Giang Duong – ID 014533857
Assignment 4 – CS 271
Professor Mark Stamp

1.

a. 2A =
[[2 0 -4]
[-4 6 2]]
b. B +C =
[[4 1]
[1 -5]
[5 0]]
c. A+B is **undefined** because A 2x3 and B is 3x2. To add matrix with another matrix we need nxm = sxt
d. AB =
[[-5 -3]
[7 -3]]
e. BA =
[[3 -3 -3]
[6 -6 -6]
[1 3 -5]]
f. BC is **undefined** because B 3x2 and C is 3x2.
To get product of 2 matrices we need m = s where B nxm = C sxt

4.

a./
[[2.5 2.5 4.]
[2.5 5. 4.5]
[4. 4.5 6.5]]

b./
Eigenvalues of C =
[12.499999999999984, 1.5000000000000004]

c./
[-0.42426407 -0.80829038 0.40824829]
[-0.70710678 0.57735027 0.40824829]

Giang Duong

HW 4.4.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \\ 2 & 3 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

a) $C = \frac{1}{2} AA^T =$

$$= \frac{1}{2}\begin{bmatrix} 1 & 2 \\ 3 & 1 \\ 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 3 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 5 & 5 & 8 \\ 5 & 10 & 9 \\ 8 & 9 & 13 \end{bmatrix} = \begin{bmatrix} 2.5 & 2.5 & 4 \\ 2.5 & 5 & 4.5 \\ 4 & 4.5 & 6.5 \end{bmatrix}$$

b) Determine λ_1, λ_2.

$$\begin{bmatrix} 2.5-\lambda & 2.5 & 4 \\ 2.5 & 5-\lambda & 4.5 \\ 4 & 4.5 & 6.5-\lambda \end{bmatrix} = (2.5-\lambda)\begin{vmatrix} 5-\lambda & 4.5 \\ 4.5 & 6.5-\lambda \end{vmatrix} + 2.5\begin{bmatrix} 2.5 & 4.5 \\ 4 & 6.5-\lambda \end{bmatrix}$$

$$+ 4\begin{bmatrix} 2.5 & 5-\lambda \\ 4 & 4.5 \end{bmatrix}$$

$$= \left[(2.5-\lambda)\left[(5-\lambda)(6.5-\lambda) - 20.25\right]\right] + 2.5\,(16.25 - 2.5\lambda - 18$$

$$+ 4\,(11.25 - 20 - 4\lambda) =$$

$$= (2.5-\lambda)\left[32.5 - 11.5\lambda - \lambda^2 - 20.25\right] + 40.625 - 6.25\lambda - 45$$

$$+ 45 - 80 - 16\lambda =$$

$$= \cancel{81.25} - 28.75\lambda + 2.5\lambda^2 - 30.625 - 12.25\lambda + 11.5\lambda^2 + \lambda^3$$

$$+ 39.375 + 22.25\lambda =$$

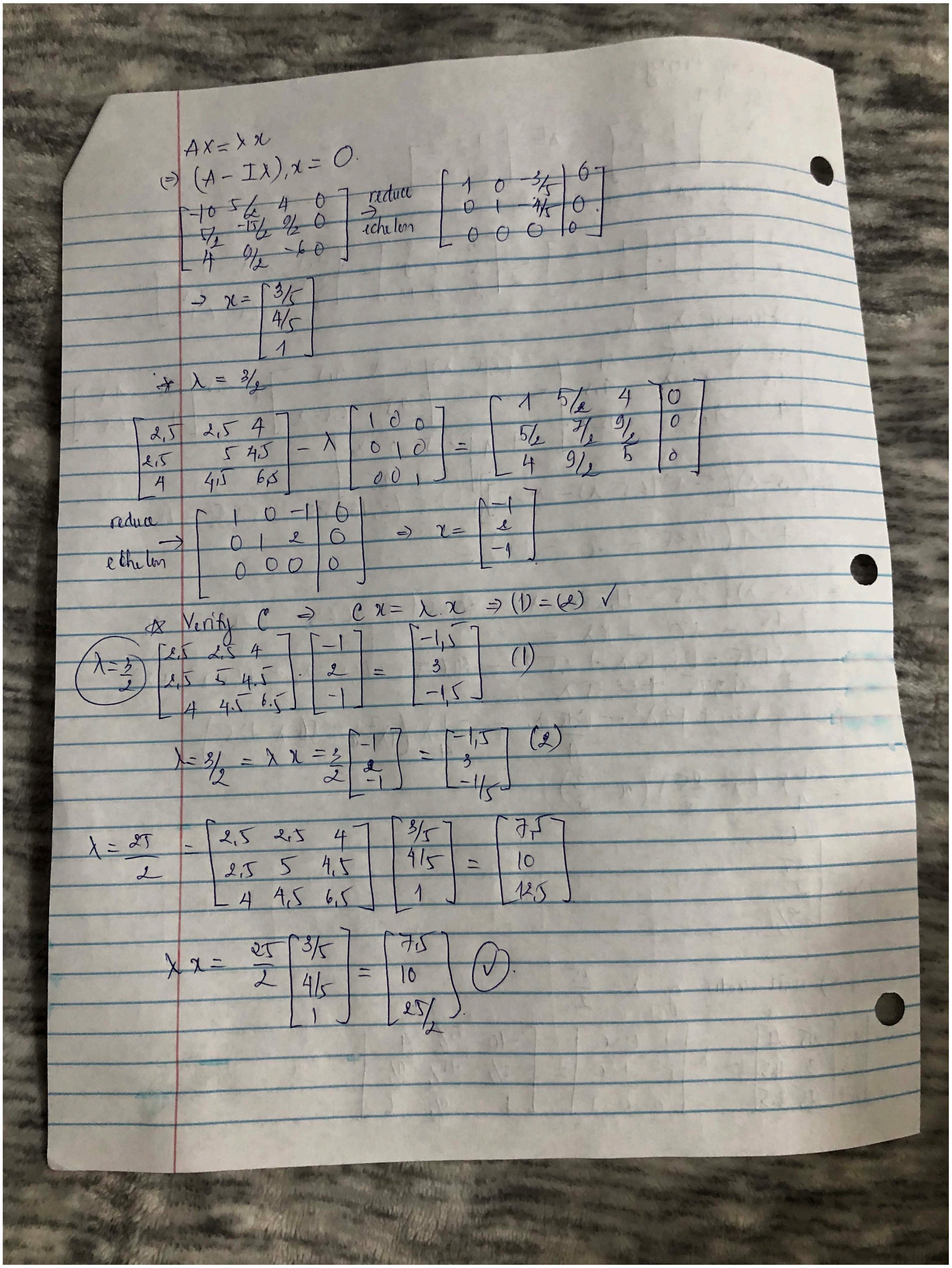
$$= \lambda^3 + 14\lambda^2 - \frac{75}{4}\lambda = \frac{-1}{4}\lambda\left(4\lambda^2 - 56\lambda + 75\right)$$

$$= \frac{-1}{4}\lambda\left(x - \frac{25}{2}\right)\left(x - \frac{3}{2}\right)$$

$$\lambda = \begin{bmatrix} 0 \\ \frac{25}{2} \\ 3/2 \end{bmatrix}$$

c) unit vector g $\lambda = \frac{25}{2}$

$$\begin{bmatrix} 2.5 & 2.5 & 4 \\ 2.5 & 5 & 4.5 \\ 4 & 4.5 & 6.5 \end{bmatrix} - \lambda \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -10 & 5/2 & 4 \\ 5/2 & -15/2 & 9/2 \\ 4 & 9/2 & -6 \end{bmatrix}$$



$Ax = \lambda x$

$\Rightarrow (A - I\lambda).x = 0$

$$\begin{bmatrix} -10 & 5/2 & 4 & 0 \\ 5/2 & -15/2 & 9/2 & 0 \\ 4 & 9/2 & -6 & 0 \end{bmatrix} \xrightarrow[\text{echelon}]{\text{reduce}} \left[\begin{array}{ccc|c} 1 & 0 & -3/5 & 0 \\ 0 & 1 & -4/5 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\Rightarrow x = \begin{bmatrix} 3/5 \\ 4/5 \\ 1 \end{bmatrix}$$

* $\lambda = 3/2$

$$\begin{bmatrix} 2.5 & 2.5 & 4 \\ 2.5 & 5 & 4.5 \\ 4 & 4.5 & 6.5 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \left[\begin{array}{ccc|c} 1 & 5/2 & 4 & 0 \\ 5/2 & 7/2 & 9/2 & 0 \\ 4 & 9/2 & 5 & 0 \end{array}\right]$$

$$\xrightarrow[\text{echelon}]{\text{reduce}} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow x = \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}$$

* Verify C $\Rightarrow$ $Cx = \lambda.x \Rightarrow (1) = (2)$ ✓

$\lambda = \frac{3}{2}$ $$\begin{bmatrix} 2.5 & 2.5 & 4 \\ 2.5 & 5 & 4.5 \\ 4 & 4.5 & 6.5 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} -1.5 \\ 3 \\ -1.5 \end{bmatrix} \quad (1)$$

$$\lambda = 3/2 = \lambda x = \frac{3}{2}\begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} -1.5 \\ 3 \\ -1.5 \end{bmatrix} \quad (2)$$

$$\lambda = \frac{25}{2} = \begin{bmatrix} 2.5 & 2.5 & 4 \\ 2.5 & 5 & 4.5 \\ 4 & 4.5 & 6.5 \end{bmatrix} \begin{bmatrix} 3/5 \\ 4/5 \\ 1 \end{bmatrix} = \begin{bmatrix} 7.5 \\ 10 \\ 12.5 \end{bmatrix}$$

$$\lambda x = \frac{25}{2}\begin{bmatrix} 3/5 \\ 4/5 \\ 1 \end{bmatrix} = \begin{bmatrix} 7.5 \\ 10 \\ 25/2 \end{bmatrix} \quad ✓$$

11.a

B matrix =

[[2. -2. -1. 3.]

```
 [-1.  3.  3. -1.]
 [ 0.  2.  3.  0.]
 [ 1.  3.  1.  3.]
 [ 1.  0. -1.  2.]
 [-3.  2.  4. -1.]
 [ 5. -1.  5.  3.]
 [ 2.  1.  2.  0.]]
[[ 1.5  -2.5  -1.5   2.5 ]
 [-2.    2.    2.   -2.  ]
 [-1.25  0.75  1.75 -1.25]
 [-1.    1.   -1.    1.  ]
 [ 0.5  -0.5  -1.5   1.5 ]
 [-3.5   1.5   3.5  -1.5 ]
 [ 2.   -4.    2.    0.  ]
 [ 0.75 -0.25  0.75 -1.25]]
rowmean [0.5, 1.0, 1.25, 2.0, 0.5, 0.5, 3.0, 1.25]
B matrix after mean = 0
 [[ 1.5  -2.5  -1.5   2.5 ]
 [-2.    2.    2.   -2.  ]
 [-1.25  0.75  1.75 -1.25]
 [-1.    1.   -1.    1.  ]
 [ 0.5  -0.5  -1.5   1.5 ]
 [-3.5   1.5   3.5  -1.5 ]
 [ 2.   -4.    2.    0.  ]
 [ 0.75 -0.25  0.75 -1.25]]
Covariance matrix =
 [[ 2.12 -2.   -1.19  0.    1.   -2.25  1.25 -0.31]
 [-2.    2.    1.25  0.   -1.    2.5  -1.    0.25]
 [-1.19  1.25  0.84 -0.12 -0.69  1.69 -0.25  0.22]
 [ 0.    0.   -0.12  0.5   0.25  0.   -1.   -0.38]
 [ 1.   -1.   -0.69  0.25  0.62 -1.25  0.   -0.31]
 [-2.25  2.5   1.69  0.   -1.25  3.62 -0.75  0.19]
 [ 1.25 -1.   -0.25 -1.    0.   -0.75  3.    0.5 ]
 [-0.31  0.25  0.22 -0.38 -0.31  0.19  0.5   0.34]]
Eigenvalues of C =
 [ 8.99  0.    0.74  3.33 -0.   -0.    0.    0.  ]
Eigenvector of C =
 [[ 0.47 -0.74 -0.48 -0.04  0.56  0.56 -0.27 -0.27]
 [-0.47 -0.36  0.11 -0.04  0.08  0.08 -0.12 -0.12]
 [-0.29 -0.15 -0.04 -0.15  0.04  0.04 -0.22 -0.22]
 [-0.02  0.19 -0.33  0.35 -0.37 -0.37  0.6   0.6 ]
 [ 0.23  0.28 -0.22  0.2   0.08  0.08 -0.31 -0.31]
 [-0.6   0.03 -0.61 -0.21  0.2   0.2  -0.04 -0.04]
 [ 0.26  0.3  -0.14 -0.84 -0.36 -0.36  0.24  0.24]
 [-0.04 -0.31  0.45 -0.23  0.51  0.51  0.02  0.02]]
U :
```

```
[[-0.47 -0.04  0.48 -0.69 -0.2   0.17 -0.04  0.01]
 [ 0.47 -0.04 -0.11 -0.57  0.05 -0.65  0.11  0.04]
 [ 0.29 -0.15  0.04 -0.19  0.36  0.41  0.26 -0.7 ]
 [ 0.02  0.35  0.33  0.17 -0.13 -0.07  0.84  0.1 ]
 [-0.23  0.2   0.22  0.01  0.89 -0.15 -0.07  0.21]
 [ 0.6  -0.21  0.61  0.11 -0.02  0.23 -0.21  0.34]
 [-0.26 -0.84  0.14  0.21  0.06 -0.3   0.25 -0.03]
 [ 0.04 -0.23 -0.45 -0.27  0.14  0.46  0.32  0.58]]
S :
 [8.48 5.16 2.44 0.  ]
V :
 [[-0.56  0.52  0.48 -0.44]
 [-0.22  0.63 -0.69  0.28]
 [-0.63 -0.29  0.22  0.69]
 [-0.5  -0.5  -0.5  -0.5 ]]
The 3 most significant eigenvector of C is :
```

[[-4.71 4.38 4.07 -3.74]
[-1.16 3.27 -3.54 1.44]
[-1.53 -0.7   0.54 1.69]

```
 [-0.    0.   -0.    0.  ]
 [ 0.    0.   -0.    0.  ]]
```

11.b

```
UVector choose:
 [[-0.465  0.47   0.291  0.018 -0.226  0.597 -0.262  0.043]
 [-0.037 -0.042 -0.154  0.353  0.197 -0.211 -0.844 -0.231]
 [ 0.483 -0.109  0.036  0.332  0.22   0.611  0.138 -0.449]]
Scoring phrase:
Y~:
 [ 0.5   4.   -0.25  3.    4.5   0.5  -2.    1.75]
W =
 [1.508 2.98  1.027]
[7.895164247569233, 3.36654399883552, 7.023789468377841, 5.507596435478498]
```

Emin = **3.36654399883552**

```
Scoring phrase:
Y~:
 [-2.5   2.    0.75  1.   -0.5   1.5  -4.   -0.25]
W =
 [ 4.384  3.266 -0.7  ]
[10.148891565092217, 1.1102230246251565e-16, 6.92820323027551, 8.660254037844386]
```

Emin = **1.1102230246251565e-16**

```
Scoring phrase:
Y~:
 [ 1.5  -4.    0.75  1.   -0.5  -0.5  -1.   -2.25]
W =
 [-2.361  1.718  1.976]
```

[5.1044628445923825, 7.4199421110830865, 8.429444876848308, 1.4337157779086354]
Emin = **1.4337157779086354**
Scoring phrase:
Y~:
[1.5 -3.   0.75 0.   -1.5   0.5 -1.   0.75]
W =
[-0.958 0.223 0.578]
[4.519521770919681, 6.279018795784652, 6.279018795784655, 3.228943641164548]
Emin = **3.228943641164548**
13.
question 13.a
B matrix =
[[1. -2. 1. 2.]
[-1. 2. 3. 3.]
[1. 2. 0. 1.]
[-1. -1. 1. 1.]
[-1. -2. 3. -2.]
[1. 2. 1. 0.]]
B matrix after mean = 0
[[-0.75 -3.75 -0.75 0.25]
[-2.75 0.25 1.25 1.25]
[-0.25 0.75 -1.25 -0.25]
[-3.   -3.   -1.   -1.]
[-3.   -4.   1.   -4.]
[0.   1.   0.   -1.]]
Covariance matrix =
[[2.54 0.08 -0.29 2.33 2.58 -0.67]
[0.08 1.79 -0.17 0.83 0.58 -0.17]
[-0.29 -0.17 0.38 0.   -0.42 0.17]
[2.33 0.83 0.   3.33 4.   -0.33]
[2.58 0.58 -0.42 4.   7.   0.]
[-0.67 -0.17 0.17 -0.33 0.   0.33]]
deltaScoreMatrix 13A :

[[-1.56 -0.82 1.99 -0.5]
[-1.15 -2.51 0.39 0.81]]
13b
B1 matrix =
[[-1. -2. -1. 0.]
[2. 1. 3. 2.]
[1. 2. 0. 3.]
[2. 3. 1. 1.]
[-1. 2. 3. 1.]
[0. 1. -1. -2.]]
B1 matrix after mean = 0
[[-2.75 -3.75 -2.75 -1.75]

[0.25 -0.75 1.25 0.25]
[-0.25 0.75 -1.25 1.75]
[0.   1.   -1.   -1.]
[-3.   0.   1.   -1.]
[-1.   0.   -2.   -3.]]

Covariance matrix =

[[5.38 -0.29 -0.29 0.12 1.21 2.25]
[-0.29 0.38 -0.29 -0.38 0.04 -0.58]
[-0.29 -0.29 0.88 0.04 -0.38 -0.42]
[0.12 -0.38 0.04 0.5   0.   0.83]
[1.21 0.04 -0.38 0.   1.83 0.67]
[2.25 -0.58 -0.42 0.83 0.67 2.33]]

deltaScoreMatrix 13B:

[[-1.7 -1.82 1.51 -0.74]
[0.26 -1.17 0.48 -1.22]]

Y1
Score from 13a:
0.8799097965132561
Score from 13b:
0.7951786277309016
Benign

Y2
Score from 13a:
0.22640558738688443
Score from 13b:
0.44946056556721437
Malware

Y3
Score from 13a:
3.3048021499024713
Score from 13b:
2.9987218693970266
Benign

Y4
Score from 13a:
2.421184489046632
Score from 13b:
2.1755883801859213
Benign

17.
U1: [0.164 0.628 0.26 0.539 0.464 0.075] U2: [0.244 0.107 0.802 0.428 0.137 0.29]

a) Vector u1 has the greatest positive impact on the projection space is 0.6278 at index 2
a) Vector u2 has the greatest negative impact on the projection space is 0.8017 at index 3
17.b
L1 = [0.332 1.269 -0.526 -1.089 0.937 0.152]
L2 = [0.272 0.119 -0.891 0.476 -0.153 -0.323]
L3 = [-0.061 0.253 0.341 0.296 0.314 -0.61]
L1^2 + L2^2 = [0.184 1.624 1.072 1.412 0.901 0.127]
L2^2 + L3^2 = [0.078 0.078 0.911 0.314 0.122 0.477]
L1^2 + L3^2 = [0.114 1.673 0.393 1.274 0.977 0.396]
17.c
[0.184 1.624 1.072 1.412 0.901 0.127]
L1^2 + L2^2 = [0.184 1.624 1.072 1.412 0.901 0.127]
The most important feature is at index 2 with value 1.6235181691720009
The less important feature is at index 6 with value 0.12735950745599997

REFERENCES

1. M. Stamp, "Introduction to Machine Learning with Applications in Information Security."
2. Steven A.Cohen and Matthew W.Granade, "Models will run the world"