map (hd, [[8,6],[7,5],[3,0,9]])
```

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ML Expressions and Variable Bindings

Mindset

- "Let go" of all programming languages you already know
- For now, treat ML as a "totally new thing"
 - Time later to compare/contrast to what you know
 - For now, "oh that seems kind of like this thing in [Java]" will confuse you, slow you down, and you will learn less
- Start from a blank file...

A very simple ML program

[The same program we just wrote in Emacs; here for conveniene if reviewing the slides]

```
(* My first ML program *)
val x = 34;
val y = 17;
val z = (x + y) + (y + 2);
val q = z + 1;
val abs of z = if z < 0 then 0 - z else z;
val abs of z simpler = abs z
```

A variable binding

```
val z = (x + y) + (y + 2); (* comment *)
```

More generally:

$$val x = e;$$

- Syntax:
 - Keyword val and punctuation = and ;
 - Variable x
 - Expression e
 - Many forms of these, most containing subexpressions

The semantics

- Syntax is just how you write something
- Semantics is what that something means
 - Type-checking (before program runs)
 - Evaluation (as program runs)
- For variable bindings:
 - Type-check expression and extend static environment
 - Evaluate expression and extend dynamic environment

So what is the precise syntax, type-checking rules, and evaluation rules for various expressions? Good question!

```
fun append (xs,ys) =
    if xs=[]
    then ys
    else (hd xs)::append(tl xs,ys)

fun map (f,xs) =
    case xs of
    [] => []
    | x::xs' => (f x)::(map(f,xs'))

val a = map (increment, [4,8,12,16])
val b = map (hd, [[8,6],[7,5],[3,0,9]])
```

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ML Rules for Expressions (Seen So Far)

A very simple ML program

This program has integers, variables, addition, if-expressions, less-than, subtraction, and calling a pre-defined function

```
(* My first ML program *)
val x = 34;
val y = 17;
val z = (x + y) + (y + 2);
val q = z + 1;
val abs of z = if z < 0 then 0 - z else z;
val abs of z simpler = abs z
```

Expressions

We have seen many kinds of expressions:

```
34 true false x e1+e2 e1 < e2 if e1 then e2 else e3
```

- Can get arbitrarily large since any subexpression can contain subsubexpressions, etc.
- Every kind of expression has
 - 1. Syntax
 - 2. Type-checking rules
 - Produces a type or fails (with a bad error message ☺)
 - Types so far: int bool unit
 - 3. Evaluation rules (used only on things that type-check)
 - Produces a value (or exception or infinite-loop)

Variables

Syntax:

sequence of letters, digits, _, not starting with digit

• Type-checking:

Look up type in current static environment

- If not there, fail
- Evaluation:

Look up value in current dynamic environment

Addition

Syntax:

e1 + e2 where e1 and e2 are expressions

• Type-checking:

If e1 and e2 have type int, then e1 + e2 has type int

Evaluation:

If e1 evaluates to v1 and e2 evaluates to v2, then e1 + e2 evaluates to sum of v1 and v2

Values

- All values are expressions
- Not all expressions are values
- Every value "evaluates to itself" in "zero steps"
- Examples:
 - 34, 17, 42 have type int
 - true, false have type bool
 - () has type unit

A slightly tougher one

What are the syntax, typing rules, and evaluation rules for conditional expressions?

Let's write it out...

Now you try one

Syntax, type-checking rules, and evaluation rules for less-than comparisons?

```
fun append (xs,ys) =
    if xs=[]
    then ys
    else (hd xs)::append(tl xs,ys)

fun map (f,xs) =
    case xs of
    [] => []
    | x::xs' => (f x)::(map(f,xs'))

val a = map (increment, [4,8,12,16])
val b = map (hd, [[8,6],[7,5],[3,0,9]])
```

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The REPL and Error Messages

Pragmatics

Last two segments have built up key conceptual foundation

But you also need some pragmatics:

- How do we run programs using the REPL?
- What happens when we make mistakes?

Work on developing resilience to mistakes

- Slow down
- Don't panic
- Read what you wrote very carefully

use

use "foo.sml" is an unusual expression

It enters bindings from the file foo.sml

Result is () bound to variable it

- Ignorable

The REPL

- Read-Eval-Print-Loop is well named
- Can just treat it as a strange/convenient way to run programs
 - But more convenient for quick try-something-out
 - Then move things over to a testing file for easy reuse
- For reasons discussed in next segment, do not use use without restarting the REPL session
 - (But using it for multiple files at beginning of session is okay)

Errors

Your mistake could be:

- Syntax: What you wrote means nothing or not the construct you intended
- Type-checking: What you wrote does not type-check
- Evaluation: It runs but produces wrong answer, or an exception, or an infinite loop

Keep these straight when debugging even if sometimes one kind of mistake appears to be another

Play around

Best way to learn something: Try lots of things and don't be afraid of errors

Maybe watching me make a few mistakes will help...

```
fun append (xs,ys) =
    if xs=[]
    then ys
    else (hd xs)::append(tl xs,ys)

fun map (f,xs) =
    case xs of
    [] => []
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val a = map (increment, [4,8,12,16])
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```

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Shadowing

Multiple bindings of same variable

Multiple variable bindings of the same variable is often poor style

Often confusing

But it's an instructive exercise

- Helps explain how the environment "works"
- Helps explain how a variable binding "works"

(Emphasize this now to lay the foundation for first-class functions)

Our example

```
val a = 10
val b = a * 2
val a = 5
val c = b
val d = a
val a = a + 1
(* val g = f - 3 *) (* does not type-check *)
val f = a * 2
```

Two reasons (either one sufficient)

```
val a = 1
val b = a (* b is bound to 1 *)
val a = 2
```

- 1. Expressions in variable bindings are evaluated "eagerly"
 - Before the variable binding "finishes"
 - Afterwards, the expression producing the value is irrelevant
- 2. There is no way to "assign to" a variable in ML
 - Can only shadow it in a later environment

use

This is why I am so insistent about not reusing **use** on a file without restarting the REPL

Else you are introducing some of the same bindings again

- May make it seem like wrong code is correct
- May make it seem like correct code is wrong
- (It's all well-defined, but we humans get confused)

```
fun append (xs,ys) =
    if xs=[]
    then ys
    else (hd xs)::append(tl xs,ys)

fun map (f,xs) =
    case xs of
      [] => []
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val a = map (increment, [4,8,12,16])
val b = map (hd, [[8,6],[7,5],[3,0,9]])
```

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Functions (Informally)

Function definitions

Functions: the most important building block in the whole course

- Like Java methods, have arguments and result
- But no classes, this, return, etc.

Example function binding:

```
(* Note: correct only if y>=0 *)
fun pow (x:int, y:int) =
  if y=0
  then 1
  else x * pow(x,y-1)
```

Note: The body includes a (recursive) function call: pow(x,y-1)

Example, extended

```
fun pow (x : int, y : int) =
   if y=0
   then 1
   else x * pow(x,y-1)

fun cube (x : int) =
   pow (x,3)

val sixtyfour = cube 4

val fortytwo = pow(2,2+2) + pow(4,2) + cube(2) + 2
```

Some gotchas

Three common "gotchas"

- Bad error messages if you mess up function-argument syntax
- The use of * in type syntax is not multiplication
 - Example: int * int -> int
 - In expressions, * is multiplication: x * pow(x,y-1)
- Cannot refer to later function bindings
 - That's simply ML's rule
 - Helper functions must come before their uses
 - Need special construct for mutual recursion (later)

Recursion

- If you're not yet comfortable with recursion, you will be soon ©
 - Will use for most functions taking or returning lists
- "Makes sense" because calls to same function solve "simpler" problems
- Recursion more powerful than loops
 - We won't use a single loop in ML
 - Loops often (not always) obscure simple, elegant solutions

```
fun append (xs,ys) =
    if xs=[]
    then ys
    else (hd xs)::append(tl xs,ys)

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val b = map (hd, [[8,6],[7,5],[3,0,9]])
```

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Functions (Informally)

Function definitions

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- Like Java methods, have arguments and result
- But no classes, this, return, etc.

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```

Note: The body includes a (recursive) function call: pow(x,y-1)

Example, extended

```
fun pow (x : int, y : int) =
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fun cube (x : int) =
   pow (x,3)

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val fortytwo = pow(2,2+2) + pow(4,2) + cube(2) + 2
```

Some gotchas

Three common "gotchas"

- Bad error messages if you mess up function-argument syntax
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    case xs of
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val a = map (increment, [4,8,12,16])
val b = map (hd, [[8,6],[7,5],[3,0,9]])
```

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Pairs and Other Tuples

Tuples and lists

So far: numbers, booleans, conditionals, variables, functions

- Now ways to build up data with multiple parts
- This is essential
- Java examples: classes with fields, arrays

Now:

- Tuples: fixed "number of pieces" that may have different types
 Coming soon:
- Lists: any "number of pieces" that all have the same type
 Later:
 - Other more general ways to create compound data

Pairs (2-tuples)

Need a way to build pairs and a way to access the pieces

Build:

- Syntax: *(e1,e2)*
- Evaluation: Evaluate e1 to v1 and e2 to v2; result is (v1, v2)
 - A pair of values is a value
- Type-checking: If e1 has type ta and e2 has type tb, then the pair expression has type ta * tb
 - A new kind of type

Pairs (2-tuples)

Need a way to *build* pairs and a way to *access* the pieces

Access:

- Syntax: #1 e and #2 e
- Evaluation: Evaluate e to a pair of values and return first or second piece
 - Example: If e is a variable x, then look up x in environment
- Type-checking: If e has type ta * tb, then #1 e has type ta and #2 e has type tb

Examples

Functions can take and return pairs

```
fun swap (pr : int*bool) =
  (#2 pr, #1 pr)
fun sum two pairs (pr1 : int*int, pr2 : int*int) =
  (#1 pr1) + (#2 pr1) + (#1 pr2) + (#2 pr2)
fun div mod (x : int, y : int) =
  (x \text{ div } y, x \text{ mod } y)
fun sort pair (pr : int*int) =
  if (#1 pr) < (#2 pr)
  then pr
  else (#2 pr, #1 pr)
```

Tuples

Actually, you can have tuples with more than two parts

A new feature: a generalization of pairs

```
• (e1,e2,...,en)
```

- ta * tb * ... * tn
- #1 e, #2 e, #3 e, ...

Homework 1 uses triples of type int*int a lot

Nesting

Pairs and tuples can be nested however you want

Not a new feature: implied by the syntax and semantics

```
fun append (xs,ys) =
    if xs=[]
    then ys
    else (hd xs)::append(tl xs,ys)

fun map (f,xs) =
    case xs of
      [] => []
      | x::xs' => (f x)::(map(f,xs'))

val a = map (increment, [4,8,12,16])
val b = map (hd, [[8,6],[7,5],[3,0,9]])
```

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Introducing Lists

Lists

 Despite nested tuples, the type of a variable still "commits" to a particular "amount" of data

In contrast, a list:

- Can have any number of elements
- But all list elements have the same type

Need ways to *build* lists and *access* the pieces...

Building Lists

The empty list is a value:

[]

 In general, a list of values is a value; elements separated by commas:

If e1 evaluates to v and e2 evaluates to a list [v1,...,vn],
 then e1::e2 evaluates to [v,...,vn]

```
e1::e2 (* pronounced "cons" *)
```

Accessing Lists

Until we learn pattern-matching, we will use three standard-library functions

- null e evaluates to true if and only if e evaluates to []
- If e evaluates to [v1, v2, ..., vn] then hd e evaluates to v1
 - (raise exception if e evaluates to [])
- If e evaluates to [v1, v2, ..., vn] then t1 e evaluates to [v2, ..., vn]
 - (raise exception if e evaluates to [])
 - Notice result is a list

Type-checking list operations

Lots of new types: For any type t, the type t list describes lists where all elements have type t

- Examples: int list bool list int list list
 (int * int) list (int list * int) list
- So [] can have type t list for any type
 - SML uses type 'a list to indicate this ("quote a" or "alpha")
- For e1::e2 to type-check, we need a t such that e1 has type t and e2 has type t list. Then the result type is t list
- null : 'a list -> bool
- hd : 'a list -> 'a
- tl : 'a list -> 'a list

```
fun append (xs,ys) =
    if xs=[]
    then ys
    else (hd xs)::append(tl xs,ys)

fun map (f,xs) =
    case xs of
    [] => []
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val a = map (increment, [4,8,12,16])
val b = map (hd, [[8,6],[7,5],[3,0,9]])
```

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List Functions

Functions over lists

Gain experience with lists and recursion by writing several functions that process and/or produce lists...

Example list functions

```
fun sum list (xs:int list) =
 if null xs
 then 0
 else hd(xs) + sum list(tl(xs))
fun countdown (x : int) =
 if x=0
 then []
 else x :: countdown (x-1)
fun append (xs:int list, ys:int list) =
 if null xs
 then ys
 else hd (xs) :: append (tl(xs), ys)
```

Recursion again

Functions over lists are usually recursive

- Only way to "get to all the elements"
- What should the answer be for the empty list?
- What should the answer be for a non-empty list?
 - Typically in terms of the answer for the tail of the list!

Similarly, functions that produce lists of potentially any size will be recursive

You create a list out of smaller lists

Lists of pairs

Processing lists of pairs requires no new features. Examples:

```
fun sum_pair list (xs: (int*int) list) =
  if null xs
  then 0
  else #1(hd xs) + #2(hd xs) + sum pair list(tl xs)
fun firsts (xs: (int*int) list) =
  if null xs
  then []
  else #1(hd xs) :: firsts(tl xs)
fun seconds (xs: (int*int) list) =
  if null xs
  then []
  else #2(hd xs) :: seconds(tl xs)
fun sum pair list2 (xs: (int*int) list) =
 (sum list (firsts xs)) + (sum list (seconds xs))
```

```
fun append (xs,ys) =
    if xs=[]
    then ys
    else (hd xs)::append(tl xs,ys)

fun map (f,xs) =
    case xs of
      [] => []
      | x::xs' => (f x)::(map(f,xs'))

val a = map (increment, [4,8,12,16])
val b = map (hd, [[8,6],[7,5],[3,0,9]])
```

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Let Expressions

Review

Huge progress already on the core pieces of ML:

- Types: int bool unit t1*...*tn t list t1*...*tn->t
 - Types "nest" (each t above can be itself a compound type)
- Variables, environments, and basic expressions
- Functions
 - Build: $fun \times 0 (x1:t1, ..., xn:tn) = e$
 - Use: e0 (e1, ..., en)
- Tuples
 - Build: (e1, ..., en)
 - Use: **#1 e**, **#2 e**, ...
- Lists
 - Build: [] e1::e2
 - Use: null e hd e tl e

Now...

The big thing we need: local bindings

For style and convenience

This segment:

Basic let-expressions

Next segments:

- A big but natural idea: nested function bindings
- For efficiency (not "just a little faster")

The construct to introduce local bindings is *just an expression*, so we can use it anywhere an expression can go

Let-expressions

3 questions:

- Syntax: let b1 b2 ... bn in e end
 - Each bi is any binding and e is any expression
- Type-checking: Type-check each bi and e in a static environment that includes the previous bindings.
 Type of whole let-expression is the type of e.
- Evaluation: Evaluate each **bi** and **e** in a dynamic environment that includes the previous bindings.
 - Result of whole let-expression is result of evaluating *e*.

Silly examples

```
fun silly1 (z : int) =
    let val x = if z > 0 then z else 34
        val y = x+z+9
    in
        if x > y then x*2 else y*y
    end
fun silly2 () =
    let val x = 1
    in
        (let val x = 2 in x+1 end) +
        (let val y = x+2 in y+1 end)
    end
```

silly2 is poor style but shows let-expressions are expressions

- Can also use them in function-call arguments, if branches, etc.
- Also notice shadowing

What's new

- What's new is **scope**: where a binding is in the environment
 - In later bindings and body of the let-expression
 - (Unless a later or nested binding shadows it)
 - Only in later bindings and body of the let-expression
- Nothing else is new:
 - Can put any binding we want, even function bindings
 - Type-check and evaluate just like at "top-level"

```
fun append (xs,ys) =
    if xs=[]
    then ys
    else (hd xs)::append(tl xs,ys)

fun map (f,xs) =
    case xs of
      [] => []
      | x::xs' => (f x)::(map(f,xs'))

val a = map (increment, [4,8,12,16])
val b = map (hd, [[8,6],[7,5],[3,0,9]])
```

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Nested Functions

Any binding

According to our rules for let-expressions, we can define functions inside any let-expression

This is a natural idea, and often good style

(Inferior) Example

```
fun countup_from1 (x : int) =
   let fun count (from : int, to : int) =
        if from = to
        then to :: []
        else from :: count(from+1,to)
   in
        count (1,x)
   end
```

- This shows how to use a local function binding, but:
 - Better version on next slide
 - count might be useful elsewhere

Better:

```
fun countup_from1_better (x : int) =
   let fun count (from : int) =
        if from = x
        then x :: []
        else from :: count(from+1)
   in
        count 1
   end
```

- Functions can use bindings in the environment where they are defined:
 - Bindings from "outer" environments
 - Such as parameters to the outer function
 - Earlier bindings in the let-expression
- Unnecessary parameters are usually bad style
 - Like to in previous example

Nested functions: style

- Good style to define helper functions inside the functions they help if they are:
 - Unlikely to be useful elsewhere
 - Likely to be misused if available elsewhere
 - Likely to be changed or removed later
- A fundamental trade-off in code design: reusing code saves effort and avoids bugs, but makes the reused code harder to change later

```
fun append (xs,ys) =
    if xs=[]
    then ys
    else (hd xs)::append(tl xs,ys)

fun map (f,xs) =
    case xs of
    [] => []
    | x::xs' => (f x)::(map(f,xs'))

val a = map (increment, [4,8,12,16])
val b = map (hd, [[8,6],[7,5],[3,0,9]])
```

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Let Expressions to Avoid Repeated Computation

Avoid repeated recursion

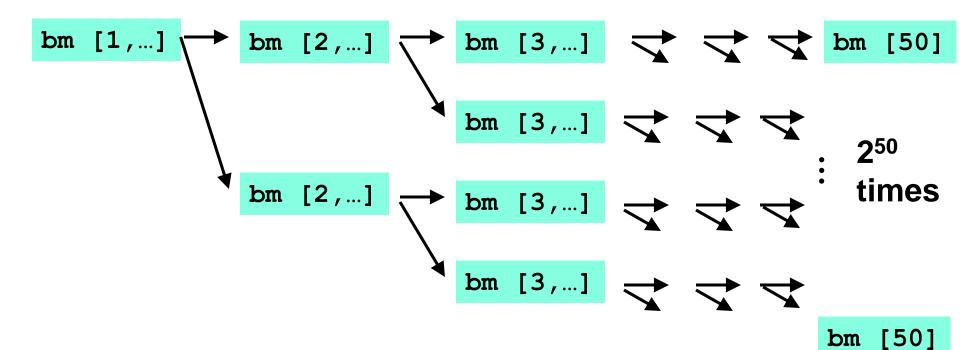
Consider this code and the recursive calls it makes

 Don't worry about calls to null, hd, and tl because they do a small constant amount of work

```
fun bad max (xs:int list) =
    if null xs
    then 0 (* horrible style; fix later *)
    else if null (tl xs)
    then hd xs
    else if hd xs > bad max (tl xs)
    then hd xs
    else bad max (tl xs)
let x = bad max [50, 49, ..., 1]
let y = bad max [1,2,...,50]
```

Fast vs. unusable

if hd xs > bad_max (tl xs)
then hd xs
else bad_max (tl xs)



Math never lies

Suppose one bad_max call's if-then-else logic and calls to hd, null, tl take 10⁻⁷ seconds

- Then bad_max [50,49,...,1] takes 50×10^{-7} seconds
- And bad_max [1,2,...,50] takes 1.12 x 108 seconds
 - (over 3.5 years)
 - bad_max [1,2,...,55] takes over 1 century
 - Buying a faster computer won't help much ©

The key is not to do repeated work that might do repeated work that might do...

Saving recursive results in local bindings is essential...

Efficient max

```
fun good max (xs:int list) =
    if null xs
    then 0 (* horrible style; fix later *)
    else if null (tl xs)
    then hd xs
   else
         let val tl ans = good max(tl xs)
         in
             if hd xs > tl ans
             then hd xs
             else tl ans
         end
```

Fast vs. fast

```
let val tl_ans = good_max(tl xs)
in
    if hd xs > tl_ans
    then hd xs
    else tl_ans
end
```

gm [50,...]
$$\rightarrow$$
 gm [49,...] \rightarrow gm [48,...] \rightarrow \rightarrow gm [1]
gm [1,...] \rightarrow gm [2,...] \rightarrow gm [3,...] \rightarrow \rightarrow gm [50]

```
fun append (xs,ys) =
    if xs=[]
    then ys
    else (hd xs)::append(tl xs,ys)

fun map (f,xs) =
    case xs of
      [] => []
      | x::xs' => (f x)::(map(f,xs'))

val a = map (increment, [4,8,12,16])
val b = map (hd, [[8,6],[7,5],[3,0,9]])
```

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Options

Motivating Options

Having max return 0 for the empty list is really awful

- Could raise an exception (future topic)
- Could return a zero-element or one-element list
 - That works but is poor style because the built-in support for options expresses this situation directly

Options

- t option is a type for any type t
 - (much like t list, but a different type, not a list)

Building:

- NONE has type 'a option (much like [] has type 'a list)
- SOME e has type t option if e has type t (much like e::[])

Accessing:

- isSome has type 'a option -> bool
- valOf has type 'a option -> 'a (exception if given NONE)

Example

```
fun better_max (xs:int list) =
   if null xs
   then NONE
   else
       let val tl_ans = better_max(tl xs)
       in
       if isSome tl_ans
            andalso valOf tl_ans > hd xs
       then tl_ans
       else SOME (hd xs)
   end
```

```
val better_max = fn : int list -> int option
```

 Nothing wrong with this, but as a matter of style might prefer not to do so much useless "valOf" in the recursion

Example variation

```
fun better max2 (xs:int list) =
    if null xs
    then NONE
    else let (* ok to assume xs nonempty b/c local *)
             fun max nonempty (xs:int list) =
               if null (tl xs)
               then hd xs
               else
                  let val tl ans = max nonempty(tl xs)
                  in
                    if hd xs > tl ans
                    then hd xs
                    else tl ans
                  end
          in
             SOME (max nonempty xs)
          end
```

```
fun append (xs,ys) =
    if xs=[]
    then ys
    else (hd xs)::append(tl xs,ys)

fun map (f,xs) =
    case xs of
      [] => []
      | x::xs' => (f x)::(map(f,xs'))

val a = map (increment, [4,8,12,16])
val b = map (hd, [[8,6],[7,5],[3,0,9]])
```

More Boolean and Comparison Expressions

Some More Expressions

Some "odds and ends" that haven't come up much yet:

- Combining Boolean expressions (and, or, not)
- Comparison operations

Boolean operations

e1 andalso e2

- Type-checking: e1 and e2 must have type bool
- Evaluation: If result of e1 is false then false else result of e2

e1 orelse e2
not e1

- Syntax in many languages is e1 && e2, e1 || e2, !e
 - && and | | don't exist in ML and ! means something different
- "Short-circuiting" evaluation means andalso and orelse are not functions, but not is just a pre-defined function

Style with Booleans

Language does not need andalso, orelse, not

```
(* e1 andalso e2 *) (* e1 orelse e2 *) (* not e1 *)
if el
then e2
else false
```

```
if el
then true
else e2
```

```
if el
then false
else true
```

Using more concise forms generally much better style

And definitely please do not do this:

```
(* just say e (!!!) *)
if e
then true
else false
```

Comparisons

For comparing int values:

You might see weird error messages because comparators can be used with some other types too:

- > < >= <= can be used with real, but not 1 int and 1 real
- = <> can be used with any "equality type" but not with real
 - Let's not discuss equality types yet

```
fun append (xs,ys) =
    if xs=[]
    then ys
    else (hd xs)::append(tl xs,ys)

fun map (f,xs) =
    case xs of
      [] => []
      | x::xs' => (f x)::(map(f,xs'))

val a = map (increment, [4,8,12,16])
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```

A Key Benefit of Immutable Data

A valuable non-feature: no mutation

Have now covered all the features you need (and should use) on hw1

Now learn a very important non-feature

- Huh?? How could the *lack* of a feature be important?
- When it lets you know things other code will not do with your code and the results your code produces

A major aspect and contribution of functional programming:

Not being able to assign to (a.k.a. *mutate*) variables or parts of tuples and lists

(This is a "Big Deal")

Cannot tell if you copy

```
fun sort_pair (pr : int * int) =
   if #1 pr < #2 pr
   then pr
   else (#2 pr, #1 pr)

fun sort_pair (pr : int * int) =
   if #1 pr < #2 pr
   then (#1 pr, #2 pr)
   else (#2 pr, #1 pr)</pre>
```

In ML, these two implementations of sort_pair are indistinguishable

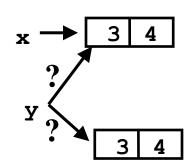
- But only because tuples are immutable
- The first is better style: simpler and avoids making a new pair in the then-branch
- In languages with mutable compound data, these are different!

Suppose we had mutation...

```
val x = (3,4)
val y = sort_pair x

somehow mutate #1 x to hold 5

val z = #1 y
```



- What is z?
 - Would depend on how we implemented sort_pair
 - Would have to decide carefully and document sort_pair
 - But without mutation, we can implement "either way"
 - No code can ever distinguish aliasing vs. identical copies
 - No need to think about aliasing: focus on other things
 - Can use aliasing, which saves space, without danger

An even better example

```
fun append (xs:int list, ys:int list) =
       if null xs
       then ys
       else hd (xs) :: append (tl(xs), ys)
  val x = [2,4]
  val y = [5,3,0]
  val z = append(x,y)
                                             (can't tell,
                                             but it's the
or
                                             first one)
```

ML vs. Imperative Languages

- In ML, we create aliases all the time without thinking about it because it is impossible to tell where there is aliasing
 - Example: t1 is constant time; does not copy rest of the list
 - So don't worry and focus on your algorithm
- In languages with mutable data (e.g., Java), programmers are obsessed with aliasing and object identity
 - They have to be (!) so that subsequent assignments affect the right parts of the program
 - Often crucial to make copies in just the right places
 - Optional Java example in next segment

```
fun append (xs,ys) =
    if xs=[]
    then ys
    else (hd xs)::append(tl xs,ys)

fun map (f,xs) =
    case xs of
      [] => []
      | x::xs' => (f x)::(map(f,xs'))

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```

Optional: Java Mutation Bug

ML vs. Imperative Languages

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 - They have to be (!) so that subsequent assignments affect the right parts of the program
 - Often crucial to make copies in just the right places
 - Optional Java example...

Java security nightmare (bad code)

```
class ProtectedResource {
   private Resource theResource = ...;
   private String[] allowedUsers = ...;
   public String[] getAllowedUsers() {
      return allowedUsers;
   public String currentUser() { ... }
   public void useTheResource() {
      for(int i=0; i < allowedUsers.length; i++) {</pre>
         if (currentUser().equals(allowedUsers[i])) {
             ... // access allowed: use it
             return;
      throw new IllegalAccessException();
```

Have to make copies

The problem:

```
p.getAllowedUsers()[0] = p.currentUser();
p.useTheResource();
```

The fix:

```
public String[] getAllowedUsers() {
    ... return a copy of allowedUsers ...
}
```

Reference (alias) vs. copy doesn't matter if code is immutable!

```
fun append (xs,ys) =
    if xs=[]
    then ys
    else (hd xs)::append(tl xs,ys)

fun map (f,xs) =
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```

The Pieces of Learning a Language

Five different things

- Syntax: How do you write language constructs?
- Semantics: What do programs mean? (Evaluation rules)
- 3. Idioms: What are typical patterns for using language features to express your computation?
- 4. Libraries: What facilities does the language (or a well-known project) provide "standard"? (E.g., file access, data structures)
- 5. Tools: What do language implementations provide to make your job easier? (E.g., REPL, debugger, code formatter, ...)
 - Not actually part of the language

These are 5 separate issues

- In practice, all are essential for good programmers
- Many people confuse them, but shouldn't

Our Focus

This course focuses on semantics and idioms

- Syntax is usually uninteresting
 - A fact to learn, like "The American Civil War ended in 1865"
 - People obsess over subjective preferences
- Libraries and tools crucial, but often learn new ones "on the job"
 - We are learning semantics and how to use that knowledge to understand all software and employ appropriate idioms
 - By avoiding most libraries/tools, our languages may look "silly" but so would any language used this way