

```
fun append (xs,ys) =  
  if xs=[]  
  then ys  
  else (hd xs)::append(tl xs,ys)  
  
fun map (f,xs) =  
  case xs of  
    [] => []  
  | x::xs' => (f x)::(map(f,xs'))  
  
val a = map (increment, [4,8,12,16])  
val b = map (hd, [[8,6],[7,5],[3,0,9]])
```

Programming Languages

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Lexical Scope and Higher-Order Functions

The rule stays the same

A function body is evaluated in the environment where the function was defined (created)

- Extended with the function argument

Nothing changes to this rule when we take and return functions

- But “the environment” may involve nested let-expressions, not just the top-level sequence of bindings

Makes first-class functions much more powerful

- Even if may seem counterintuitive at first

Example: Returning a function

```
(* 1 *) val x = 1
(* 2 *) fun f y =
(* 2a *)   let val x = y+1
(* 2b *)   in fn z => x+y+z end
(* 3 *) val x = 3
(* 4 *) val g = f 4
(* 5 *) val y = 5
(* 6 *) val z = g 6
```

- Trust the rule: Evaluating line 4 binds to **g** to a closure:
 - Code: “take **z** and have body **x+y+z**”
 - Environment: “**y** maps to 4, **x** maps to 5 (shadowing), ...”
 - So this closure will always add 9 to its argument
- So line 6 binds 15 to **z**

Example: Passing a function

```
(* 1 *) fun f g = (* call arg with 2 *)  
(* 1a *)      let val x = 3  
(* 1b *)      in g 2 end  
(* 2 *) val x = 4  
(* 3 *) fun h y = x + y  
(* 4 *) val z = f h
```

- Trust the rule: Evaluating line 3 binds **h** to a closure:
 - Code: “take **y** and have body **x+y**”
 - Environment: “**x** maps to **4**, **f** maps to a closure, ...”
 - So this closure will always add **4** to its argument
- So line 4 binds **z** to **6**
 - Line 1a is as stupid and irrelevant as it should be